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Lattice QCD studies on the decay of light 1^{-+} hybrids

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Outline

I. Introduction
II. Masses of light hybrids from lattice QCD
IV. Decays of light hybrids
V. Summary and perspectives

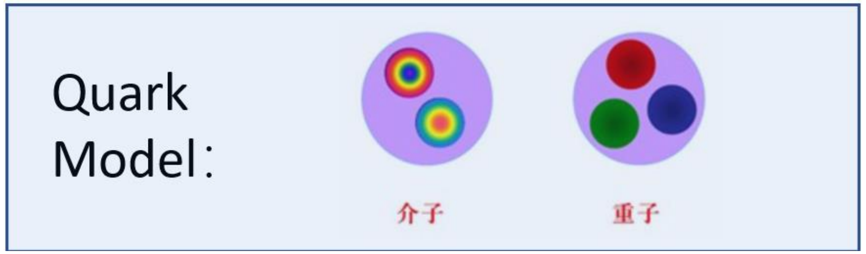
I. Introduction

1. Conventional and exotic light hadrons

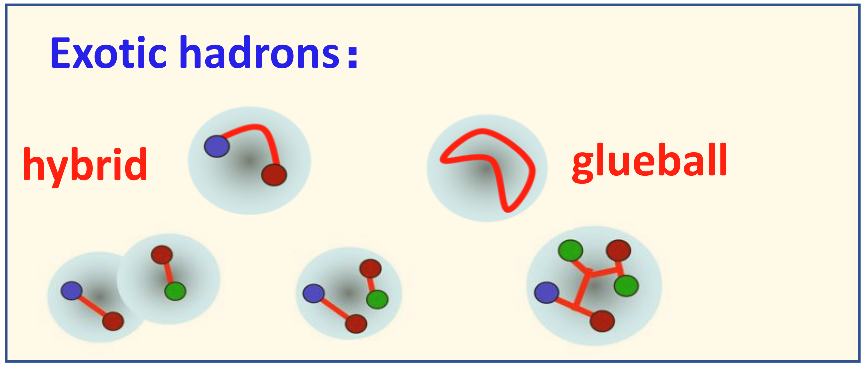
- Quark model sort light hadrons into
  $\bar{q}q$ multiplets (mesons)
  qqq multiplets (baryons)
- Gluons are also fundamental degrees of freedom of QCD, therefore there may exist glueballs $(gg\cdots)$ and hybrids $(\bar{q}qg)$.
- Exotic quantum numbers that $q\bar{q}$ mesons cannot have:
  $1^{-+}, 0^{+-}, 0^{--}, 2^{+-}$ etc.
- The lightest hybrid meson may be the 1^{-+} state.

2. Experimental candidates for 1^{-+} hybrids

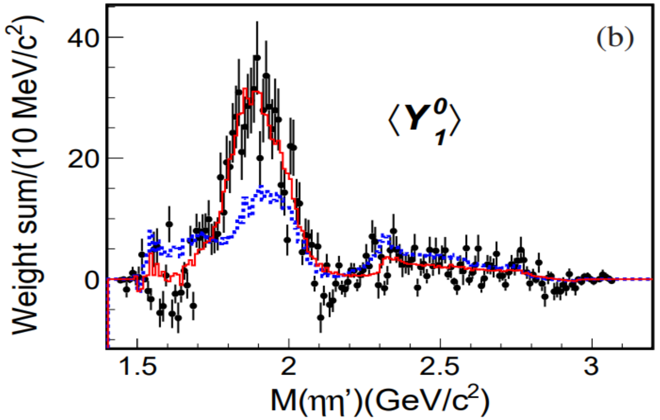
- Isovector ($I^G J^{PC} = 1^- 1^{-+}$): $\pi_1(1400)$ (?), $\pi_1(1600)$
  They are now taken as the same state $\pi_1(1600)$.
  $$\pi_1(1600) \rightarrow b_1\pi, \rho\pi, f_1\pi, \eta(\eta')\pi$$
- Isoscalar ($I^G J^{PC} = 0^+ 1^{-+}$): $\eta_1(1855)$
  In 2022, BESIII observed $\eta_1(1855)$ in the decay process
  $$J/\psi \rightarrow \gamma\eta_1(1855) \rightarrow \gamma\eta\eta'$$



Quark Model:



Exotic hadrons:



$\eta_1(1855)$, PRL129,192002(2022)

3. Formalism of Lattice QCD

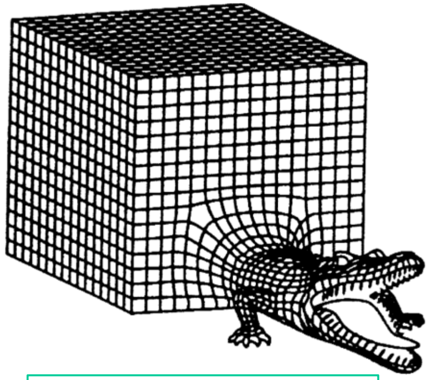
- **Path integral quantization on finite Euclidean spacetime lattices**

$$Z = \int DAD\psi D\bar{\psi}\, e^{iS[A,\psi,\bar{\psi}]} \rightarrow \int DU \det M[U]\; e^{-S_g[U]}$$

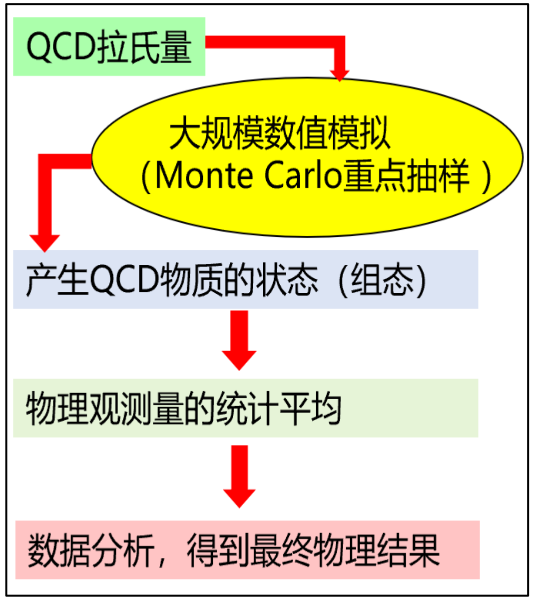
$$\langle \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int DU \;\; \det M[U]\; e^{-S_g[U]} \;\; \mathcal{O}[U]$$

Green’s functions

Product of fields



Spacetime discretization



QCD拉氏量

大规模数值模拟
（Monte Carlo重点抽样 ）

产生QCD物质的状态（组态）

物理观测量的统计平均

数据分析，得到最终物理结果

- **Very similar to a statistical physics system**
- **Monte Carlo simulation——importance sampling according to** $\mathcal{P}[U] \propto \det M[U]\; e^{-S_g[U]}$

Gauge ensemble: $\{U_i(\text{spacetime}), i = 1, \dots, N\}$ ➡ $\langle \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{N}\sum_i \mathcal{O}[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$

II. Masses of light hybrids from lattice QCD

- Experiments: π_1 observed by E852, Crystal Barrel, Compass;
  BESIII observed $\eta_1(1855)$ in 2022.

  PDG (2024) : $m_{\pi_1} = 1645^{+40}_{-17}$ MeV, $\Gamma \approx 370^{+50}_{-60}$ MeV
  (non-$\eta\pi$ mode)
  $m_{\eta_1} = 1855^{+11}_{-9}$ MeV, $\Gamma \approx 188 \pm 19$ MeV

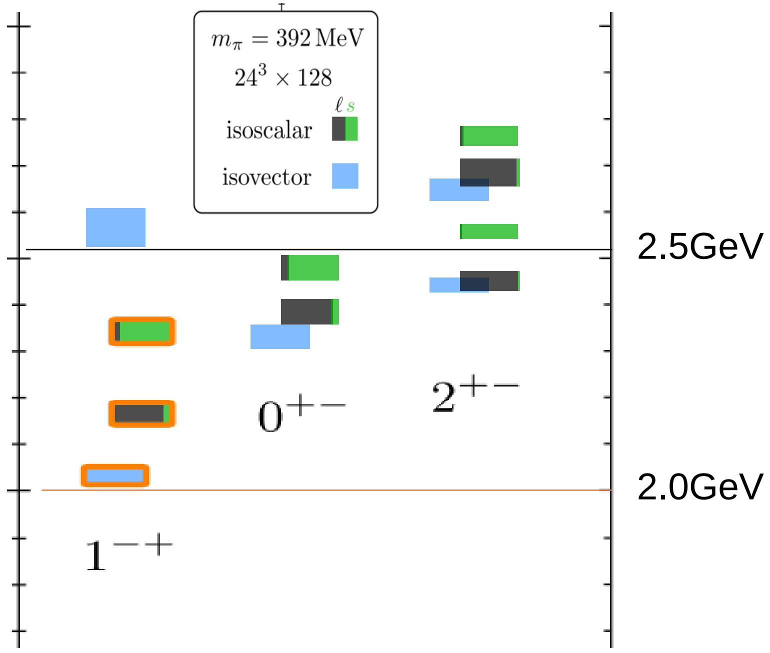
- Lattice QCD studies predict:

$$m_{\pi_1} \sim 1.7 - 2.1\ \mathrm{GeV}$$

$$m_{\eta_1}^{(L)} \sim 2.1\ \mathrm{GeV},$$

$$m_{\eta_1}^{(H)} \sim 2.3\ GeV$$

- It is puzzling that the lattice predictions of light hybrid masses are higher than that of the experimental candidates.



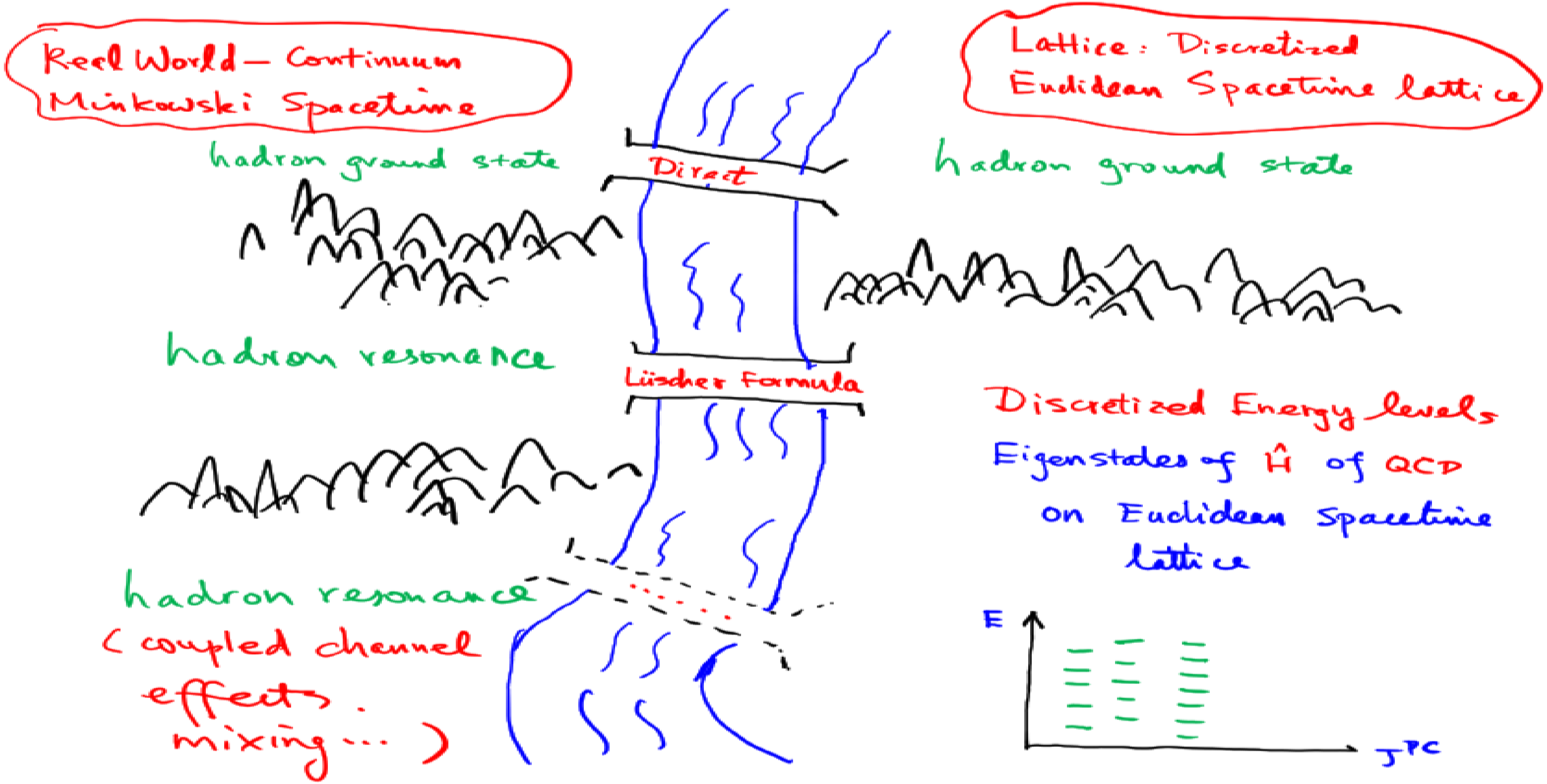
$$\begin{pmatrix} |\eta_1^{(L)}\rangle \\ |\eta_1^{(H)}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |l\bar{l}\rangle \\ |s\bar{s}\rangle \end{pmatrix}$$

$$\alpha \approx 22.7°$$

J. Dudek et al. (HSC), PRD 88(2013) 094505

IV. Decays of light hybrids

1. The connection of the lattice spectroscopy with that of the real world

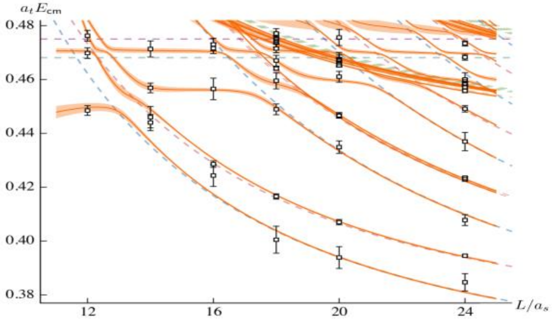


2. Bound states and resonances from hadron-hadron scatterings on the lattice

State-of-art Approach——Lΰscher's formalism

(see R. Briceno et al., Rev. Mod. Phys. 90 (2018) 025001 for a review).

$$\det\left[F^{-1}\left(\vec{P},E,L\right)+\mathcal{M}(E)\right]=0$$



$E_n(L)$: Eigen-energies of lattice Hamiltonian.

- **Interpolation field operator set for a given J^{PC}**

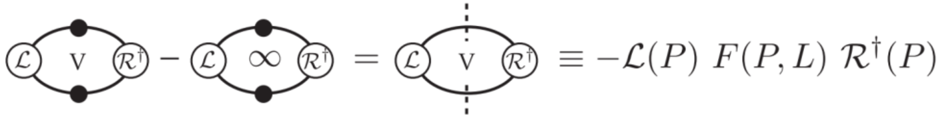
  $\mathcal{O}_i$: $\bar{q}_1\Gamma q_2$ $[\bar{q}_1\Gamma_1 q][\bar{q}\Gamma_2 q_2]$ $[q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T]$, …
- **Correlation function matrix —— Observables**

$$C_{ij}(t)\& = \left\langle\Omega\middle|\mathcal{O}_i(t)\mathcal{O}_j^{+}(0)\middle|\Omega\right\rangle$$

$$=\sum_n\langle\Omega|\mathcal{O}_i|n\rangle\left\langle n\middle|\mathcal{O}_j^{+}\middle|\Omega\right\rangle e^{-E_n t}$$

All the energy levels $E_n(L)$ are discretized.

$F\left(\vec{P},E,L\right)$: Mathematically known function matrix in the channel space (the explicit expression omitted



$\mathcal{M}(E)$: Scattering matrix.

- Unitarity requires

$$\mathcal{M}_{ab}^{-1}=(\mathcal{K}^{-1})_{ab}-i\delta_{ab}\frac{q_a^*}{8\pi\,E_{cm}}$$

- $\mathcal{K}$ is a real function of s for real energies above kinematic threshold.
- The pole singularities of $\mathcal{M}(s)$ in the complex s-plane correspond to bound states, virtual states, resonances, etc.

$$\mathcal{M}_{ab}(s)\sim\frac{g_a g_b}{s_0-s}$$

- The pole couplings g_a will give the partial decay widths:

3. Two-body decays of π_1 in Lüscher's formalism

A.J. Woss (HSC Collaboration), Phys.Rev.D 103 (2021) 054502 , arXiv:2009.10034(hep-lat)

- Worked on **6 lattices** with different lattice sizes
  —— many lattice energy levels.
- Starting from the **exact $SU_F(3)$ flavor symmetry with $m_u = m_d = m_s \approx m_s^{\text{phys}}$**
  —— a simpler spectrum

$(L/a_s)^3 \times (T/a_t)$	N_{vecs}	N_{cfgs}	N_{tsrcs}
$12^3 \times 96$	48	219	24
$14^3 \times 128$	64	397	16
$16^3 \times 128$	64	529	4
$18^3 \times 128$	96	358	4
$20^3 \times 128$	128	501	4
$24^3 \times 128$	160	607	4

✓ The flavor symmetry decomposition of M_1M_2 system

$$\pi_1 \to M_1M_2$$

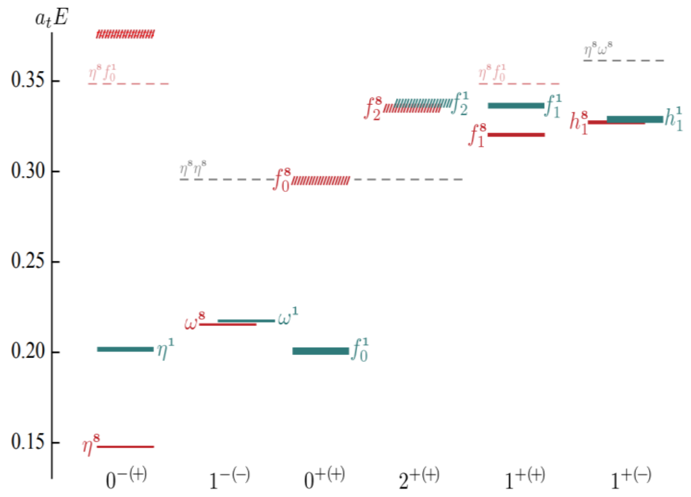
$$8 \otimes 1, \quad 1 \otimes 8$$

$$8 \otimes 8 \to 1 \oplus 8_1 \oplus 8_2 \oplus 10 \oplus 10^* \oplus 27$$

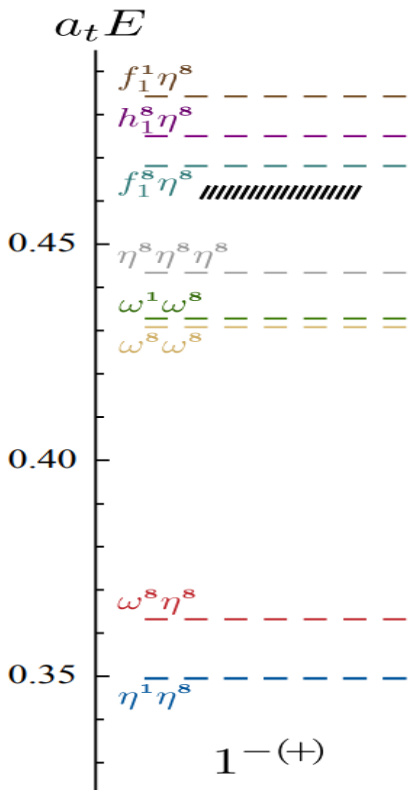
✓ Possible combinantions of the final states M_1M_2

$$\eta^1\eta^8, \quad \omega^8\eta^8, \quad \omega^8\omega^8, \omega^1\omega^8, \quad f_1^8\omega^8, h_1^8\eta^8, f_1^1\eta^8$$

$$\eta(\eta')\pi, \quad \rho\pi, \quad \rho\omega(\phi), \quad a_1\rho,\ b_1\pi,\ f_1\pi$$



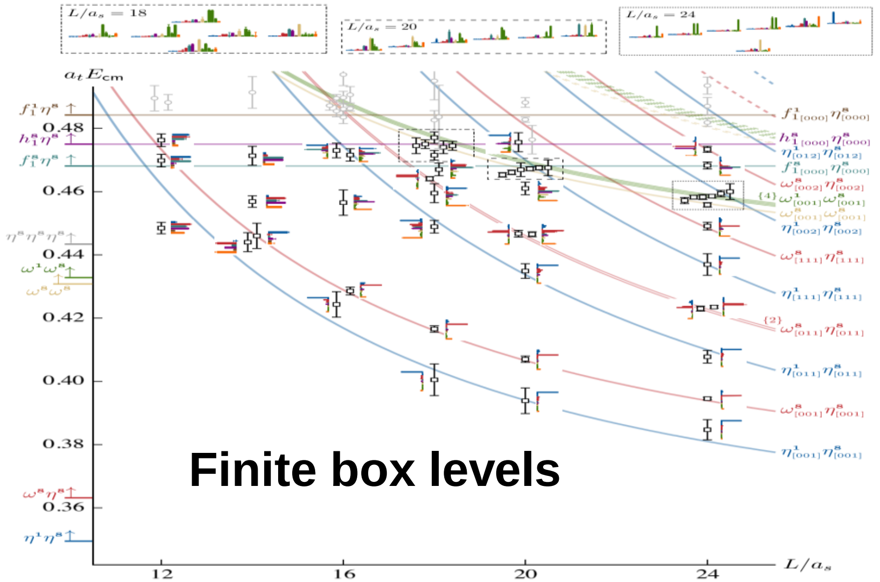
The light hadrons involved



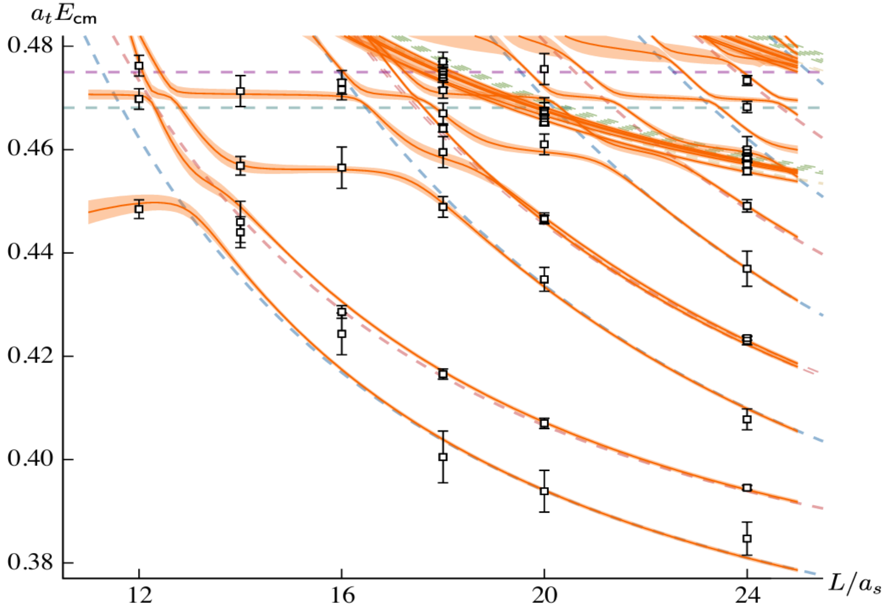
$$(\bar{\psi}\mathbf{\Gamma}\psi)_i = \underbrace{\epsilon_{ijk}(\bar{\psi}\gamma_j\psi)\,B_k}_{1^{--}\otimes 1^{+-}\to 1^{-+}}\,,$$

$L/a_s = 12$	$L/a_s = 14$	$L/a_s = 16$	$L/a_s = 18$	$L/a_s = 20$	$L/a_s = 24$
$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$
$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[001]}\eta^{\mathbf{8}}_{[001]}$
$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$
$\omega^{\mathbf{8}}_{[001]}\eta^{\mathbf{8}}_{[001]}$	$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\eta^{\mathbf{1}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\eta^{\mathbf{1}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\eta^{\mathbf{1}}_{[011]}\eta^{\mathbf{8}}_{[011]}$
$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\eta^{\mathbf{1}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\{2\}\,\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\{2\}\,\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\{2\}\,\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$
$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[111]}\eta^{\mathbf{8}}_{[111]}$
	$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\omega^{\mathbf{8}}_{[111]}\eta^{\mathbf{8}}_{[111]}$
	$\{4\}\,\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\{2\}\,\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\{4\}\,\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$
	$\eta^{\mathbf{1}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\{4\}\,\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$\eta^{\mathbf{1}}_{[111]}\eta^{\mathbf{8}}_{[111]}$	$\{4\}\,\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$
	$\{2\}\,\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$	$\{4\}\,\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$	$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\eta^{\mathbf{1}}_{[002]}\eta^{\mathbf{8}}_{[002]}$
			$\eta^{\mathbf{1}}_{[111]}\eta^{\mathbf{8}}_{[111]}$	$\omega^{\mathbf{8}}_{[111]}\eta^{\mathbf{8}}_{[111]}$	$f_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$
			$\omega^{\mathbf{8}}_{[111]}\eta^{\mathbf{8}}_{[111]}$	$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$	$\omega^{\mathbf{8}}_{[002]}\eta^{\mathbf{8}}_{[002]}$
				$\omega^{\mathbf{8}}_{[002]}\eta^{\mathbf{8}}_{[002]}$	$h_1^{\mathbf{8}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$
					$f_1^{\mathbf{1}}{}_{[000]}\eta^{\mathbf{8}}_{[000]}$
					$\eta^{\mathbf{1}}_{[012]}\eta^{\mathbf{8}}_{[012]}$

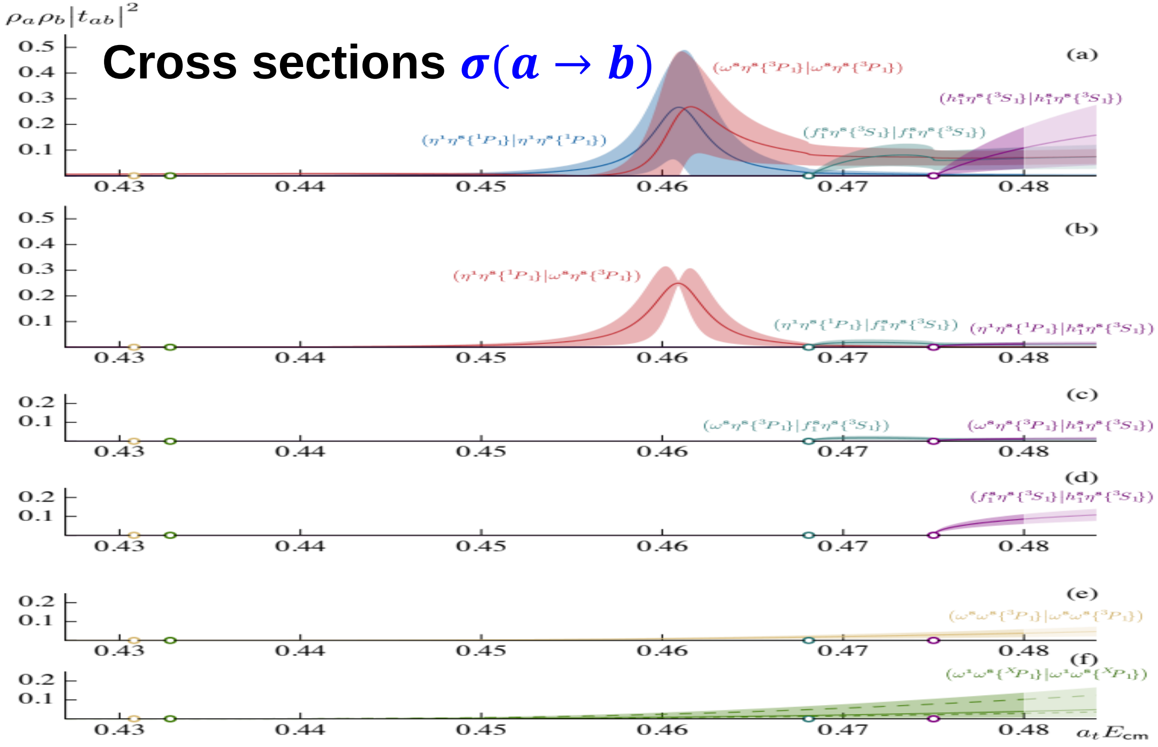
Operator sets on the six lattices

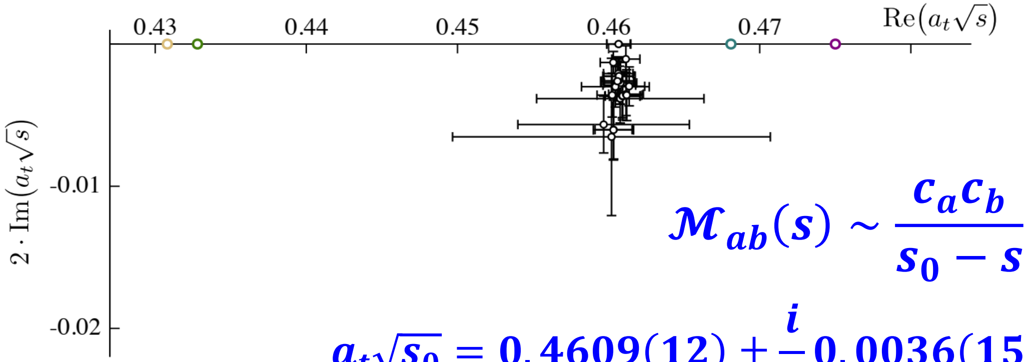


Finite box levels



Cross sections $\sigma(a \to b)$





$$\mathcal{M}_{ab}(s) \sim \frac{c_a c_b}{s_0 - s}$$

$$a_t\sqrt{s_0} = 0.4609(12) \pm \frac{i}{2} 0.0036(15)$$

$$a_t^{-1} = 4.655\ \mathrm{GeV}$$

$$\left|a_t c_{\eta^1\eta^8\{^1P_1\}}\right| = 0 \to 0.055$$
$$\left|a_t c_{\omega^8\eta^8\{^3P_1\}}\right| = 0 \to 0.060$$
$$\left|a_t c_{\omega^8\omega^8\{^3P_1\}}\right| = 0 \to 0.020$$
$$\left|a_t c_{\omega^1\omega^8\{^XP_1\}}\right| \lesssim 0.020$$
$$\left|a_t c_{f_1^8\eta^8\{^3S_1\}}\right| = 0 \to 0.21$$
$$\left|a_t c_{h_1^8\eta^8\{^3S_1\}}\right| = 0.21 \to 0.41$$

$$\left|c_{\eta^1\eta^8\{^1P_1\}}\right| = 0 \to 256\ \mathrm{MeV}$$
$$\left|c_{\omega^8\eta^8\{^3P_1\}}\right| = 0 \to 279\ \mathrm{MeV}$$
$$\left|c_{\omega^8\omega^8\{^3P_1\}}\right| = 0 \to 93\ \mathrm{MeV}$$
$$\left|c_{\omega^1\omega^8\{^XP_1\}}\right| \lesssim 93\ \mathrm{MeV}$$
$$\left|c_{f_1^8\eta^8\{^3S_1\}}\right| = 0 \to 978\ \mathrm{MeV}$$
$$\left|c_{h_1^8\eta^8\{^3S_1\}}\right| = 978 \to 1909\ \mathrm{MeV}$$

$$a_t\sqrt{s_0} = 0.4609(12) \pm \frac{i}{2}0.0036(15)$$
$$\sqrt{s_0} = 2145(6) \pm \frac{i}{2}17(7)\ \mathrm{MeV}$$

- By assuming the insensitivity to m_π and using the physical kinematics the partial decay widths are estimated to be

	thr./MeV	$\left\|c_i^{\mathrm{phys}}\right\|$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\overline{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
		$\Gamma = \sum_i \Gamma_i = 139 \to 590$	

4. An alternative method (C. McNeile & C. Michael, Phys. Lett. B 556 (2003) 177)

- For the two-body decay $h \to AB$, in the space spanned by $|h\rangle$ and $|AB\rangle$ $(m_h > E_{AB})$

$$|h\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |AB\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

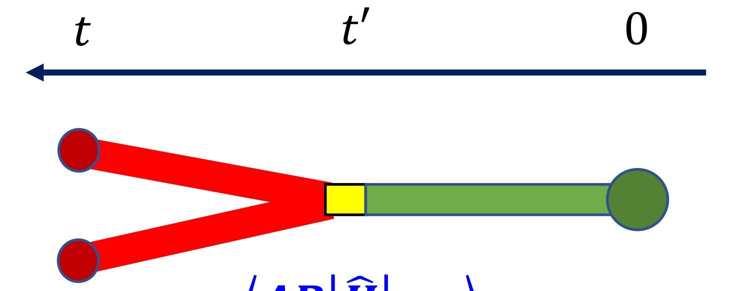
$$\hat{H} = \begin{pmatrix} m_h & x \\ x & E_{AB} \end{pmatrix}$$

$$\hat{T}(a) = e^{-a\hat{H}} = e^{-a\bar{E}} \begin{pmatrix} e^{-a\Delta/2} & ax \\ ax & e^{a\Delta/2} \end{pmatrix}$$

$$\bar{E} = \frac{m_h + E_{AB}}{2}, \qquad \Delta = m_h - E_{AB}$$

The transition takes place at any t' between 0 and t:

$$\langle\Omega|\mathcal{O}_{AB}|h\rangle \approx 0 \qquad \langle\Omega|\mathcal{O}_h|AB\rangle \approx 0$$



$$\langle AB|\hat{H}|\eta_{c1}\rangle = ax$$

$$\mathcal{C}_{h,AB}(t) = \langle\Omega|\mathcal{O}_{AB}(t)\mathcal{O}_h^{+}(0)|\Omega\rangle$$
$$= \langle\Omega|\mathcal{O}_{AB}(0)e^{-t\,a\hat{H}}\mathcal{O}_h^{+}(0)|\Omega\rangle$$
$$\to -axt\; e^{-t\,a\bar{E}}\langle\Omega|\mathcal{O}_{AB}|AB\rangle\,\langle h|\mathcal{O}_h^{+}|\Omega\rangle$$

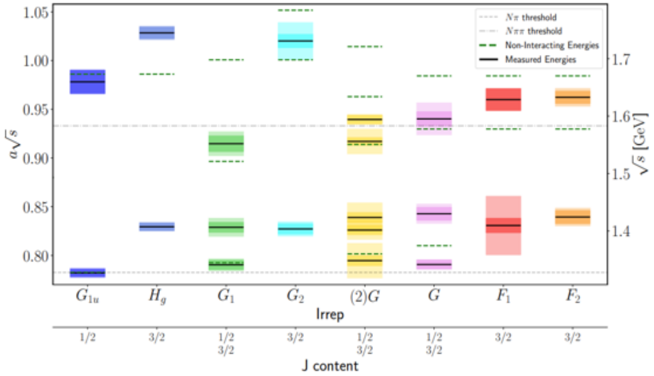
$$\langle\Omega|\mathcal{O}_{AB}|AB\rangle \approx \langle\Omega|\mathcal{O}_A|A\rangle\langle\mathcal{O}|\mathcal{O}_B|B\rangle$$

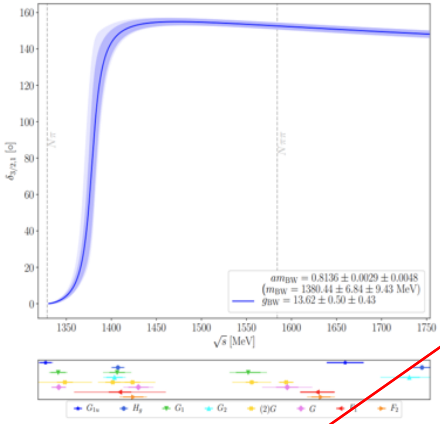
$$\frac{\mathcal{C}_{h,AB}(t)}{\sqrt{\mathcal{C}_h(t)\mathcal{C}_A(t)\mathcal{C}_B(t)}} \to -ax\,t\left(1 + \frac{1}{24}(a\Delta\,t)^2\right)$$

$N\pi$ scattering and the Δ resonance

G. Silvi et al., PRD103 (2021) 094508 (arXiv:2101.00689) and references therein

$N_s^3 \times N_t$	$24^3 \times 48$
β	3.31
$am_{u,d}$	-0.09530
am_s	-0.040
a [fm]	0.1163(4)
L [fm]	2.791(9)
m_π [MeV]	255.4(1.6)
$m_\pi L$	3.61(2)
N_{config}	600
N_{meas}	9600





K-matrix rescaled: $K = \rho^{1/2}\widehat{K}\rho^{1/2}$

K relates to the phase shift: $K^{Jl} = \tan\delta_{Jl}$

$$m_\Delta = (1378.3 \pm 6.6 \pm 9.0)\ \text{MeV}$$

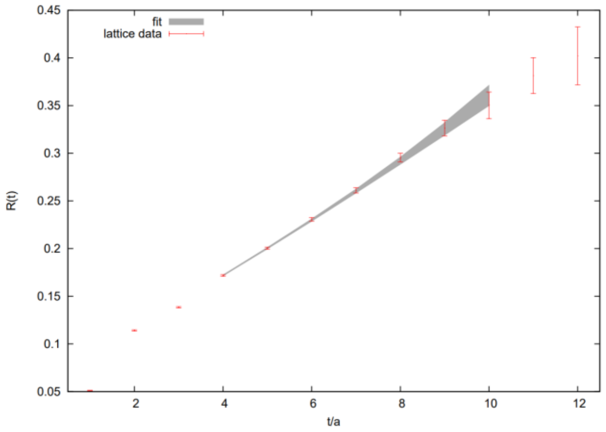
$$\Gamma_\Delta = (\ \ 16.4 \pm 1.0 \pm 1.4)\ \text{MeV}$$

Breit Wigner: $\widehat{K}^{(3/2,1)} = \dfrac{\sqrt{s}\Gamma(s)}{(m_{BW}^2 - s)\rho}$

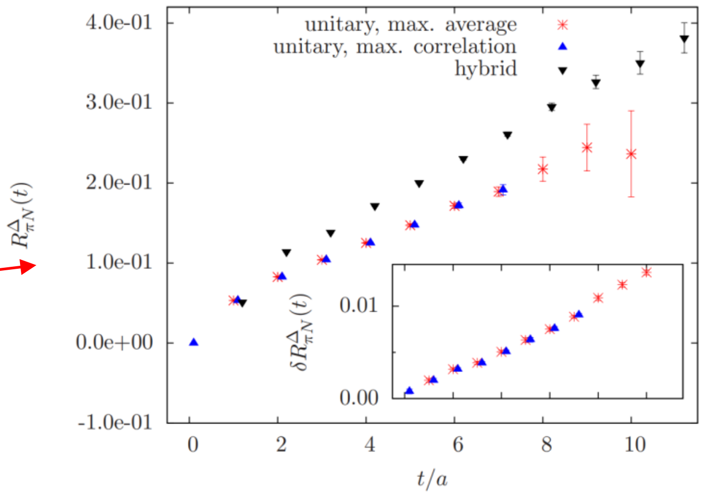
$$\Gamma(s) = \frac{g_{BW}^2}{6\pi}\frac{k^3}{s}$$

$$\Gamma_{\rm EFT}^{\rm LO} = \frac{g^2_{\Delta\text{-}\pi N}}{48\pi}\frac{E_N + m_N}{E_N + E_\pi}\frac{k^3}{m_N^2}$$

Collaboration	m_π [MeV]	Methodology	m_Δ [MeV]	$g_{\Delta\text{-}\pi N}$
Verduci 2014 [38]	266(3)	Distillation, Lüscher	$1396(19)_{\rm BW}$	19.90(83)
Alexandrou et al. 2013 [37]	360	Michael, McNeile	1535(25)	27.0(0.6)(1.5)
Alexandrou et al. 2016 [39]	180	Michael, McNeile	1350(50)	23.7(0.7)(1.1)
Andersen et al. 2018 [41]	280	Stoch. distillation, Lüscher	$1344(20)_{\rm BW}$	37.1(9.2)
Our result	255.4(1.6)	Smeared sources, Lüscher	$1380(7)(9)_{\rm BW}$, $1378(7)(9)_{\rm pole}$	23.8(2.7)(0.9)
Physical value [5]	139.5704(2)	phenomenology, K-matrix	$1232(1)_{\rm BW}$, $1210(1)_{\rm pole}$	29.4(3) [79], 28.6(3) [80]



C. Alexandrou et al.,
Phys. Rev. D 88 (2013) 031501



C. Alexandrou et al.,
Phys. Rev. D 93 (2016) 114515

- **The effective Lagrangian for the two-body decay $h \to AB$ (J. Liang et al, arXiv:2409.14410 [hep-lat])**

✓ **The tree-level amplitudes:**

$$\begin{aligned}\mathcal{M}_{AP}^{\lambda'\lambda} =& g_{AP} m_h \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}),\\ \mathcal{M}_{PP}^{\lambda} =& 2 g_{PP} \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{k},\\ \mathcal{M}_{VP}^{\lambda'\lambda} =& g_{VP} \vec{\epsilon}_\lambda(\vec{0}) \cdot (\vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}) \times \vec{k}),\\ \mathcal{M}_{VV}^{\lambda''\lambda'\lambda} =& 2 g_{VV} \vec{\epsilon}_\lambda(\vec{0}) \cdot \left(\vec{k} \times \left[\vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}) \times \vec{\epsilon}^{\,*}_{\lambda''}(-\vec{k})\right]\right)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\pi_1^0 \to b_1 \pi} &= g_{\pi b_1} m_{\pi_1} \pi_1^{0,\mu} \frac{1}{\sqrt{2}} \left(b_{1,\mu}^{+} \pi^{-} - b_{1,\mu}^{-} \pi^{+}\right)\\ \mathcal{L}_{\pi_1^0 \to \rho \pi} &= \frac{g_{\rho\pi} \epsilon^{\mu\nu\rho\sigma}}{\sqrt{2} m_{\pi_1}} \left(\partial_\mu \pi_{1,\nu}^0\right) \left(\partial_\rho \rho_\sigma^{+} \pi^{-} - \partial_\rho \rho_\sigma^{-} \pi^{+}\right)\\ \mathcal{L}_{\pi_1^0 \to f_1 \pi} &= g_{f_1 \pi} \pi_1^{0,\mu} f_{1,\mu} \pi^0\\ \mathcal{L}_{\pi_1^0 \to a_1 \eta} &= g_{a_1 \eta} \pi_1^{0,\mu} a_{1,\mu}^0 \eta\\ \mathcal{L}_{\pi_1^0 \to \pi \eta} &= i g_{\pi\eta} \pi_1^{0,\mu} (\eta \overleftrightarrow{\partial}_\mu \pi^0)\\ \mathcal{L}_{\eta_1 \to a_1 \pi} &= g_{a_1 \pi} \eta_1^\mu \frac{1}{\sqrt{3}} \left(a_{1,\mu}^{+} \pi^{-} + a_{1,\mu}^{0} \pi^{0} + a_{1,\mu}^{-} \pi^{+}\right)\\ \mathcal{L}_{\eta_1 \to f_1 \eta} &= g_{f_1 \eta} \eta_1^\mu f_{1,\mu} \eta.\end{aligned}$$

✓ **The relation between $\mathcal{M}_{AB}$ and x_{AB}**

$$x_{AB} = \frac{\mathcal{M}_{AB}}{(8 L^3 m_h E_A(k) E_B(k))^{1/2}}$$

✓ **After x_{AB} is derived, we can use the relations above to extract the effective couplings g_{AB}**

$$\Gamma(h \to AB) = \frac{c}{8\pi} \frac{k_{\rm ex}}{m_h^2} \overline{|\mathcal{M}(h \to AB)|^2}$$

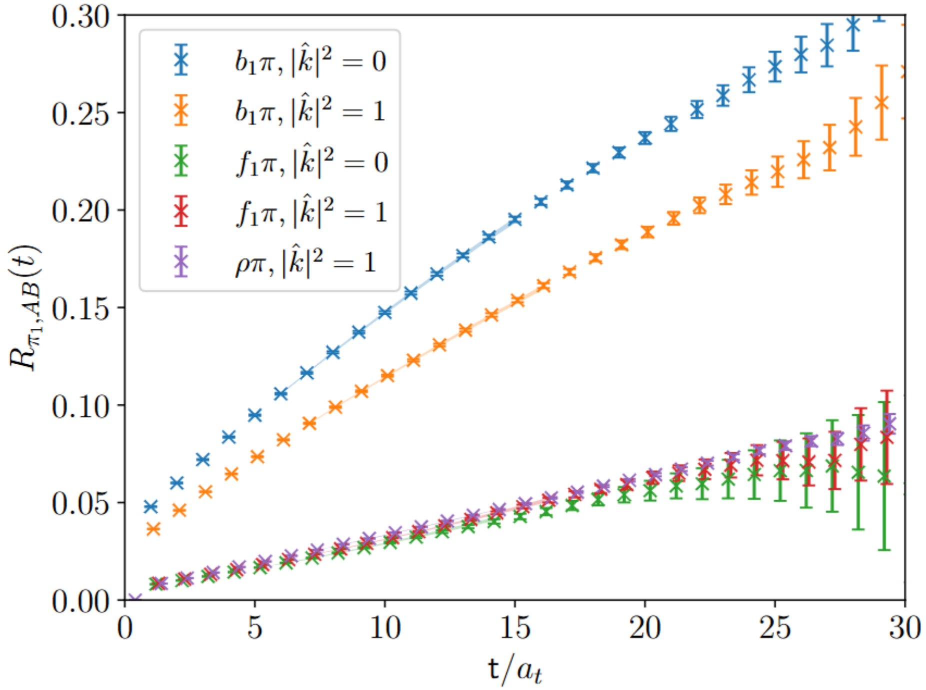
$$\begin{aligned}\overline{|\mathcal{M}(h \to AP)|^2} =& \frac{1}{3} g_{AP}^2 m_h^2 (3 + \frac{k_{\rm ex}^2}{m_A^2})\\ \overline{|\mathcal{M}(h \to PP)|^2} =& \frac{4}{3} g_{PP}^2 k_{\rm ex}^2,\\ \overline{|\mathcal{M}(h \to VP)|^2} =& \frac{2}{3} g_{VP}^2 k_{\rm ex}^2,\\ \overline{|\mathcal{M}(h \to VV)|^2} =& \frac{4}{3} g_{VV}^2 k_{\rm ex}^2 \frac{m_h^2}{m_V^2}.\end{aligned}$$

- ✓ In practice, we use the following functions with the r_0 and r_2 terms accounting for excited state contaminations.

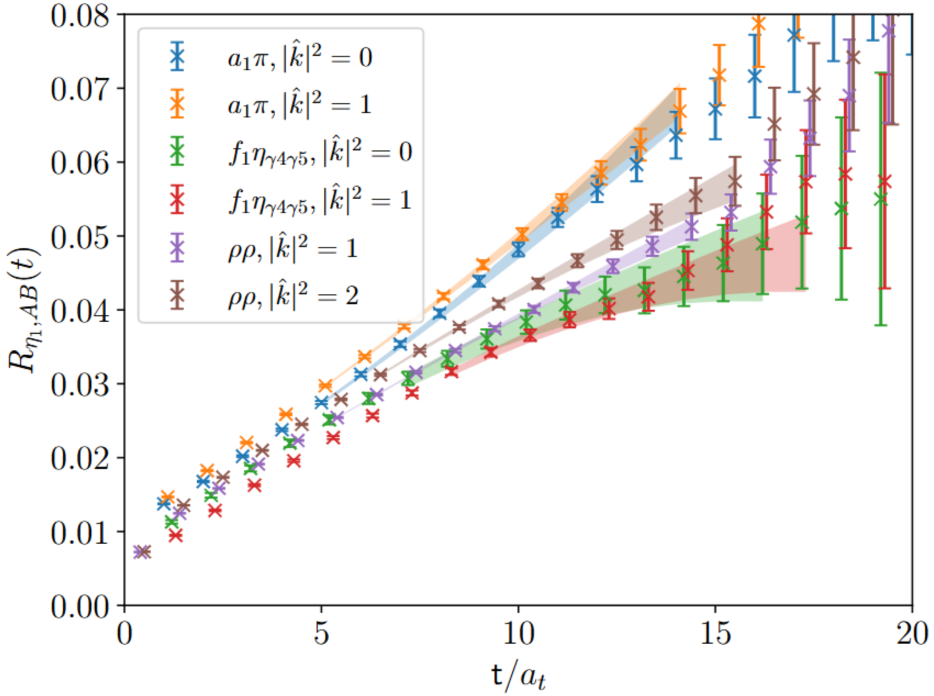
$m_{\pi_1} = 1.980(21)$ GeV $m_{\eta_1} = 2.250(54)$ GeV

$$r_{AB} = \sum_{\lambda,\lambda'} \frac{\mathcal{M}_{AB}^{(\lambda'\lambda'')\lambda}[\epsilon_{(\lambda')}^{(i)}(\vec{k})\epsilon_{(\lambda'')}^{(j)}(\vec{k})]\epsilon_{\lambda}^{3*}(\vec{0})}{\sqrt{(8L^3 m_h E_A(k)E_B(k))\mathcal{P}_A^{(ii)}(\vec{k})\mathcal{P}_B^{(jj)}(-\vec{k})}}$$

$$R_{AB}(t) = \frac{\mathcal{C}_{h,AB}(t)}{\sqrt{\mathcal{C}_h(t)\mathcal{C}_A(t)\mathcal{C}_B(t)}} \approx r_0 + r_{AB}\,t + r_2 t^2$$



π_1 decay relevant $R_{AB}(t)$



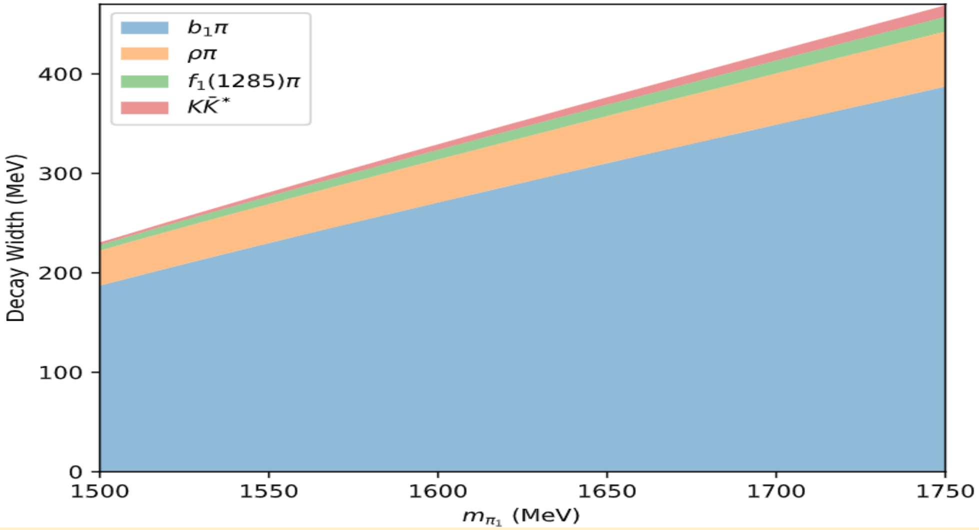
η_1 decay relevant $R_{AB}(t)$

Effective couplings g_{AB} for π_1 and η_1 decays

mode	g_{AB}	$\bar{g}_{AB}$
$\pi_1 \to b_1\pi(\hat{k}^2 = 0)$	4.81(46)	4.71(52)
$\pi_1 \to b_1\pi(\hat{k}^2 = 1)$	4.61(38)	
$\pi_1 \to f_1\pi(\hat{k}^2 = 0)$	0.80(6)	0.98(32)
$\pi_1 \to f_1\pi(\hat{k}^2 = 1)$	1.16(20)	
$\pi_1 \to \rho\pi(\hat{k}^2 = 1)$	4.34(32)	4.34(32)
$\eta_1 \to a_1\pi(\hat{k}^2 = 0)$	1.10(28)	1.42(53)
$\eta_1 \to a_1\pi(\hat{k}^2 = 1)$	1.64(25)	
$\eta_1 \to f_1\eta(\hat{k}^2 = 0)$	2.22(62)	2.12(70)
$\eta_1 \to f_1\eta(\hat{k}^2 = 1)$	2.02(61)	
$\eta_1 \to \rho\rho(\hat{k}^2 = 1)$	2.76(31)	2.93(60)
$\eta_1 \to \rho\rho(\hat{k}^2 = 2)$	3.10(56)	

$$\bar{g}_{AB} = \frac{1}{2}\left(g_{AB}(p=0) + g_{AB}(p=1)\right),$$
$$\delta\bar{g}_{AB} = \frac{1}{2}\left(\max(g_{AB} + \delta g_{AB}) - \min(g_{AB} - \delta g_{AB})\right)$$

Γ_i	Γ_{AB}(MeV)	Γ_{AB}(MeV)[49]
$\Gamma(\pi_1 \to b_1\pi)$	323(72)	139-529
$\Gamma(\pi_1 \to f_1(1285)\pi)$	$\mathcal{O}(10)$	0-24
$\Gamma(\pi_1 \to f_1(1420)\pi)$	$\mathcal{O}(1)$	0-2
$\Gamma(\pi_1 \to \rho\pi)$	48(7)	0-20
$\Gamma(\pi_1 \to K\bar{K}^*)$	7.9(1.3)	0-2
$\sum_i \Gamma_i$	$\sim 375(90)$	139-590



- PDG 2024: $\Gamma \approx \mathbf{370^{+50}_{-60}}$ MeV
- COMPASS (2018): $\Gamma \approx 580^{+100}_{-230}$ MeV
- B852 (2005): $\Gamma \approx 185 \pm 25 \pm 28$ MeV
- B852 (2004): $\Gamma \approx 403 \pm 80 \pm 115$ MeV
- B852 (2001): $\Gamma \approx 340 \pm 40 \pm 50$ MeV

5. The partial decay widths of $\eta_1(1855)$ and its mass partner (possible $\eta_1(2200)$)

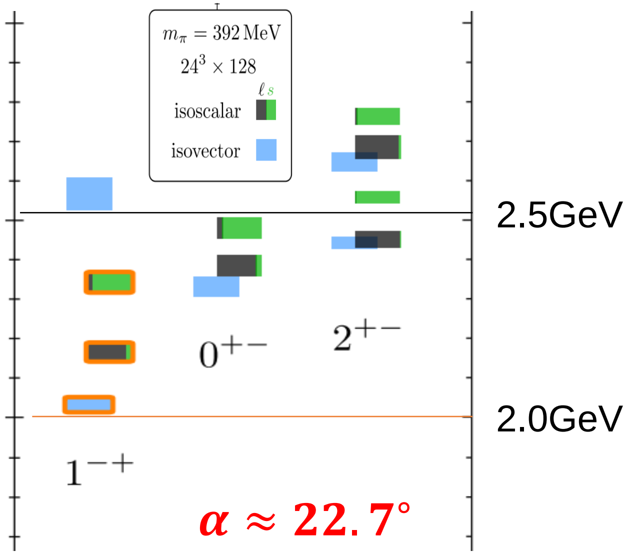
- Extension from isospin SU(2) to flavor SU(3):

Meson nonet, $X = H, A, B$ ($\eta_X^{(l)} \sim (u\bar{u} + d\bar{d})/\sqrt{2},\ \eta_X^{(s)} \sim s\bar{s}$)

$$X = \begin{pmatrix} \frac{\pi_X^0 + \eta_X^{(l)}}{\sqrt{2}} & \pi_X^+ & K_X^+ \\ \pi_X^- & \frac{-\pi_X^0 + \eta_X^{(l)}}{\sqrt{2}} & K_X^0 \\ K_X^- & \bar{K}_X^0 & \eta_X^{(s)} \end{pmatrix}$$

Obviously, there should be two mass eigenstates of η_1, similar to η and η'.

J. Dudek et al. (HSC), PRD 88(2013) 094505



$\alpha \approx 22.7°$

- The flavor structure of the effective Lagrangian:

a) If $C'(A)C'(B) = -$, this is the case for $\rho\pi, b_1\pi, K_{1B}\bar{K}$, then

$$\boxed{\mathcal{L}_{HAB}^{(-)} = \frac{g^{(-)}}{2}\mathrm{Tr}(H[A,B])}$$

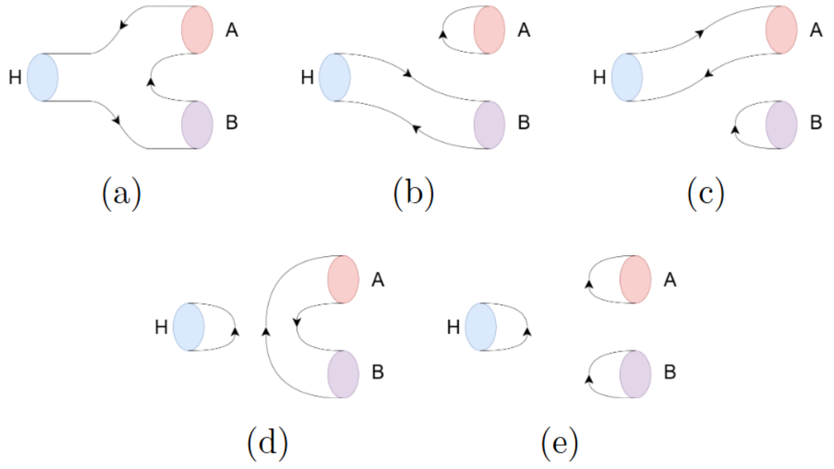
$$\begin{pmatrix} |\eta_1^{(L)}\rangle \\ |\eta_1^{(H)}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |\eta_1^{(l)}\rangle \\ |\eta_1^{(s)}\rangle \end{pmatrix}$$

$$\begin{pmatrix} |\eta_1^{(L)}\rangle \\ |\eta_1^{(H)}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_1^{8}\rangle \\ |\eta_1^{1}\rangle \end{pmatrix}$$

$$\theta = 54.7° - \alpha$$

b) If $C'(A)C'(B) = +$, this is the case for $f_1\pi, \eta(\eta')\pi, a_1\pi, VV, K_{1A}\bar{K}$, then

$$\begin{aligned}\mathcal{L}_{HAB}^{(+)} &= \frac{g}{2}\,\mathrm{Tr}(H\{A,B\}) - g_H\,\mathrm{Tr}\,H\,\mathrm{Tr}(AB)\\ &\quad -g_A\,\mathrm{Tr}\,A\,\mathrm{Tr}(BH) - g_B\,\mathrm{Tr}\,B\,\mathrm{Tr}(HA)\\ &\quad +g_3\,\mathrm{Tr}\,H\,\mathrm{Tr}\,A\,\mathrm{Tr}\,B,\end{aligned}$$

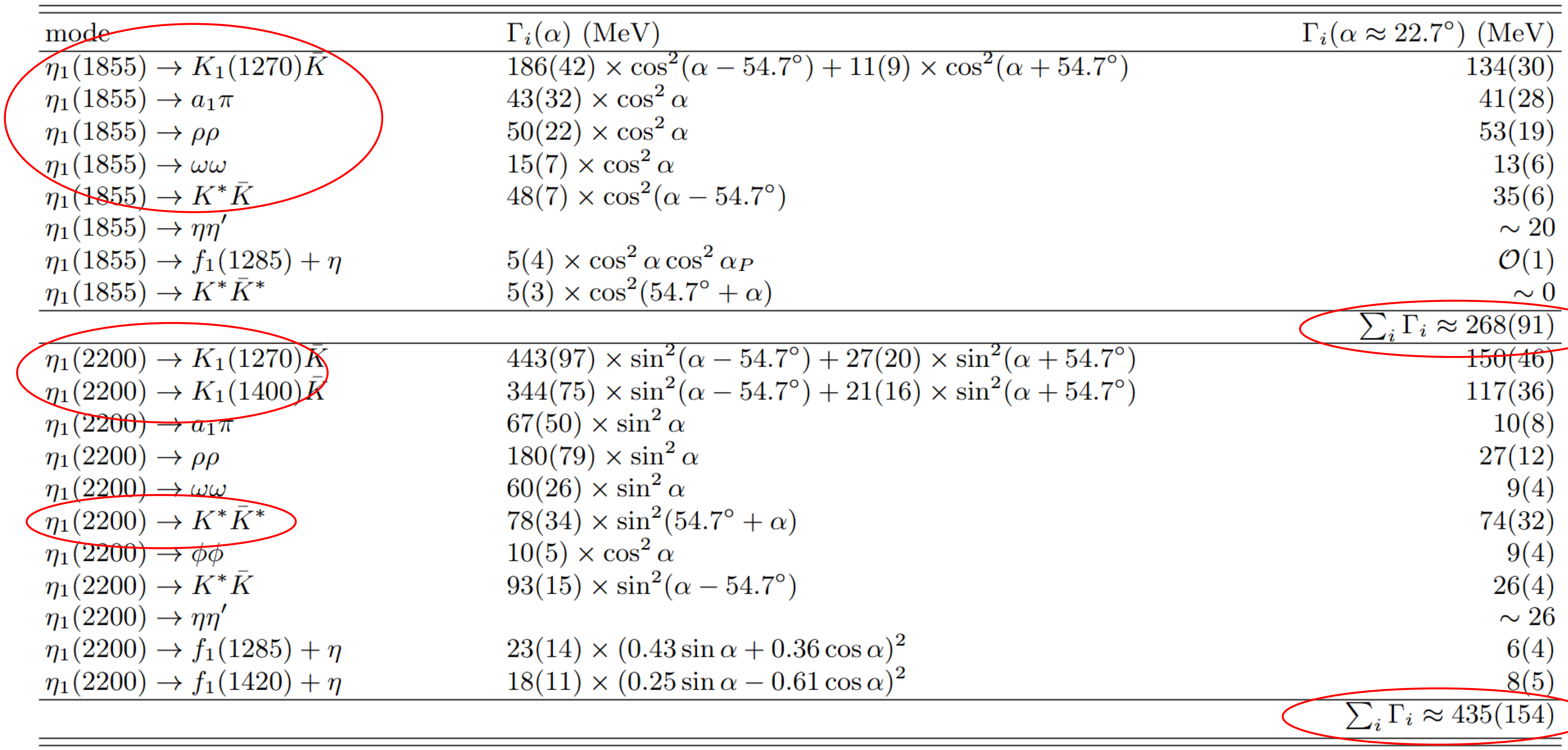


- **The flavor SU(3) symmetry indicates**

$$\begin{aligned}
g_{\eta_1^{(l)}(K_1\bar{K})_0^-} &= \frac{1}{\sqrt{2}}g^{(-)} \approx \frac{1}{\sqrt{2}}\bar{g}_{b_1\pi}\\
g_{\eta_1^{(l)}a_1\pi} &= \sqrt{\frac{3}{2}}(g-2g_H) \approx \bar{g}_{a_1\pi}\\
g_{\eta_1^{(l)}(K_1\bar{K})_0^+} &= \frac{1}{\sqrt{2}}(g-4g_H) \approx \frac{1}{\sqrt{3}}\bar{g}_{a_1\pi}\\
g_{\eta_1^{(l)}\rho\rho} &= \sqrt{\frac{3}{2}}(g-2g_H) \approx \bar{g}_{\rho\rho}\\
g_{\eta_1^{(l)}(K^*\bar{K}^*)_0^+} &= \frac{1}{\sqrt{2}}(g-4g_H) \approx \frac{1}{\sqrt{3}}\bar{g}_{\rho\rho}\\
g_{\eta_1^{(l)}\omega\omega} &= \frac{1}{2}(g-2g_H-\ldots) \approx \frac{1}{\sqrt{3}}\bar{g}_{\rho\rho}\\
g_{\eta_1^{(l)}K^*\bar{K}} &= \frac{1}{\sqrt{2}}g^{(-)} \approx \frac{1}{\sqrt{2}}\bar{g}_{\rho\pi},
\end{aligned}$$

$$\begin{aligned}
g_{\eta_1^{(s)}(K_1\bar{K})_0^-} &= -g^{(-)} \approx -\bar{g}_{b_1\pi}\\
g_{\eta_1^{(s)}a_1\pi} &= -\sqrt{3}g_H \approx 0\\
g_{\eta_1^{(s)}(K_1\bar{K})_0^+} &= (g-2g_H) \approx \sqrt{\frac{2}{3}}\bar{g}_{a_1\pi}\\
g_{\eta_1^{(s)}\rho\rho} &= -\sqrt{3}g_H \approx 0\\
g_{\eta_1^{(s)}(K^*\bar{K}^*)_0^+} &= (g-2g_H) \approx \sqrt{\frac{2}{3}}\bar{g}_{\rho\rho}\\
g_{\eta_1^{(s)}\phi\phi} &= (g-g_H-\ldots) \approx \sqrt{\frac{2}{3}}\bar{g}_{\rho\rho}\\
g_{\eta_1^{(s)}K^*\bar{K}} &= -g^{(-)} \approx -\bar{g}_{\rho\pi}.
\end{aligned}$$

- **The partial decay widths of $\eta_1(1855)$ and $\eta_1(2200)$ with respect to the mixing angle α**



mode	$\Gamma_i(\alpha)$ (MeV)	$\Gamma_i(\alpha \approx 22.7^\circ)$ (MeV)
$\eta_1(1855) \to K_1(1270)\bar{K}$	$186(42) \times \cos^2(\alpha - 54.7^\circ) + 11(9) \times \cos^2(\alpha + 54.7^\circ)$	134(30)
$\eta_1(1855) \to a_1\pi$	$43(32) \times \cos^2\alpha$	41(28)
$\eta_1(1855) \to \rho\rho$	$50(22) \times \cos^2\alpha$	53(19)
$\eta_1(1855) \to \omega\omega$	$15(7) \times \cos^2\alpha$	13(6)
$\eta_1(1855) \to K^*\bar{K}$	$48(7) \times \cos^2(\alpha - 54.7^\circ)$	35(6)
$\eta_1(1855) \to \eta\eta'$		~ 20
$\eta_1(1855) \to f_1(1285) + \eta$	$5(4) \times \cos^2\alpha \cos^2\alpha_P$	$\mathcal{O}(1)$
$\eta_1(1855) \to K^*\bar{K}^*$	$5(3) \times \cos^2(54.7^\circ + \alpha)$	~ 0
		$\sum_i \Gamma_i \approx 268(91)$
$\eta_1(2200) \to K_1(1270)\bar{K}$	$443(97) \times \sin^2(\alpha - 54.7^\circ) + 27(20) \times \sin^2(\alpha + 54.7^\circ)$	150(46)
$\eta_1(2200) \to K_1(1400)\bar{K}$	$344(75) \times \sin^2(\alpha - 54.7^\circ) + 21(16) \times \sin^2(\alpha + 54.7^\circ)$	117(36)
$\eta_1(2200) \to a_1\pi$	$67(50) \times \sin^2\alpha$	10(8)
$\eta_1(2200) \to \rho\rho$	$180(79) \times \sin^2\alpha$	27(12)
$\eta_1(2200) \to \omega\omega$	$60(26) \times \sin^2\alpha$	9(4)
$\eta_1(2200) \to K^*\bar{K}^*$	$78(34) \times \sin^2(54.7^\circ + \alpha)$	74(32)
$\eta_1(2200) \to \phi\phi$	$10(5) \times \cos^2\alpha$	9(4)
$\eta_1(2200) \to K^*\bar{K}$	$93(15) \times \sin^2(\alpha - 54.7^\circ)$	26(4)
$\eta_1(2200) \to \eta\eta'$		~ 26
$\eta_1(2200) \to f_1(1285) + \eta$	$23(14) \times (0.43 \sin\alpha + 0.36 \cos\alpha)^2$	6(4)
$\eta_1(2200) \to f_1(1420) + \eta$	$18(11) \times (0.25 \sin\alpha - 0.61 \cos\alpha)^2$	8(5)
		$\sum_i \Gamma_i \approx 435(154)$

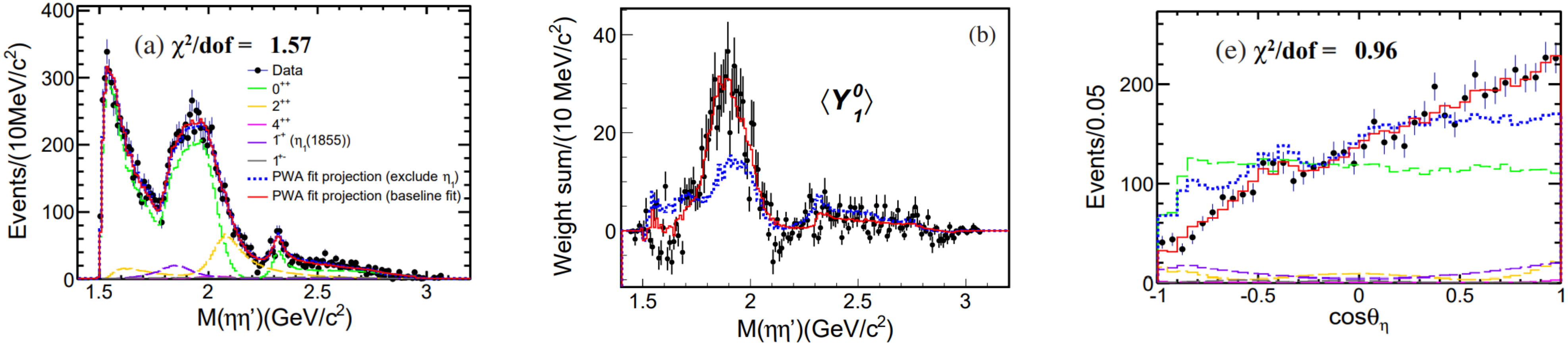
Here we assume $K_1(1270)$ and $K_1(1400)$ are mixed from K_{1A} and K_{1B} equally.

6. Implication for BESIII search for $\eta_1(1855)$ and possible $\eta_1(2200)$

$\eta_1(1855)$ ($I^G J^{PC} = 0^+ 1^{-+}$) observed by BESIII

(BESIII, Phys. Rev. Lett. 129, 192002 (2022),arXiv:2202.00621(hep-ex))

- Partial wave analysis of the process $J/\psi \to \gamma\eta\eta'$



Blue dot lines: PWA fit excluding $\eta_1(1855)$

- Resonance parameters of $\eta_1(1855)$: $m_{\eta_1} = 1855 \pm 9^{+6}_{-1}$ MeV, $\Gamma_{\eta_1} = 188 \pm 18^{+3}_{-8}$ MeV
  Combined branching fraction: $\mathrm{Br}(J/\psi \to \gamma\eta_1 \to \gamma\eta\eta') = \left(2.70 \pm 0.41^{+0.16}_{-0.35}\right) \times 10^{-6}$

- The first candidate for isoscalar 1^{-+} hybrid.

• The radiative decay of J/ψ into η_1 from lattice QCD (F. Chen et al., Phys. Rev. D 107, 054511 (2023))

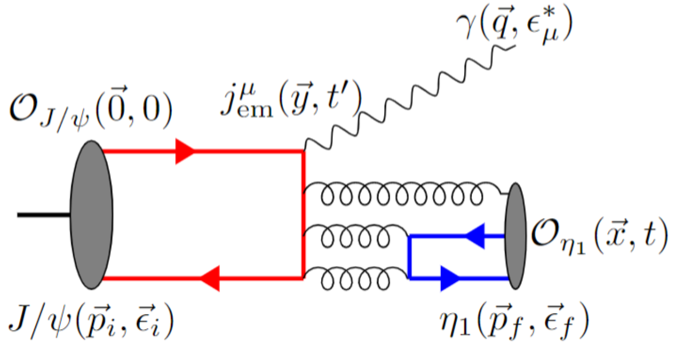
$$\Gamma(J/\psi \to \gamma\eta_1) = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m_\psi^2}\left(M_1^2(0) + E_2^2(0)\right)$$

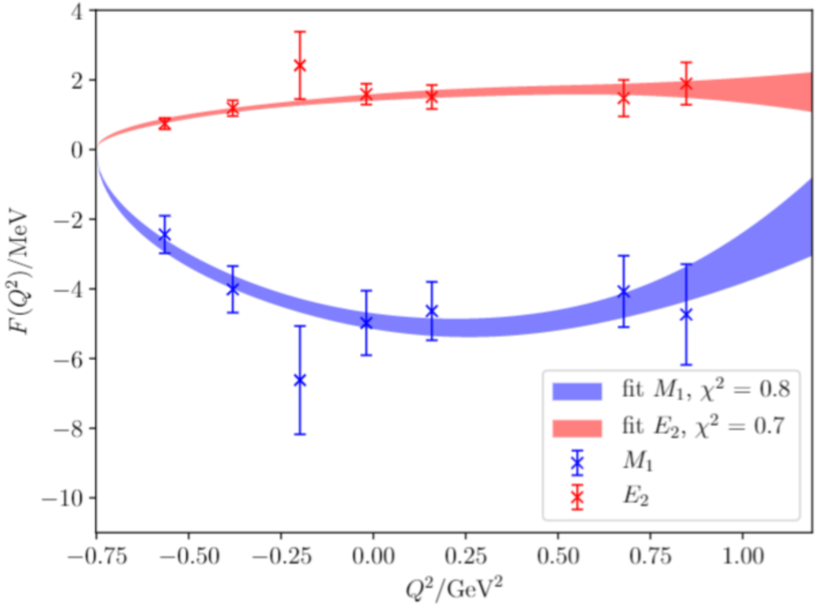
Extraction of the form factors $M_1(Q^2)$ and $E_2(Q^2)$

$$\Gamma^{(3)}_{i\mu j} = \frac{1}{T}\sum_{\tau}^{T}\langle \mathcal{O}^i_{\eta_1}(0, t+\tau) G_{\mu j}(\vec{p}, \vec{p}; t'+\tau, \tau)\rangle$$

$$\boxed{\begin{aligned} M_1(0) &= -4.73(74)\ \text{MeV} \\ E_2(0) &= \quad 1.18(22)\ \text{MeV} \end{aligned}}$$

$$\boxed{\Gamma(J/\psi \to \gamma\eta_1) = 2.04(61)\ \text{eV}}$$





Intuitively, gluons in J/ψ radiative decay couple to flavor singlets. Therefore

$$\Gamma\left(J/\psi \to \gamma\eta_1^{(l)}\right) = \sin^2\theta\ \Gamma(J/\psi \to \gamma\eta_1^1)\,\chi^{(l)}$$

$$\Gamma\left(J/\psi \to \gamma\eta_1^{(h)}\right) = \cos^2\theta\ \Gamma(J/\psi \to \gamma\eta_1^1)\chi^{(h)}$$

$$\chi^{(x)} = \frac{m_{\eta_1}^2 \left|\vec{p}_\gamma\left(\eta_1^{(x)}\right)\right|^3}{m_{\eta_1^{(x)}}^2 \left|\vec{p}_\gamma(\eta_1)\right|^3}$$

On the other hand, $\eta\eta'$ only appears as a flavor octet, so $\eta_1^{(h,l)} \to \eta\eta'$ must take place through its octet component:

$$\left\langle \eta\eta'|H_I|\eta_1^{(l)}\right\rangle = \cos\theta\left\langle \eta\eta'|H_I|\eta_1^{(8)}\right\rangle \equiv g\cos\theta\ \left|\vec{k}^{(l)}\right|$$

$$\left\langle \eta\eta'|H_I|\eta_1^{(h)}\right\rangle = \sin\theta\left\langle \eta\eta'|H_I|\eta_1^{(8)}\right\rangle \equiv g\sin\theta\ \left|\vec{k}^{(h)}\right|$$

$$\Gamma\left(\eta^{(l)} \to \eta\eta'\right) \propto g^2 \frac{\left|\vec{k}^{(l)}\right|^3}{m_{\eta_1^{(l)}}^2}\cos^2\theta$$

$$\Gamma\left(\eta^{(l)} \to \eta\eta'\right) \propto g^2 \frac{\left|\vec{k}^{(h)}\right|^3}{m_{\eta_1^{(h)}}^2}\sin^2\theta$$

$$\boxed{r = \frac{\mathrm{Br}(J/\psi \to \gamma\eta_1^{(l)} \to \gamma\eta\eta')}{Br(J/\psi \to \gamma\eta_1^{(h)} \to \gamma\eta\eta')} = \frac{\chi^{(l)}\left|\vec{k}^{(l)}\right|^3 m_{\eta_1^{(h)}}^2}{\chi^{(h)}\left|\vec{k}^{(h)}\right|^3 m_{\eta_1^{(l)}}^2}\frac{\Gamma_{\eta_1^{(h)}}}{\Gamma_{\eta_1^{(l)}}} \sim \frac{\Gamma_{\eta_1^{(h)}}}{\Gamma_{\eta_1^{(l)}}}\mathcal{O}(1)}$$

BESIII observation: $\mathbf{Br}(J/\psi \rightarrow \gamma\eta_1(1855) \rightarrow \gamma\eta\eta') = \left(2.70 \pm 0.41^{+0.16}_{-0.35}\right) \times 10^{-6}$

If $\eta_1(1855)$ is the $\eta_1^{(l)}$, then

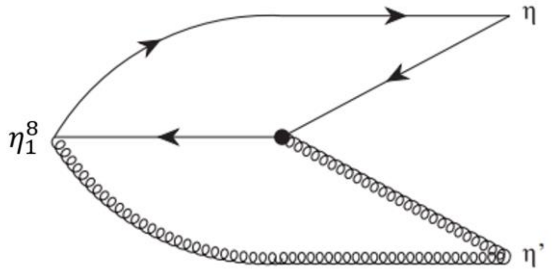
$$\boxed{\begin{aligned} \Gamma(J/\psi \rightarrow \gamma\eta_1(1855)) &= (2.0 \pm 0.7)\ \text{eV} \\ \text{Br}(J/\psi \rightarrow \gamma\eta_1(1855)) &= (2.1 \pm 0.7) \times 10^{-5} \\ \text{Br}(\eta_1(1855) \rightarrow \eta\eta') &= (13 \pm 5)\% \end{aligned}}$$

If $\eta_1(1855)$ is the $\eta_1^{(h)}$, then

$$\boxed{\begin{aligned} \Gamma(J/\psi \rightarrow \gamma\eta_1(1855)) &= (5.0 \pm 1.6)\ \text{eV} \\ \text{Br}(J/\psi \rightarrow \gamma\eta_1(1855)) &= (5.4 \pm 1.8) \times 10^{-5} \\ \text{Br}(\eta_1(1855) \rightarrow \eta\eta') &= (5.0 \pm 1.9)\% \end{aligned}}$$

Both cases implies $\mathbf{Br}\left(\eta_1^8 \rightarrow \eta\eta'\right) \sim 20\%$,

$$\mathbf{Br}\left(\eta_1^8 \rightarrow \eta\eta'\right) \sim \frac{\mathbf{Br}(\eta_1(1855) \rightarrow \eta\eta')}{\cos^2\theta} \sim 18(7)\%$$



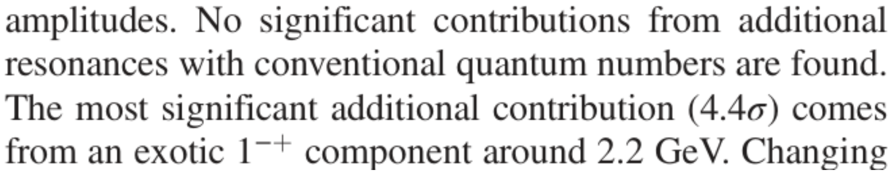
$U_A(1)$ anomaly

$U_A(1)$ anomaly may play a role here (but too large a value!)

L. Qiu and Q. Zhao, Chin. Phys. C 051001 (2022), arXiv:2202.00904 (hep-ph);
H. Chen, N. Su, S.L. Zhu, Chin. Phys. Lett. 39, 051201 (2022), arXiv:2202.04918 (hep-ph)

$$\mathbf{Br}(J/\psi \to \gamma\eta_1 \to \gamma\eta\eta') = \left(2.70 \pm 0.41^{+0.16}_{-0.35}\right) \times 10^{-6}$$ (21.4σ)

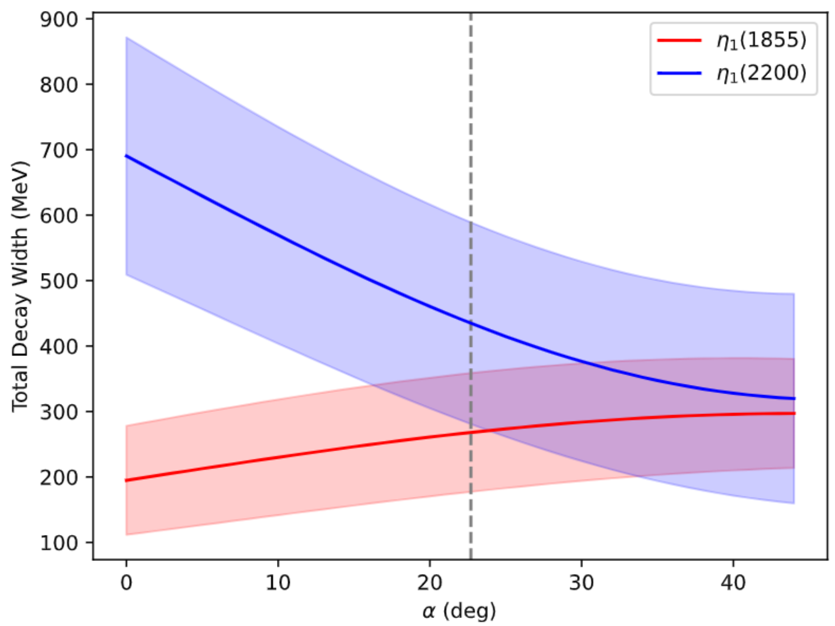
BESIII Phys.Rev.D106, 072012 (2022)



amplitudes. No significant contributions from additional resonances with conventional quantum numbers are found. The most significant additional contribution (4.4σ) comes from an exotic 1^{-+} component around 2.2 GeV. Changing

$$\boxed{r = \frac{\mathbf{Br}(J/\psi \to \gamma\eta_1^{(L)} \to \gamma\eta\eta')}{\mathbf{Br}(J/\psi \to \gamma\eta_1^{(H)} \to \gamma\eta\eta')} \sim \frac{\Gamma_{\eta_1^{(H)}}}{\Gamma_{\eta_1^{(L)}}}\mathcal{O}(1)}$$

$$\frac{\Gamma(\eta_1(2200))}{\Gamma(\eta_1(1855))} \approx 1.6 \quad (\alpha \approx 22.7^\circ)$$



$\boldsymbol{\alpha}$-dependence of the total widths of $\boldsymbol{\eta_1(1855)}$ and $\boldsymbol{\eta_1(2200)}$.

- ✓ The major decay modes of $\eta_1(1855)$ are $K_1(1270)\bar{K}$, $\rho\rho$, $a_1\pi$, $K^*\bar{K}$.
- ✓ The major decay modes of $\eta_1(2200)$ are $K_1(1270)\bar{K}$, $K_1(1400)\bar{K}$, $K^*\bar{K}^*$, $K^*\bar{K}$, and $\rho\rho$.
- ✓ We suggest BESIII to search for $\eta_1(1855)$ and $\eta_1(2200)$ in these systems.
- ✓ If $\eta_1(2200)$ is dominated by a $s\bar{s}g$ component, a good place to find it is $\psi(3686) \to \phi\eta_1(2200)$

V. Summary and perspectives

- **QCD expects the existence of hybrid mesons**
- **There do exist several experimental candidates for $\mathbf{1^{-+}}$ light hybrids such as $\boldsymbol{\pi_1(1600)}$ and $\boldsymbol{\eta_1(1855)}$.**
- **We calculate the production rate of $\boldsymbol{\eta_1(1855)}$ in the $\boldsymbol{J/\psi}$ radiative decay.**
- **We calculate the partial decay widths of $\boldsymbol{\pi_1(1600)}$ which are compatible with experiments and previous lattice results using the Luescher's method.**
- **We predict the partial decay withs of $\boldsymbol{\eta_1(1855)}$ and its mass partner (possibly $\boldsymbol{\eta_1(2200)}$).**

Thank you for your Attention!