正负电子对湮灭过程中的碎裂函数研究

王晓玉

郑州大学物理学院

xiaoyuwang@zzu.edu.cn

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Collaborated with Zhun Lu@SEU, Tianbo Liu, Shuailiang Yang@SDU, Yongliang Yang@QDU, Hui Li@SXNU
Yanli Li, Keyang She, Zhi Cao@ZZU

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Outline

1. Introduction
2. The transverse polarization of Lambda
3. The parametrization of Lambda FF
4. Summary & Outlook

Fundamental property of QCD

Asymptotic freedom

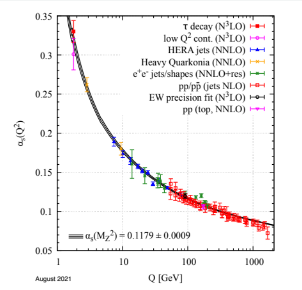


图: Strong coupling constant as the function of the energy scale Q

- High Q—perturbative QCD
- Low Q—non-perturbative

Confinement

- All of the observed particles are color-singlet, quarks and gluons can not be seen in isolation.
- It is a challenge to quantify the quark-gluon structure of hadrons.
- The emergence of hadrons from quarks and gluons also remains challenge.

QCD factorization

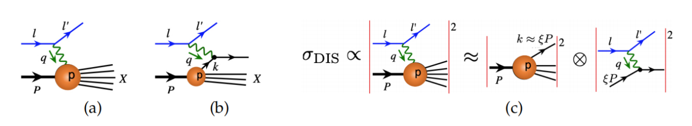
- Choose a factorization scale to separate the high energy (short range) and low energy (long range).
- high energy (short range)—perturbative QCD
- low energy (long range)—non-perturbative—Model/Parametrization/LQCD......
- The universality of PDF/FF makes it possible to determine them from known experimental data and ensure predictive power of QCD.

QCD along with its factorization formalism has been extremely successful in interpreting all available data from high energy scatterings with larger than 2 GeV momentum transfer in the collision, which has provided us the confidence and the tools to discover the Higgs particle at the LHC and to explore the new physics beyond QCD and the Standard Model in high energy hadronic collisions.[1]

[1] **Boussarie:2023izj**.

QCD factorization

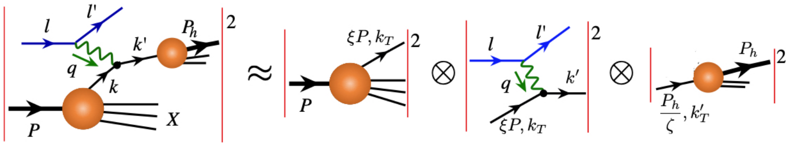
Parton Model



- $Q^2 = -q^2, Q \gg 1/R$, short-distance EM probe for charged point-like particles inside proton
- PDF $f_{i/p}(\xi)$ —probability distribution density to find i point-like particle inside fast moving proton carrying momentum fraction ξ, $\sigma_{ep\to e'X} \equiv \sum_i \int d\xi f_{i/p}(\xi)\hat{\sigma}_{ei\to e'i}$
- Interpret the DIS data from SLAC[2] and verify the existence of spin-$\frac{1}{2}$ point-like particles inside the proton and provide a way to extract the PDFs.

[2] Bloom:1969kc.

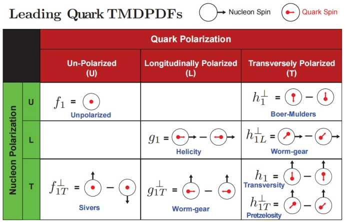
SIDIS process

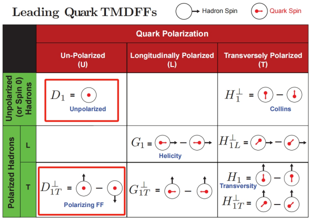
$\sigma_{\text{SIDIS}} \propto$ 

- The cross section @ SIDIS: $\sigma_{ep\to e'hX} = \sum_i f_{i/p} \otimes D_{h/i} \otimes \hat{\sigma}_{ei\to e'i}$
  - $f_{i/p}(\xi, \mathbf{p_T})$: PDF
  - $D_{h/i}(\zeta, \mathbf{k'_T})$: Fragmentation Function
    —parton i hadronize into an observed hadron h carrying the momentum fraction ζ of the fragmenting quark momentum
  - $\otimes$: convolution of both longitudinal momentum fraction and transverse momenta
- Final state hadron can be a probe to separate the flavor dependence of PDF

TMD PDF & FF

- Introducing the transverse momentum of the parton inside the proton and the final-state hadron respective to the fragmented parton
- Introducing the spin of the parton as well as the hadrons
- PDF & FF become the transverse momentum dependent PDF & FF (Figures from TMD handbook: arXiv: 2304.03302)





Why polarized fragmentation function?

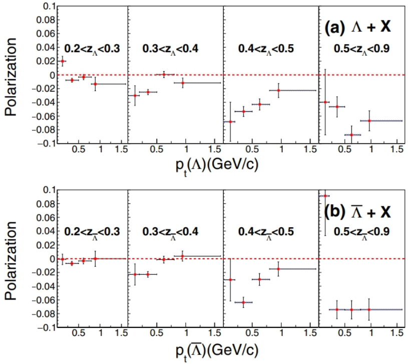
- In 70s, the production of a polarized hyperon from unpolarized *pp* collisions has been observed and it formed a long-standing challenge in hadron physics and spin physics.
- Anselmino, Boer *et al.*[3] suggested that a polarizing fragmentation function (PFF) $D_{1T}^{\perp}$ can account for the polarization of the Λ production.
- As a time-reversal-odd and transverse momentum dependent (TMD) fragmentation function, $D_{1T}^{\perp}$ describes the fragmentation of an unpolarized quark to a transversely polarized hadron (the analog of the Sivers function).
- $e^+e^- \to \Lambda^\uparrow + h + X$ and semi-inclusive deep inelastic scattering (SIDIS) $\ell\, p \to \ell' + \Lambda^\uparrow + X$ were also suggested[4] to study the Λ polarization.

[3] **Anselmino:2000vs**.

[4] **Boer:1997mf**.

Why polarized fragmentation function?

- Significant signal on the transverse polarization of Λ hyperon has not been observed in the single inclusive $e^+ e^-$ annihilation (SIA) experiment performed by OPAL at LEP[a].
- Nonzero transverse polarization of Λ production in SIA and semi-inclusive $e^+e^- \to \Lambda(\bar{\Lambda}) + K^{\pm}(\pi^{\pm}) + X$ process was measured by the Belle Collaboration[b], making the extraction[c] of the polarized fragmentation function of the Λ possible.



[a] Ackerstaff:1997nh.
[b] Guan:2018ckx.
[c] DAlesio:2020wjq, Callos:2020qtu.

Transverse polarization in $e^+e^- \to \Lambda(\bar{\Lambda}) + K^{\pm}(\pi^{\pm}) + X$ process

- Setup the basic framework based on the TMD factorization framework
- Compare different parametrization form of non-perturbative Sudakov form factor
- Estimate the transverse polarization in semi-inclusive $e^+e^- \to \Lambda(\bar{\Lambda}) + K^{\pm}(\pi^{\pm}) + X$ process at the kinematics region of Belle and compare with the experimental measurement

What have we done?

Theoretical Framework

- The transverse-spin dependent differential cross section can be expressed as

$$\frac{d\sigma(S_\perp)}{dz_1 dz_2 d(\cos\theta) d^2 q_T} = \frac{N_c \pi \alpha_{em}^2}{2Q^2}(1+\cos^2\theta) z_1^2 z_2^2 \{\mathcal{F}[D_1 \bar{D}_1] + |S_\perp| \sin(\phi_S - \phi)\mathcal{F}[\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \frac{D_{1T}^\perp \bar{D}_1}{M_\Lambda}], \quad (1)$$

- The transverse polarization $P_{\Lambda T}$ in $e^+e^- \to \Lambda^\uparrow hX$ can be defined as

$$P_{\Lambda T} = \frac{d\Delta\sigma}{d\sigma} = \frac{\mathcal{F}[\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \frac{D_{1T}^\perp \bar{D}_1}{M_\Lambda}]}{2\mathcal{F}[D_1 \bar{D}_1]}, \quad (2)$$

$\mathcal{F}[\omega D\bar{D}] = \sum_q e_q^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^2(\boldsymbol{q}_T - \boldsymbol{k}_T - \boldsymbol{p}_T)\omega(\boldsymbol{p}_T, \boldsymbol{k}_T) D^{\Lambda/q}(z_1, z_1^2 \boldsymbol{k}_T^2) \bar{D}^{h/\bar{q}}(z_2, z_2^2 \boldsymbol{p}_T^2)$

What have we done?

Theoretical Framework-Structure Function

$$\mathcal{F}[\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \frac{D_{1T}^{\perp} \bar{D}_1}{M_\Lambda}] = -\frac{1}{z_1^3} \frac{1}{z_2^2} \sum_q e_q^2 \int \frac{d^2 \boldsymbol{b}_\perp}{(2\pi)^2} e^{i\boldsymbol{P}_{\Lambda\perp} \cdot \boldsymbol{b}_\perp / z_1} \hat{\boldsymbol{h}}_\alpha \tilde{D}_{1T}^{\perp \Lambda^\uparrow / q(\alpha)}(z_1, b; Q) \tilde{\bar{D}}_1^{h/\bar{q}}(z_2, b; Q), \quad (3)$$

$$\mathcal{F}[D_1 \bar{D}_1] = \sum_q e_q^2 \int d^2 \boldsymbol{k}_T d^2 \boldsymbol{p}_T \delta^2(\boldsymbol{q}_T - \boldsymbol{k}_T - \boldsymbol{p}_T) D_1^{\Lambda/q}(z_1, z_1^2 \boldsymbol{k}_T^2; Q) \bar{D}_1^{h/\bar{q}}(z_2, z_2^2 \boldsymbol{p}_T^2; Q)$$
$$= \frac{1}{z_1^2} \frac{1}{z_2^2} \sum_q e_q^2 \int_0^\infty \frac{dbb}{(2\pi)} J_0(P_{\Lambda\perp} b / z_1) \tilde{D}_1^{\Lambda/q}(z_1, b; Q) \tilde{\bar{D}}_1^{h/\bar{q}}(z_2, b; Q), \quad (4)$$

What have we done?

Theoretical Framework-TMD evolution

The key purpose of the TMD evolution is to deal with the energy dependence of the TMD fragmentation functions in the b space.

$$\frac{\partial \ln \tilde{D}(z, b; \mu, \zeta_D)}{\partial \sqrt{\zeta_D}} = \tilde{K}(b; \mu), \tag{5}$$

while the μ dependence is derived from the renormalization group equation as

$$\frac{d \tilde{K}}{d \ln \mu} = -\gamma_K(\alpha_s(\mu)), \tag{6}$$

$$\frac{d \ln \tilde{D}(z, b; \mu, \zeta_D)}{d \ln \mu} = \gamma_D(\alpha_s(\mu); \frac{\zeta_D^2}{\mu^2}), \tag{7}$$

What have we done?

Theoretical Framework-TMD evolution

After solving the above evolution equations, the overall structure of the solutions are identical to each other in all TMD factorization schemes, and the evolution effects are incorporated into the exponential form factors as

$$\tilde{D}(z, b; Q) = \mathcal{D}(Q) \times e^{-S(Q,b)} \times \tilde{D}(z, b, \mu_i). \tag{8}$$

A b-dependent function $b_*(b)$ was introduced to have the property $b_* \approx b$ at small b value and $b_* \approx b_{\max}$ at large b value. In the original CSS approach it has the following form

$$b_* = b/\sqrt{1 + b^2/b_{\max}^2}\ , \ b_{\max} < 1/\Lambda_{\rm QCD}. \tag{9}$$

What have we done?

- Perturbative Sudakov form factor

$$S_{\rm P}(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A(\alpha_s(\bar{\mu})) \ln \frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})) \right], \tag{10}$$

where the coefficients A and B can be expanded as the series of α_s/π:

$$A = \sum_{n=1}^{\infty} A^{(n)} (\frac{\alpha_s}{\pi})^n, B = \sum_{n=1}^{\infty} B^{(n)} (\frac{\alpha_s}{\pi})^n. \tag{11}$$

- Non-perturbative Sudakov form factor

$$S_{\rm NP}^{D_1}(b,Q) = b^2 (g_1^{\rm ff} + \frac{g_2}{2} \ln \frac{Q}{Q_0}). \tag{12}$$

What have we done?

In the perturbative region $1/Q \ll b \ll 1/\Lambda$, the TMD fragmentation functions can be expressed as the convolution of the perturbatively calculable coefficients and the corresponding collinear counterparts of the TMD fragmentation functions

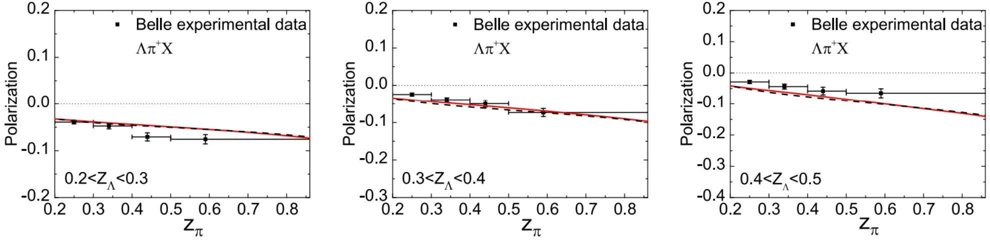
$$\tilde{D}(z, b; \mu_b) = \sum_i \int_z^1 \frac{d\xi}{\xi} C_{q\leftarrow i}(z/\xi, b; \mu) D_{i/H}(\xi, \mu). \tag{13}$$

at the fixed energy scale μ_b that related to b_* through $\mu_b = c/b_*$, with $c = 2e^{-\gamma_E}$.

$$\begin{aligned}
\tilde{D}_1^{h/q}(z, b; Q) &= e^{-\frac{1}{2}S_{\rm P}(Q,b_*) - S_{\rm NP}^{D_1^{h/q}}(Q,b)} D_1^{h/q}(z, \mu_b), \\
\tilde{D}_1^{\Lambda/q}(z, b; Q) &= e^{-\frac{1}{2}S_{\rm P}(Q,b_*) - S_{\rm NP}^{D_1^{\Lambda/q}}(Q,b)} D_1^{\Lambda/q}(z, \mu_b), \\
\tilde{D}_{1T}^{\perp\Lambda/q(\alpha)}(z, b; Q) &= (\frac{ib^\alpha}{2}) e^{-\frac{1}{2}S_{\rm P}(Q,b_*) - S_{\rm NP}^{D_{1T}^{\perp\Lambda/q}}(Q,b)} \hat{D}_{1T}^{\perp(3)}(z, z, \mu_b).
\end{aligned} \tag{14}$$

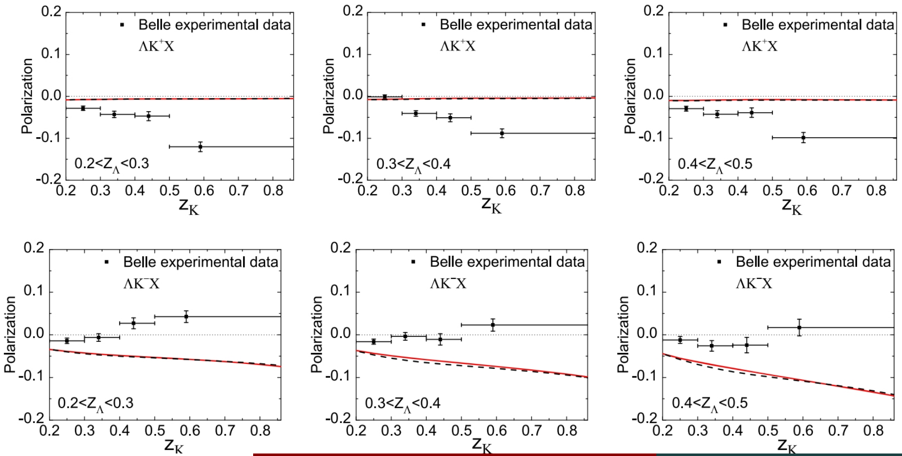
The comparison with experimental measurement

The Λ transverse polarization calculated in the process $e^+e^- \to \Lambda^\uparrow + \pi^+ + X$.



The comparison with experimental measurement

The Λ transverse polarization calculated in the process $e^+e^- \to \Lambda^\uparrow + K^\pm + X$.



Summary

- Different choices of nonperturbative Sudakov form factors in the TMD evolution formalism lead to similar results for transverse polarization of Λ in process e^+e^- annihilation.
- Within the framework of TMD evolution, our prediction for the polarization in $\Lambda^{\uparrow}\pi^{+}$ production agrees with the Belle data.
- There are large discrepancies between the Belle data and our predictions for $\Lambda^{\uparrow}\pi^{-}$ production as well as $\Lambda^{\uparrow}K^{\pm}$ production.
- A comparison with our model input for the PFF with the available parameterizations indicates that a positive $D_{1T}^{\perp\Lambda/u}$ and sizable sea PFF are essential to describe the many aspects of the Belle data.

Unpolarized Integrated Fragmentation function $D_{1,h/i}$

- FFs describe the fragmentation of an unpolarized parton of type *i* into an unpolarized hadron of type *h*, where the hadron carries the fraction ζ of the parton momentum
- Universal among different processes
- Plays crucial role in SIA: $e^+e^- \to hX$, SIDIS: $ep \to ehX$, $h_1 h_2 \to hX$ process
- The constituent quarks for Λ contain *uds*, which may be a tool to study the flavor dependence of proton PDF
- Makes Lambda production SIDIS process important in studying the nucleon structure
- The Lambda FF is the fundamental input in Lambda production SIDIS process

The existed parametrization of Lambda FF $D_{1,\Lambda/i}$

The existed parametrization of Lambda FF $D_{1,\Lambda/i}$

para.	considered process	# of data	χ^2 (χ^2_{DF})
DSV98[a]	$e^+e^- \to \Lambda(\bar{\Lambda})X$	103	103.55(NLO), 104.29(LO)
AKK05[b]	$e^+e^- \to \Lambda(\bar{\Lambda})X$	129	1.39
AKK08[c]	$e^+e^- \to \Lambda(\bar{\Lambda})X$, $p\bar{p} \to \Lambda(\bar{\Lambda})X$	188	1.45
SAK20[d]	$e^+e^- \to \Lambda(\bar{\Lambda})X$	137	1.601(NLO) , 1.602(NNLO)

[a] **deFlorian:1997zj**.
[b] **Albino:2005mv**.
[c] **Albino:2008fy**.
[d] **Soleymaninia:2020ahn**.

The opportunity

Some issues

- Lack of experimental data from SIDIS process
- Without consistent theoretical framework of 3 processes in global fitting

What can we do?

- Including much more data from SIDIS process
- Considering the consistent theoretical framework of all 3 processes to do the global fitting

What have we done up to now?

Data

We have collected most of the data in three different processes: $e^+e^- \to \Lambda X$, $ep \to e'\Lambda X$, $pp \to \Lambda X$.

Collaboration	process	CM energy (GeV)	observable	No. of points
TASSO 81	$e^+e^- \to \Lambda(\bar{\Lambda})X$	33.3	$\frac{d\sigma}{dp}$	5
TASSO 85	$e^+e^- \to \Lambda(\bar{\Lambda})X$	14, 22, 34	$\frac{s}{\beta}\frac{d\sigma}{dx_E}$	3, 4, 7
TASSO 89	$e^+e^- \to \Lambda(\bar{\Lambda})X$	34.8, 42.1	$\frac{s}{\beta}\frac{d\sigma}{dx_E}$	10, 5
ARGUS 88	$e^+e^- \to \Lambda X$	10.0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	15
CELLO 89	$e^+e^- \to \Lambda(\bar{\Lambda})X$	35	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_E}$	7
JADE 81	$e^+e^- \to \bar{\Lambda}X$	34	$\frac{d\sigma}{dp}$	2
MARK II 85	$e^+e^- \to \Lambda(\bar{\Lambda})X$	29	$\frac{d\sigma}{dx_E}$	15
MARK II 85	$e^+e^- \to \Lambda(\bar{\Lambda})X$	29	$\frac{d\sigma}{dp_\perp^2}$	15

What have we done up to now?

data

HRS 86	$e^+e^- \to \Lambda(\bar{\Lambda})X$	29	$\frac{s}{\beta}\frac{d\sigma}{dx_E}$	12
HRS 92	$e^+e^- \to \Lambda(\bar{\Lambda})X$	29	$\frac{s}{\beta}\frac{d\sigma}{dx_E}$	7
CLEO 85	$e^+e^- \to \Lambda(\bar{\Lambda})X$	10.49	$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$	11
ALEPH	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	22
ALEPH	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dp_T}$	30
ALEPH 98	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	25
OPAL	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	13
OPAL 97	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	18
DELPHI	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	6
DELPHI 93	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	11
DELPHI 2000	$e^+e^- \to \Lambda X$	183	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi_p}$	7

What have we done up to now?

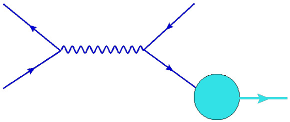
data

DELPHI 2000	$e^+e^- \to \Lambda X$	189	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi_p}$	8
SLD	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	15
SLD-uds tagged	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	8
SLD-c tagged	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	8
SLD-b tagged	$e^+e^- \to \Lambda(\bar{\Lambda})X$	91.2	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	8
CLEO 84	$e^+e^- \to \Lambda(\bar{\Lambda})X$	10.49	$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$	15
TPC 85	$e^+e^- \to \Lambda(\bar{\Lambda})X$	29	$\frac{1}{\sigma_h \beta}\frac{d\sigma}{dx_E}$	9
ALICE	$pp \to \Lambda X$	13000,7000		
CMS	$pp \to \Lambda X$	900,5000,7000,13000		
ZEUS	$ep \to e\Lambda X$	296,318		
H1	$ep \to e\Lambda X$	319		

What have we done up to now?

Physics

- We have setup the basic formula to calculate the cross section in $e^+e^- \to \Lambda X$, $e^-p \to e\Lambda X$, $pp \to \Lambda X$ process.
- Take $e^+e^- \to \Lambda X$ as an example



- The cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dz} = F^h(z, Q^2)$, $\sigma_{tot} = N_c \Sigma_q e_q^2 \frac{4\pi\alpha^2}{3Q^2}(1 + \frac{\alpha_s}{\pi})$.

$$F^h(z, Q^2) = \frac{1}{\Sigma_q e_q^2}(2F_1^h(z, Q^2) + F_L^h(z, Q^2)), \tag{15}$$

What have we done up to now?

Physics

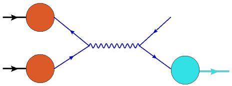
$$2F_1^h(z,Q^2) = \sum_q e_q^2 \Big[\big(D_1^{h/q}(z,Q^2) + D_1^{h/\bar{q}}(z,Q^2)\big) + \frac{\alpha_s(Q^2)}{2\pi} \Big(C_1^q \otimes (D_1^{h/q} + D_1^{h/\bar{q}}) + C_1^g \otimes D_1^{h/g} \Big)(z,Q^2),$$

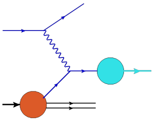
$$F_L^h(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_q e_q^2 \Big((C_L^q \otimes \big(D_1^{h/q} + D_1^{h/\bar{q}}\big) + C_L^g \otimes D_1^{h/g})(z,Q^2) \Big), \qquad (16)$$

What have we done up to now?

Physics

- We have setup the basic formula to calculate the cross section in $e^+e^- \to \Lambda X$, $e^-p \to \Lambda X$, $pp \to \Lambda X$ process.





- $pp \to \Lambda X$ process
  - Cross section: $\sigma = \sum_q PDF \otimes PDF \otimes \hat{\sigma} \otimes FF$
  - Dependent on the initial PDFs
  - May access the gluon FF
- SIDIS process
  - Cross section: $\sigma = \sum_q PDF \otimes \hat{\sigma} \otimes FF$
  - Dependent on the initial PDF
  - Flavor dependence of PDF can be accessed by FF

What have we done up to now?

Physics

- DGLAP evolution equations of FF specify the Q^2 dependence of the fragmentation function

$$\frac{\partial D_{1,q}(z,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_0^x \frac{dy}{y}[D_{1,i}(y,Q^2)P_{qi}(\frac{x}{y}) + D_{1,g}(y,Q^2)P_{qg}(\frac{x}{y})]$$
$$\frac{\partial D_{1,g}(z,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_0^x \frac{dy}{y}[D_{1,i}(y,Q^2)P_{gq}(\frac{x}{y}) + D_{1,g}(y,Q^2)P_{gg}(\frac{x}{y})] \quad (17)$$

- Mellin Transformation can be introduced to simplify the evaluation of DGLAP.

What have we done up to now?

Physics

- The Mellin transform of a function $f(x)$ is defined as:

$$f(n) = \int_0^1 x^{n-1} f(x)\, dx \tag{18}$$

where n is a complex parameter.

- The convolution can be simplified to product of the Mellin transformed function

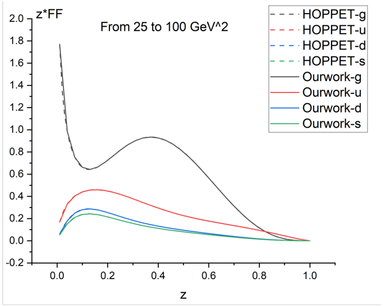
$$\int_0^1 dx x^{n-1} \int_x^1 \frac{dy}{y} h(y) g(\frac{x}{y}) = h(n) \cdot g(n). \tag{19}$$

DGLAP evolution effect takes a very simple form.

What have we done up to now?

Code

- We have set the standard format of the data and normalized the data
- We have written the pseudo fragmentation function
- We have performed the Mellin Transformation and the Inverse Mellin Transformation of the pseudo fragmentation function
- We have performed the DGLAP evolution in the Mellin space at LO
- The DGLAP code can perfectly reproduce the evolution effect performed by HOPPET



What shall we do in the future?

Summary

- We have collected most of the data considering the Lambda FF
- We have setup the basic theoretical framework to calculate the experimental observables
- We have written and done some test of the pseudo fragmentation function

Future Plan

- **Physics**: Derive the evolution in the Mellin moment space at higher order
- **Code**: Perform the evolution in the Mellin moment space at higher order and transform into the z space and compare with HOPPET.
- **Global Fitting**: Determine the form of the parametrization, perform the fitting procedure and compare the prediction power of different forms

Summary & Outlook

Summary

- We have done some preliminary work on the parametrization of integrated FF of Lambda
- We can reproduce the widely used evolution package HOPPET

Outlook

- Much more data, much more uncertainty analysis
- Polarized FF of Lambda, Collins function of Lambda etc...
- TMD FF of Lambda

Thank you!

TMD Factorization

- TMD Factorization depends on the two energy scale
  - Hard scale $Q_1 \gg \Lambda_{QCD} \approx 1/R$
  - Soft Scale $Q_2 \ll Q$
- Typical two-scale processes
  - SIDIS
  - Drell-Yan
  - e^+e^- annihilation to produce two hadrons