# 类粲偶素X(3700)的理论研究

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# Outline

- **Motivation**
- $e^+e^- \to J/\psi D\bar{D}$ (EPJA57(2021)38)
- $\gamma\gamma \to D\bar{D}$ (PRD103(2021)054008)
- $\Lambda_b \to D\bar{D}\Lambda$ (PRD103(2021)114013)
- $B^- \to K^-\eta\eta_c$(PRD109(2024)094014)
- **Summary**

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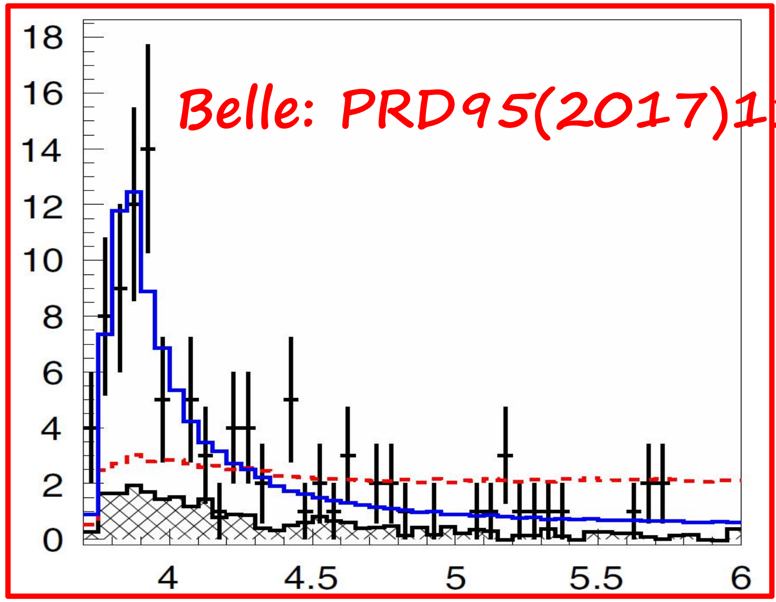
# Belle: $e^+e^- \to J/\psi D\bar{D}$

- In 2016, Belle has observed a $\chi_{c0}(3860)$ in $e^+e^- \to J/\psi D\bar{D}$, and its signal is described by the BW amplitude.




Belle: PRD95(2017)112003

| $J^{PC}$ | Mass, MeV/$c^2$ | Width, MeV | Significance |
|---|---|---|---|
| $0^{++}$ | $3862^{+26}_{-32}$ | $201^{+154}_{-67}$ | $9.1\sigma$ |
| $2^{++}$ | $3879^{+20}_{-17}$ | $171^{+129}_{-62}$ | $8.0\sigma$ |
| $2^{++}$ | $3879^{+17}_{-17}$ | $148^{+108}_{-50}$ | $8.0\sigma$ |
| $2^{++}$ | $3883^{+26}_{-24}$ | $227^{+201}_{-125}$ | $8.0\sigma$ |

**$\chi_{c0}(3860)$** $I^G(J^{PC}) = 0^+(0^{++})$

PDG2018

OMITTED FROM SUMMARY TABLE

The assignment $J^P = 0^+$ is preferred over $2^+$ by 2.5 sigma.

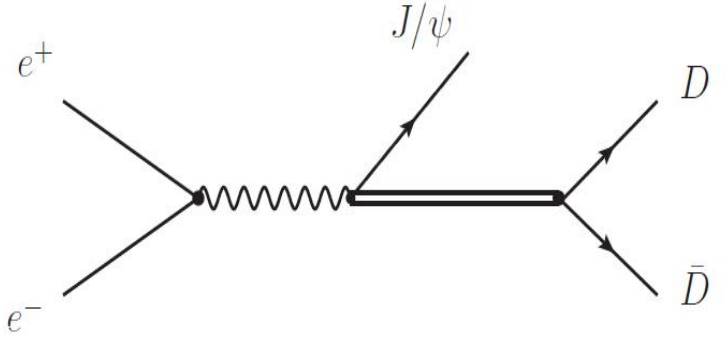
Observed by CHILIKIN 17 using full amplitude analysis of the process $e^+e^- \to J/\psi D\bar{D}$, where $D = D^0$, $D^+$.

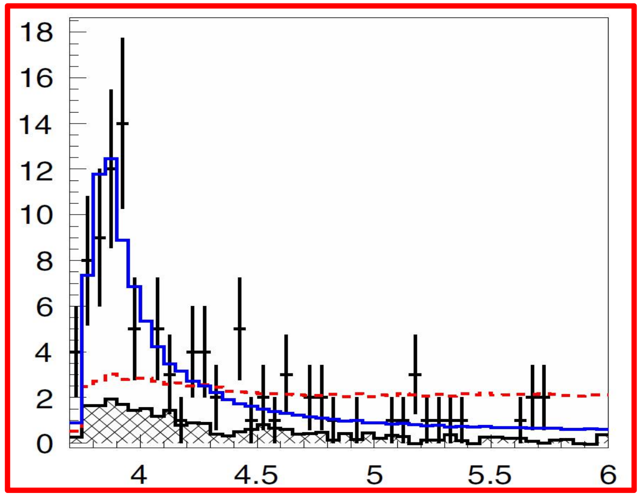
- The $J^{PC} = 0^{++}$ hypothesis is favored over the $2^{++}$ at the level of 2.5$\sigma$.
- The lineshape of resonance can not be described by BW near the threshold.
- Large uncertainties of the experimental data.

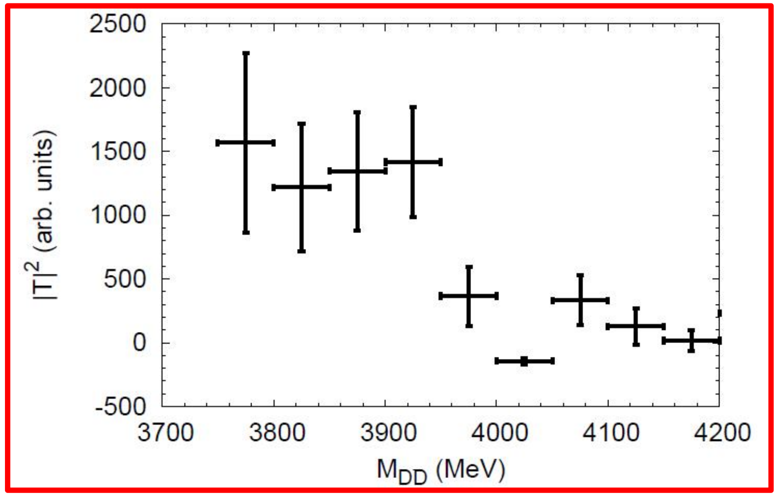
# Amplitude squared

## □Theometical model

$$\frac{d\sigma}{dM_{\rm inv}(D\bar{D})} = \mathcal{C}\,\frac{1}{(2\pi)^3}\,\frac{m_e^2}{s\sqrt{s}}\,|\vec{p}|\,|\tilde{k}|\,|T|^2,$$



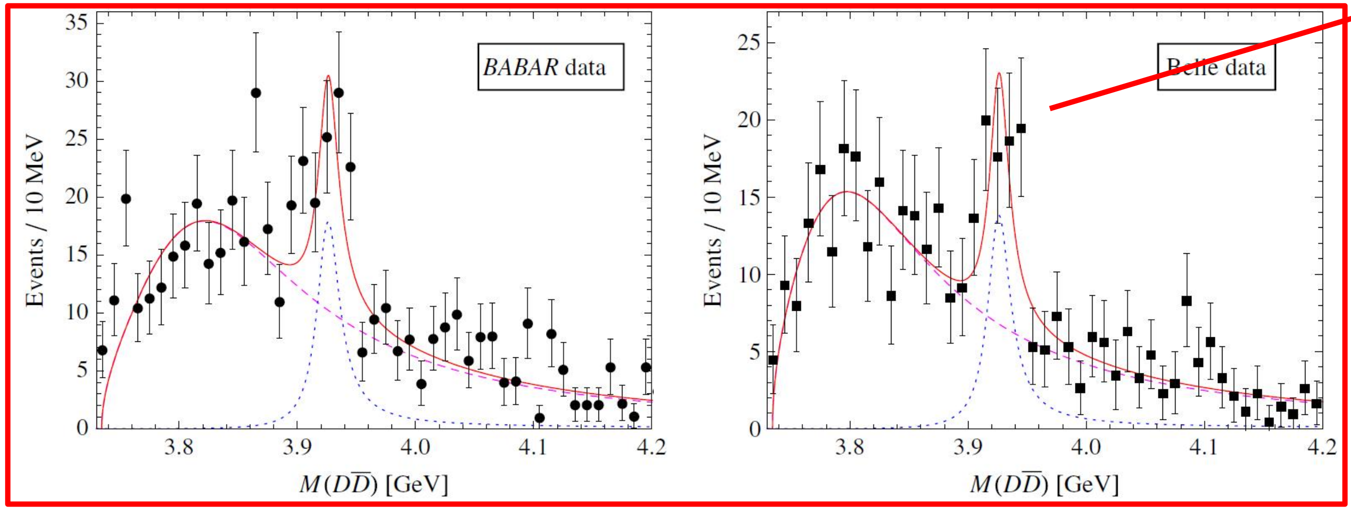




**No significant signal around 3860MeV!!!**

# Belle&BABAR: $\gamma\gamma \to D\bar{D}$

## □FKGuo-Meissner, PRD86(2012)091501




X(3915)

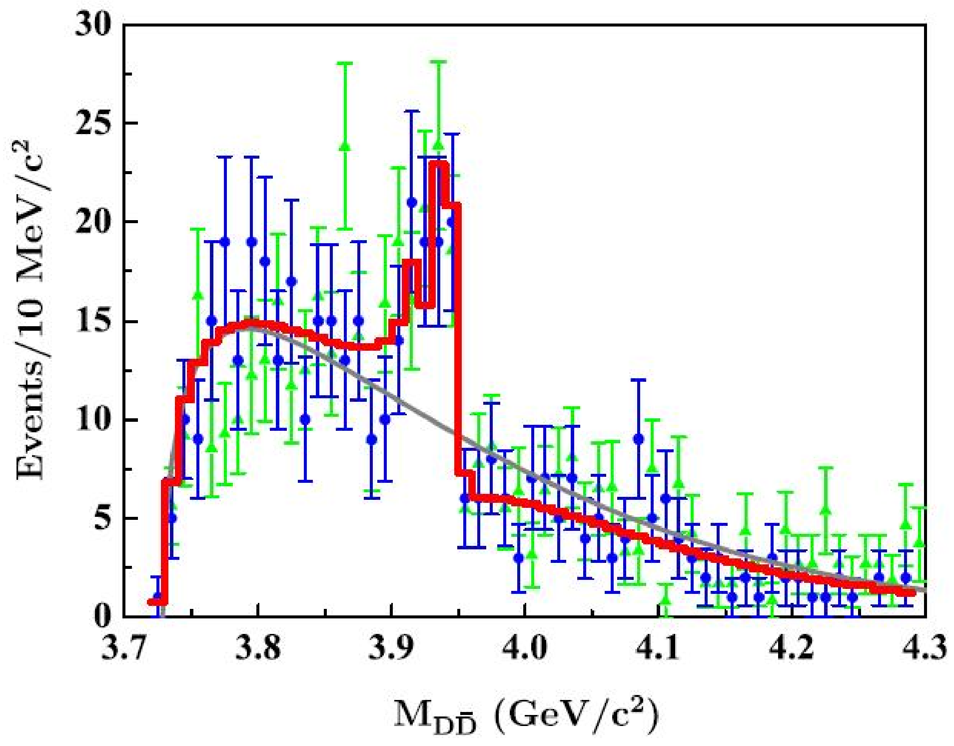
$$B_L(w) = \left(\frac{p}{p_0}\right)^{2L+1} \frac{M}{w} \frac{F_L^2(w)}{(w^2 - M^2)^2 + \Gamma^2(w) M^2}$$

$$M_{\chi_{c0}(2P)} = (3837.6 \pm 11.5)\ \text{MeV},$$
$$\Gamma_{\chi_{c0}(2P)} = (221 \pm 19)\ \text{MeV}.$$

$X(3915)$ is probably of exotic nature. There is an indication that the present data of the $\gamma\gamma \to D\bar{D}$ process already contain signals of the $\chi_{c0}(2P)$ with a mass and width around 3840 and 200 MeV, respectively. More refined analysis of the data with higher statistics is definitely necessary to confirm our assertion. In addition to the

# Belle&BABAR: $\gamma\gamma \to D\bar{D}$

## ☐Analysis without X(3860)




Eur. Phys. J. C (2012) 72:2226
DOI 10.1140/epjc/s10052-012-2226-4

THE EUROPEAN PHYSICAL JOUR

Regular Article - Theoretical Physics

**Does the enhancement observed in $\gamma\gamma \to D\bar{D}$ contain two $P$-wave higher charmonia?**

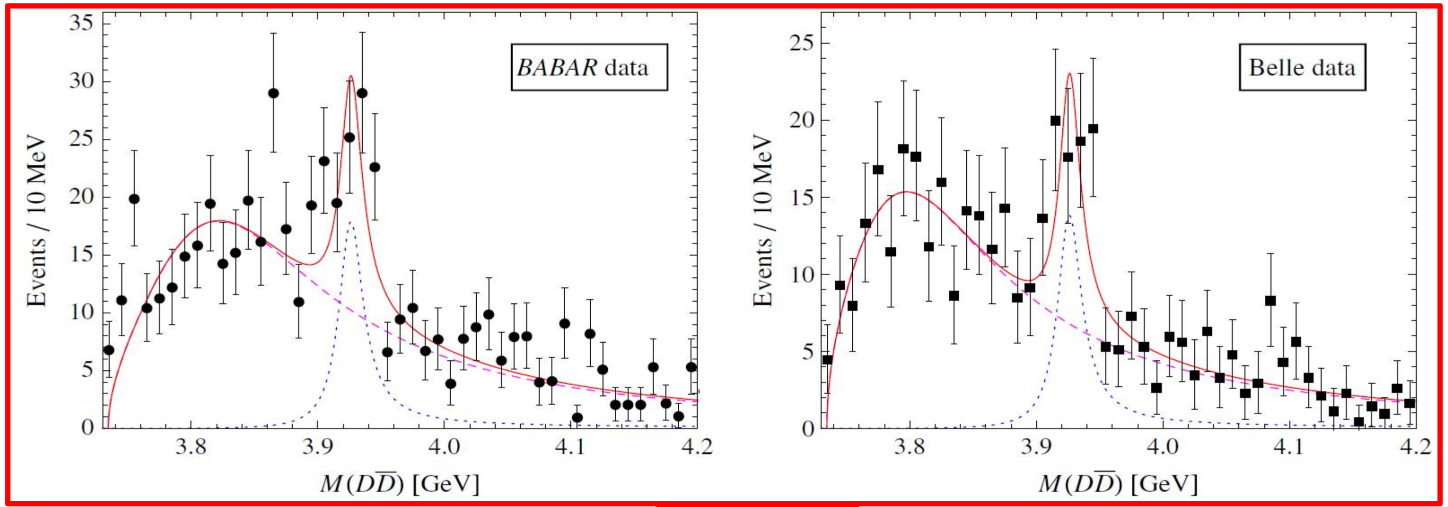
Dian-Yong Chen[1,3,a], Jun He[1,3,b], Xiang Liu[1,2,c], Takayuki Matsuki[4,d]

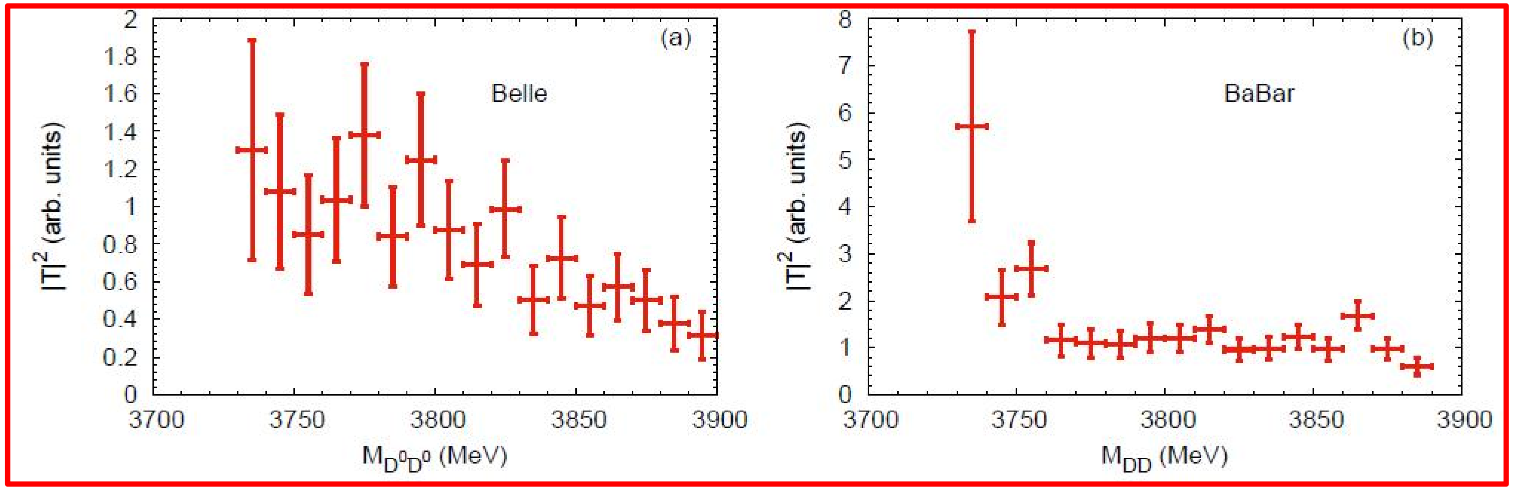
$\cos\theta^*$ distribution of $\gamma\gamma \to D\bar{D}$. Our study has illustrated that the experimental data of these quantities can be well reproduced when considering both $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$ in the $\gamma\gamma \to D\bar{D}$ process, which supports our conjecture of $Z(3930)$ structure. As indicated in Ref. [13], BaBar is carrying out the spin–parity analysis of $X(3915)$, which will be helpful to clarify whether it is suitable to explain $X(3915)$ as $P$-wave higher charmonium $\chi_{c0}(2P)$.

# Belle&BABAR: $\gamma\gamma \to D\bar{D}$

- Theoretical model

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\frac{|\vec{p}'|}{|\vec{p}|}\bar{\sum}|\mathcal{M}|^2$$




**No significant signal around 3840MeV!!!**

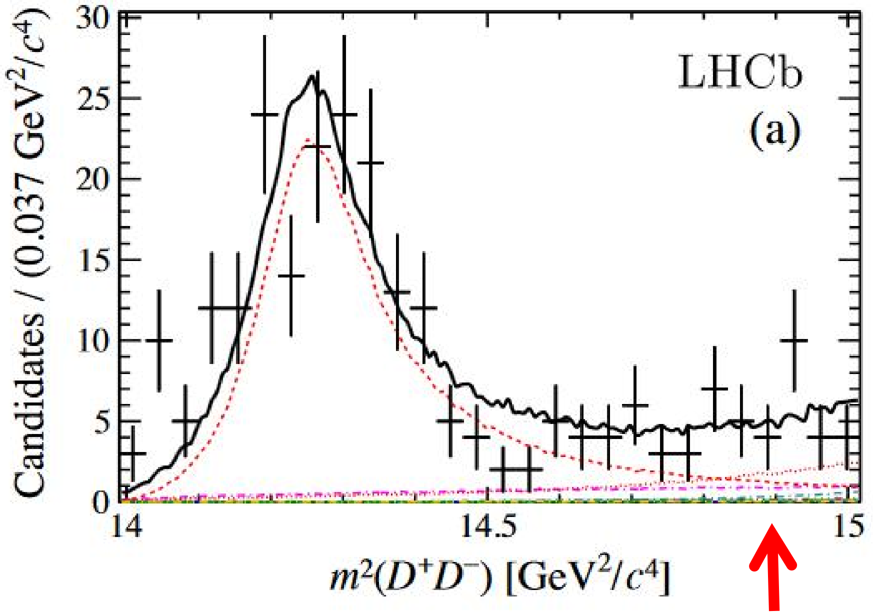
# Prediction of $\chi_{c0}(2P)$

- **3.92 GeV,** Godfrey-Isgur, PRD32(1985)189
- **3852 MeV(NR,LP), 3916MeV(GI),** Barnes-Godfrey-Swason, PRD72(2005)054026
- **3842 MeV, screening potentials,** BQLi-KTChao, PRD79(2009)094004
- **3869 MeV (LP), 3848MeV(SP), new paras,** WJDeng-Gui-Zhong,PRD95(2017)034026
- **Narrow width~20 MeV:** Gui-Lv-Zhong-Zhao, PRD98(2019)016010, Barnes-PRD72(2005)054026, Xiang Liu PRD101(2020)054029

**$\chi_{c0}(3860)$ still needs to be confirmed by more experiments.**

# LHCb: $B^+ \to D^+ D^- h$

- ➢No evidence of the $\chi_{c0}(3860)$ in the $B^+ \to D^+ D^- h$
  - LHCb,PRD102(2020)112003,PRL125(2020)242001
- ➢A large strength of observed at the $D\bar{D}$ threshold.




S wave in the baseline model. The broad $\chi_{c0}(3860)$ state reported by the Belle Collaboration [53] has been included in alternative fit models but is disfavored. Fits in which

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

PDG2024

**$\chi_{c0}(3860)$** $I^G(J^{PC}) = 0^+(0^{++})$

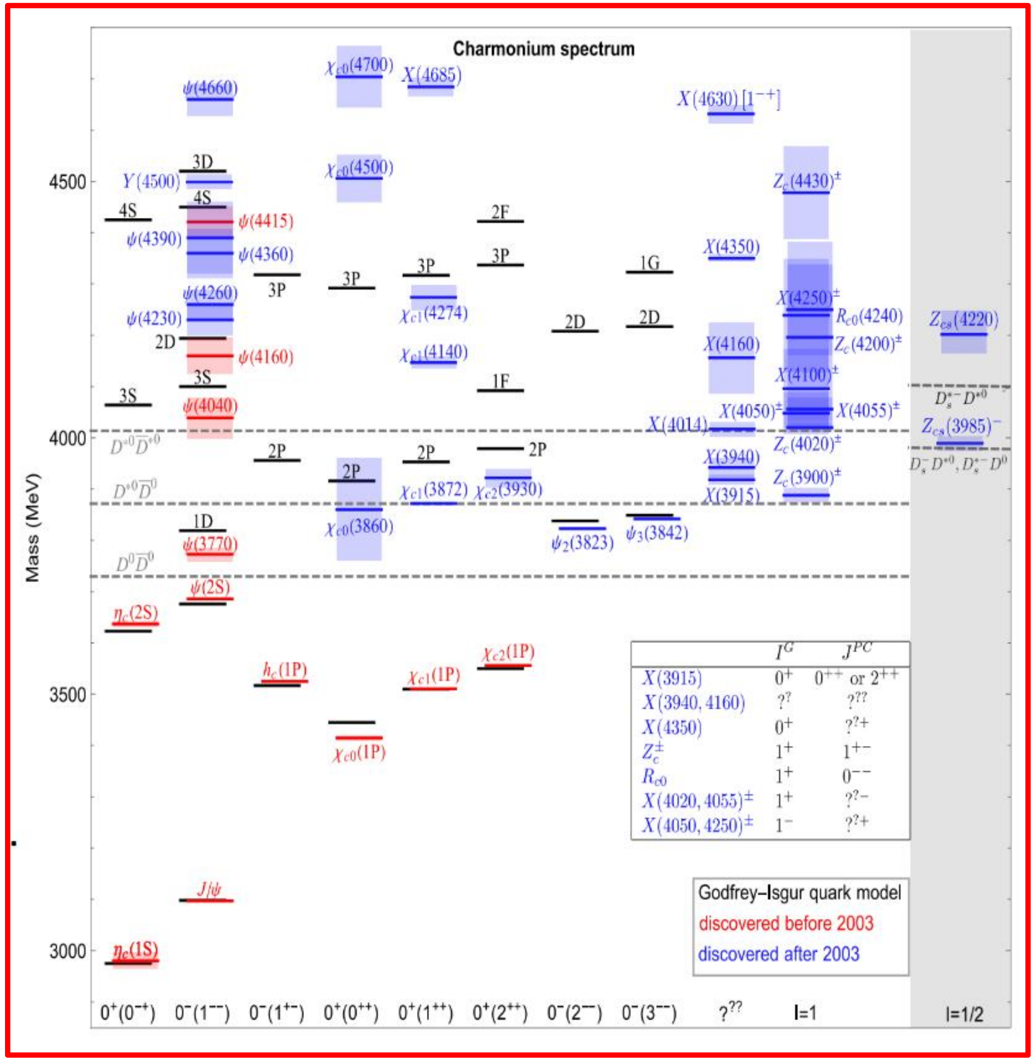
OMITTED FROM SUMMARY TABLE

The assignment $J^P = 0^+$ is preferred over $2^+$ by 2.5 sigma.

Observed by CHILIKIN 17 using full amplitude analysis of the process $e^+e^- \to J/\psi D\bar{D}$, where $D = D^0$, $D^+$. Not seen by AAIJ 20AI in the decay $B^+ \to D^+ D^- K^+$.

# Hadron-harond threshold

- X(3872) ~$D\bar{D}^*$
- $T_{cc}$ ~$DD^*$



# Prediction of $D\bar{D}$ state X(3700)

- $D\bar{D}$, $0^{++}$
  - 3700-3730 MeV Nieves, PRD86(2012) 056004, PRD87(2013)076006
  - 3720-3730MeV,Gamermann-Oset, PRD76(2007)074016, EPJA41(2009)85
  - B=4 MeV, LQCD, JHEP06(2021)035
  - 3734MeV, Dong-Guo-Zou, Progr.Phys.4(2021)45
  - 3739MeV, PPShi, ZHZhang-FKGuo-ZYang, PRD105(2022) 034024
  - 3.73GeV HXChen, PRD105(2022)094003
  - ~3730MeV, FZPeng-MJYan-Pavon, PRD108(2023)114001
  - 3700~3730MeV, PNShen-BSZou, PRD74(2006)014013(2006)

# Search for the $D\bar{D}$ bound state

- $B \to D\bar{D}K$, Dai-Xie-Oset, EPJC76(2016)121
- $\psi(3770) \to \gamma X(3700) \to \gamma\eta\eta'$, CWXiao-Oset, EPJA49(2013)52
- $\psi(4040) \to \gamma X(3700) \to \gamma\eta\eta'$ , CWXiao-Oset, EPJA49(2013)52
- $e^+e^- \to J/\psi X(3700) \to J/\psi\eta\eta'$, CWXiao-Oset, EPJA49(2013)52
- $\Lambda_b \to D\bar{D}\Lambda$ , EWang-Xie-DMLi, PRD103(2021)114013
- $B^- \to K^-\eta\eta_c$ , Li-Liu-Wang-Wei(PRD109(2024)094014), Xie-Liu-Geng, PRD107(2023)016003
- $B^+ \to K^+\eta\eta$, Brandao-Song-Oset, PRD108 (2023) 054004
- $e^+e^- \to J/\psi D\bar{D}, \gamma\gamma \to D\bar{D}$

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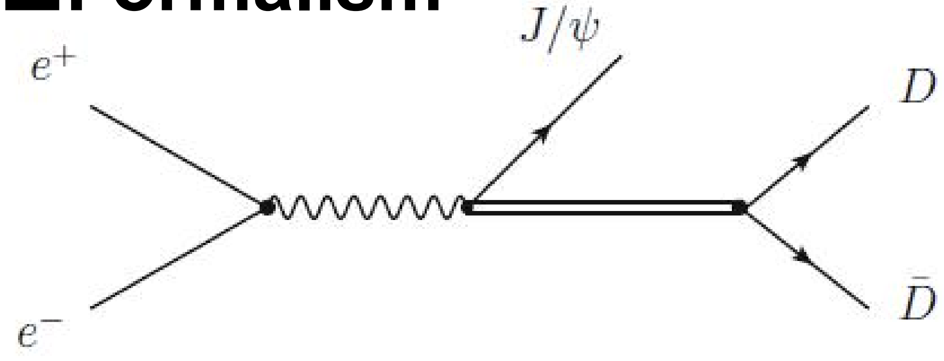
# Mechanism for $e^+e^- \to J/\psi D\bar{D}$

## ◻Formalism




$$\frac{d\sigma}{dM_{\rm inv}(D\bar{D})} = \mathcal{C}\frac{1}{(2\pi)^3}\frac{m_e^2}{s\sqrt{s}}|\vec{p}\,||\tilde{k}||T|^2,$$

$$T = [1 - VG]^{-1}V,$$

$$\mathcal{L} = \frac{1}{12f^2}(\mathrm{Tr}(J_{88\mu}J^{\mu}_{88} + 2J_{3\bar{3}\mu}J^{\mu}_{88} + J_{3\bar{3}_\mu}J^{\mu}_{3\bar{3}}) + \frac{8}{3}\gamma J_{\bar{3}1\mu}J^{\mu}_{13} + \frac{4}{\sqrt{3}}\gamma(J_{\bar{3}1\mu}J^{\mu}_{83} + J_{\bar{3}8\mu}J^{\mu}_{13}) + 2\gamma J_{\bar{3}8\mu}J^{\mu}_{83} + \psi_5 J_{\bar{3}3\mu}J^{\mu}_{\bar{3}3} + \mathcal{L}_{\rm mass}).$$

Phys. Rev. D **76**, 074016 (2007)

$$G_{ii} = i\int\frac{dq^4}{(2\pi)^4}\frac{1}{q^2 - m_1^2 + i\epsilon}\frac{1}{(P-q)^2 - m_2^2 + i\epsilon} = \tag{30}$$

$$\frac{1}{16\pi^2}\Big(\alpha_i + \log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\log\frac{m_2^2}{m_1^2} + \frac{p}{\sqrt{s}} \times\Big(\log\frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{-s + m_2^2 - m_1^2 + 2p\sqrt{s}} + \log\frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}}\Big)\Big). \tag{31}$$

$\alpha = -1.3$

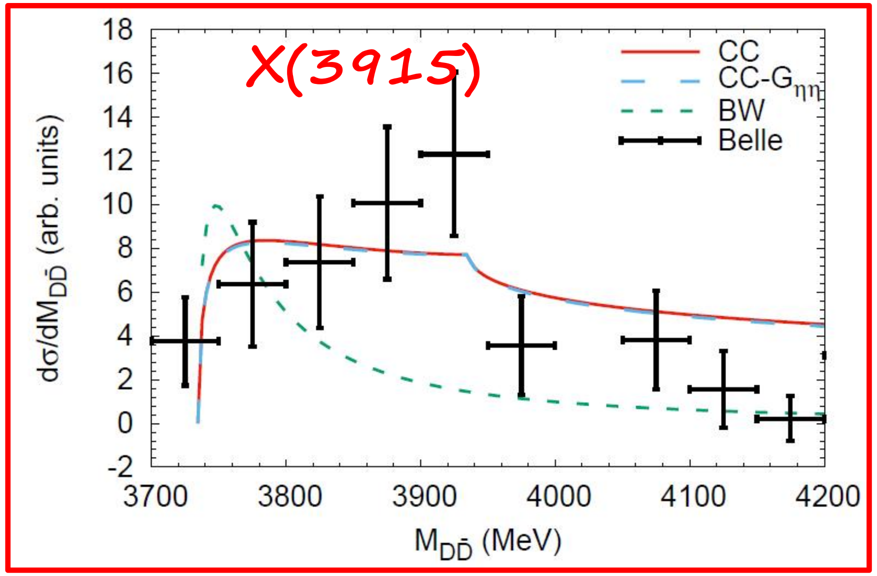
$\mu = 1500$ MeV,

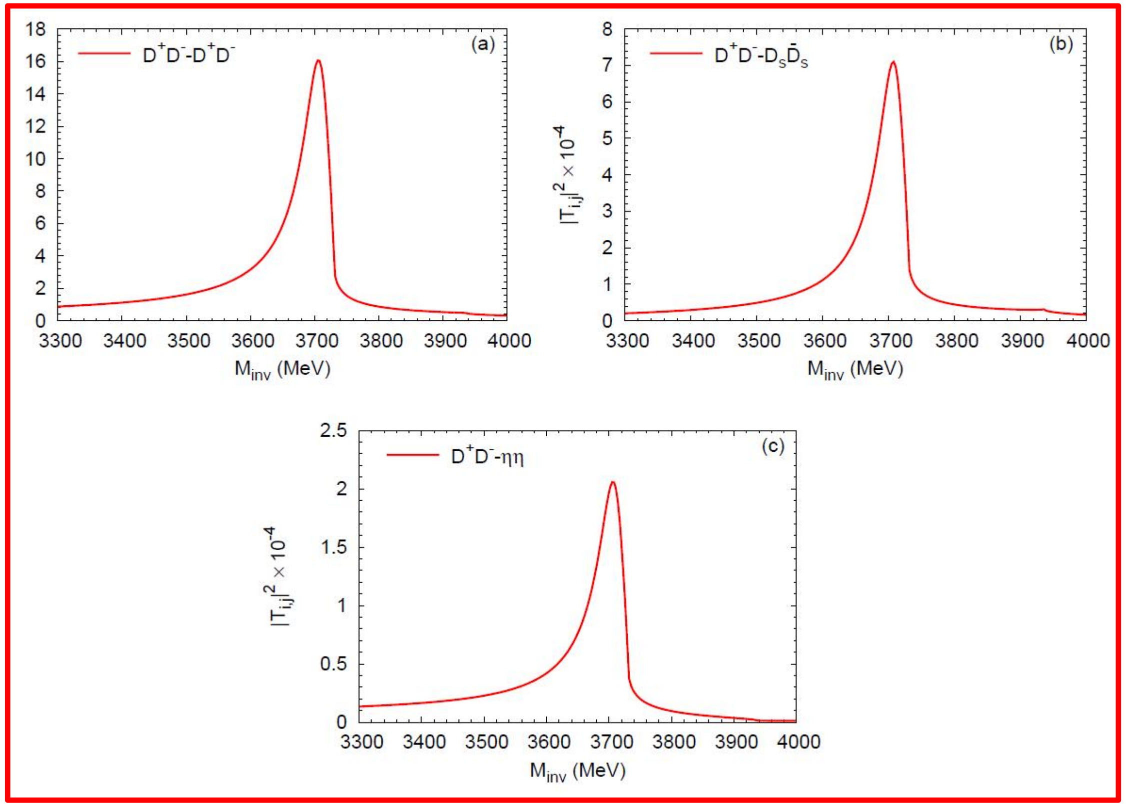
TABLE VI. Residues for the poles in the C = 0, S = 0, I = 0 sector.

| Channel | $f_0$ (GeV) | $\sigma$ (GeV) | Heavy singlet (GeV) |
|---|---|---|---|
| $\pi\pi$ | 1.37 | 3.00 | 0.16 ± 0.05 |
| $K\bar{K}$ | 3.80 | 1.25 | 0.05 ± 0.03 |
| $\eta\eta$ | 3.14 | 0.36 | 0.01 ± 0.01 |
| $D\bar{D}$ | 0.73 | 4.14 | 11.44 ± 4.42 |
| $D_s\bar{D}_s$ | 3.73 | 0.49 | 7.55 ± 2.97 |
| $\eta\eta_c$ | 1.97 | 0.98 | 0.12 ± 0.09 |

# Results for $e^+e^- \rightarrow J/\psi D\bar{D}$

## □Results with $\alpha = -1.3$

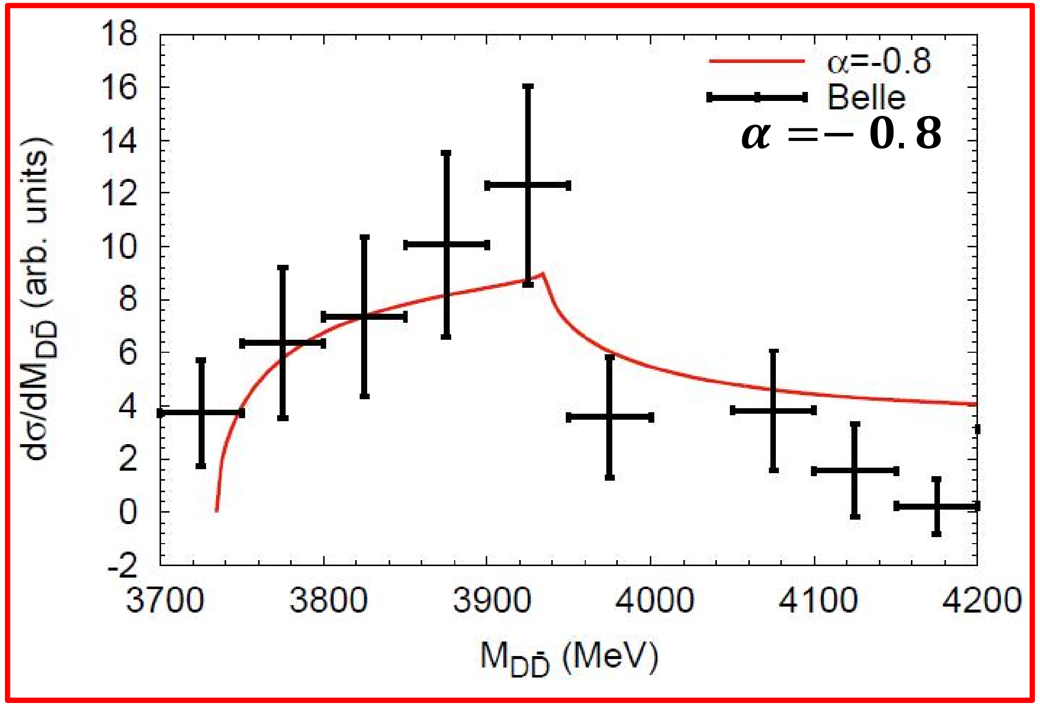


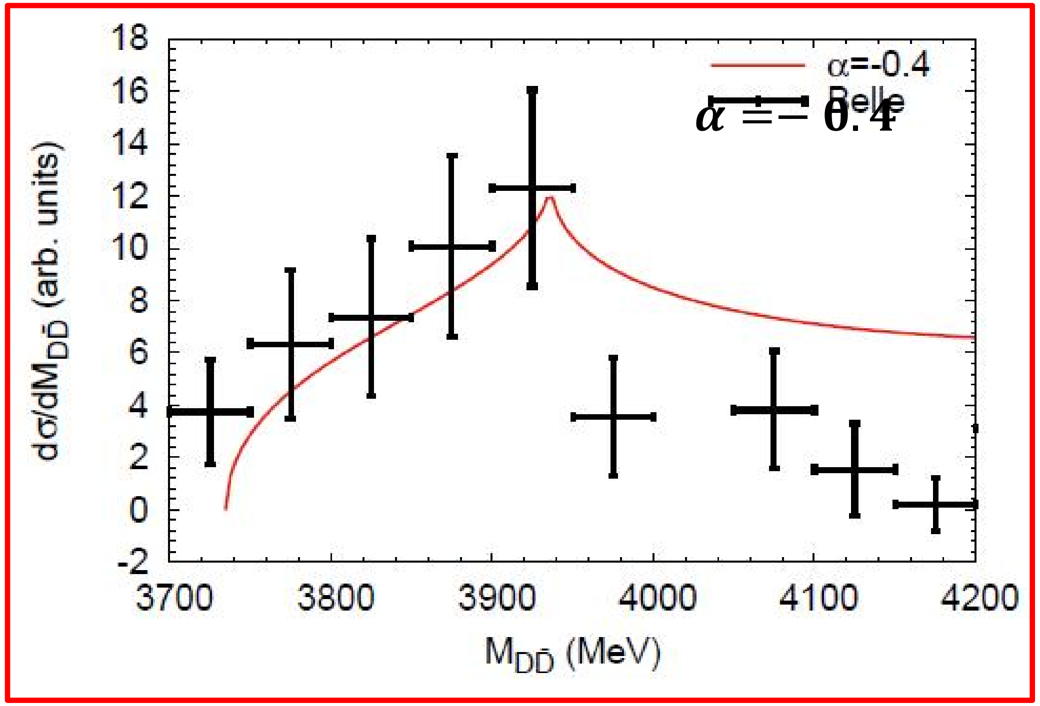




- **Near-threshold enhancement**
- **Peak around 3720~MeV**

# Results for $e^+e^- \rightarrow J/\psi D\bar{D}$

## □Results with different values of $\alpha$




- **The Belle data could be well described by considering the dynamically generated $D\bar{D}$ bound state in the process $e^+e^- \rightarrow J/\psi\, D\bar{D}$.**

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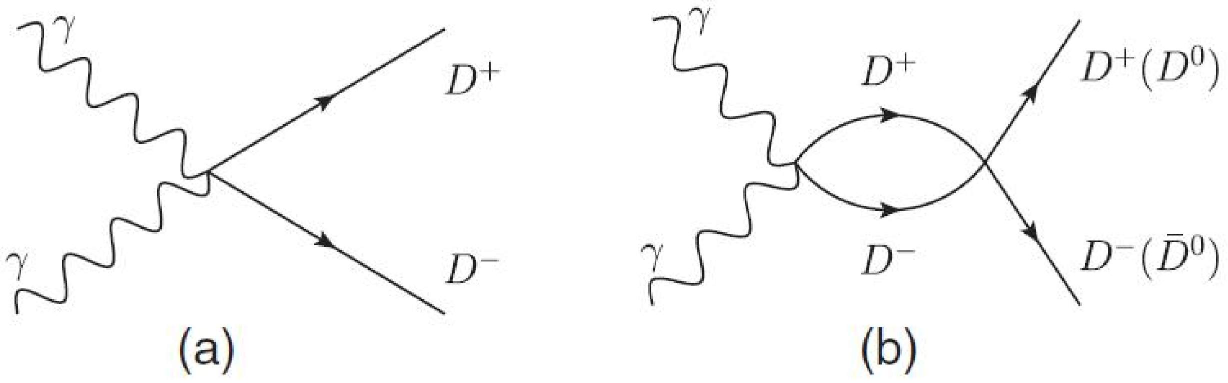
# Mechanism for $\gamma\gamma \to D\bar{D}$

## □Tree diagram+ FSI



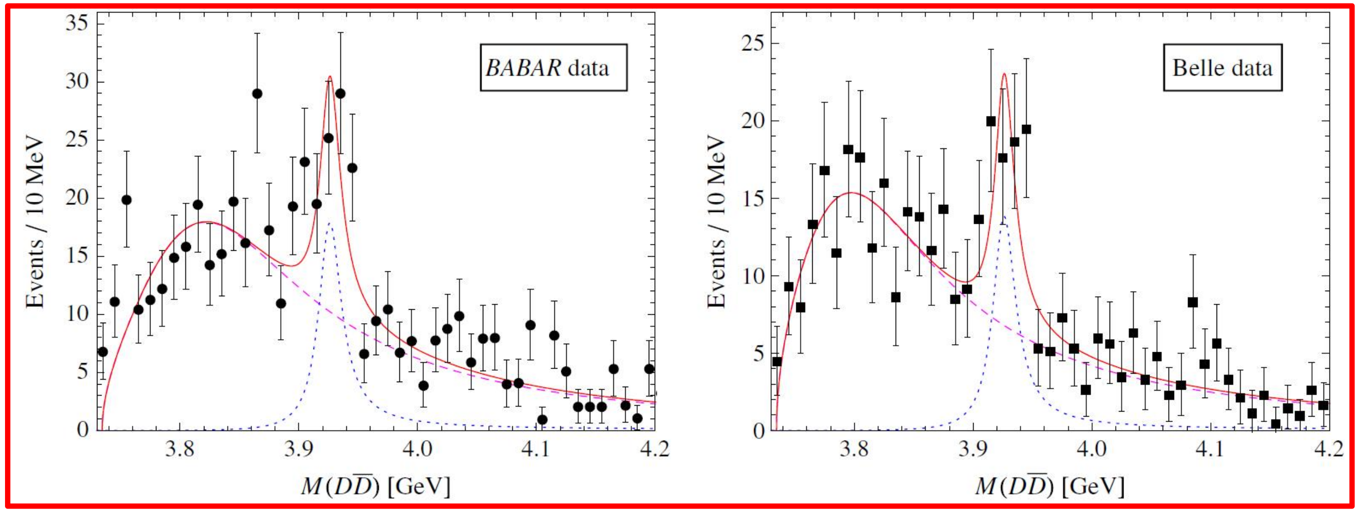

$$|t_{\rm Belle}|^2 = \mathcal{C}_{\rm Belle}|t_{D^0\bar{D}^0}|^2,$$

$$|t_{BABAR}|^2 = \mathcal{C}_{BABAR}(|t_{D^0\bar{D}^0}|^2 + B|t_{D^+D^-}|^2),$$

$$\mathcal{M} = 2e^2\vec{\epsilon}_1 \cdot \vec{\epsilon}_2 \times t,$$

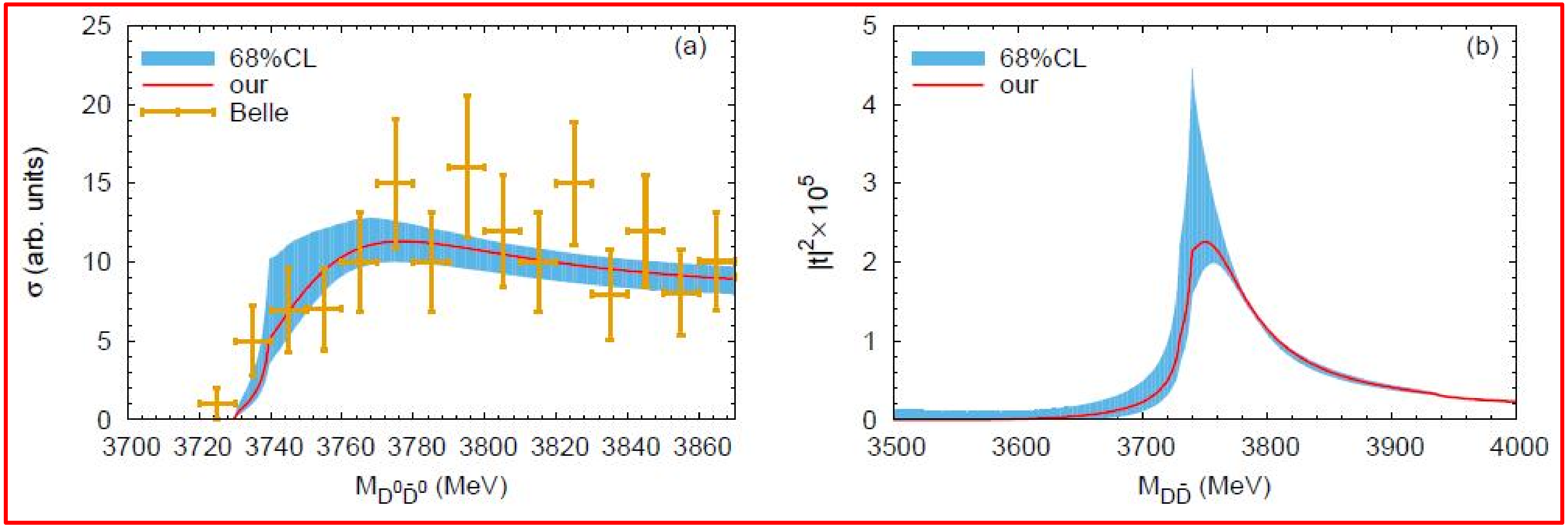
$$t_{D^+D^-} = \left(1 + \frac{1}{2}G_{D^+D^-}t^{I=0}_{D\bar{D},D\bar{D}}\right),$$

$$t_{D^0\bar{D}^0} = \frac{1}{2}G_{D^+D^-}t^{I=0}_{D\bar{D},D\bar{D}}.$$



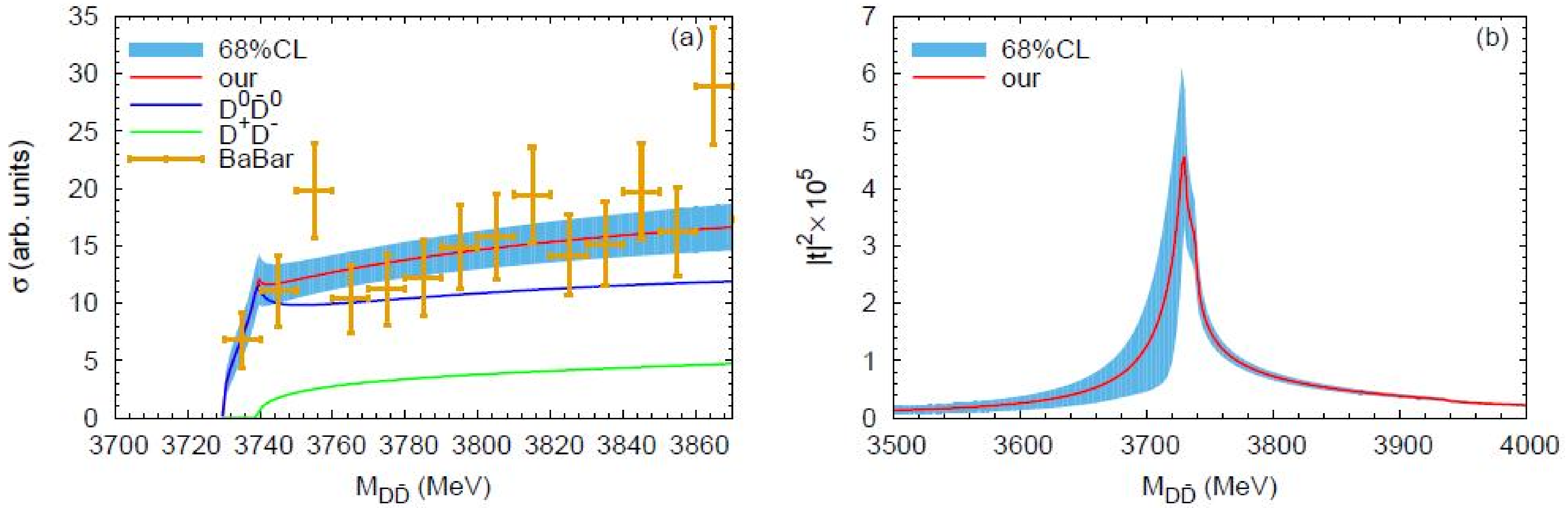
# Results for $\gamma\gamma \to D\bar{D}$

## □Fit to Belle Data



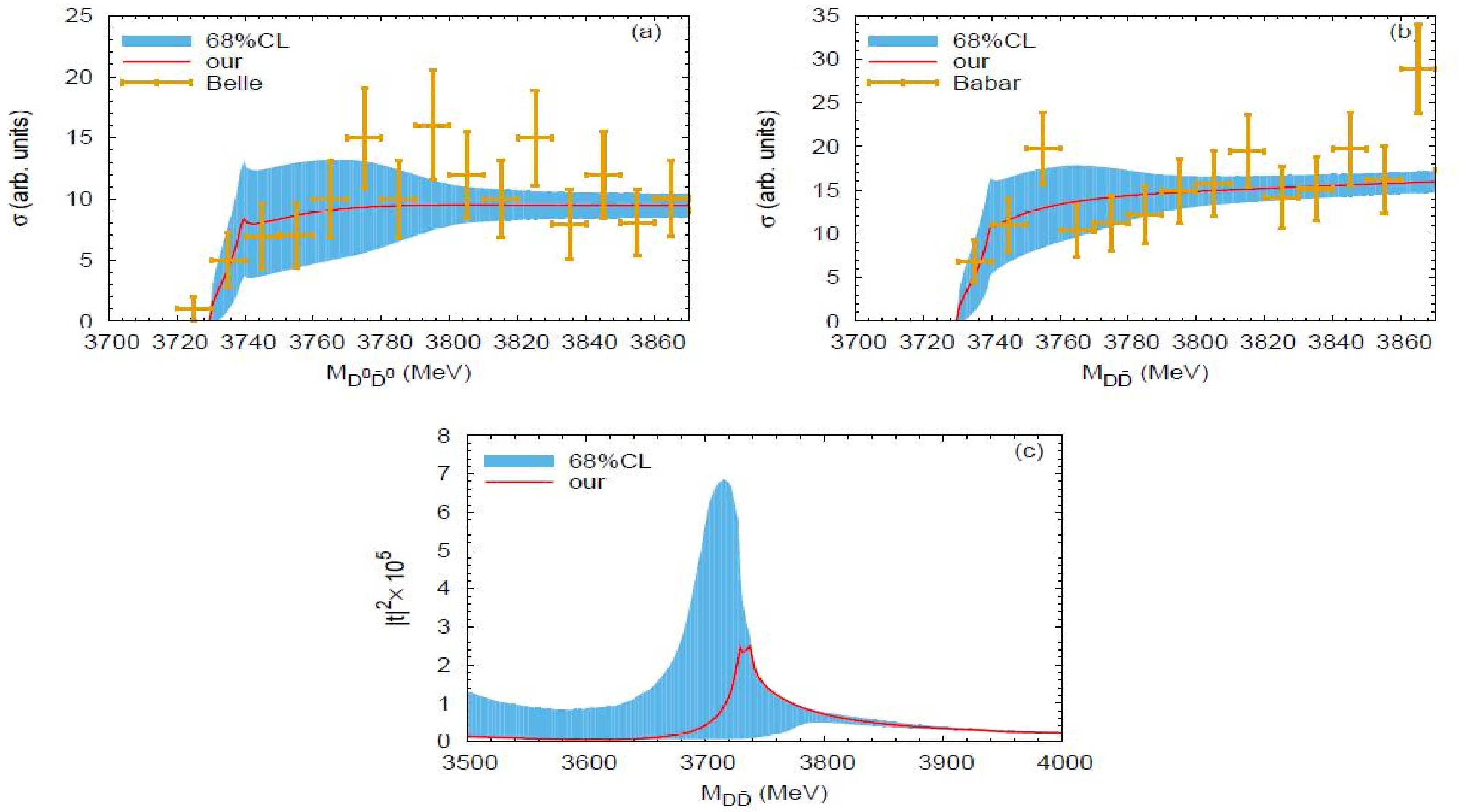
# Results for $\gamma\gamma \to D\bar{D}$

## □Fit to BaBar Data



# Fit C-Both Belle and BABAR

## □Fit to Belle and BaBar data



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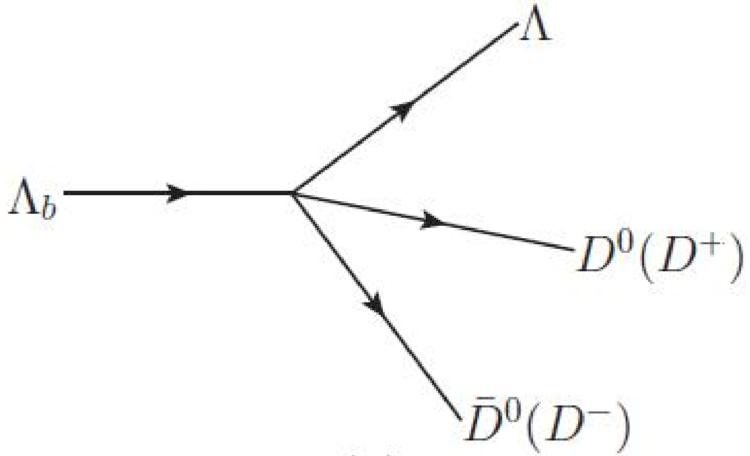
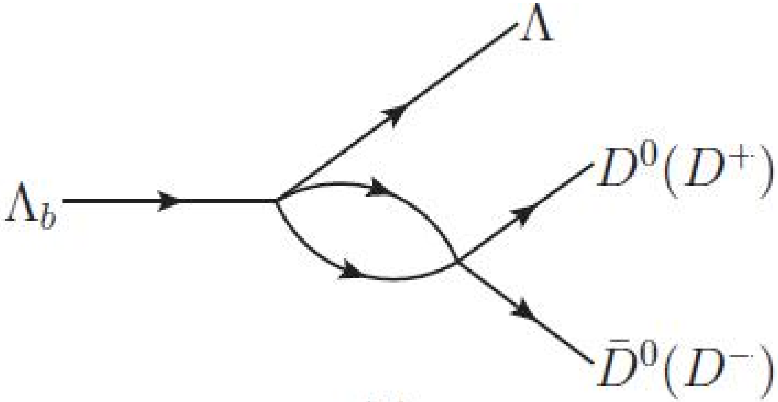
# Mechanism for $\Lambda_b \to D\bar{D}\Lambda$

## □Formalism

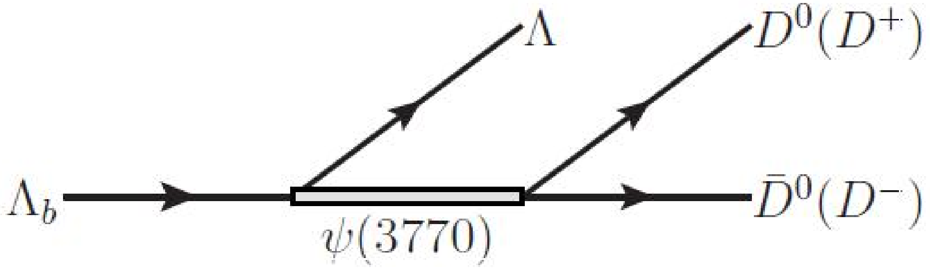



(a)




(b)

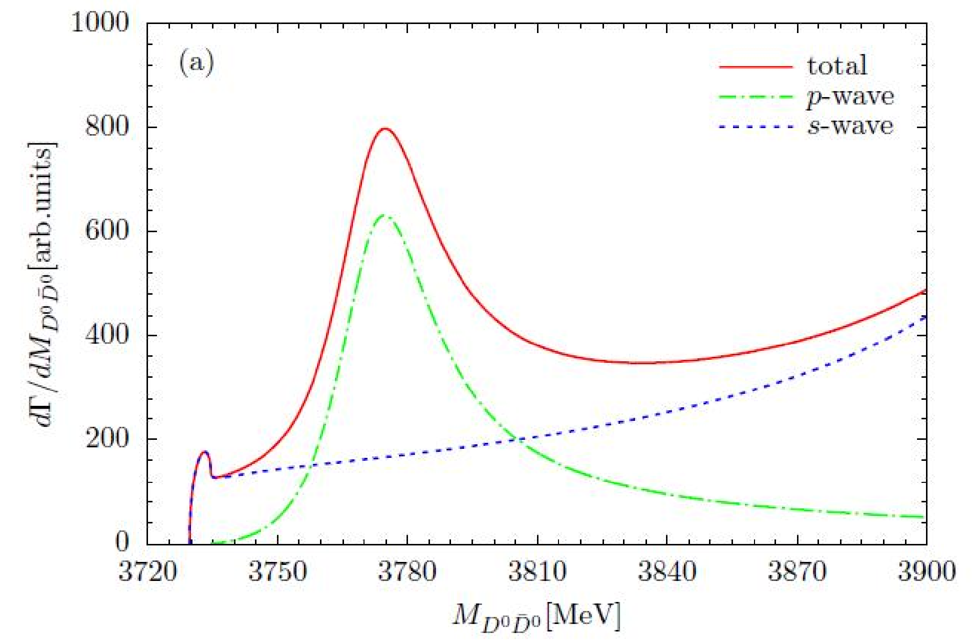
$$
\begin{aligned}
t^{s\text{-wave}}_{\Lambda_b\to\Lambda D^0\bar{D}^0} &= V_p\,[1 + G_{D^+D^-}t_{D^+D^-\to D^0\bar{D}^0} \\
&\quad + G_{D^0\bar{D}^0}t_{D^0\bar{D}^0\to D^0\bar{D}^0} \\
&\quad + (1+C)G_{D_s^+D_s^-}t_{D_s^+D_s^-\to D^0\bar{D}^0}\Big], \\
t^{s\text{-wave}}_{\Lambda_b\to\Lambda D^+D^-} &= V_p\,[1 + G_{D^+D^-}t_{D^+D^-\to D^+D^-} \\
&\quad + G_{D^0\bar{D}^0}t_{D^0\bar{D}^0\to D^+D^-} \\
&\quad + (1+C)G_{D_s^+D_s^-}t_{D_s^+D_s^-\to D^+D^-}\Big],
\end{aligned}
$$

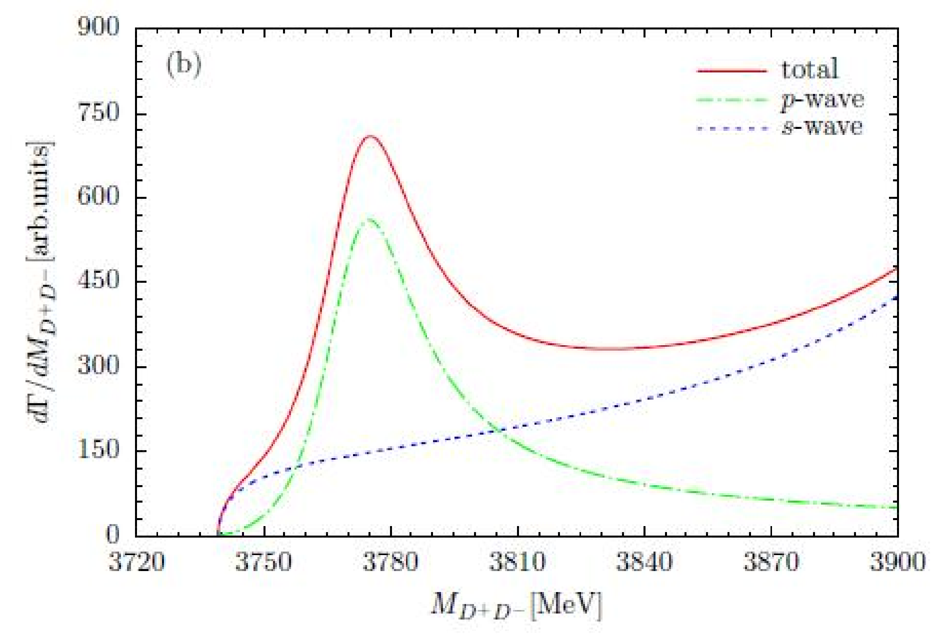



$$
t^{p\text{-wave}} = \frac{\beta V_p \times M_{\psi(3770)}\tilde{p}_D}{M^2_{D\bar{D}} - M^2_{\psi(3770)} + iM_{\psi(3770)}\tilde{\Gamma}_{\psi(3770)}},
$$

# Results for $\Lambda_b \to D\bar{D}\Lambda$

- $M(D^+D^-) = 3739$ **MeV,** $M(D^0\bar{D}^0) = 3730$ **MeV**



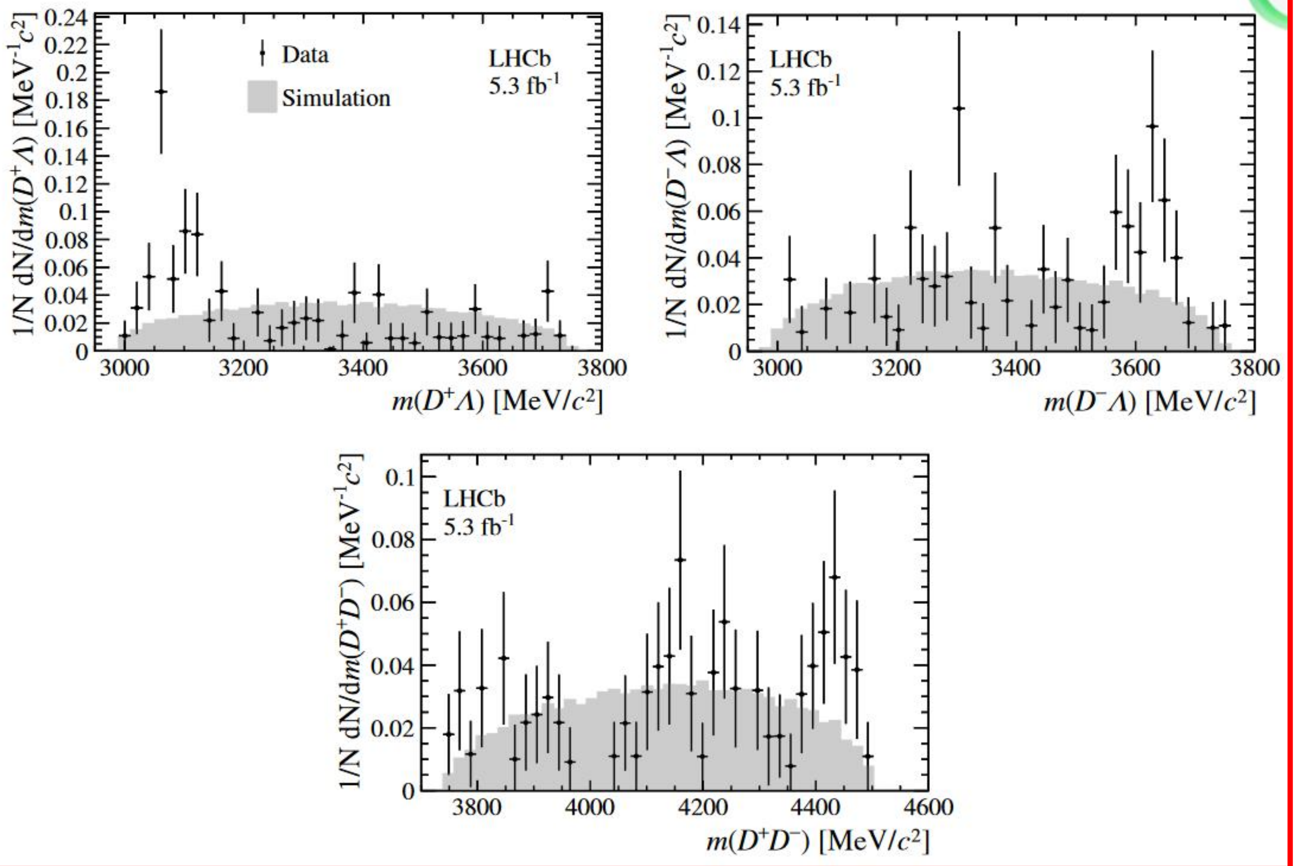




**The enhancement near the $D^0\bar{D}^0$ threshold is more significant.**

# LHCb data of $\Lambda_b \to D\bar{D}\Lambda$

## □LHCb measurements

$$\mathcal{B}(\Lambda_b^0 \to D^+ D^- \Lambda) = (1.24 \pm 0.15 \pm 0.10 \pm 0.28 \pm 0.11) \times 10^{-4}$$




LHCb: JHEP07(2024)140

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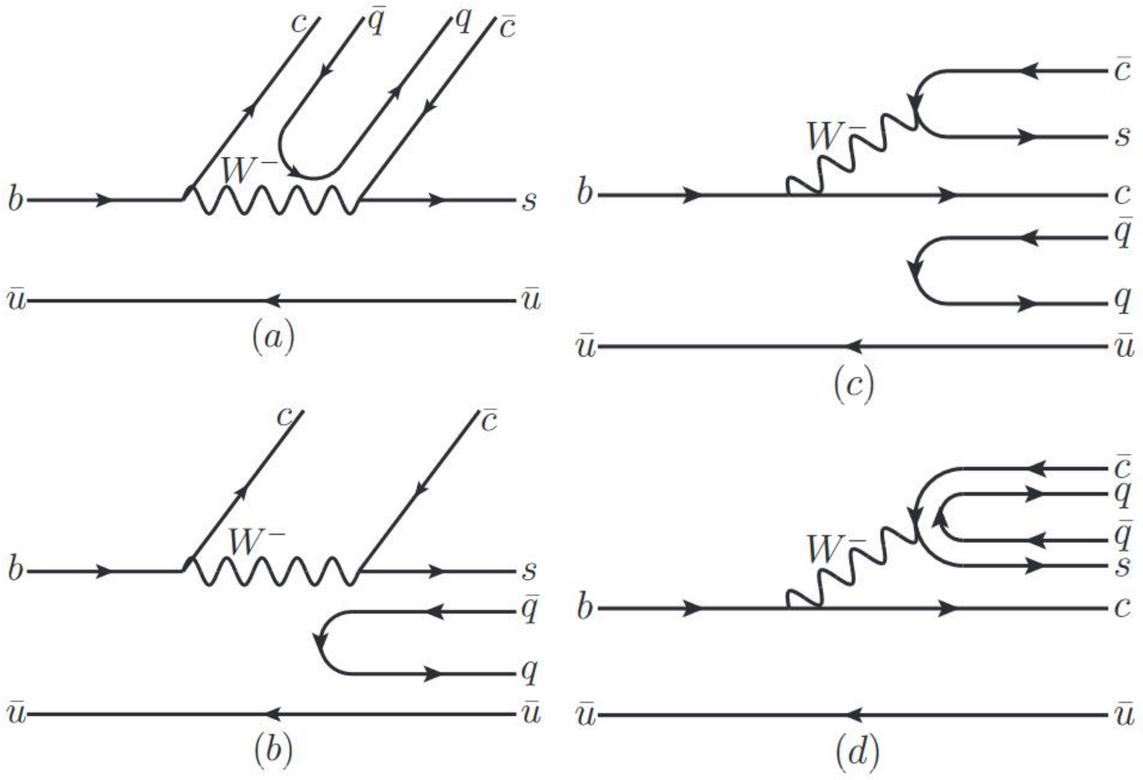
# Mechanism for $B^- \to K^-\eta\eta_c$

## □Formalism

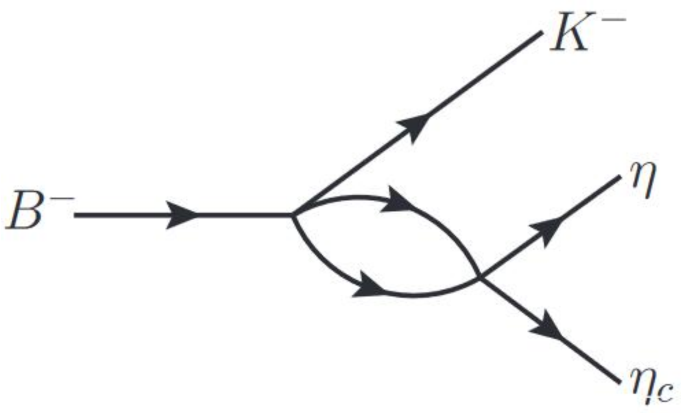



$$
\begin{aligned}
|H\rangle^{\mathrm{a}} &= V_p V_{cb} V_{cs}^* c(\bar{u}u + \bar{d}d + \bar{s}s)\bar{c}s\bar{u} \\
&= V_p V_{cb} V_{cs}^* \left(M^2\right)_{44} K^- \\
&= V_p V_{cb} V_{cs}^* \left(D^0\bar{D}^0 + D^+D^- + D_s^+D_s^-\right) K^-, \\
|H\rangle^{\mathrm{b}} &= V_p V_{cb} V_{cs} c\bar{c}s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{u} \\
&= V_p V_{cb} V_{cs}^* \left(M^2\right)_{31} \eta_c \\
&= V_p V_{cb} V_{cs}^* \left(\frac{1}{\sqrt{2}}K^-\pi^0 + \frac{3}{\sqrt{6}}K^-\eta'\right)\eta_c,
\end{aligned}
$$

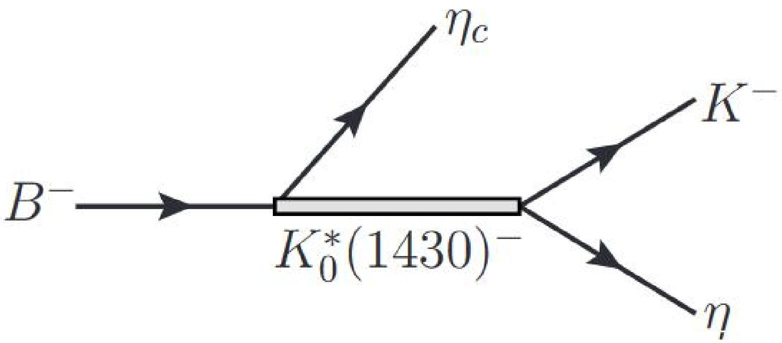
$$
\begin{aligned}
|H\rangle^{\mathrm{c}} &= V_p V_{cb} V_{cs}^* \times C \times \left(K^- D_s^+\right) D_s^-, \\
|H\rangle^{\mathrm{d}} &= V_p V_{cb} V_{cs}^* \times C \times \left(K^- \bar{D}^0\right) D^0,
\end{aligned}
$$

# Mechanism for $B^- \to K^-\eta\eta_c$

- **Final state interactions**




$$\mathcal{T}_X = V_p V_{cb} V_{cs}^* \Big[ G_{D^+D^-}\, t_{D^+D^-\to\eta\eta_c} + (1+C)\times G_{D^0\bar{D}^0}\, t_{D^0\bar{D}^0\to\eta\eta_c} + (1+C)\times G_{D_s^+D_s^-}\, t_{D_s^+D_s^-\to\eta\eta_c} + \frac{3}{\sqrt{6}}\times G_{\eta'\eta_c}\, t_{\eta'\eta_c\to\eta\eta_c} \Big],$$

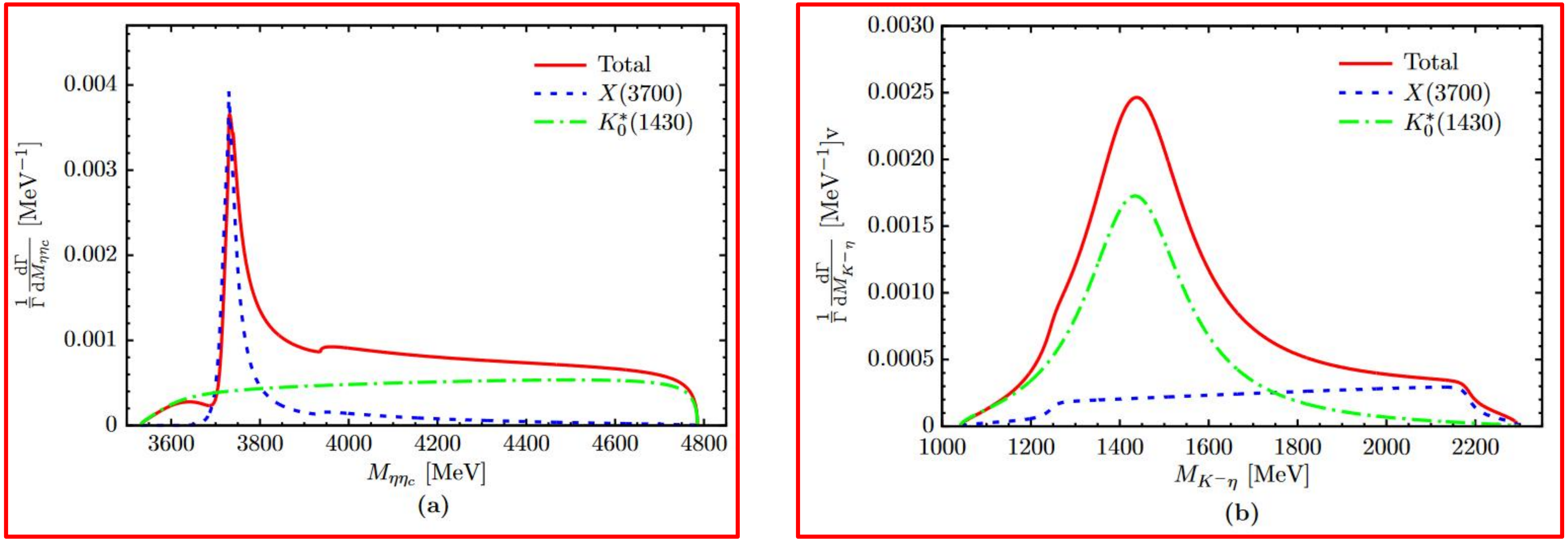



$$\mathcal{T}_{K_0^*} = \frac{V_p \times \beta \times M^2_{K_0^*(1430)}}{M^2_{K^-\eta} - M^2_{K_0^*(1430)} + i M_{K_0^*(1430)} \Gamma_{K_0^*(1430)}};$$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}M_{\eta\eta_c}\mathrm{d}M_{K^-\eta}} = \frac{1}{(2\pi)^3}\frac{M_{\eta\eta_c}M_{K^-\eta}}{8M^3_{B^-}}\left|\mathcal{T}_X + \mathcal{T}_{K_0^*}\right|^2.$$

# Results for $B^- \to K^-\eta\eta_c$

## □Invariant mass distribution

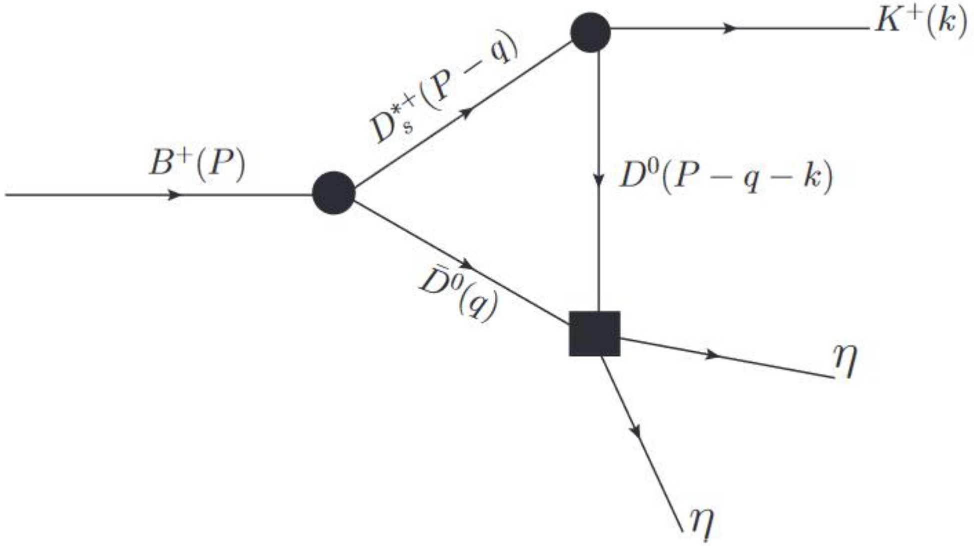


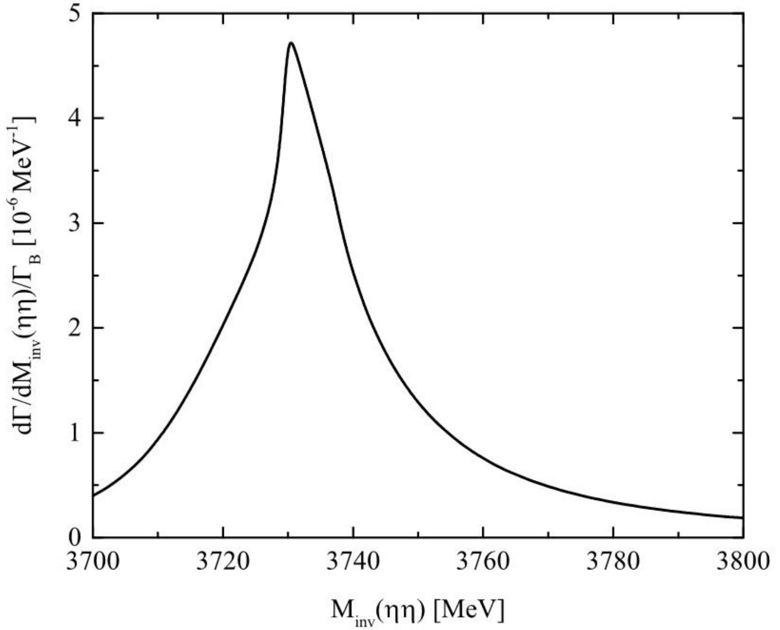

| | | | |
|---|---|---|---|
| $B^+ \to (X_{q\bar{q}} \to \eta_c\eta)K^+$ | $(0.9 \sim 6.7) \times 10^{-4}$ | | $\mathrm{Br}(B^+ \to \eta_c\eta K^+) < 2.2 \times 10^{-4}$ |

- **JMXie-Mzliu-LSGeng, Phys.Rev.D 107 (2023) 1, 016003**
- **Belle: JHEP06(2015)132, 772 × $10^6$ $B\bar{B}$ pairs at $\Upsilon(4S)$**
- **Belle II: $50ab^{-1}$**

# Study of $B^+ \to K^+\eta\eta$

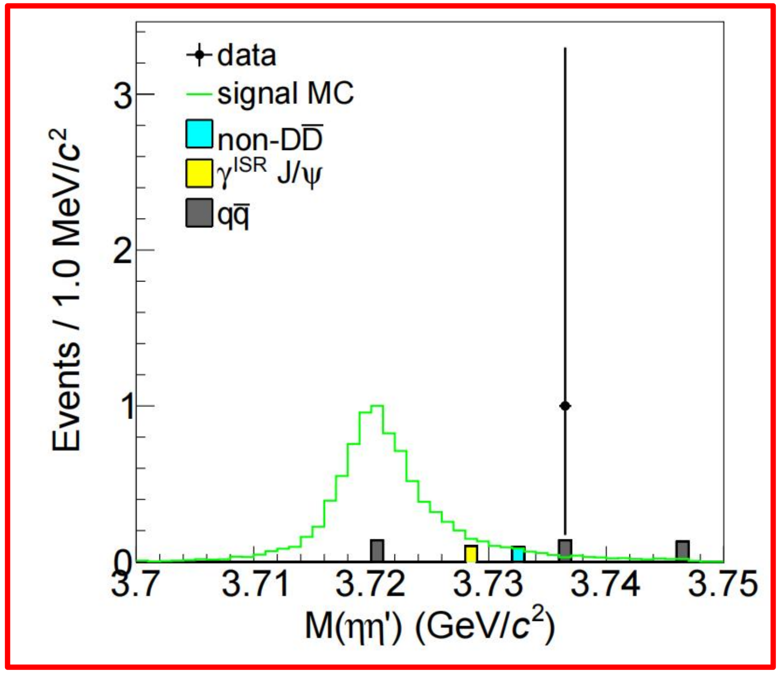
- **2307.05351,** Brandao, J. Song, Abreu, Oset

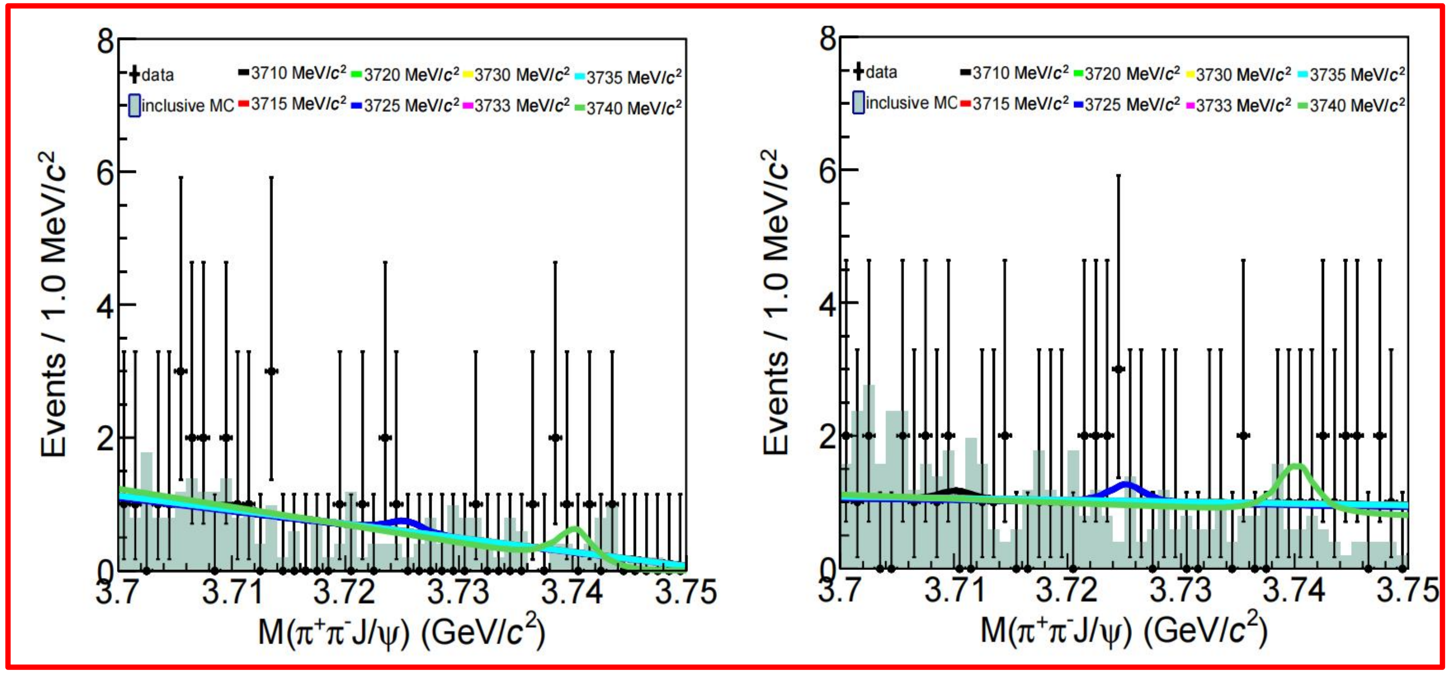



$$Br[B^+ \to K^+ D\bar{D}|_b;\ D\bar{D}|_b \to \eta\eta] = 1.47 \times 10^{-4}$$

# BESIII Data for $\psi(3770) \to \gamma X(3700)$




| $M(X(3700))$ (MeV/$c^2$) | | 3710 | 3715 | 3720 | 3725 | 3730 | 3733 | 3735 | 3740 |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(3770) \to \gamma\eta\eta'$ | $\varepsilon_0$ (%) | 10.35 | 10.32 | 9.94 | 9.58 | 9.11 | 8.74 | 8.49 | 7.93 |
| | $\mathcal{B}^{up}(\times 10^{-6})$ | 7.9 | 8.0 | 8.2 | 8.6 | 9.1 | 16 | 17 | 18 |
| $\psi(3770) \to \gamma\pi^+\pi^- J/\psi$ | $\varepsilon_0^{ee}$ (%) | 15.68 | 15.59 | 15.88 | 15.50 | 15.38 | 15.32 | 14.83 | 13.76 |
| | $\varepsilon_0^{\mu\mu}$ (%) | 24.10 | 23.97 | 24.02 | 23.93 | 23.67 | 23.54 | 23.16 | 21.80 |
| | $\mathcal{B}^{up}(\times 10^{-5})$ | 2.2 | 1.2 | 1.8 | 3.0 | 0.86 | 1.0 | 1.3 | 3.4 |

The photon energies are about 30~40 MeV for M(eta eta')=3720 MeV。

BESIII: PRD108 (2023) 052012

# Summary

- The $D\bar{D}$ bound state was predicted in many theoretical works.
- The present experimental data of $e^+e^- \to J/\psi D\bar{D}$ and $\gamma\gamma \to D\bar{D}$ support the existentce of the $\mathrm{D\bar{D}}$ state.
- Some processes are proposed to search for $\mathrm{D\bar{D}}$ state
  - $\Lambda_b \to D\bar{D}\Lambda$
  - $B^- \to K^-\eta\eta_c$
  - $B^+ \to K^+\eta\eta$

## Thanks for your attention!