

应用QCD求和规则研究五夸克态

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1 引言

In 2015, the LHCb collaboration observed the $P_c(4380)$ and $P_c(4450)$ in the $J/\psi p$ mass spectrum in the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays. The preferred spin-parity assignments are $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^+$, respectively.

In 2019, the LHCb collaboration studied the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays, and observed the $P_c(4312)$ in the $J/\psi p$ mass spectrum. And confirmed the $P_c(4450)$, which consists of two narrow peaks $P_c(4440)$ and $P_c(4457)$.

In 2021, the LHCb collaboration reported an evidence for the $P_{cs}(4459)$ with the strangeness $S = -1$ in the $J/\psi\Lambda$ mass spectrum in the $\Xi_b^- \rightarrow J/\psi K^- \Lambda$ decays.

In 2022, the LHCb collaboration observed an evidence for the $P_c(4337)$ in the $J/\psi p$ and $J/\psi \bar{p}$ systems in the $B_s^0 \rightarrow J/\psi p \bar{p}$ decays.

In 2023, the LHCb collaboration observed an evidence for the $P_{cs}(4338)$ in the $J/\psi\Lambda$ mass spectrum in the $B^- \rightarrow J/\psi \Lambda \bar{p}$ decays. The favored spin-parity is $J^P = \frac{1}{2}^-$.

2 重子QCD求和规则一般计算步骤

首先构造重子流 (for details: arXiv:2502.11351)

质子流是典型的重子流，五夸克态具有分数自旋，可以归结为重子。

质子流：最简单的流

$$J(x) = \varepsilon^{ijk} u_i^T(x) C \gamma_\alpha u_j(x) \gamma^\alpha \gamma_5 d_k(x). \quad (1)$$

The $J(x)$ has the spin-parity $J^P = \frac{1}{2}^+$, then the $i\gamma_5 J(x)$ would have the spin-parity $J^P = \frac{1}{2}^-$, as multiplying $i\gamma_5$ changes the parity of the $J(x)$.

其次，写出关联函数，完成算符乘积展开。

$$\Pi_{\pm}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\pm}(x) \bar{J}_{\pm}(0) \} | 0 \rangle, \quad (2)$$

where we add the subscripts \pm to denote the positive and negative parity, respectively, $J_- = i\gamma_5 J_+$.

We decompose the correlation functions $\Pi_{\pm}(p)$,

$$\Pi_{\pm}(p) = \not{p} \Pi_1(p^2) \pm \Pi_0(p^2), \quad (3)$$

according to Lorentz covariance, because

$$\Pi_-(p) = -\gamma_5 \Pi_+(p) \gamma_5. \quad (4)$$

The currents J_+ couple to both the positive- and negative-parity baryons,

$$\langle 0 | J_+ | B^\pm \rangle \langle B^\pm | \bar{J}_+ | 0 \rangle = -\gamma_5 \langle 0 | J_- | B^\pm \rangle \langle B^\pm | \bar{J}_- | 0 \rangle \gamma_5 ,$$

where the B^\pm denote the positive and negative parity baryons, respectively.

$$\begin{aligned} \langle 0 | J_\pm(0) | B^\pm(p) \rangle &= \lambda_\pm U^\pm(p, s) , \\ \langle 0 | J_\pm(0) | B^\mp(p) \rangle &= \lambda_\mp i\gamma_5 U^\mp(p, s) . \end{aligned} \quad (5)$$

正负宇称的重子，可能互相污染。

为了区分正负宇称重子的贡献，这三篇原始文献，提出了半解析方法。

Y. Chung, H. G. Dosch, M. Kremer and D. Schall, *Baryon Sum Rules and Chiral Symmetry Breaking*, Nucl. Phys. **B197** (1982) 55.

E. Bagan, M. Chabab, H. G. Dosch and S. Narison, *Baryon sum rules in the heavy quark effective theory*, Phys. Lett. **B301**, 243 (1993).

D. Jido, N. Kodama and M. Oka, *Negative parity nucleon resonance in the QCD sum rule*, Phys. Rev. **D54** (1996) 4532.

Then

$$\Pi_+(p) = \lambda_+^2 \frac{p \not{+} M_+}{M_+^2 - p^2} + \lambda_-^2 \frac{p \not{-} M_-}{M_-^2 - p^2} + \dots \quad (6)$$

If we take $\vec{p} = 0$, then

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{\text{Im} \Pi_+(p_0 + i\epsilon)}{\pi} &= \lambda_+^2 \frac{\gamma_0 + 1}{2} \delta(p_0 - M_+) + \\ \lambda_-^2 \frac{\gamma_0 - 1}{2} \delta(p_0 - M_-) + \dots &= \gamma_0 A(p_0) + B(p_0) + \dots, \end{aligned}$$

where

$$\begin{aligned} A(p_0) &= \frac{1}{2} \left[\lambda_+^2 \delta(p_0 - M_+) + \lambda_-^2 \delta(p_0 - M_-) \right], \\ B(p_0) &= \frac{1}{2} \left[\lambda_+^2 \delta(p_0 - M_+) - \lambda_-^2 \delta(p_0 - M_-) \right]. \quad (7) \end{aligned}$$

$A(p_0) + B(p_0)$ ($A(p_0) - B(p_0)$) contains contributions from the positive parity (negative parity) states only.

$$\int_{\Delta}^{\sqrt{s_0}} dp_0 [A(p_0) \pm B(p_0)] \exp \left[-\frac{p_0^2}{T^2} \right] = \int_{\Delta}^{\sqrt{s_0}} dp_0 [\rho_{QCD}^A(p_0) \pm \rho_{QCD}^B(p_0)] \exp \left[-\frac{p_0^2}{T^2} \right],$$

$$\int_{\Delta}^{\sqrt{s_0}} dp_0 [A(p_0) \pm B(p_0)] p_0^2 \exp \left[-\frac{p_0^2}{T^2} \right] = \int_{\Delta}^{\sqrt{s_0}} dp_0 [\rho_{QCD}^A(p_0) \pm \rho_{QCD}^B(p_0)] p_0^2 \exp \left[-\frac{p_0^2}{T^2} \right].$$

我们改造上述方法，拓展成解析方法，[Eur. Phys. J. C76 (2016) 70]，适用于五夸克态与传统重子。

Setting $\Pi(p^2) = \Pi_-(p)$, we obtain the spectral densities through the dispersion relation,

$$\begin{aligned} \frac{\text{Im}\Pi(s)}{\pi} &= \not{p} \left[\lambda_-^2 \delta(s - M_-^2) + \lambda_+^2 \delta(s - M_+^2) \right] \\ &\quad + \left[M_- \lambda_-^2 \delta(s - M_-^2) - M_+ \lambda_+^2 \delta(s - M_+^2) \right], \\ &= \not{p} \rho_H^1(s) + \rho_H^0(s), \end{aligned} \tag{8}$$

where the subscript H denotes the hadron side.

Then we introduce the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules at the hadron side,

$$2M_{\mp}\lambda_{\mp}^2 \exp\left(-\frac{M_{\mp}^2}{T^2}\right) = \int_{\Delta^2}^{s_0} ds \left[\sqrt{s}\rho_H^1(s) \pm \rho_H^0(s) \right] \exp\left(-\frac{s}{T^2}\right) .$$

$$M_{\mp}^2 = \frac{-\int_{4m_c^2}^{s_0} ds \frac{d}{d(1/T^2)} \left[\sqrt{s}\rho_{QCD}^1(s) \pm \rho_{QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds \left[\sqrt{s}\rho_{QCD}^1(s) \pm \rho_{QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)} . \quad (9)$$

对于隐粲(或隐美或双重)五夸克流 $J(x)$,

$$\Pi(p^2) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J(x) \bar{J}(0) \} | 0 \rangle, \quad (10)$$

做维克收缩, 得到五个完全传播子, 两个重夸克传播子, 三个轻夸克传播子。如果每个重夸克传播子贡献一个胶子, 每个轻夸克传播子贡献一个夸克对, 则得到一个维度为13的算符, 所以算符乘积展开应该到维度为13的真空凝聚。但实际计算, 一般展开到维度为8或者10。

算符乘积展开如果达不到指定的维度, 影响计算的准确度。对于高维真空凝聚, 采取因子化的假设, 因子化为低维真空凝聚。

3 QCD求和规则中参数的选取

The correlation functions $\Pi(p^2)$ do not depend on the energy scale μ , that is

$$\frac{d}{d\mu}\Pi(p^2) = 0, \quad (11)$$

至少对裸关联函数如此，但不能保证基态贡献不依赖能标， $\rho_{QCD}(s, \mu) = \frac{\text{Im}\Pi(s)}{\pi}$,

$$\frac{d}{d\mu} \int_{4m_Q^2(\mu)}^{s_0} ds \frac{\rho_{QCD}(s, \mu)}{s - p^2} \rightarrow 0, \quad (12)$$

due to the following two reasons inherited from the QCD sum rules:

- 微扰修正项被略去，高维真空凝聚因子化为低维真空凝聚，高维真空凝聚的能标依赖性被修正了；
- 引入截断 s_0 ，阈值 $4m_Q^2(\mu)$ 和连续态阈值 s_0 之间的关联是未知的，强子-夸克对偶只是一个假设。

我们得不到不依赖于能标的QCD求和规则，但我们提出一个能标公式，可以协调地把QCD谱密度的能标定下来。

We perform the Borel transformation with respect to the variable $P^2 = -p^2$ and obtain

$$\int_{4m_Q^2(\mu)}^{s_0} ds \frac{\rho_{QCD}(s, \mu)}{s - p^2} \rightarrow \int_{4m_Q^2(\mu)}^{s_0} ds \frac{\rho_{QCD}(s, \mu)}{T^2} \exp\left(-\frac{s}{T^2}\right). \quad (13)$$

The integrals are sensitive to the heavy quark masses m_Q .

重夸克质量的变化，或者说能标的变化，可以引起积分区间 $\underbrace{4m_Q^2(\mu) - s_0}$ 和QCD谱密度 $\underbrace{\rho_{QCD}(s, \mu)}$ 的变化，这也就引起布莱尔窗口的变化，并由此产生强子质量和极点留数的变化。具体的计算表明：微小的重夸克质量 m_Q 变化，可以起比较大强子质量变化。

从上面的分析，我们可以得出结论：能标的选取很重要，对结果影响很大。

对于隐粲(隐美或双重)四(五)夸克态的QCD求和规则，我们区分轻重自由度，提出能标公式

$$\mu = \sqrt{M_{X/Y/Z/P}^2 - (2M_Q)^2 - \kappa M_s}, \quad (14)$$

the κ is the number of the s -quark.

我们首次研究了四夸克态 $q\bar{q}Q\bar{Q}$ 的QCD求和规则的能标依赖性，发现能标公式适用于所有四夸克系统 $q\bar{q}Q\bar{Q}$ 与五夸克系统 $qqqQ\bar{Q}$ 。

我们把所有夸克质量和真空凝聚演化到这个特定的能标 μ ，然后提取强子质量 $M_{X/Y/Z/P}$ 和极点留数。或者说 μ 和 $M_{X/Y/Z/P}$ 满足一个特定的关系，参数 M_Q 是一定的，对所有过程适用。能标公式既能显著提高极点项贡献，又能显著改善算符乘积展开收敛性，并首次使隐粲五夸克态的极点贡献达到 $(40 - 60)\%$ 。

The vacuum condensates are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2\langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$.

并考虑随能标的演化：

$$\begin{aligned} \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(1\text{GeV}) \left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\ \langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(1\text{GeV}) \left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\ \langle \bar{q}g_s\sigma Gq \rangle(\mu) &= \langle \bar{q}g_s\sigma Gq \rangle(1\text{GeV}) \left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\ \langle \bar{s}g_s\sigma Gs \rangle(\mu) &= \langle \bar{s}g_s\sigma Gs \rangle(1\text{GeV}) \left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \end{aligned} \quad (15)$$

We take the \overline{MS} masses $\underbrace{m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}}$, $\underbrace{m_b(m_b) = (4.18 \pm 0.03) \text{ GeV}}$ and $\underbrace{m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV}}$ from the Particle Data Group, and take into account the energy-scale dependence of the \overline{MS} masses from the renormalization group equation,

$$\begin{aligned}
 m_Q(\mu) &= m_Q(m_Q) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right]^{\frac{12}{33-2n_f}}, \\
 m_s(\mu) &= m_s(2\text{GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33-2n_f}}, \\
 \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right]. \quad (16)
 \end{aligned}$$

4 QCD求和规则对双夸克-双夸克-反夸克型五夸克态质量谱的计算

首先给出夸克结构、 J^P 、布莱尔参数、QCD谱密度能标(满足能标公式)、阈值参数、极点贡献、最高维凝聚贡献

其次给出质量的理论值以及对现有P粒子的可能确认。**还有实验检验。**

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$[qq'][q''c]\bar{c} (S_L, S_H, J_{LH}, J)$	J^P	Currents
$[ud][uc]\bar{c} (0, 0, 0, \frac{1}{2})$	$\frac{1}{2}^-$	$J^1(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{1}{2})$	$\frac{1}{2}^-$	$J^2(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{1}{2})$	$\frac{1}{2}^-$	$J^3(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 0, \frac{1}{2})$	$\frac{1}{2}^-$	$J^4(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{3}{2})$	$\frac{3}{2}^-$	$J_\mu^1(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{3}{2})$	$\frac{3}{2}^-$	$J_\mu^2(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{3}{2})$	$\frac{3}{2}^-$	$J_\mu^3(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{3}{2})$	$\frac{3}{2}^-$	$J_\mu^4(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{5}{2})$	$\frac{5}{2}^-$	$J_{\mu\nu}^1(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{5}{2})$	$\frac{5}{2}^-$	$J_{\mu\nu}^2(x)$

The S_L and S_H denote the spins of the light and heavy diquarks respectively, $\vec{J}_{LH} = \vec{S}_L + \vec{S}_H$, $\vec{J} = \vec{J}_{LH} + \vec{J}_{\bar{c}}$, the $\vec{J}_{\bar{c}}$ is the angular momentum of the \bar{c} -quark.

$$J^1(x) = \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} u_j^T(x) C \gamma_5 d_k(x) u_m^T(x) C \gamma_5 c_n(x) C \bar{c}_a^T(x),$$

$$J^2(x) = \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} u_j^T(x) C \gamma_5 d_k(x) u_m^T(x) C \gamma_\mu c_n(x) \gamma_5 \gamma^\mu C \bar{c}_a^T(x),$$

$$J^3(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} \left[u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_5 c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_5 c_n(x) \right] \gamma_5 \gamma^\mu C \bar{c}_a^T(x),$$

$$J^4(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} \left[u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma^\mu c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma^\mu c_n(x) \right] C \bar{c}_a^T(x),$$

$$J_\mu^1(x) = \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} u_j^T(x) C \gamma_5 d_k(x) u_m^T(x) C \gamma_\mu c_n(x) C \bar{c}_a^T(x),$$

$$J_\mu^2(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} \left[u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_5 c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_5 c_n(x) \right] C \bar{c}_a^T(x),$$

$$J_\mu^3(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} \left[u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_\alpha c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_\alpha c_n(x) \right] \gamma_5 \gamma^\alpha C \bar{c}_a^T(x),$$

$$J_\mu^4(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} \left[u_j^T(x) C \gamma_\alpha u_k(x) d_m^T(x) C \gamma_\mu c_n(x) + 2u_j^T(x) C \gamma_\alpha d_k(x) u_m^T(x) C \gamma_\mu c_n(x) \right] \gamma_5 \gamma^\alpha C \bar{c}_a^T(x),$$

$$J_{\mu\nu}^1(x) = \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{6}} \left[u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_\nu c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_\nu c_n(x) \right] C \bar{c}_a^T(x) + (\mu \leftrightarrow \nu),$$

$$J_{\mu\nu}^2(x) = \frac{1}{\sqrt{2}} \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} u_j^T(x) C \gamma_5 d_k(x) \left[u_m^T(x) C \gamma_\mu c_n(x) \gamma_5 \gamma_\nu C \bar{c}_a^T(x) + u_m^T(x) C \gamma_\nu c_n(x) \gamma_5 \gamma_\mu C \bar{c}_a^T(x) \right],$$

(17)

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	$T^2\text{GeV}^2$	$\sqrt{s_0}(\text{GeV})$	$\mu(\text{GeV})$	pole	D_{13}
$J^1(x)$	3.1 – 3.5	4.96 ± 0.10	2.3	(41 – 62)%	< 1%
$J^2(x)$	3.2 – 3.6	5.10 ± 0.10	2.6	(42 – 63)%	< 1%
$J^3(x)$	3.2 – 3.6	5.11 ± 0.10	2.6	(42 – 63)%	$\ll 1\%$
$J^4(x)$	2.9 – 3.3	5.00 ± 0.10	2.4	(40 – 64)%	$\leq 1\%$
$J_\mu^1(x)$	3.1 – 3.5	5.03 ± 0.10	2.4	(42 – 63)%	$\leq 1\%$
$J_\mu^2(x)$	3.3 – 3.7	5.11 ± 0.10	2.6	(40 – 61)%	$\ll 1\%$
$J_\mu^3(x)$	3.4 – 3.8	5.26 ± 0.10	2.8	(42 – 62)%	$\ll 1\%$
$J_\mu^4(x)$	3.3 – 3.7	5.17 ± 0.10	2.7	(41 – 61)%	< 1%
$J_{\mu\nu}^1(x)$	3.2 – 3.6	5.03 ± 0.10	2.4	(40 – 61)%	$\leq 1\%$
$J_{\mu\nu}^2(x)$	3.1 – 3.5	5.03 ± 0.10	2.4	(42 – 63)%	$\leq 1\%$

成功实现极点项贡献(40 – 60)%。

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$[qq'][q''c]\bar{c} (S_L, S_H, J_{LH}, J)$	$M(\text{GeV})$	$\lambda(10^{-3}\text{GeV}^6)$	Assignments	Currents
$[ud][uc]\bar{c} (0, 0, 0, \frac{1}{2})$	4.31 ± 0.11	1.40 ± 0.23	? $P_c(4312)$	$J^1(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{1}{2})$	4.45 ± 0.11	3.02 ± 0.48	? $P_c(4440/4457)$	$J^2(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{1}{2})$	4.46 ± 0.11	4.32 ± 0.71	? $P_c(4440/4457)$	$J^3(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 0, \frac{1}{2})$	4.34 ± 0.14	3.23 ± 0.61	? $P_c(4312/\mathbf{4337})$	$J^4(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{3}{2})$	4.39 ± 0.11	1.44 ± 0.23	? $P_c(4440)$	$J_\mu^1(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{3}{2})$	4.47 ± 0.11	2.41 ± 0.38	? $P_c(4440/4457)$	$J_\mu^2(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{3}{2})$	4.61 ± 0.11	5.13 ± 0.79		$J_\mu^3(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{5}{2})$	4.52 ± 0.11	4.49 ± 0.72		$J_\mu^4(x)$
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{7}{2})$	4.39 ± 0.11	1.94 ± 0.31	? $P_c(4440)$	$J_{\mu\nu}^1(x)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{5}{2})$	4.39 ± 0.11	1.44 ± 0.23	? $P_c(4440)$	$J_{\mu\nu}^2(x)$

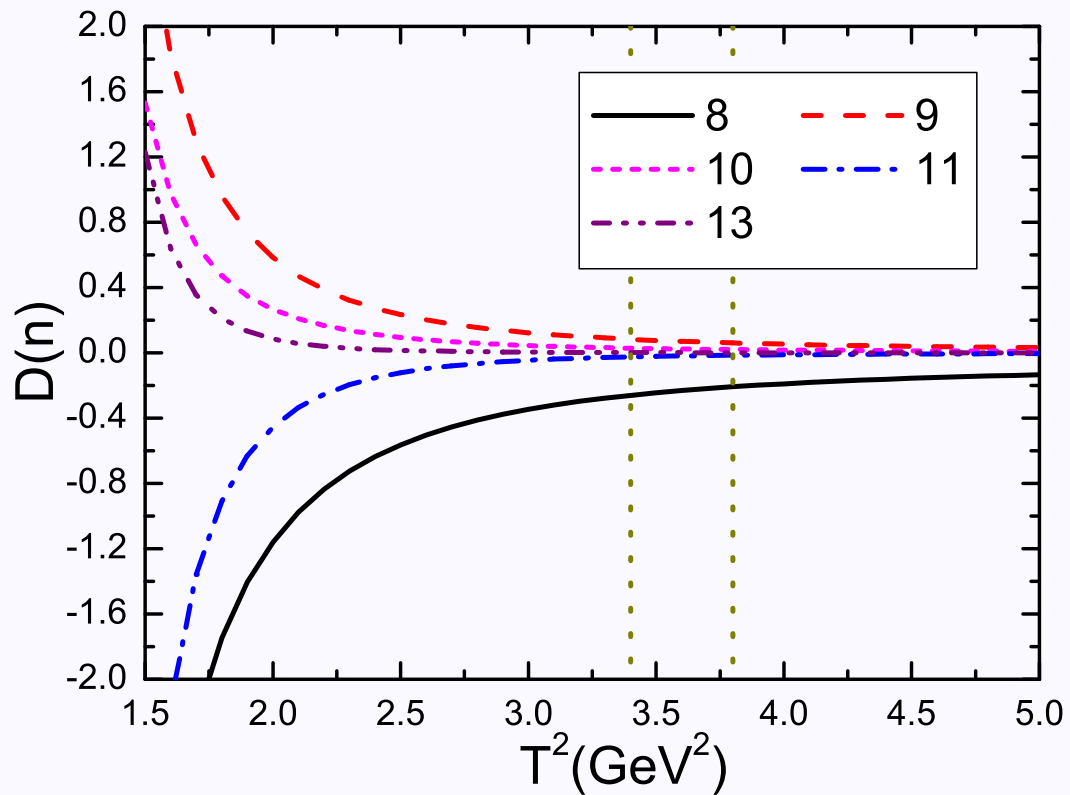
We predict a pentaquark state with the mass $4.34 \pm 0.14 \text{ GeV}$ and $J^P = \frac{1}{2}^-$ before the LHCb's observation of the $P_c(4337)$.

- Int. J. Mod. Phys. A36 (2021) 2150071, based on the $SU(3)$ symmetry.

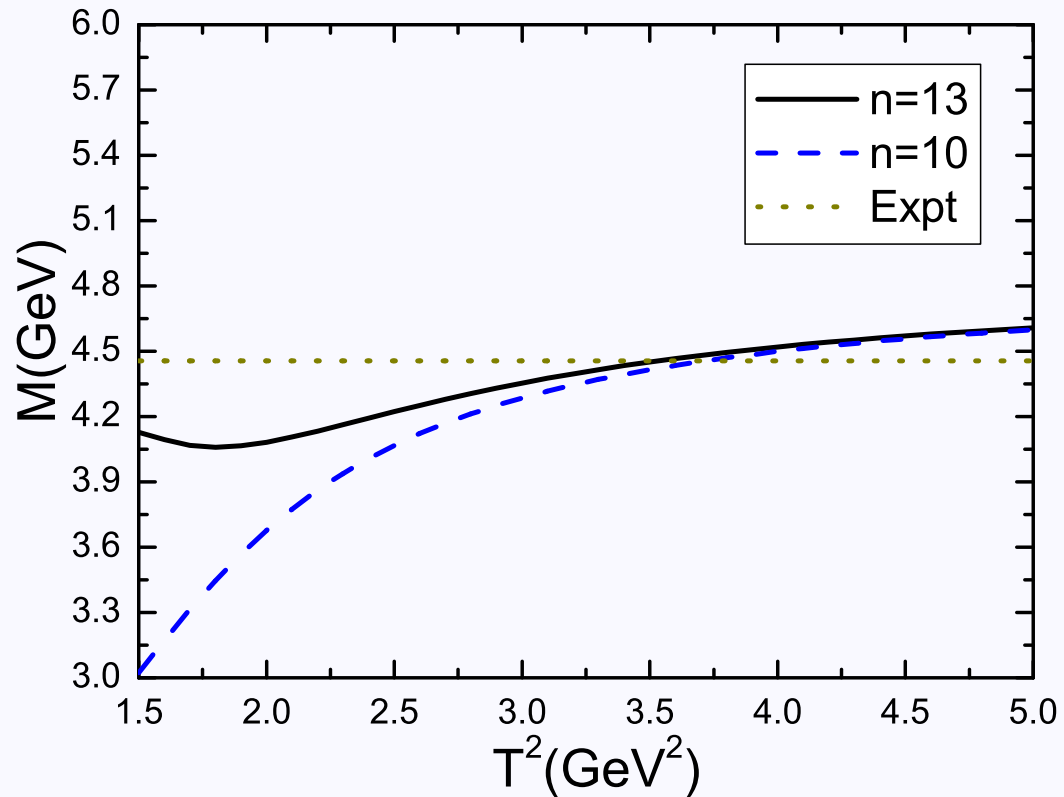
$[qq'][q''c]\bar{c} (S_L, S_H, J_{LH}, J)$	New analysis	u or $d \rightarrow s$	Assignments
$[ud][uc]\bar{c} (0, 0, 0, \frac{1}{2})$	4.31 ± 0.11	4.46 ± 0.11	? $P_{cs}(4459)$
$[ud][uc]\bar{c} (0, 1, 1, \frac{1}{2})$	4.45 ± 0.11	4.60 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{1}{2})$	4.46 ± 0.11	4.61 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 0, \frac{1}{2})$	4.34 ± 0.14	4.49 ± 0.14	
$[ud][uc]\bar{c} (0, 1, 1, \frac{3}{2})$	4.39 ± 0.11	4.54 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{3}{2})$	4.47 ± 0.11	4.62 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{1}{2})$	4.61 ± 0.11	4.76 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{3}{2})$	4.52 ± 0.11	4.67 ± 0.11	
$[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 2, \frac{5}{2})$	4.39 ± 0.11	4.54 ± 0.11	
$[ud][uc]\bar{c} (0, 1, 1, \frac{5}{2})$	4.39 ± 0.11	4.54 ± 0.11	

There is no room for the $P_{cs}(4338)$. For more literatures, see: arXiv:2502.11351.

Higher dimensional vacuum condensates for the $P_{cs}(4459)_0$.



Mass with variation of the Borel parameter T^2 for the $P_{cs}(4459)_o$.



5 QCD求和规则对色单态-色单态型五夸克态质量谱的计算

利用QCD求和规则做计算，用的是定域流。对于色单态-色单态型的五夸克流，有两个色中性的集团，每个集团和一个介子或重子有相同的量子数，虽然这个集团，我们也用介子或重子描述，但并不是真正的物理介子与重子。我们说的分子态，确切地说，应该叫做色单态-色单态型五夸克态。

首次区分同位旋，用标准的介子流与Ioffe流，构造五夸克流

$$\begin{aligned}
 J_{\frac{1}{2}}^{\bar{D}\Sigma_c}(x) &= \frac{1}{\sqrt{3}}J^{\bar{D}^0}(x)J^{\Sigma_c^+}(x) - \sqrt{\frac{2}{3}}J^{\bar{D}^-}(x)J^{\Sigma_c^{++}}(x), \\
 J_{\frac{3}{2}}^{\bar{D}\Sigma_c}(x) &= \sqrt{\frac{2}{3}}J^{\bar{D}^0}(x)J^{\Sigma_c^+}(x) + \frac{1}{\sqrt{3}}J^{\bar{D}^-}(x)J^{\Sigma_c^{++}}(x), \\
 J_{\frac{1}{2};\mu}^{\bar{D}\Sigma_c^*}(x) &= \frac{1}{\sqrt{3}}J^{\bar{D}^0}(x)J_{\mu}^{\Sigma_c^{*+}}(x) - \sqrt{\frac{2}{3}}J^{\bar{D}^-}(x)J_{\mu}^{\Sigma_c^{*++}}(x), \\
 J_{\frac{3}{2};\mu}^{\bar{D}\Sigma_c^*}(x) &= \sqrt{\frac{2}{3}}J^{\bar{D}^0}(x)J_{\mu}^{\Sigma_c^{*+}}(x) + \frac{1}{\sqrt{3}}J^{\bar{D}^-}(x)J_{\mu}^{\Sigma_c^{*++}}(x),
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 J_{\frac{1}{2};\mu}^{\bar{D}^*\Sigma_c}(x) &= \frac{1}{\sqrt{3}}J_{\mu}^{\bar{D}^{*0}}(x)J^{\Sigma_c^+}(x) - \sqrt{\frac{2}{3}}J_{\mu}^{\bar{D}^{*-}}(x)J^{\Sigma_c^{++}}(x), \\
 J_{\frac{3}{2};\mu}^{\bar{D}^*\Sigma_c}(x) &= \sqrt{\frac{2}{3}}J_{\mu}^{\bar{D}^{*0}}(x)J^{\Sigma_c^+}(x) + \frac{1}{\sqrt{3}}J_{\mu}^{\bar{D}^{*-}}(x)J^{\Sigma_c^{++}}(x), \\
 J_{\frac{1}{2};\mu\nu}^{\bar{D}^*\Sigma_c^*}(x) &= \frac{1}{\sqrt{3}}J_{\mu}^{\bar{D}^{*0}}(x)J_{\nu}^{\Sigma_c^{*+}}(x) - \sqrt{\frac{2}{3}}J_{\mu}^{\bar{D}^{*-}}(x)J_{\nu}^{\Sigma_c^{*++}}(x) + (\mu \leftrightarrow \nu), \\
 J_{\frac{3}{2};\mu\nu}^{\bar{D}^*\Sigma_c^*}(x) &= \sqrt{\frac{2}{3}}J_{\mu}^{\bar{D}^{*0}}(x)J_{\nu}^{\Sigma_c^{*+}}(x) + \frac{1}{\sqrt{3}}J_{\mu}^{\bar{D}^{*-}}(x)J_{\nu}^{\Sigma_c^{*++}}(x) + (\mu \leftrightarrow \nu),
 \end{aligned} \tag{19}$$

$$\begin{aligned}
J_0^{\bar{D}\Xi'_c}(x) &= \frac{1}{\sqrt{2}} J^{\bar{D}^0}(x) J^{\Xi'_c{}^0}(x) - \frac{1}{\sqrt{2}} J^{\bar{D}^-}(x) J^{\Xi'_c{}^+}(x), \\
J_1^{\bar{D}\Xi'_c}(x) &= \frac{1}{\sqrt{2}} J^{\bar{D}^0}(x) J^{\Xi'_c{}^0}(x) + \frac{1}{\sqrt{2}} J^{\bar{D}^-}(x) J^{\Xi'_c{}^+}(x), \\
J_{0;\mu}^{\bar{D}\Xi_c^*}(x) &= \frac{1}{\sqrt{2}} J^{\bar{D}^0}(x) J_{\mu}^{\Xi_c^*{}^0}(x) - \frac{1}{\sqrt{2}} J^{\bar{D}^-}(x) J_{\mu}^{\Xi_c^*{}^+}(x), \\
J_{1;\mu}^{\bar{D}\Xi_c^*}(x) &= \frac{1}{\sqrt{2}} J^{\bar{D}^0}(x) J_{\mu}^{\Xi_c^*{}^0}(x) + \frac{1}{\sqrt{2}} J^{\bar{D}^-}(x) J_{\mu}^{\Xi_c^*{}^+}(x),
\end{aligned} \tag{20}$$

$$\begin{aligned}
J_{0;\mu}^{\bar{D}^*\Xi'_c}(x) &= \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^0}(x) J^{\Xi'_c{}^0}(x) - \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^-}(x) J^{\Xi'_c{}^+}(x), \\
J_{1;\mu}^{\bar{D}^*\Xi'_c}(x) &= \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^0}(x) J^{\Xi'_c{}^0}(x) + \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^-}(x) J^{\Xi'_c{}^+}(x), \\
J_{0;\mu\nu}^{\bar{D}^*\Xi_c^*}(x) &= \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^0}(x) J_{\nu}^{\Xi_c^*{}^0}(x) - \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^-}(x) J_{\nu}^{\Xi_c^*{}^+}(x) + (\mu \leftrightarrow \nu), \\
J_{1;\mu\nu}^{\bar{D}^*\Xi_c^*}(x) &= \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^0}(x) J_{\nu}^{\Xi_c^*{}^0}(x) + \frac{1}{\sqrt{2}} J_{\mu}^{\bar{D}^*{}^-}(x) J_{\nu}^{\Xi_c^*{}^+}(x) + (\mu \leftrightarrow \nu),
\end{aligned} \tag{21}$$

$$\begin{aligned}
J_0^{\bar{D}\Xi_c}(x) &= \frac{1}{\sqrt{2}}J^{\bar{D}^0}(x)J^{\Xi_c^0}(x) - \frac{1}{\sqrt{2}}J^{\bar{D}^-}(x)J^{\Xi_c^+}(x), \\
J_1^{\bar{D}\Xi_c}(x) &= \frac{1}{\sqrt{2}}J^{\bar{D}^0}(x)J^{\Xi_c^0}(x) + \frac{1}{\sqrt{2}}J^{\bar{D}^-}(x)J^{\Xi_c^+}(x), \\
J_{\frac{1}{2}}^{\bar{D}\Lambda_c}(x) &= J^{\bar{D}^0}(x)J^{\Lambda_c^+}(x), \\
J_{\frac{1}{2}}^{\bar{D}_s\Xi_c}(x) &= J^{\bar{D}_s^-}(x)J^{\Xi_c^+}(x), \\
J_0^{\bar{D}_s\Lambda_c}(x) &= J^{\bar{D}_s^-}(x)J^{\Lambda_c^+}(x),
\end{aligned} \tag{22}$$

$$\begin{aligned}
J_{0;\mu}^{\bar{D}^*\Xi_c}(x) &= \frac{1}{\sqrt{2}}J_\mu^{\bar{D}^{*0}}(x)J^{\Xi_c^0}(x) - \frac{1}{\sqrt{2}}J_\mu^{\bar{D}^{*-}}(x)J^{\Xi_c^+}(x), \\
J_{1;\mu}^{\bar{D}^*\Xi_c}(x) &= \frac{1}{\sqrt{2}}J_\mu^{\bar{D}^{*0}}(x)J^{\Xi_c^0}(x) + \frac{1}{\sqrt{2}}J_\mu^{\bar{D}^{*-}}(x)J^{\Xi_c^+}(x), \\
J_{\frac{1}{2};\mu}^{\bar{D}^*\Lambda_c}(x) &= J_\mu^{\bar{D}^{*0}}(x)J^{\Lambda_c^+}(x), \\
J_{\frac{1}{2};\mu}^{\bar{D}_s^*\Xi_c}(x) &= J_\mu^{\bar{D}_s^{*-}}(x)J^{\Xi_c^+}(x), \\
J_{0;\mu}^{\bar{D}_s^*\Lambda_c}(x) &= J_\mu^{\bar{D}_s^{*-}}(x)J^{\Lambda_c^+}(x),
\end{aligned} \tag{23}$$

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	IJ^P	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	$\mu(\text{GeV})$	PC
$\bar{D}\Sigma_c$	$\frac{1}{2}\frac{1}{2}^-$	3.2 – 3.8	5.00 ± 0.10	2.2	(42 – 60)%
$\bar{D}\Sigma_c$	$\frac{3}{2}\frac{1}{2}^-$	2.8 – 3.4	4.98 ± 0.10	2.2	(44 – 65)%
$\bar{D}\Sigma_c^*$	$\frac{1}{2}\frac{3}{2}^-$	3.3 – 3.9	5.06 ± 0.10	2.3	(42 – 60)%
$\bar{D}\Sigma_c^*$	$\frac{3}{2}\frac{3}{2}^-$	2.9 – 3.5	5.03 ± 0.10	2.4	(44 – 64)%
$\bar{D}^*\Sigma_c$	$\frac{1}{2}\frac{3}{2}^-$	3.3 – 3.9	5.12 ± 0.10	2.5	(42 – 60)%
$\bar{D}^*\Sigma_c$	$\frac{3}{2}\frac{3}{2}^-$	3.0 – 3.6	5.10 ± 0.10	2.5	(41 – 61)%
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}\frac{5}{2}^-$	3.2 – 3.8	5.08 ± 0.10	2.5	(43 – 60)%
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}\frac{5}{2}^-$	3.0 – 3.6	5.24 ± 0.10	2.8	(42 – 61)%
$\bar{D}\Xi'_c$	$0\frac{1}{2}^-$	3.4 – 4.0	5.12 ± 0.10	2.2	(41 – 58)%
$\bar{D}\Xi'_c$	$1\frac{1}{2}^-$	3.2 – 3.8	5.14 ± 0.10	2.3	(43 – 61)%
$\bar{D}\Xi_c^*$	$0\frac{3}{2}^-$	3.4 – 4.0	5.15 ± 0.10	2.3	(43 – 60)%
$\bar{D}\Xi_c^*$	$1\frac{3}{2}^-$	3.3 – 3.9	5.22 ± 0.10	2.4	(44 – 62)%

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	IJ^P	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	$\mu(\text{GeV})$	PC
$\bar{D}^* \Xi'_c$	$0\frac{3}{2}^-$	3.5 – 4.1	5.26 ± 0.10	2.5	(42 – 59)%
$\bar{D}^* \Xi'_c$	$1\frac{3}{2}^-$	3.4 – 4.0	5.31 ± 0.10	2.6	(43 – 60)%
$\bar{D}^* \Xi_c^*$	$0\frac{5}{2}^-$	3.6 – 4.2	5.31 ± 0.10	2.6	(42 – 58)%
$\bar{D}^* \Xi_c^*$	$1\frac{5}{2}^-$	3.4 – 4.0	5.35 ± 0.10	2.6	(44 – 61)%
$\bar{D} \Xi_c$	$0\frac{1}{2}^-$	3.2 – 3.8	5.00 ± 0.10	2.1	(41 – 60)%
$\bar{D} \Xi_c$	$1\frac{1}{2}^-$	3.1 – 3.7	5.09 ± 0.10	2.3	(42 – 61)%
$\bar{D} \Lambda_c$	$\frac{1}{2}\frac{1}{2}^-$	3.2 – 3.8	5.11 ± 0.10	2.5	(42 – 60)%
$\bar{D}_s \Xi_c$	$\frac{1}{2}\frac{1}{2}^-$	3.2 – 3.8	5.15 ± 0.10	2.2	(41 – 59)%
$\bar{D}_s \Lambda_c$	$0\frac{1}{2}^-$	3.2 – 3.8	5.13 ± 0.10	2.3	(43 – 61)%
$\bar{D}^* \Xi_c$	$0\frac{3}{2}^-$	3.2 – 3.8	5.10 ± 0.10	2.3	(43 – 61)%
$\bar{D}^* \Xi_c$	$1\frac{3}{2}^-$	3.3 – 3.9	5.27 ± 0.10	2.6	(43 – 61)%
$\bar{D}^* \Lambda_c$	$\frac{1}{2}\frac{3}{2}^-$	3.3 – 3.9	5.23 ± 0.10	2.7	(41 – 61)%
$\bar{D}_s^* \Xi_c$	$\frac{1}{2}\frac{3}{2}^-$	3.3 – 3.9	5.28 ± 0.10	2.4	(42 – 59)%
$\bar{D}_s^* \Lambda_c$	$0\frac{3}{2}^-$	3.2 – 3.8	5.14 ± 0.10	2.4	(42 – 60)%

成功实现极点项贡献(40 – 60)%

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	IJ^P	$M(\text{GeV})$	$\lambda(10^{-3}\text{GeV}^6)$	Assignments	Thresholds (MeV)
$\bar{D}\Sigma_c$	$\frac{1}{2}\frac{1}{2}^-$	$4.31_{-0.07}^{+0.07}$	$3.25_{-0.41}^{+0.43}$	$P_c(4312)$	4321
$\bar{D}\Sigma_c$	$\frac{3}{2}\frac{1}{2}^-$	$4.33_{-0.08}^{+0.09}$	$1.97_{-0.26}^{+0.28}$		4321
$\bar{D}\Sigma_c^*$	$\frac{1}{2}\frac{3}{2}^-$	$4.38_{-0.07}^{+0.07}$	$1.97_{-0.24}^{+0.26}$	$P_c(4380)$	4385
$\bar{D}\Sigma_c^*$	$\frac{3}{2}\frac{3}{2}^-$	$4.41_{-0.08}^{+0.08}$	$1.24_{-0.16}^{+0.17}$		4385
$\bar{D}^*\Sigma_c$	$\frac{1}{2}\frac{3}{2}^-$	$4.44_{-0.08}^{+0.07}$	$3.60_{-0.44}^{+0.47}$	$P_c(4440)$	4462
$\bar{D}^*\Sigma_c$	$\frac{3}{2}\frac{3}{2}^-$	$4.47_{-0.09}^{+0.09}$	$2.31_{-0.31}^{+0.33}$		4462
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}\frac{5}{2}^-$	$4.46_{-0.08}^{+0.08}$	$4.05_{-0.50}^{+0.54}$	$P_c(4457)$	4527
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}\frac{5}{2}^-$	$4.62_{-0.09}^{+0.09}$	$2.40_{-0.35}^{+0.37}$		4527
$\bar{D}\Xi'_c$	$0\frac{1}{2}^-$	$4.43_{-0.07}^{+0.07}$	$3.02_{-0.37}^{+0.39}$		4446
$\bar{D}\Xi'_c$	$1\frac{1}{2}^-$	$4.45_{-0.08}^{+0.07}$	$2.50_{-0.31}^{+0.33}$		4446
$\bar{D}\Xi_c^*$	$0\frac{3}{2}^-$	$4.46_{-0.07}^{+0.07}$	$1.71_{-0.21}^{+0.22}$	$P_{cs}(4459)$	4513
$\bar{D}\Xi_c^*$	$1\frac{3}{2}^-$	$4.53_{-0.07}^{+0.07}$	$1.56_{-0.19}^{+0.20}$		4513

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	IJ^P	$M(\text{GeV})$	$\lambda(10^{-3}\text{GeV}^6)$	Assignments	Thresholds (MeV)
$\bar{D}^* \Xi'_c$	$0\frac{3}{2}^-$	$4.57^{+0.07}_{-0.07}$	$3.41^{+0.43}_{-0.41}$		4588
$\bar{D}^* \Xi'_c$	$1\frac{3}{2}^-$	$4.62^{+0.08}_{-0.08}$	$3.05^{+0.39}_{-0.37}$		4588
$\bar{D}^* \Xi_c^*$	$0\frac{5}{2}^-$	$4.64^{+0.07}_{-0.07}$	$4.36^{+0.54}_{-0.51}$		4655
$\bar{D}^* \Xi_c^*$	$1\frac{5}{2}^-$	$4.67^{+0.08}_{-0.08}$	$3.25^{+0.41}_{-0.39}$		4655
$\bar{D} \Xi_c$	$0\frac{1}{2}^-$	$4.34^{+0.07}_{-0.07}$	$1.43^{+0.19}_{-0.18}$? $P_{cs}(4338)$	4337
$\bar{D} \Xi_c$	$1\frac{1}{2}^-$	$4.46^{+0.07}_{-0.07}$	$1.37^{+0.19}_{-0.18}$		4337
$\bar{D} \Lambda_c$	$\frac{11}{22}^-$	$4.46^{+0.07}_{-0.08}$	$1.47^{+0.20}_{-0.18}$		4151
$\bar{D}_s \Xi_c$	$\frac{11}{22}^-$	$4.54^{+0.07}_{-0.07}$	$1.58^{+0.21}_{-0.20}$		4437
$\bar{D}_s \Lambda_c$	$0\frac{1}{2}^-$	$4.48^{+0.07}_{-0.07}$	$1.57^{+0.21}_{-0.20}$		4255
$\bar{D}^* \Xi_c$	$0\frac{3}{2}^-$	$4.46^{+0.07}_{-0.07}$	$1.55^{+0.20}_{-0.19}$? $P_{cs}(4459)$	4479
$\bar{D}^* \Xi_c$	$1\frac{3}{2}^-$	$4.63^{+0.08}_{-0.08}$	$1.69^{+0.22}_{-0.21}$		4479
$\bar{D}^* \Lambda_c$	$\frac{13}{22}^-$	$4.59^{+0.08}_{-0.08}$	$1.67^{+0.22}_{-0.21}$		4293
$\bar{D}_s^* \Xi_c$	$\frac{13}{22}^-$	$4.65^{+0.08}_{-0.08}$	$1.66^{+0.22}_{-0.21}$		4580
$\bar{D}_s^* \Lambda_c$	$0\frac{3}{2}^-$	$4.50^{+0.07}_{-0.07}$	$1.52^{+0.21}_{-0.19}$		4398

There is no room for the $P_c(4337)$.

6 总结

- 我们采用能标公式提高基态贡献，成功实现了极点为主与算符乘积展开收敛；这两个条件很难同时满足。
- 五夸克态不能容纳 $P_{cs}(4338)$ 。
- 五夸克分子态不能容纳 $P_c(4337)$ 。
- 理论与实验都有待提高，目前难以有确切结论。

谢谢大家， 欢迎批评指正！