QCD LCDA of Heavy Mesons from bHQET

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Beneke, Finauri, Vos and **YBW**: 2305.06401

Deng, Wang, **YBW** and Zeng: 2409.00632

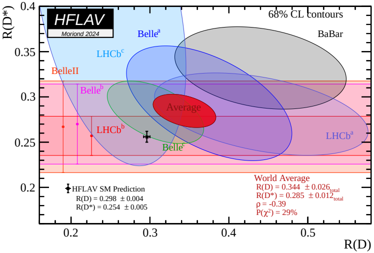
Outline

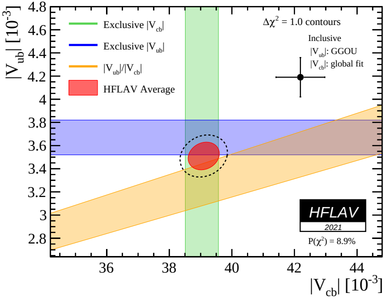
- LCDA of heavy mesons
- Introduction to bHQET
- Factorization of QCD LCDA
- Numeric applications

B Physics

New physics beyond the SM

- Direct search: new particles
- Indirect search: flavour physics
  CPV, $R(D^{(*)})$, $|V_{ub}|$, $|V_{cb}|$, $\cdots$





- BaBar, Belle
- LHC, Belle-II
- HL-LHC

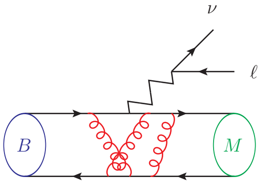
HQET LCDA of heavy meson

The leading-twist heavy-meson LCDA in HQET [Grozin and Neubert, 96']

$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\,\widetilde{f}_H(\mu)\,n_+\cdot v\int_0^\infty d\omega\, e^{i\omega tn_+\cdot v}\varphi_+(\omega)$$

Most important long-distance function in exclusive B decays

- Non-leptonic decays: $B\to\pi\pi,\ B\to\pi K\cdots$
- Semi-leptonic decays: $B\to D^{(*)}\ell\nu$,
  $B\to K^{(*)}\ell\nu\ \cdots$
- Radiative decay: $B\to\gamma\ell\nu,\ B\to\gamma\gamma\ \cdots$



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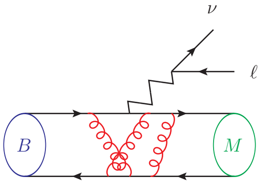

⁂ Inverse moments: [Braun, Ivanov and Korchemsky, 03'], [Belle, 18'], [Han, et al. 24']

⁂ RG evolution properties: [Lange and Neubert, 03'], [Braun, Ji and Manashov. 19']

⁂ asymptotic behavior, generalized LCDA, EOM ...

Non-perturbative LCDA of hadrons

Light-cone distribution amplitudes (LCDA): non-perturbative physics

- Calculate with LQCD [LPC collaboration, 22']
- Extract from the experiments: clean process $B \to \gamma \ell \nu$

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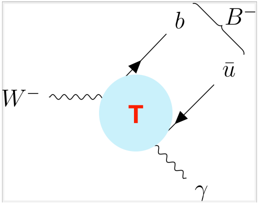
Lattice: LaMET [Ji, 13']: quasi DA $\to$ QCD LCDA $\to$ HQET LCDA

Rare decay $W^- \to B^- \gamma$ [Grossman, König and Neubert, 15']

$$A(W^- \to B^- \gamma) = \int_0^1 du\, T(u)\, \phi(u)$$



- Hard function T: m_W
- non-perturbative QCD LCDA ϕ: large logs $\ln(\Lambda_{\rm QCD}/m_Q)$

$W \to B\gamma$

The branch ratio of $W \to B\gamma$ decay

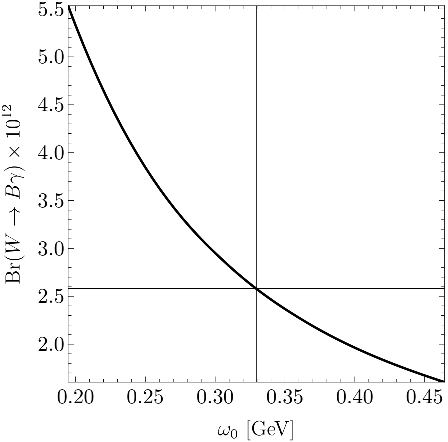
[Grossman, König and Neubert, 15']:
$m_Q \sim \mathcal{O}(\Lambda_{\rm QCD})$

$$\text{Br} = (1.99^{+2.48}_{-0.80}\,\lambda_B) \cdot 10^{-12}$$

Our result: with $\ln \Lambda_{\rm QCD}/m_Q$ resummation

$$\text{Br} = (2.58^{+2.95}_{-0.98}\,\lambda_B) \cdot 10^{-12}$$

30% increase

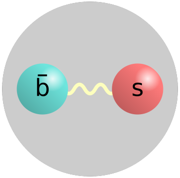


LCDA of heavy meson

The heavy meson QCD LCDA [Braun and Filyanov, 89']

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+\cdot p_H\int_0^1 du\, e^{iutn_+\cdot p_H}\,\phi(u)$$

- Only large $n_+\cdot p_H$ component appears in the hard function T
- ϕ: two scales m_Q and $\Lambda_{\rm QCD}$



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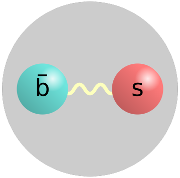

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Factorization of the QCD LCDA

- Momentum space matching of QCD to HQET @ NLO [Ishaq, Jia, Xiong and Yang, 19']

$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)$$

Tree level

$$\mathcal{J}^{(0)}(u,\omega) = \delta\Big(u - \frac{\omega}{\omega + m_Q}\Big)$$

ω and m_Q have different power counting

NLO from [Bell and Feldmann, 08']

- Coordinate space matching @NLO [Zhao, 19']

Boost invariance: decay constant

- Rest frame: QCD to HQET

$$\underbrace{\bar{Q}\gamma^\alpha\gamma^5 q}_{f_H} = C_V(\mu)\,\underbrace{\bar{h}_v\gamma^\alpha\gamma^5 q_s}_{\tilde{f}_H} + C_S(\mu)\,v^\alpha\,\underbrace{\bar{h}_v\gamma^5 q_s}_{\tilde{f}_H}$$

Then one derive the relation

$$f_H = K(\mu)\,\tilde{f}_H(\mu)$$

- Boosted frame: SCET to bHQET

$$\underbrace{\bar{\xi}_C^{(Q)}\not{n}_+\gamma^5\xi_C}_{f_H} = C_+(\mu)\,\underbrace{\bar{h}_n\not{n}_+\gamma^5\xi_{sc}}_{\hat{f}_H} + C_-(\mu)\,\underbrace{\bar{h}_n\not{n}_+\frac{i\not{D}_\perp}{in_+D}\gamma^5\xi_{sc}}_{\hat{f}_H}$$

with

$$f_H = K(\mu)\,\hat{f}_H(\mu)$$

Boost invariance: $\hat{f}_H(\mu) = \tilde{f}_H(\mu)$

Momentum modes

A given momentum in the light-cone coordinate

$$l^\mu = (n_+ l,\ l_\perp,\ n_- l)$$

In the rest frame of the heavy meson

- heavy quark (hard): $(1,1,1)m_Q$
- light degree of freedom (soft): $(1,1,1)\Lambda_{\mathrm{QCD}}$

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A given momentum in the light-cone coordinate

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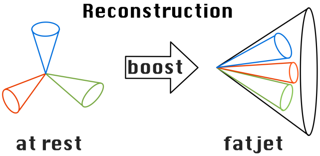

For a boosted heavy meson: $b \sim \frac{m_Q}{Q}$

- heavy quark (hard-collinear): $(\frac{1}{b}, 1, b)m_Q = (Q, m_Q, \frac{m_Q^2}{Q})$
- light degree of freedom (soft-collinear): $(\frac{Q}{m_Q}, 1, \frac{m_Q}{Q})\Lambda_{\rm QCD}$

Boosted HQET

The heavy quark field in bHQET [Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21']

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ v}} e^{i m_Q v \cdot x} \xi^{(Q)}(x)$$

The relation between the HQET and bHQET heavy quark field

$$h_v(x) = \sqrt{\frac{n_+ v}{2}} \frac{1 + \not{v}}{2} \left(1 - \frac{\not{n}_+}{2} \frac{i\not{D}_\perp + m_Q \not{v}_\perp - m_Q}{i n_+ D + m_Q n_+ v}\right) h_n(x)$$

The bHQET Lagrangian could be derived from the HQET one

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(x) i v \cdot D h_v(x) = \underbrace{\bar{h}_n(x) i v \cdot D \frac{\not{n}_+}{2} h_n(x)}_{\mathcal{L}_{\rm bHQET}} + \mathcal{O}(\lambda)$$

Leading power bHQET operators

The operators must preserve the reparameterization invariance [Beneke, Chapovsky, Diehl and Feldmann, 02']

$$n_- \to \alpha\, n_- \,, \qquad n_+ \to \frac{1}{\alpha}\, n_+ \quad (\alpha \text{ real})$$

Then one find out the LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v}\sqrt{\frac{n_+ v}{2}}\bar{h}_n \not{n}_+ \left(n_+ v \frac{i\not{D}_\perp}{in_+ D}\right)^k \gamma^5 \xi_{sc}$$

From the EOM of the light quark

$$\not{n}_+ i\not{D}_\perp \frac{1}{in_+ D} i\not{D}_\perp \xi_{sc} = -\not{n}_+ in_- D \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP

Inhomogeneous power counting of ϕ

For the matching scale μ ($\delta \sim \Lambda_{\mathrm{QCD}}/m_H$)

- $\mu \gg m_Q$: $\phi(u)$ is symmetric under $u \leftrightarrow 1-u$
- $\mu \lesssim m_Q$: $\phi(u)$ develops a peak at $u \sim \mathcal{O}(\delta)$
  $\phi(u)$ is suppressed at $u \sim \mathcal{O}(1)$
- Normalization condition

$$\phi(u) \sim \begin{cases} \delta^{-1}\,, & \text{for } u \sim \delta \quad (\text{“peak”}) \\ 1\,, & \text{for } u \sim 1 \quad (\text{“tail”}) \end{cases}$$

For consistent power counting: separate $u \sim \mathcal{O}(1)$ and $u \sim \mathcal{O}(\delta)$ region

Factorization of the QCD LCDA

The factorization formula

$$\phi(u) = \begin{cases} \mathcal{J}_p(u,\omega) \otimes \varphi_+(\omega)\,, & u \sim \delta\,, \\ \mathcal{J}_{\text{tail}}(u)\,, & u \sim 1\,, \end{cases}$$

The jet function $\mathcal{J}$: $\mathcal{O}(m_Q)$, HQET LCDA φ_+: $\mathcal{O}(\Lambda_{\text{QCD}})$

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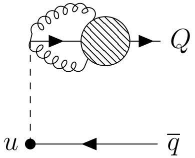
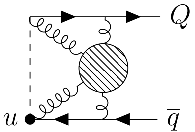


Peak region:

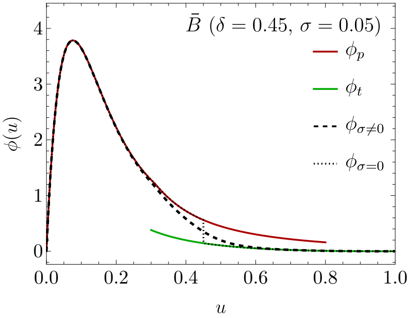
$$\mathcal{J}_p(u,\omega) = \theta(m_Q-\omega)\delta\Big(u-\frac{\omega}{m_Q}\Big)\Big[1 + \frac{\alpha_s C_F}{4\pi}\Big(\frac{L^2}{2}+\frac{L}{2}+\frac{\pi^2}{12}+2\Big)\Big], \quad L = \ln\frac{\mu^2}{m_Q^2}$$



Tail region:

$$\mathcal{J}_{\rm tail}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\Big((1+u)[L-2\ln u]-u+1\Big)$$

QCD LCDA from HQET LCDA



Merging the tail and peak contributions

$$\phi(u) = \begin{cases} \phi_p(u) = \mathcal{J}_p(u,\omega) \otimes \varphi_+(\omega) \\ \phi_t(u) = \mathcal{J}_{\text{tail}}(u) \end{cases}$$

$$\phi_p(u)|_{u\sim 1} = \phi_t(u)|_{u\ll 1}$$

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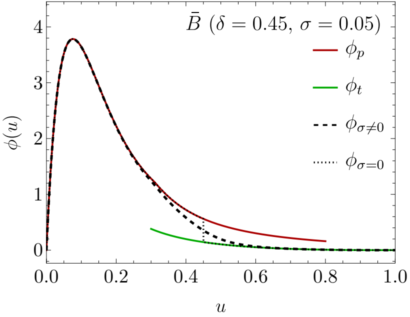

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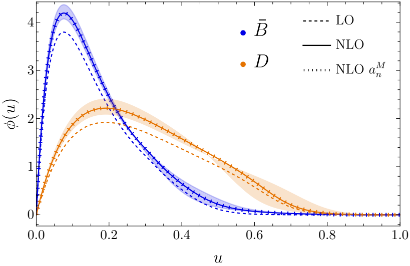
The Gegenbauer parameterization

$$\phi(u) = 6u\bar{u}[1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2u-1)]$$

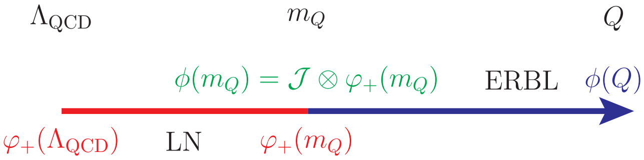
with a_n from the merged LCDA

$$a_n^{\bar{B}}(\mu_b) = \{-1.08, 0.83, -0.51, 0.28, \dots\}$$



Evolution of the LCDA

Two evolution steps: $\Lambda_{\mathrm{QCD}} \to m_Q \to Q$



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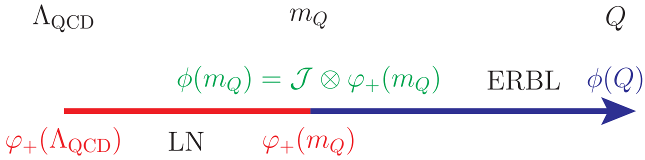
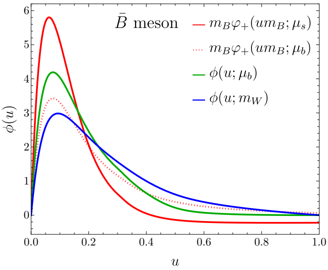


ERBL kernel: $\phi(m_Q) \to \phi(Q)$:
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{2\beta_0}}$$



$$a_n^{\bar{B}}(\mu_b) = \{-1.082, 0.826, -0.513, 0.288, -0.157, 0.078, -0.030, \dots\}$$
$$a_n^{\bar{B}}(m_W) = \{-0.826, 0.542, -0.302, 0.156, -0.079, 0.037, -0.014, \dots\}$$

Heavy-quark spin symmetry

Heavy-quark limit [Grozin and Neubert, 97']: the HQET operators define the same HQET LCDA φ_+: pseudoscale meson and vector meson

$$\langle H|\bar{h}_n(0)\not{n}_+\gamma_5 q_{sc}(tn_+)|0\rangle, \quad \langle H^*|\bar{h}_n(0)\not{n}_+ q_{sc}(tn_+)|0\rangle, \quad \langle H^*|\bar{h}_n(0)\not{n}_+\gamma_\perp^\mu q_{sc}(tn_+)|0\rangle$$

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Determine φ_+ from different QCD LCDAs

$$\langle H^*|\bar{Q}(0)\not{n}_+ q(tn_+)|0\rangle \quad \Rightarrow \quad \phi_\parallel$$

$$\langle H^*|\bar{Q}(0)\not{n}_+\gamma_\perp^\mu q(tn_+)|0\rangle \quad \Rightarrow \quad \phi_\perp$$

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Factorization formulas

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} m_H \mathcal{J}^i_{\rm peak}(m_H)\varphi_+(um_H), \qquad i = P,\ \parallel,\ \perp$$

EOM of the quarks, method of region [Deng, Wang, **YBW** and Zeng, 24']

$$\mathcal{J}^i_{\rm peak}(u,\omega) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right) + \mathcal{O}(\alpha_s^2)$$

I: HQET LCDA from quasi-DA

At the peak region: QCD LCDA to HQET LCDA

$$\phi(u) = \frac{\tilde{f}_H}{f_H} m_H \mathcal{J}_{\rm peak}(m_H) \varphi_+(u m_H), \qquad u \sim \mathcal{O}(\delta)$$

LaMET [Ji, 13']: quasi DA to QCD LCDA

$$\tilde{\phi}(x, P^z; m_H) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu; m_H) + \cdots$$

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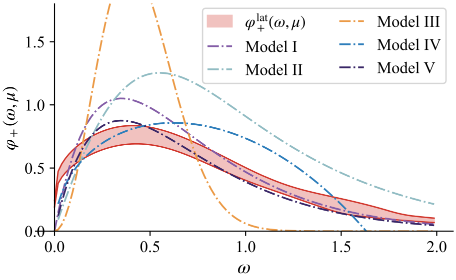


Quasi DA from lattice

HQET LCDA from quasi DA: [Han, et al. 24']

First inverse moment $\lambda_B = 0.449(42)$ GeV

II: $W \to B\gamma$ (non-perturbative input)

HQET LCDA with radiative tail [Lee and Neubert, 05']

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s(\mu_s) C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\rm mod}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\rm asy}(\omega;\mu_s)$$

Two parameter models

$$\begin{aligned}
\varphi_+^{(\rm I)}(\omega;\mu_s) &= \left[1-\beta+\frac{\beta}{2-\beta}\frac{\omega}{\omega_0}\right]\varphi_+^{\rm exp}\big(\omega,(1-\beta/2)\omega_0;\mu_s\big)\,, \quad \text{for} \quad 0\le\beta\le 1\,,\\
\varphi_+^{(\rm II)}(\omega;\mu_s) &= \frac{(1+\beta)^\beta}{\Gamma(2+\beta)}\left(\frac{\omega}{\omega_0}\right)^\beta \varphi_+^{\rm exp}\left(\omega,\frac{\omega_0}{1+\beta};\mu_s\right)\,, \quad \text{for} \quad -\frac{1}{2}<\beta<1\,,\\
\varphi_+^{(\rm III)}(\omega;\mu_s) &= \frac{\sqrt{\pi}}{2\Gamma(3/2+\beta)}U\left(-\beta,\frac{3}{2}-\beta,(1+2\beta)\frac{\omega}{\omega_0}\right)\\
&\quad\times\varphi_+^{\rm exp}\left(\omega,\frac{\omega_0}{1+2\beta};\mu_s\right)\,, \quad \text{for} \quad 0\le\beta<\frac{1}{2}
\end{aligned}$$

II: $W \to B\gamma$ (branch ratio)

The branch ratio

$$\text{Br}(W \to B\gamma) = \frac{\Gamma(W \to B\gamma)}{\Gamma_W} = \frac{\alpha_{\text{em}} m_W f_B^2}{48 v^2 \Gamma_W} |V_{ub}|^2 \left(|F_1^B|^2 + |F_2^B|^2 \right)$$

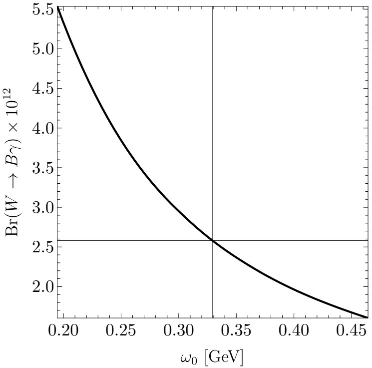
[Grossman, König and Neubert, 15']:
$m_Q \sim \Lambda_{\text{QCD}}$

$$\text{Br} = (1.99 \pm 0.17_{\text{in}} \,{}^{+0.03}_{-0.06}\,{}_{\mu_h} \,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}) \cdot 10^{-12}$$

Our result: with $\ln \Lambda_{\text{QCD}}/m_Q$ resummation

$$\text{Br} = (2.58 \pm 0.21_{\text{in}} \,{}^{+0.05}_{-0.08}\,{}_{\mu_h} \,{}^{+2.95}_{-0.98}\,{}_{\lambda_B}) \cdot 10^{-12}$$

30% increase



Summary

※ Introduction to the SCET and bHQET

※ Match QCD LCDA to the HQET LCDA

- Peak region: $\mathcal{O}_C(u) = \mathcal{J}_p(u,\omega) \otimes \mathcal{O}_h(\omega)$
- Tail region: $\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$

※ $W^- \to B^-\gamma$ decay: 30% enhancement
HQCD LCDA from Lattice

Thank you!

Introduction to SCET

The collinear fields [Bauer, Fleming, Pirjol and Stewart, 01'], [Beneke, Chapovsky, Diehl and Feldmann, 02'], [Becher, Broggio and Ferroglia, 14']

$$q(x) = \frac{\not{n}_- \not{n}_+}{4} q(x) + \frac{\not{n}_+ \not{n}_-}{4} q(x) = \xi(x) + \eta(x)$$

The power counting of the soft and collinear fields $\lambda = \sqrt{m_Q/Q}$

$$\begin{aligned} &\xi \sim \lambda, \quad \eta \sim \lambda^2, \quad q_s \sim \lambda^3, \quad A_s \sim \lambda^2 \\ &n_+ A_c \sim 1, \quad A_{\perp c} \sim \lambda, \quad n_- A_c \sim \lambda^2, \end{aligned}$$

The Lagrangian also has definite power counting

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}^{(0)} + \mathcal{L}_{\xi}^{(1)} + \mathcal{L}_{\xi}^{(2)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi q}^{(2)} + \cdots$$

The QCD operator defining ϕ

$$\bar{Q}(0) \not{n}_+ \gamma^5 [0, tn_+] q(tn_+) = \bar{\xi}^{(Q)}(0) \not{n}_+ \gamma^5 [0, tn_+] \xi(tn_+)$$

Quasi DA

The quasi DA defined as

$$\tilde{\phi}(x, P^z; m_H) = \int \frac{dz}{2\pi} e^{-ixP^z z} \frac{\tilde{M}^0(z, P^z; \gamma^z\gamma_5, m_H)}{\tilde{M}^0(z, 0; \gamma^t\gamma_5, m_H)}$$

P^z denotes the momentum along the z direction.

The involved matrix element is defined as

$$\tilde{M}^0(z, P^z; \Gamma, m_H) = \langle 0 \,|\bar{q}(z)\Gamma\, W_c(z,0)\, Q(0)|\, H(P^z)\rangle$$

The peak region $u \sim \mathcal{O}(\delta)$

The operator level matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\, \mathcal{O}_h(\omega)$$

The SCET and bHQET non-local operators are

$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p} \bar{\xi}^{(Q)}(0)\, \not{n}_+ \gamma^5\, [0, tn_+] \xi(tn_+)$$

$$\mathcal{O}_h(\omega) = \frac{1}{m_H} \int \frac{dt}{2\pi} e^{-i\omega t n_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n(0)\, \not{n}_+ \gamma^5\, [0, tn_+] \xi_{sc}(tn_+) \sim \hat{\mathcal{O}}_0$$

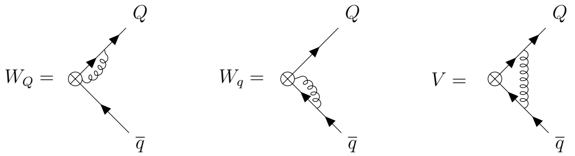
$\mathcal{O}_C$ and $\mathcal{O}_h$ have the same IR but different UV behaviour

On-shell partonic states

$$\langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}_C(u)|0\rangle_{\rm SCET} = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega) \langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}_h(\omega)|0\rangle_{\rm bHQET}$$

The peak region: matching procedure

$$\langle \mathcal{O}_C(u) \rangle_{\text{SCET}} = \frac{1}{n_+ p_H} \bar{u}(p_Q) \not{n}_+ \gamma^5 v(p_q) \left\{ \delta(u-s) + \frac{\alpha_s C_F}{4\pi} M^{(1)}(u,s) \right\}$$



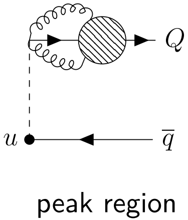
Perform the UV renormalization with the ERBL kernel, $M^{(1)}$ only has IR divergence [Lepage and Brodsky, 79'], [Efremov and Radyushkin, 80']

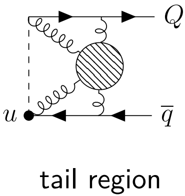
$$\langle \mathcal{O}_h(\omega) \rangle_{\text{bHQET}} = \frac{1}{n_+ p_H} \bar{u}(p_Q) \not{n}_+ \gamma^5 v(p_q) \left\{ \delta\left(\frac{n_+ p_q}{n_+ v} - \omega \right) + \frac{\alpha_s C_F}{4\pi} N^{(1)}\left(\omega, \frac{n_+ p_q}{n_+ v} \right) \right\}$$

Perform the UV renormalization with the LN kernel, $N^{(1)}$ only has IR divergence [Lange and Neubert, 03']

The peak region: matching procedure

- The UV renormalized $N^{(1)}$ cancels the IR contribution of $M^{(1)}$
- The jet function is free of both UV and IR divergences





The hard-clooinear jet function up to NLO is

$$\mathcal{J}_p(u,\omega) = \theta(m_Q - \omega)\delta\left(u - \frac{\omega}{m_Q}\right)\left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right) + \mathcal{O}(\alpha_s^2)\right]$$

with $L = \ln \frac{\mu^2}{m_Q^2}$

The tail region $u \sim \mathcal{O}(1)$

The matching coefficient will not dependent on ω, the SCET operator will match onto local operators

$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

The SCET matrix element starts at NLO

$$\langle \mathcal{O}_C(u) \rangle_{\text{SCET}} = \frac{\alpha_s C_F}{4\pi} \sum_{\pm} M_{\pm}^{(1)}(u) \langle \mathcal{O}_\pm \rangle$$

The jet function at NLO

$$\mathcal{J}_{\text{tail}}(u) = \mathcal{J}_+(u) + \mathcal{J}_-(u) = \frac{\alpha_s C_F}{4\pi} \frac{2\bar{u}}{u} \Big((1+u)[L - 2\ln u] - u + 1 \Big)$$

- $\mathcal{J}_{\text{tail}}$ will depend on both u and m_Q