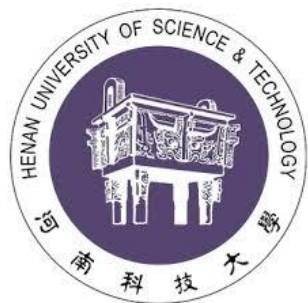


# Neutrino-Nucleus scattering with *ab initio* No-Core Shell Model

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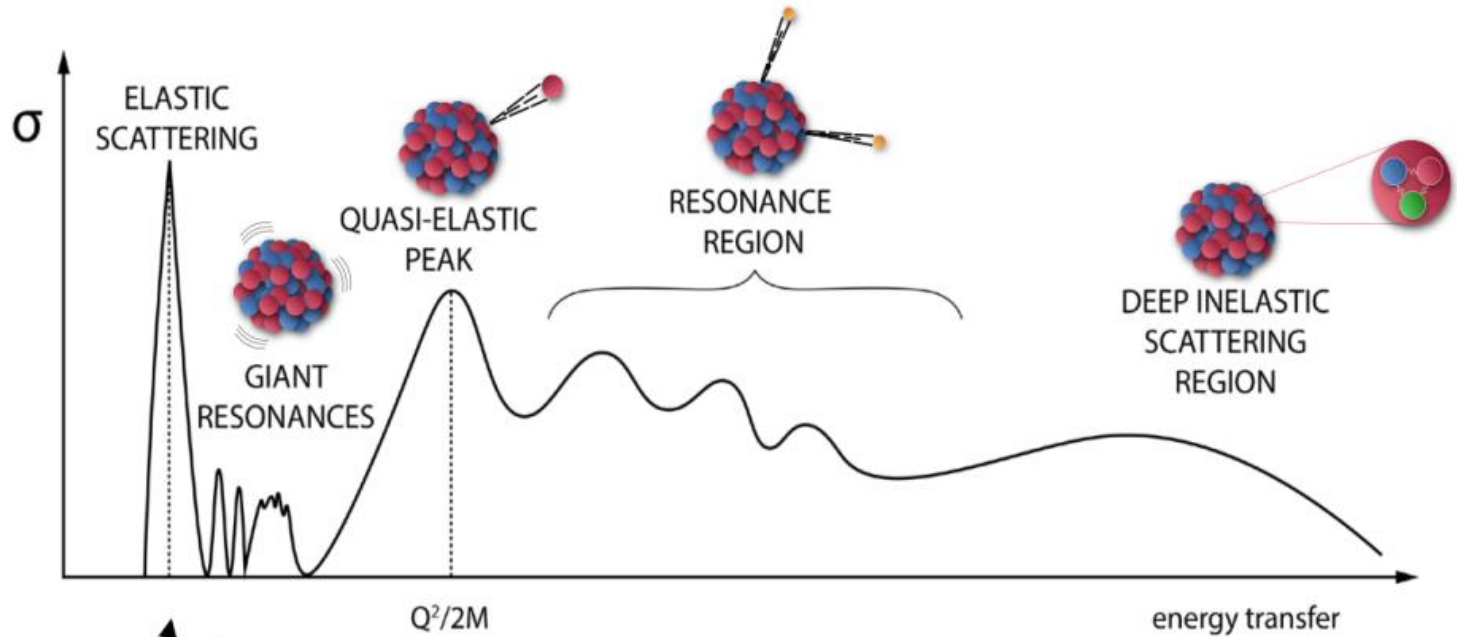
Pieter Maris, James P. Vary (ISU)

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# Neutrino-nucleus interactions



Inelastic scattering

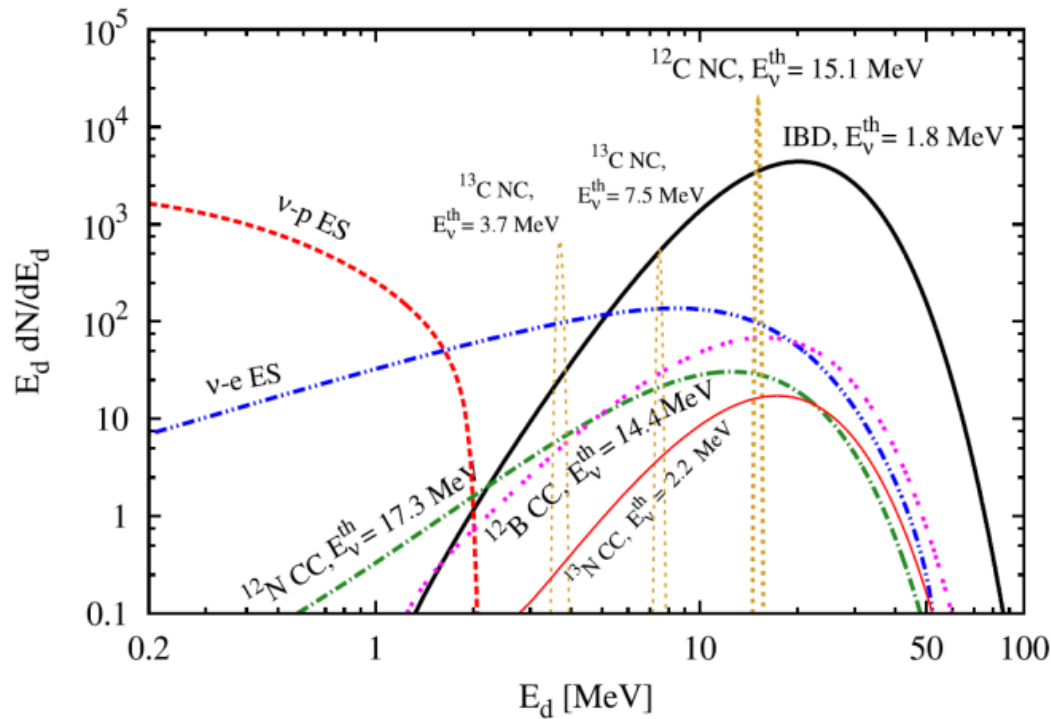
**Elastic scattering: CEvNS**  
(coherent elastic neutrino-nucleus scattering)

e.g. Supernovae neutrinos

Long-baseline experiments  
(DUNE, HyperK)

- Nuclear response at different energy transfer regions is crucial to make the predictions.

# (Quasi-)elastic $\nu$ -nucleus CC/NC interactions



Experiment	Nuclear Target	Reaction	$\sigma_0$ [ $10^{-46} \text{cm}^2$ ]	$\Delta E_{\text{nucl}}$ [MeV] (no det. Thres.)
GALLEX/GNO SAGE	$^{71}\text{Ga}_{33}$	$\nu_e + ^{71}\text{Ga} \rightarrow e^- + ^{71}\text{Ge}$	$8.611 \pm 0.4\%$ (GT)	0.2327
HOMESTAKE	$^{37}\text{Cl}_{17}$	$\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}$	1.725 (F)	0.814
SNO	$^2\text{H}_1$	$\nu_e + ^2\text{H} \rightarrow e^- + p + p$	(GT)	1.442
DUNE, ICARUS, etc.	$^{40}\text{Ar}_{18}$	$\nu_e + ^{40}\text{Ar} \rightarrow e^- + ^{40}\text{K}^*$	148.58 (F) ... 44.367 (GT <sub>2</sub> ) ... 41.567 (GT <sub>6</sub> ) ...	1.505 +

From Kevin McFarland

➤ Important for solar & supernova neutrino detection

JUNO, Prog.Part.Nucl.Phys. 123 (2022) 103927

Channels	Threshold [MeV]	Signal	Event numbers	
			[200 kt × yrs]	after cuts
CC $\nu_e + ^{13}\text{C} \rightarrow e^- + ^{13}\text{N} (\frac{1}{2}^-; \text{gnd})$	2.2 MeV	$e^- + ^{13}\text{N}$ decay	3929	647
NC $\nu_x + ^{13}\text{C} \rightarrow \nu_x + ^{13}\text{C} (\frac{3}{2}^-; 3.685 \text{ MeV})$	3.685 MeV	$\gamma$	3032	738
ES $\nu_x + e \rightarrow \nu_x + e$	0	$e^-$	$3.0 \times 10^5$	$6.0 \times 10^4$

JUNO, Astrophys.J. 965 (2024) 2, 122

# Phenomenological models calculations

## ❖ Neutrinos- $^{12}\text{C}$ & $^{13}\text{C}$ cross-sections calculations with phenomenological models

	$(\nu_\mu, \mu^-)DIF$ $\langle \sigma \rangle_f (10^{-40} \text{ cm}^2)$	$(\nu_e, e^-)DAR$ $\langle \sigma \rangle_f (10^{-42} \text{ cm}^2)$
SM(HO wf) $(0 + 1 + 2)\hbar\omega$	0.70	8.42
SM(HF wf) $(0 + 1 + 2)\hbar\omega$	0.65	8.11
SM(WS wf) $(0 + 1 + 2)\hbar\omega$	0.58	8.4
RPA	2.09	49.47
QRPA	1.97	42.92
CRPA	1.06(1.03)	13.88(12.55)
EXP	$0.66 \pm 1.0 \pm 1.0$ [a]	$10.5 \pm 1.0 \pm 1.0$ [b] $9.1 \pm 0.4 \pm 0.9$ [c] $9.1 \pm 0.5 \pm 0.8$ [d]

### Neutrinos- $^{12}\text{C}$ cross-sections from different models

Shell Model (SM)

Random Phase Approximation (RPA)

Quasi-particle RPA (QRPA)

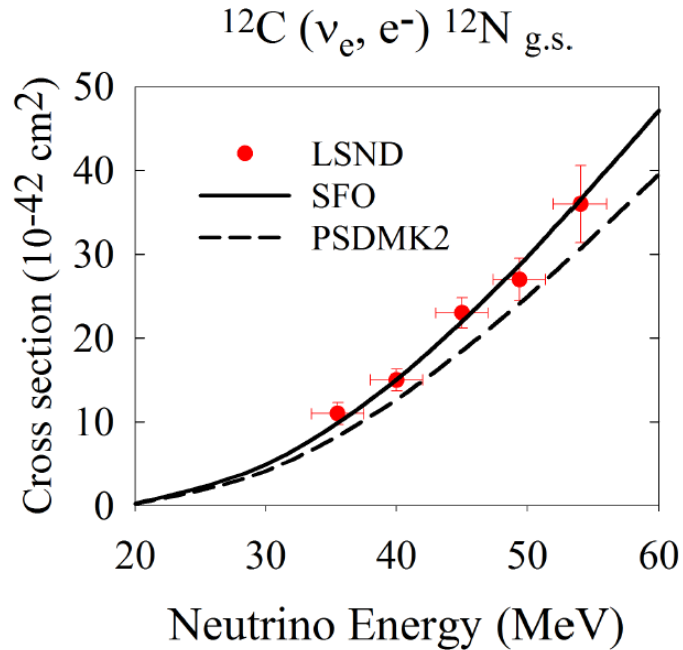
Continuum RPA (CRPA)

- a: C.Athanassopoulos and the LSND collaboration, Phys. Rev. C (1997),  
 b: R.C.Allen et al., Phys. Rev. Lett (1990),  
 c: C.Athanassopoulos and the LSND collaboration, Phys. Rev. C (1997),  
 d: B.E.Bodmann and the KARMEN collaboration, Phys. Lett. B (1994)

**The results from RPA calculations are far away from experimental data, while the SM calculations are much closer to experimental data**

# Phenomenological models calculations

## ❖ Neutrinos- $^{12}\text{C}$ & $^{13}\text{C}$ cross-sections calculations with shell model



Neutrino- $^{12}\text{C}$  charge current cross-sections from shell model with SFO interaction had a good agreement with experimental data

$B(GT: ^{13}\text{C} \rightarrow ^{13}\text{N})$			SFO	CK	EXP.
$^{13}\text{N}$	$J^\pi$	$E_x$ (MeV)			
	$1/2^-$	0.0	0.284	0.420	$0.411 \pm 0.004$ $0.398 \pm 0.008$
	$1/2^-$	8.92	0.569	0.524	
	$3/2^-$	3.50	2.103	2.14	$1.64 \pm 0.10$
	$3/2^-$	9.46	0.500	0.260	
$B(M1) (\mu_N^2)$					
$^{13}\text{C}$	$(3/2^-: 3.68 \text{ MeV}) \rightarrow ^{13}\text{C}$	$(1/2_{g.s.}^-)$	0.878	1.17	$0.698 \pm 0.072$

B(M1) and B(GT) strengths of  $^{13}\text{C}$  from shell model with SFO & CK interaction were still **a little far away from experimental data**

# What is Ab Initio in nuclear physics?

Few-Body Syst (2023) 64:77  
https://doi.org/10.1007/s00601-023-01857-2



**R. Machleidt**

## What is *ab initio*?

Nuclear structure theory at its basic level is not about fitting data to get “good” results. Fundamental nuclear structure theory is about answering the question:

*Do the same nuclear forces that explain free-space scattering experiments also explain the properties of finite nuclei and nuclear matter when applied in nuclear many-body theory?*



### OPEN ACCESS

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## What is *ab initio* in nuclear many-body theory?

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*Ab initio* has been used as a label in nuclear theory for over two decades. Its meaning has evolved and broadened over the years. We present our interpretation, briefly review its historical use, and discuss its present-day relation to theoretical uncertainty quantification.

# Challenges

- **Self bound quantum many-body problem**, with  $3A$  degrees of freedom in coordinate (or momentum) space, as well as spin degree of freedom
- **Strong interactions**, with both short-range and long-range pieces
- Not only 2-body interactions, but also **intrinsic 3-body interactions** and possibly 4- and higher N-body interactions
- **Uncertainty quantification** for calculations needed
  - for comparison with experiments
  - for comparison between different methods

# Computational Methods for Nuclear Structure

## ➤ Few-body methods

- Faddeev Equation for  $A=3$  system
  - typically in momentum space
- Faddeev-Yakuboski Equations for  $A=4$  system
  - can nowadays be pushed to  $A=5$  and  $6$  (Lazauskas)
- Hyperspherical Harmonics
  - Up to  $A=6$

## ➤ Many-body methods

- Variational Monte-Carlo ( $A \leq 12$ )
- Green's Function Monte-Carlo ( $A \leq 12$ )
- Configuration Interaction (CI) methods (NCSM ( $A \leq 20$ ), Coupled Cluster ( $A \leq 100$ ))
- Nuclear Lattice Simulations ( $A \leq 32$ )

➤ All few- and many-body methods need some levels of **High-Performance Computing**



# Nuclear Interactions

$$\hat{H}_{\text{rel}} = \hat{T}_{\text{rel}} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

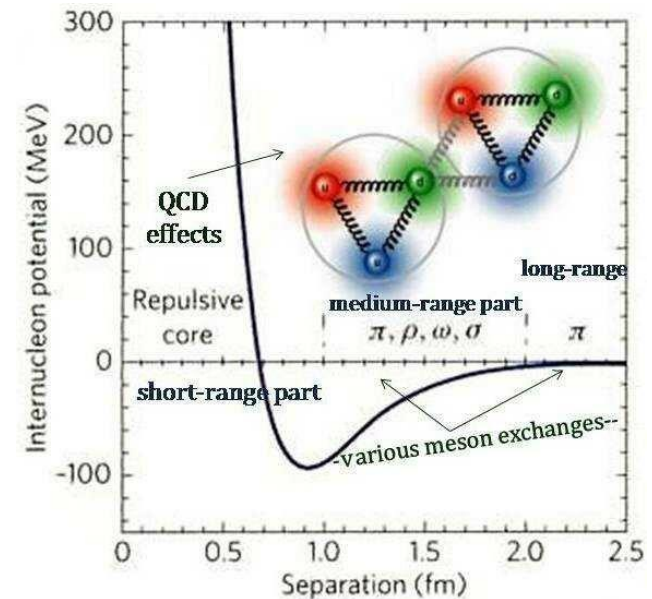
## ➤ Nuclear interactions not well-determined

- In principle calculable from QCD
- In practice constrained by (fitting to) experimental ( $NN$  scattering) data

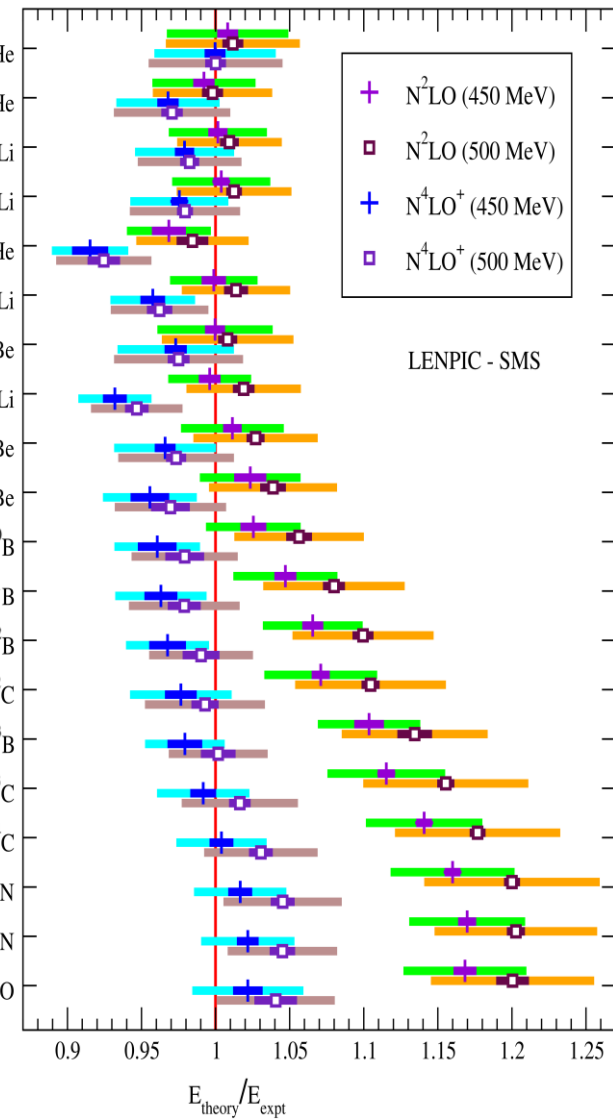
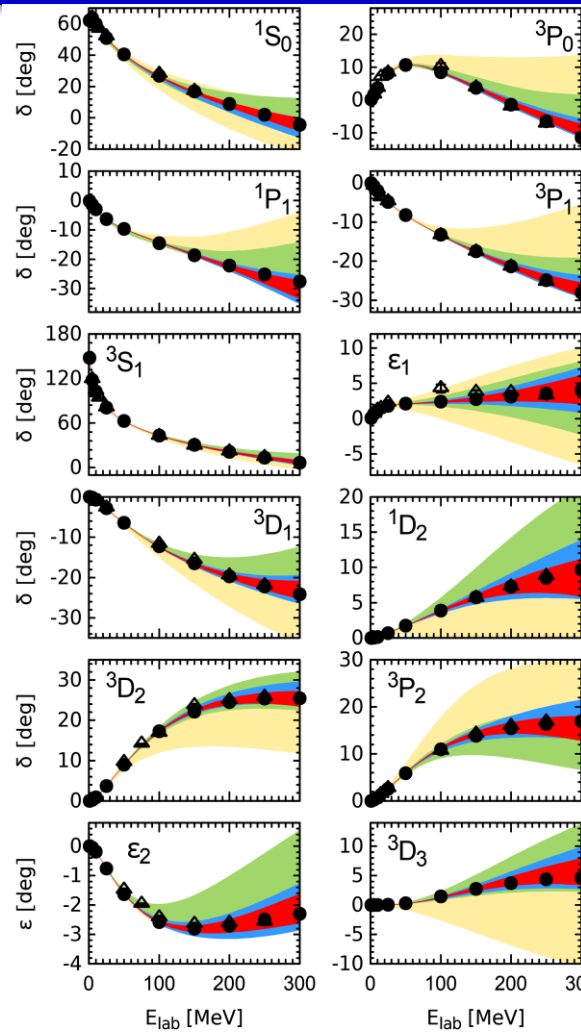
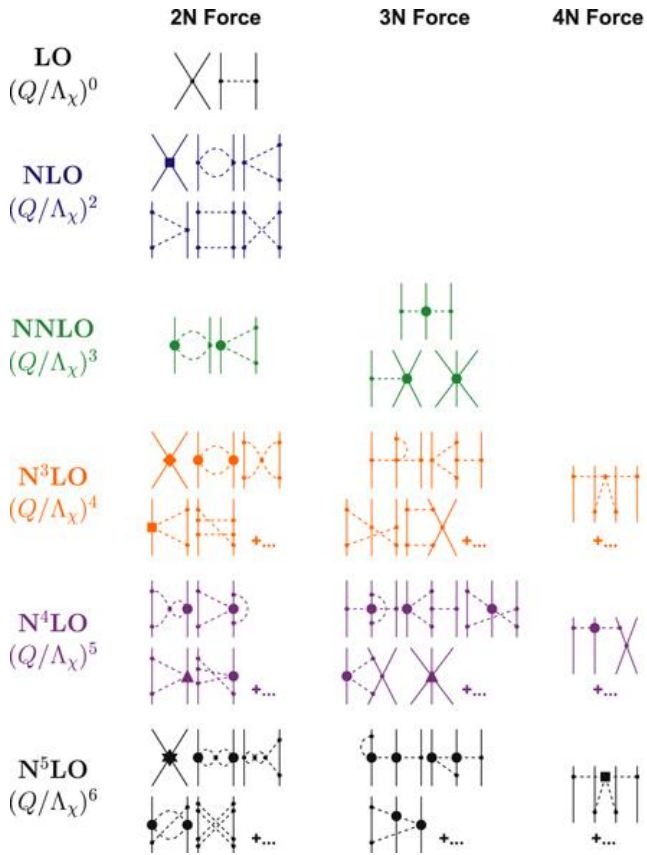
## ➤ Alphabet of realistic interactions

- Argonne potentials
- Bonn potentials
- Chiral EFT interactions
- Daejeon16 (based on Idaho-N3LO)
- ...

## ➤ Most $NN$ potentials need $3N$ forces for agreement with experiments



# LENPIC chiral EFT NN potential up to N<sup>4</sup>LO



LENPIC Collaboration,  
 Phys. Rev. C 106, 064002 (2022)

# Introduction of NCSM

## ❖ No-Core Configuration Interaction / No-Core Shell Model calculations

- Given a Hamiltonian operator

$$\hat{H} = \sum_{i < j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i < j}^A V_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

- Solve the eigenvalue problem for wavefunction of A nucleons

$$\hat{H}\Psi_{\alpha_1 \dots \alpha_A} = E\Psi_{\alpha_1 \dots \alpha_A}$$

- Expand wavefunction in basis states  $|\psi\rangle = \sum \hat{a}_j |\phi_i\rangle$

- Express Hamiltonian in basis  $\langle \phi_j | \hat{H} | \phi_i \rangle = H_{ij}$

- Diagonalize Hamiltonian matrix  $H_{ij}$

- **No-Core: All A nucleons are treated equally**

- **Complete basis ---- exact result**

- In practice

1. truncate basis

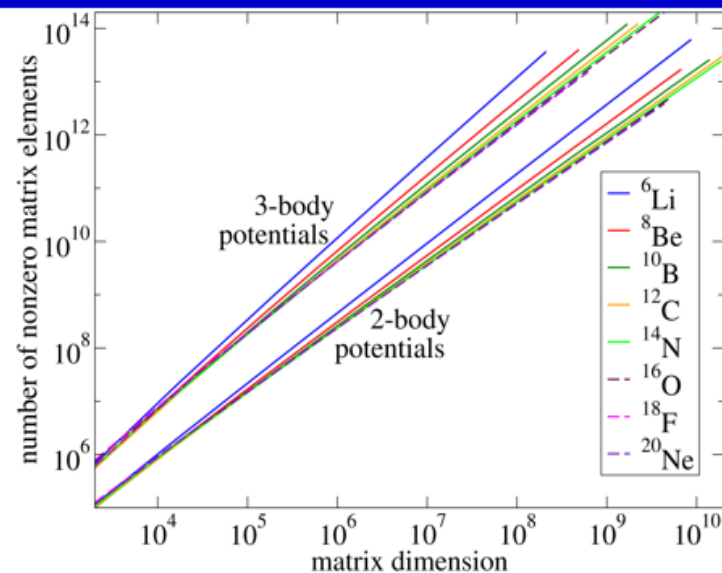
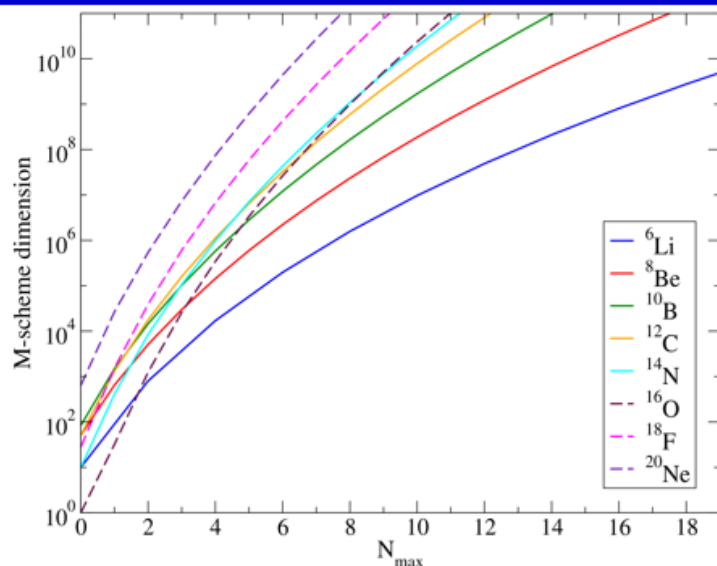
2. study behavior of observables as function of truncation

- **Computational challenge**

1. construct large ( $10^{10} \times 10^{10}$ ) sparse symmetric matrix  $H_{ij}$

2. obtain lowest eigenvalues & eigenvectors corresponding to low-lying spectrum and eigenstates

# Main Challenges



- Increase of basis space dimension with  $A$  and  $N_{\max}$ 
  - Need calculations up to at least  $N_{\max}=8$ , preferably  $N_{\max}=10$  for meaningful extrapolation and numerical error estimates
- More relevant measure for computational needs
  - Number of nonzero matrix elements
  - Current limit  $10^{14}$  (Perlmutter)



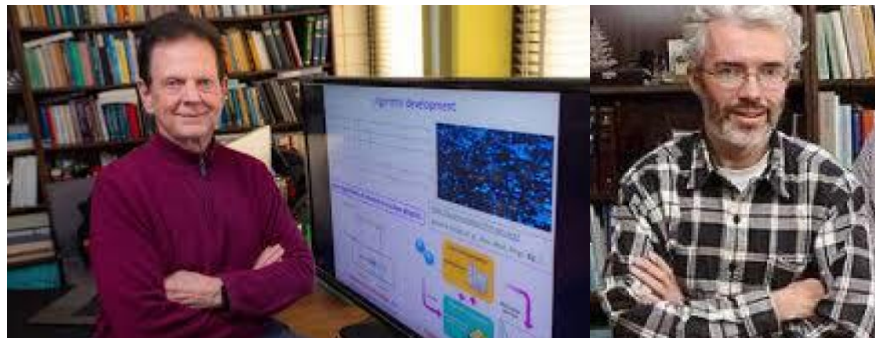
National Energy Research  
Scientific Computing Center

# MFDn in the SciDAC era

[https://nuclei.mps.ohio-state.edu/nuclei\\_home.php](https://nuclei.mps.ohio-state.edu/nuclei_home.php)

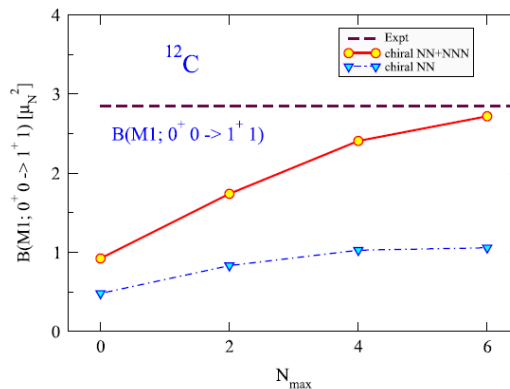
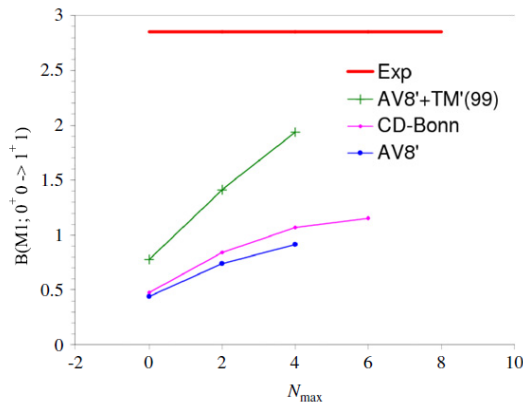


- ▶ Collaborative DOE grants for domain scientists (i.e. nuclear physicists), computer scientists, and applied mathematicians to enable scientific progress through high-performance computing
- ▶ Support for students and postdocs to do the hard & tedious work
- ▶ UNEDF (2007-2012), NUCLEI(2012-2017, 2017-2022)
  - ▶ MFDn: collaboration between ISU (James Vary), Ames Lab (Masha Sosonkina), and Berkeley Lab (Esmond Ng, Chao Yang)
  - ▶ Ab Initio: Coupled Cluster, VMC & GFMC, lattice EFT, ...
  - ▶ Density Functional Theory for heavy nuclei
  - ▶ BIGSTICK: complementary/competing NCCI code (Calvin Johnson)
  - ▶ Valence Shell Model (NuShellX, BIGSTICK) (not in NUCLEI)



# Previous calculations from ab initio NCSM

## ❖ Neutrino- $^{12}\text{C}$ cross-sections calculations with NCSM



AV8': NN potential  
 CD-Bonn: NN potential  
 AV8' + TM'(99):  
 NN potential + 3N NN potential

The B(M1) calculated values from AV8', CD-Bonn, AV8' + TM'(99) were still a little far away from experimental data

### Experimental and calculated B(M1; $0^+0 \rightarrow 1^+1$ ) values for $^{12}\text{C}$

Interaction	CD-Bonn			AV8' + TM'(99)	Experiment
	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$4\hbar\Omega$	
$(\nu_e, e^-)$	2.27	3.2	3.69	6.8	$8.9 \pm 0.3 \pm 0.9$ [a]
$(\nu_\mu, \mu^-)$	0.168	0.275	0.312	0.537	$0.56 \pm 0.08 \pm 0.1$ [b]
$\mu$ -capture	1.46	2.07	2.38	4.43	$6.0 \pm 0.4$ [c]

**Predicted weak interaction rates of  $^{12}\text{C}$ . The units are  $10^{-42} \text{ cm}^2$  for the  $(\nu_e, e^-)$  DAR cross section,  $10^{-42} \text{ cm}^2$  for the  $(\nu_\mu, \mu^-)$  DIF cross section, and  $10^3 \text{ sec}^{-1}$  for muon capture**

a:LSND Collaboration, L. B. Auerbach et al., Phys. Rev. C 64, 065501 (2001).

b:LSND Collaboration, L. B. Auerbach et al., Phys. Rev. C 66, 015501 (2002).

c: G. H. Miller et al., Phys. Lett. B 41, 50 (1972);

A. C. Hayes, P. Navrátil, and J. P. Vary, Phys. Rev. Lett. 91, 012502 (2003)

Barrett et al., , Progress in Particle and Nuclear Physics (2013)

# Improvements of our calculations

From the previous NCSM calculations of neutrino- $^{12}\text{C}$  scattering, we can infer that the precision of neutrino- $^{12}\text{C}$  cross-sections with Ab initio NCSM/NCCI, could be improved by **applying new interaction(with the contributions from 3N force)**, **increasing the basis space**, **including more reaction channels** and **excited states**.

## ❖ Highlights in our calculations

1. New interaction – Daejeon16

Dimensions of many body matrix in  
Nmax6:  $3.26 \times 10^7$ ; Nmax10:  $7.83 \times 10^9$

2. Larger basis space for calculations(up to Nmax=10)

3. Higher excitation energy (at least 20 MeV~ 30 states)

4. More channels(**neutral current**, charged current, **nucleon knock-out**)

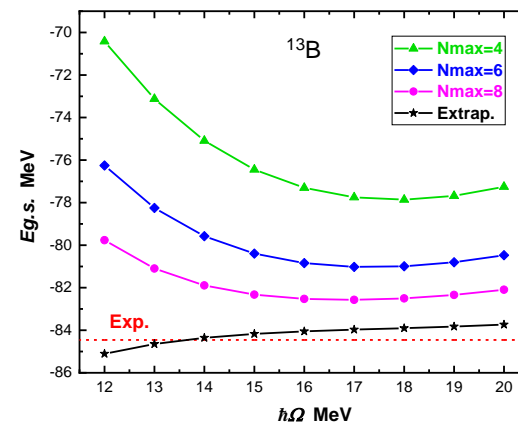
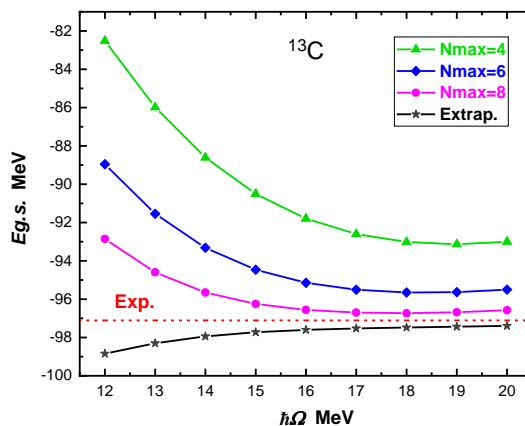
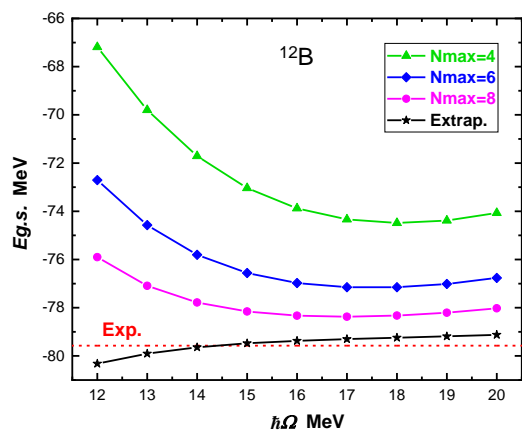
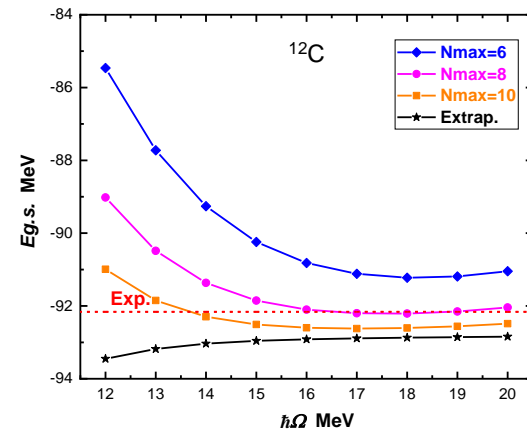
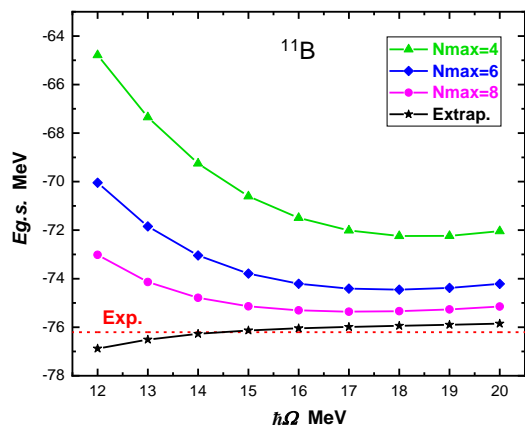
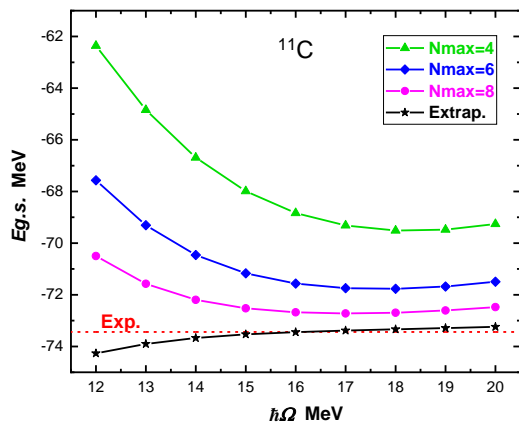
5. Theoretical uncertainty

6. Extrapolation tools (Traditional and machine learning ANN)

7. Calculations of cross sections at **reactor neutrino energies**

# Current progress

## ❖ Ground state energy from Daejeon16



**NCSM+Daejeon16 has good performance on the calculations of binding energies**

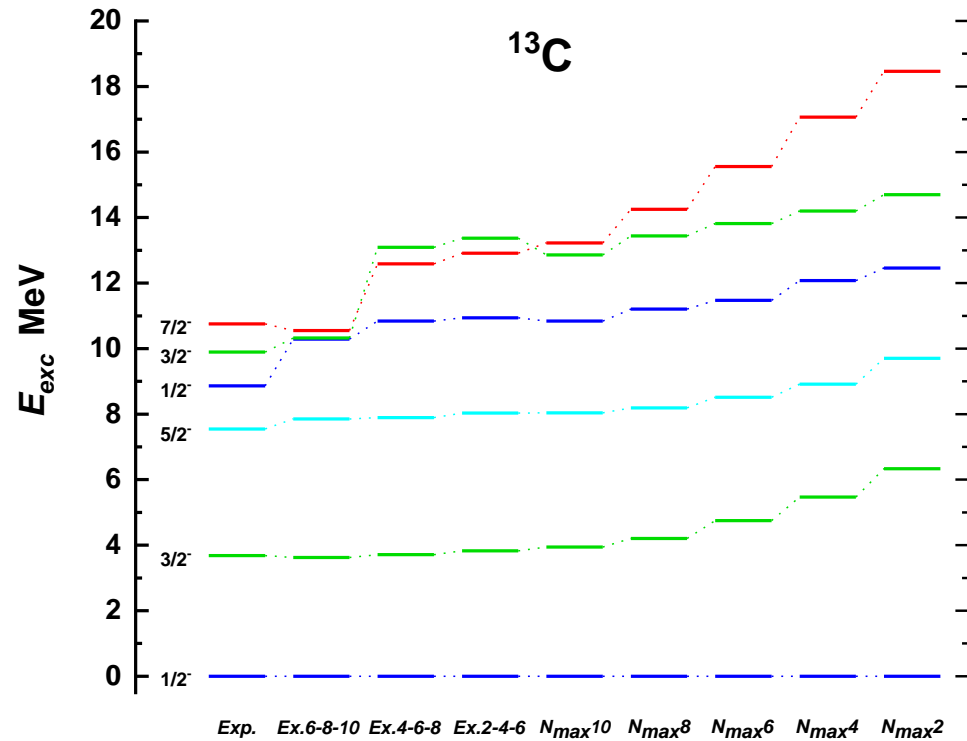
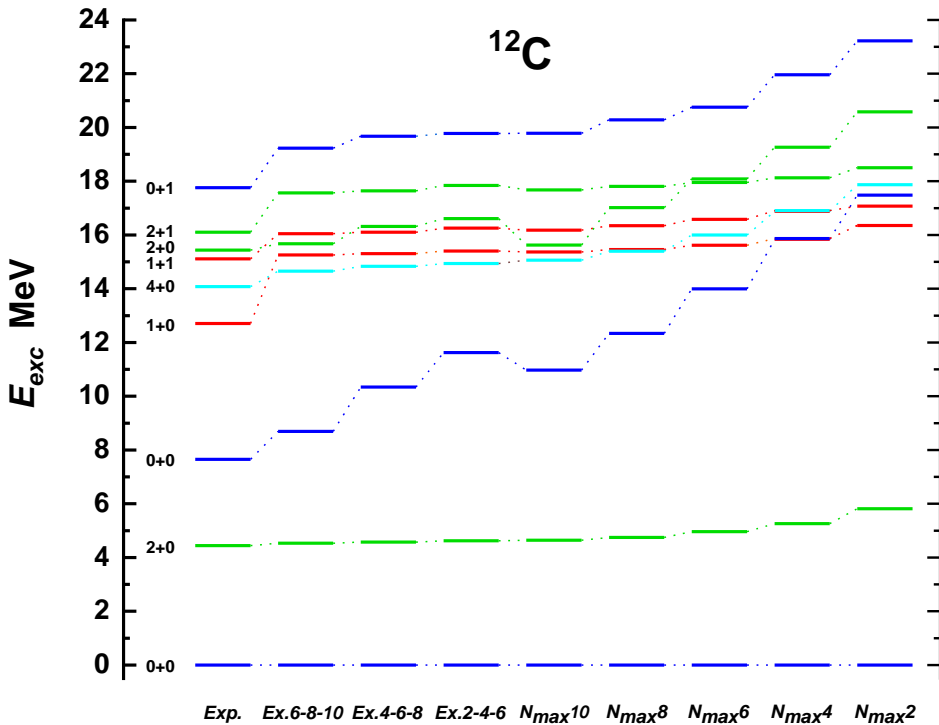
**We use a simple 3-point exponential extrapolation from the reference paper**

$$E^{\hbar\omega}(N_{\max}) = E_{\infty}^{\hbar\omega} + a e^{(-b N_{\max})}$$



# Present Progress

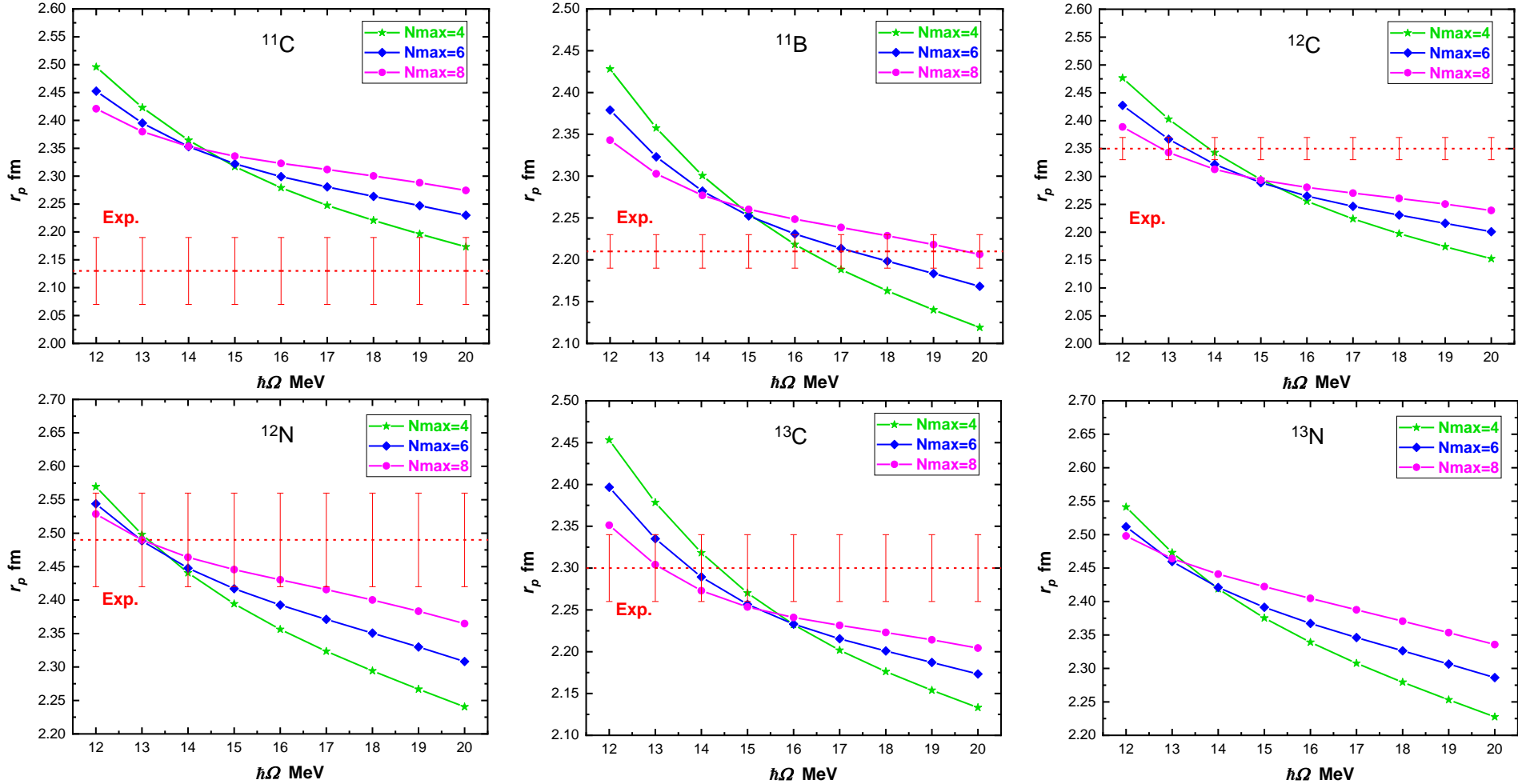
## ❖ Energy spectra from Daejeon16



The low lying spectra of  $^{12}\text{C}$  &  $^{13}\text{C}$  from Daejeon16 have good agreement with experimental data

# Present Progress

## ❖ Point Proton Radii of ground state from Daejeon16



We get the estimations and uncertainties of point proton radii from the “crossing points”  
The point proton radii values from Daejeon16 quite close to experimental data

# Present Progress

- ❖ Comparisons on ground state energies and point proton radii between Daejeon16 and other Chiral EFT interactions

		Daej16-Extrap.	NN+NNN <sub>2007</sub>	N <sup>2</sup> LO(450) <sub>2021</sub>	N <sup>2</sup> LO(500) <sub>2021</sub>
<sup>11</sup> C	Exp.				
E <sub>g.s.</sub> MeV	-73.441	-73.39(51)			
r <sub>p</sub> fm	2.13(6)	2.36(2)			
<sup>12</sup> C	Exp.	Daej16-Extrap.	NN+NNN <sub>2007</sub>	N <sup>2</sup> LO(450) <sub>2021</sub>	N <sup>2</sup> LO(500) <sub>2021</sub>
E <sub>g.s.</sub> MeV	-92.162	-92.93(64)	-95.57	-98.7(4)	-101.8(4)
r <sub>p</sub> fm	2.35(2)	2.30(1)	2.172		
<sup>13</sup> C	Exp.	Daej16-Extrap.	NN+NNN <sub>2007</sub>	N <sup>2</sup> LO(450) <sub>2021</sub>	N <sup>2</sup> LO(500) <sub>2021</sub>
E <sub>g.s.</sub> MeV	-97.108	-97.53(72)	-74.716	-108.3(4)	112.2(4)
r <sub>p</sub> fm	2.29(3)	2.25(1)	2.135		

**Ground state energies and point proton radii of Carbon-11, 12, 13 from Daejeon16 in good agreement with experiment, even better than NN + NNN interaction.**

# Present Progress

- ❖ Comparisons on ground state energies and point proton radii between Daejeon16 and other Chiral EFT interactions

$^{12}\text{N}$	Exp.	Daejeon16-Extrap.	$\text{N}^3\text{LO}$	$\text{N}^2\text{LO}_{opt}$
$E_{g.s.}(1^+)$ MeV	-74.041	-73.99(56)		
$r_p$ fm	2.49(7)	2.49(2)		
$^{13}\text{N}$	Exp.	Daejeon16-Extrap.	LO	LO*
$E_{g.s.}(1/2^-)$ MeV	-94.105	-94.62(46)		
$r_p(1/2_1^+)$ fm	NA	2.47(4)	2.52	5.85

$^{11}\text{B}$	Exp.	Daejeon16-Extrap.	$\text{N}^2\text{LO}(450)_{2021}$	$\text{N}^2\text{LO}(500)_{2021}$
$E_{g.s.}(3/2^-)$ MeV	-76.205	-75.99(51)	-79.8(4)	-82.3(4)
$r_p$ fm	2.21(2)	2.27(1)		
$^{12}\text{B}$	Exp.	Daejeon16-Extrap.	$\text{N}^2\text{LO}(450)_{2021}$	$\text{N}^2\text{LO}(500)_{2021}$
$E_{g.s.}(1^+)$ MeV	-79.575	-79.30(59)	-84.8(4)	-87.5(4)
$r_p$ fm	2.31(7)	2.27(1)		
$^{13}\text{B}$	Exp.	Daejeon16-Extrap.	$\text{N}^2\text{LO}(450)_{2021}$	$\text{N}^2\text{LO}(500)_{2021}$
$E_{g.s.}(3/2^-)$ MeV	-84.454	-83.97(69)	-92.8(5)	-95.4(5)
$r_p$ fm	2.48(3)	2.28(1)		

**Ground state energies and point proton radii of Boron 11, 12, 13 & Nitrogen 12, 13 from Daejeon16 in good agreement with experiment, even better than chiral N3LO interaction.**

# Present Progress

## ❖ Comparisons on M1 and E2 transitions between NCCI-Daejeon16 and other calculations

The CK-POT, CK, SFO interactions were performed in shell model calculations

Model space	$^{12}\text{C}$ (1900)	Daejeon16 Extrapolation	Chiral NN+3N $6\hbar\Omega$	Chiral NN+3N $8\hbar\Omega$	CK-POT $2\hbar\Omega$	SFO $2\hbar\Omega$
B(M1; $1^+0 \rightarrow 0^+0$ )	0.0145(21)	0.074(1)	0.006	0.0078	0.0048	0.0044
B(M1; $1^+1 \rightarrow 0^+0$ )	0.951(20)	0.712(5)	0.913	1.109	0.771	0.838

---

Model Space	$^{13}\text{C}$	Daejeon16 $6\hbar\Omega$	Chiral NN+3N $6\hbar\Omega$	Chiral NN $6\hbar\Omega$	CK $2\hbar\Omega$	SFO $2\hbar\Omega$
B(M1; $3/2^-: 3.68 \text{ MeV} \rightarrow 1/2^-_{\text{g.s.}}$ )	0.698(72)	0.969	0.402	1.148	1.17	0.878
B(E2; $3/2^-: 3.68 \text{ MeV} \rightarrow 1/2^-_{\text{g.s.}}$ )	6.4(8)	5.388	2.659	2.659		

Hw = 15 MeV in Daejeon16 and chiral interactions

**Electromagnetic transitions of Carbon12 & 13 from Daejeon16 are not so close to experiment, but reasonable.**

*T. Suzuki et al., Phys. Rev. C (2012)*  
*P. Navrátil et al., Phys. Rev. Lett (2007)*  
*H. Sagawa et al., EPJ Web of Conferences (2018)*

# Next plan

## 1. Cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\nu_e \bar{\nu}_e} = \frac{2}{\pi} G_F^2 \cos^2 \theta_c F(Z, \epsilon) \frac{\epsilon^2}{2J_i + 1} \cos^2 \frac{\theta}{2} \left\{ \sum_{J=0}^{\infty} |\langle J_f || \hat{M}_J + \frac{\omega}{q} \hat{L}_J || J_i \rangle|^2 \right. \\ \left. + \left[ -\frac{q_\mu^2}{2q^2} + \tan^2 \frac{\theta}{2} \right] \sum_{J=1}^{\infty} [|\langle J_f || \hat{T}_J^{\text{el}} || J_i \rangle|^2 + |\langle J_f || \hat{T}_J^{\text{mag}} || J_i \rangle|^2] \right. \\ \left. \mp 2 \tan \frac{\theta}{2} \left[ -\frac{q_\mu^2}{q^2} + \tan^2 \frac{\theta}{2} \right]^{1/2} \sum_{J=1}^{\infty} \text{Re}(\langle J_f || \hat{T}_J^{\text{mag}} || J_i \rangle \langle J_f || \hat{T}_J^{\text{el}} || J_i \rangle^*) \right\},$$

## 3. The multipole operators in the equation of charged-current cross section can be expressed in terms of the 7 basic operators

$$M_{JM_J}^\pm(q\mathbf{x}) = F_1^{(1)}(q_\mu^2) M_J^{MJ}(q\mathbf{x}) \tau_\pm, \\ T_{JM_J}^{\text{el}\pm}(q\mathbf{x}) = \frac{q}{M_N} \left( F_1^{(1)}(q_\mu^2) \Delta_J'^{MJ}(q\mathbf{x}) + \frac{1}{2} \mu^{(1)}(q_\mu^2) \Sigma_J^{MJ}(q\mathbf{x}) \right) \tau_\pm, \\ T_{JM_J}^{\text{mag}\pm}(q\mathbf{x}) = -\frac{iq}{M_N} \left( F_1^{(1)}(q_\mu^2) \Delta_J^{MJ}(q\mathbf{x}) - \frac{1}{2} \mu^{(1)}(q_\mu^2) \Sigma_J'^{MJ}(q\mathbf{x}) \right) \tau_\pm, \\ M_{JM_J}^{5\pm}(q\mathbf{x}) = \frac{iq}{M_N} \left( F_A^{(1)}(q_\mu^2) \Omega_J'^{MJ}(q\mathbf{x}) + \frac{1}{2} \omega F_P^{(1)}(q_\mu^2) \Sigma_J''^{MJ}(q\mathbf{x}) \right) \tau_\pm, \\ L_{JM_J}^{5\pm}(q\mathbf{x}) = i \left( F_A^{(1)}(q_\mu^2) - \frac{q^2}{2M_N} F_P^{(1)}(q_\mu^2) \right) \Sigma_J''^{MJ}(q\mathbf{x}) \tau_\pm, \\ T_{JM_J}^{\text{el}5\pm}(q\mathbf{x}) = i F_A^{(1)}(q_\mu^2) \Sigma_J'^{MJ}(q\mathbf{x}) \tau_\pm, \\ T_{JM_J}^{\text{mag}5\pm}(q\mathbf{x}) = F_A^{(1)}(q_\mu^2) \Sigma_J^{MJ}(q\mathbf{x}) \tau_\pm,$$

## 2. The multipole operators in the equation of charged-current cross section are defined by

$$\hat{M}_{JM_J; TM_T} \equiv \int d\mathbf{x} \mathbf{M}_J^{MJ}(q\mathbf{x}) \hat{\mathcal{J}}_0(\mathbf{x})_{TM_T} = \hat{M}_{JM_J; TM_T} + \hat{M}_{JM_J; TM_T}^5, \\ \hat{L}_{JM_J; TM_T} \equiv \frac{i}{q} \int d\mathbf{x} [\vec{\nabla} M_J^{MJ}(q\mathbf{x})] \cdot \hat{\mathcal{J}}(\mathbf{x})_{TM_T} = \hat{L}_{JM_J; TM_T} + \hat{L}_{JM_J; TM_T}^5, \\ \hat{T}_{JM_J; TM_T}^{\text{el}} \equiv \frac{1}{q} \int d\mathbf{x} [\vec{\nabla} \times \mathbf{M}_{JJ}^{MJ}(q\mathbf{x})] \cdot \hat{\mathcal{J}}(\mathbf{x})_{TM_T} = \hat{T}_{JM_J; TM_T}^{\text{el}} + \hat{T}_{JM_J; TM_T}^{\text{el}5}, \\ \hat{T}_{JM_J; TM_T}^{\text{mag}} \equiv \int d\mathbf{x} \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \cdot \hat{\mathcal{J}}(\mathbf{x})_{TM_T} = \hat{T}_{JM_J; TM_T}^{\text{mag}} + \hat{T}_{JM_J; TM_T}^{\text{mag}5},$$

## 4. 7 basic operators)

$$M_J^{MJ}(q\mathbf{x}), \\ \Delta_J^{MJ}(q\mathbf{x}) \equiv \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \cdot \frac{1}{q} \vec{\nabla}, \\ \Delta_J'^{MJ}(q\mathbf{x}) \equiv -i \left[ \frac{1}{q} \vec{\nabla} \times \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \right] \cdot \frac{1}{q} \vec{\nabla} = [J]^{-1} [-J^{1/2} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + (J+1)^{1/2} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x})] \cdot \frac{1}{q} \vec{\nabla}, \\ \Sigma_J^{MJ}(q\mathbf{x}) \equiv \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \cdot \vec{\sigma}, \\ \Sigma_J'^{MJ}(q\mathbf{x}) \equiv -i \left[ \frac{1}{q} \vec{\nabla} \times \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \right] \cdot \vec{\sigma} = [J]^{-1} [-J^{1/2} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + (J+1)^{1/2} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x})] \cdot \vec{\sigma}, \\ \Sigma_J''^{MJ}(q\mathbf{x}) \equiv \left[ \frac{1}{q} \vec{\nabla} M_J^{MJ}(q\mathbf{x}) \right] \cdot \vec{\sigma} = [J]^{-1} [(J+1)^{1/2} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + J^{1/2} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x})] \cdot \vec{\sigma}, \\ \Omega_J^{MJ}(q\mathbf{x}) \equiv M_J^{MJ}(q\mathbf{x}) \vec{\sigma} \cdot \frac{1}{q} \vec{\nabla},$$

# Computer Resources

- 1. Perlmutter (3072 CPU nodes, 1536 GPU nodes)
- 2. Nova (ISU, ~250 CPU nodes)
- 3. ¥ 7.5 million (HAUST, 70-80 CPU nodes)
- Future ?

Partition	# of nodes	CPU	GPU
GPU	1536	1x <a href="#">AMD EPYC 7763</a>	4x <a href="#">NVIDIA A100</a> (40GB)
	256	1x <a href="#">AMD EPYC 7763</a>	4x <a href="#">NVIDIA A100</a> (80GB)
CPU	3072	2x <a href="#">AMD EPYC 7763</a>	-
Login	40	1x <a href="#">AMD EPYC 7713</a>	1x <a href="#">NVIDIA A100</a> (40GB)

Number of Nodes	Processors per Node	Cores per Node	Memory per Node	Interconnect	Local \$TMPDIR Disk	Accelerator Card	Job Constraint Flags
72	Two 18-Core Intel Skylake 6140	36	192 GB	100G IB	1.5 TB	N/A	nova18, intel, skylake, avx512
40	Two 18-Core Intel Skylake 6140	36	384 GB	100G IB	1.5 TB	N/A	nova18, intel, skylake, avx512
28	Two 24-Core Intel Skylake 8260	48	384 GB	100G IB	1.5 TB	N/A	nova18, intel, skylake, avx512
2	Two 18-Core Intel Skylake 6140	36	192 GB	100G IB	1.5 TB	2x NVIDIA Tesla V100-32GB	nova18, intel, skylake, avx512
1	Two 18-Core Intel Skylake 6140	36	192 GB	100G IB	1.5 TB	one NVIDIA Tesla V100-32GB	nova18, intel, skylake, avx512
2	Two 18-Core Intel Skylake 6140	36	384 GB	100G IB	1.5 TB	2x NVIDIA Tesla V100-32GB	nova18, intel, skylake, avx512
1	Four 16-Core Intel 6130	64	3 TB	100G IB	11 TB	N/A	nova18, intel, skylake, avx512
2	Four 24-Core Intel 8260	96	3 TB	100G IB	1.5 TB	N/A	nova18, intel, skylake, avx512
40	Two 32-Core AMD EPYC 7502	64	512 GB	100G IB	1.5 TB	N/A	nova21, amd, epyc-7502
15	Two 32-Core AMD EPYC 7502	64	512 GB	100G IB	1.5 TB	four NVidia A100 80GB	nova21, amd, epyc-7502
56	Two 32-Core Intel Icelake 8358	64	512GB	100G IB	1.6TB	N/A	nova22, intel, icelake, avx512
5	Two 24-Core AMD EPYC 7413	48	512GB	100G IB	960GB	eight NVidia A100 80GB	nova22, amd
16	Two 32-Core Intel Icelake8358	64	512GB	100G IB	1.5TB	N/A	nova23, intel, icelake, avx512
14	Two 96-Core AMD EPYC 9654	192	768GB	100G IB	1.7TB	N/A	nova24, amd, epyc-9654, avx512
3	Two 96-Core AMD EPYC 9684x	192	768GB	100G IB	1.7TB	N/A	nova24, amd, epyc-9684x, avx512

# Conclusions

- **Ground state energies and spectra of stable p-shell nuclei from Daejeon16 in good agreement with experiment; electromagnetic transitions are reasonable**
- **Improve the accuracy of calculations: increasing the basis space**
- **Demanding computer resources**
- **Theoretical uncertainty**
- **Comparison with future experimental data**