第十届XYZ研讨会

Study of the $D_{s0}^*(2317)$ in $\Lambda_b \to \Lambda_c\,(\bar{D}\bar{K})^-$ decays

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arXiv: 2411.17098

2025.4. 长沙

Outline

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2. Formalism
3. Results
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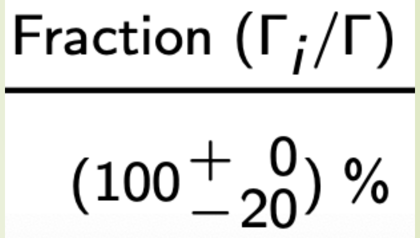
§1. Introduction

$D^*_{s0}(2317)$

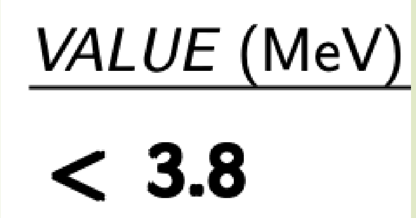
$I(J^P) = 0(0^+)$

Mainly decay mode $D_s^+\pi^0$



Fraction (Γ_i/Γ)
$(100^{+\ 0}_{-20})$ %

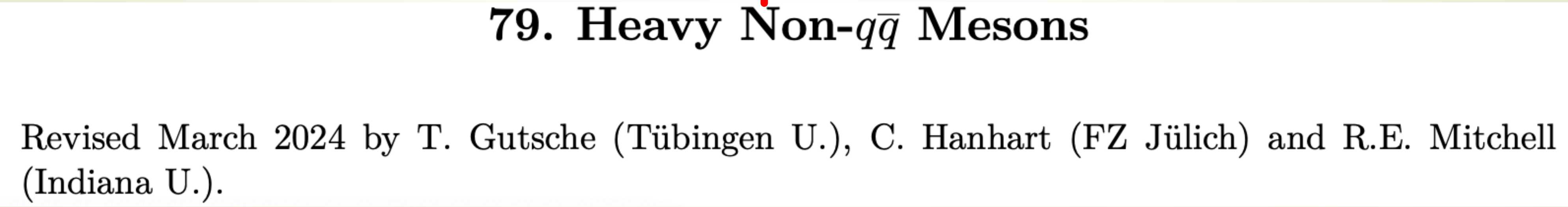
Isospin $I = 1$



VALUE (MeV)
< 3.8

theoretical predicted widths from 10 keV to 100 keV ↔

PDG review on “Heavy Non-$q\bar{q}$ Mesons”



79. Heavy Non-$q\bar{q}$ Mesons

Revised March 2024 by T. Gutsche (Tübingen U.), C. Hanhart (FZ Jülich) and R.E. Mitchell (Indiana U.).

DK molecular picture--most favored :

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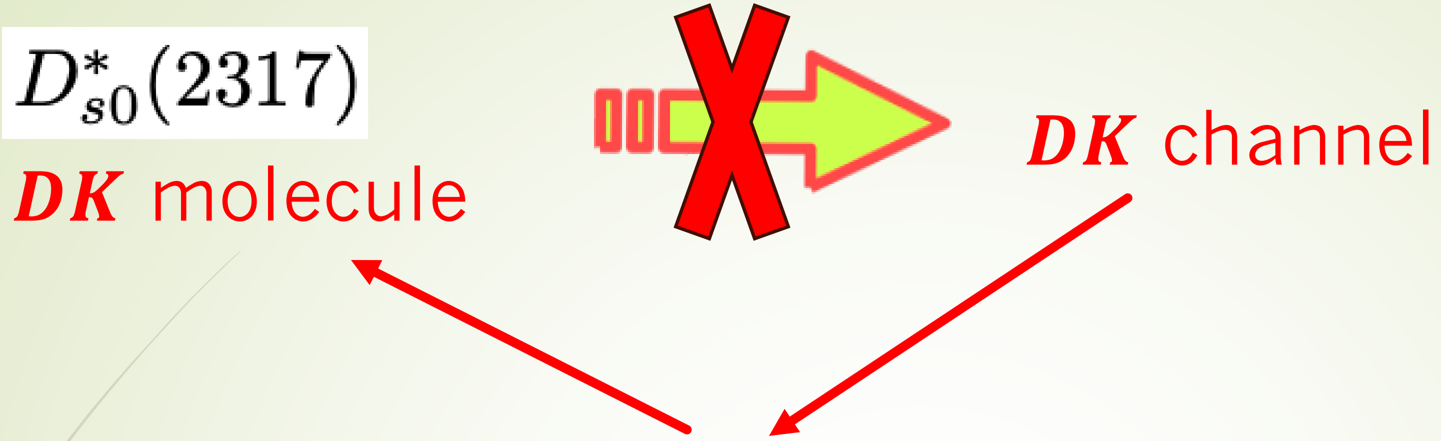
C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D 90, 034510 (2014)
A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos, JHEP 05, 153 (2015)
G. S. Bali, S. Collins, A. Cox, and A. Schäfer, Phys. Rev. D 96, 074501 (2017)
G. K. C. Cheung, C. E. Thomas, D. J. Wilson, G. Moir, M. Peardon, and S. M. Ryan (Hadron Spectrum), JHEP 02, 100 (2021)
Z. Yang, G.-J. Wang, J.-J. Wu, M. Oka, and S.- L. Zhu, Phys. Rev. Lett. 128, 112001 (2022)



$D_{s0}^*(2317)$

DK molecule

DK channel

Enhancement close to threshold
(how strong)

the LHCb collaboration had observed the reaction

$$\Lambda_b \to \Lambda_c \bar{D}^0 K^-$$

R. Aaij et al. (LHCb), Eur. Phys. J. C 84, 575 (2024)

What can we learn from the ***DK*** invariant mass distribution in the future measurement?

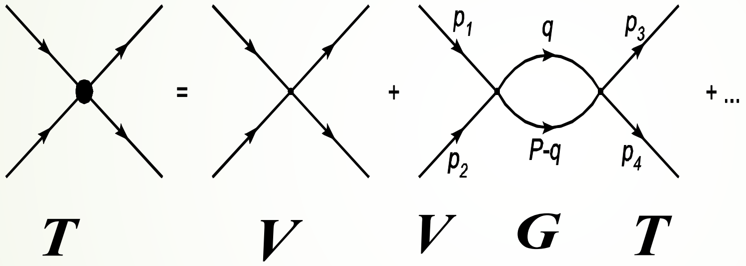
$D_{s0}^*(2317)$

§2. Formalism

(1) Coupled channel interaction from the chiral unitary approach

- **Chiral Unitary Approach**： solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where V matrix (potentials) can be evaluated from the interaction Lagrangians.

$$V_{ij} = C_{ij}\,g^2\,(p_1 + p_3)\cdot(p_2 + p_4)$$

$$C_{ij} = \begin{array}{cc} \begin{pmatrix} -\frac{1}{2}\left(\frac{1}{M_\rho^2} + \frac{1}{M_\omega^2}\right) & -\frac{1}{M_\rho^2} & \frac{2}{\sqrt{3}}\frac{1}{M_{K^*}^2} \\ & -\frac{1}{2}\left(\frac{1}{M_\rho^2} + \frac{1}{M_\omega^2}\right) & \frac{2}{\sqrt{3}}\frac{1}{M_{K^*}^2} \\ & & 0 \end{pmatrix} & \begin{matrix} \bar{D}^0K^- \\ D^-\bar{K}^0 \\ D_s^-\eta \end{matrix} \end{array}$$

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
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J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2-m_{l1}^2+i\varepsilon}\,\frac{1}{q^2-m_{l2}^2+i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

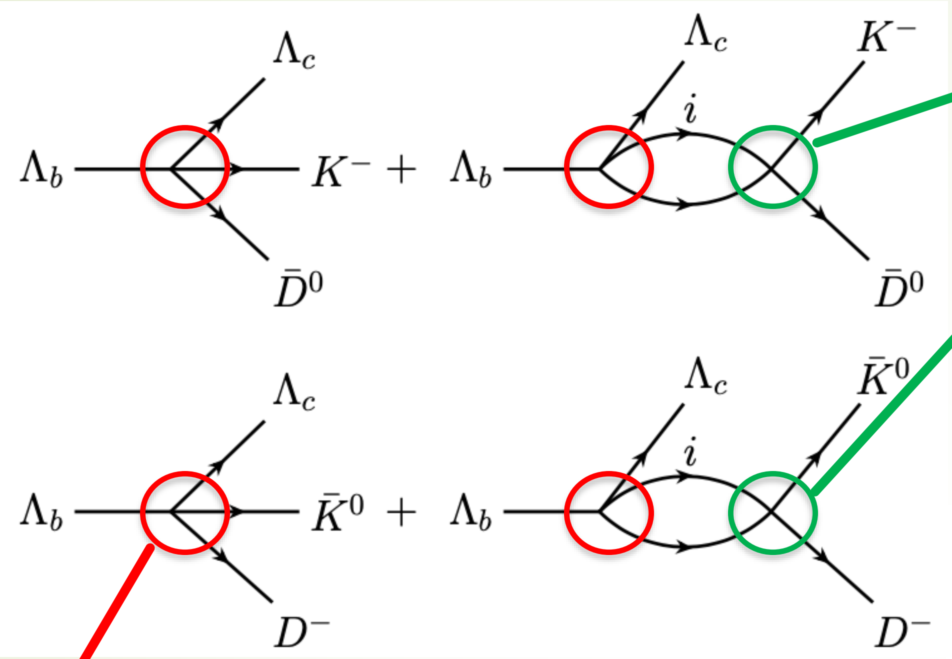
$$\mathrm{Im}\;T_{ij} = T_{in}\,\sigma_{nn}\,T_{nj}^{*}$$

$$\sigma_{nn} \equiv \mathrm{Im}\;G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s-(m_1+m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

(2) Final state interaction



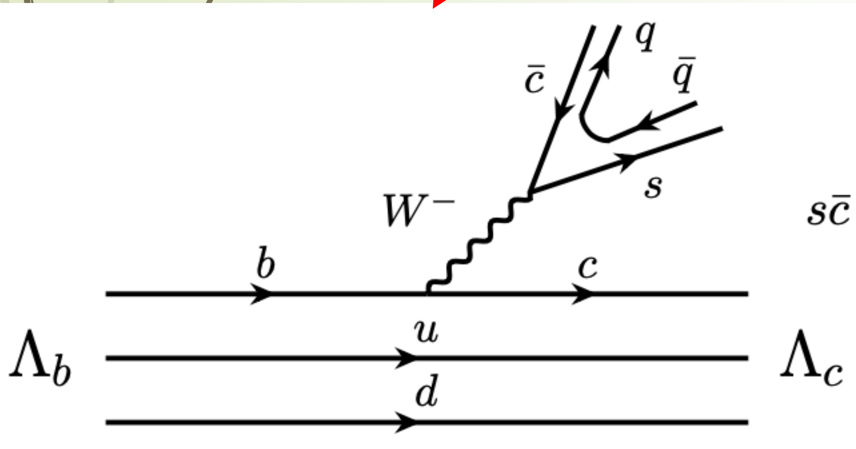
$$T = [1 - VG]^{-1}V$$

S-wave

The final state interaction at the hadron level

The weak decay process at the quark level

$$\mathcal{P} = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}$$



$$s\bar{c} \to \sum_i s\bar{q}_i q_i \bar{c}, \qquad q_i = u, d, s, c.$$

$$s\bar{c} \to \sum_i \mathcal{P}_{3i}\mathcal{P}_{i4} = (\mathcal{P}^2)_{34}$$
$$= \underline{K^-\bar{D}^0 + \bar{K}^0 D^-} - \frac{\eta}{\sqrt{3}} D_s^-$$

$$t_{K^-\bar{D}^0} = A\Big\{1 + G_{K^-\bar{D}^0}\, t_{K^-\bar{D}^0, K^-\bar{D}^0} + G_{\bar{K}^0 D^-}\, t_{\bar{K}^0 D^-, K^-\bar{D}^0} - \frac{1}{\sqrt{3}} G_{\eta D_s^-}\, t_{\eta D_s^-, K^-\bar{D}^0}\Big\}$$

$$t_{\bar{K}^0 D^-} = A\Big\{1 + G_{K^-\bar{D}^0}\, t_{K^-\bar{D}^0, \bar{K}^0 D^-} + G_{\bar{K}^0 D^-}\, t_{\bar{K}^0 D^-, \bar{K}^0 D^-} - \frac{1}{\sqrt{3}} G_{\eta D_s^-}\, t_{\eta D_s^-, \bar{K}^0 D^-}\Big\}$$

$$\Downarrow$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\text{inv}}} = \frac{1}{(2\pi)^3}\,\frac{1}{4M_{\Lambda_b}^2}\, p_{\Lambda_c}\,\tilde{p}_{\bar{K}}\,\overline{\sum}\sum |t|^2$$

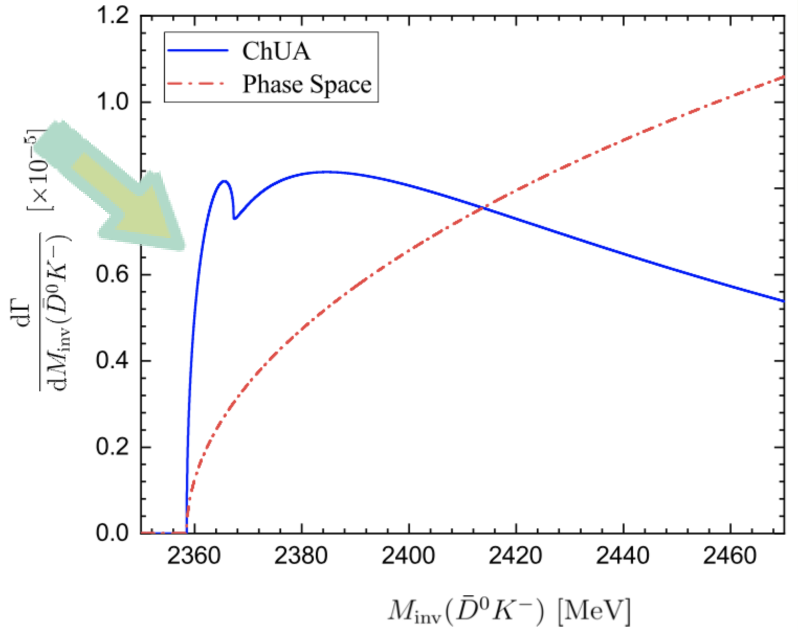
$$p_{\Lambda_c} = \frac{\lambda^{1/2}\left(M_{\Lambda_b}^2, M_{\Lambda_c}^2, M_{\text{inv}}^2(\bar{K}\bar{D})\right)}{2M_{\Lambda_b}}$$

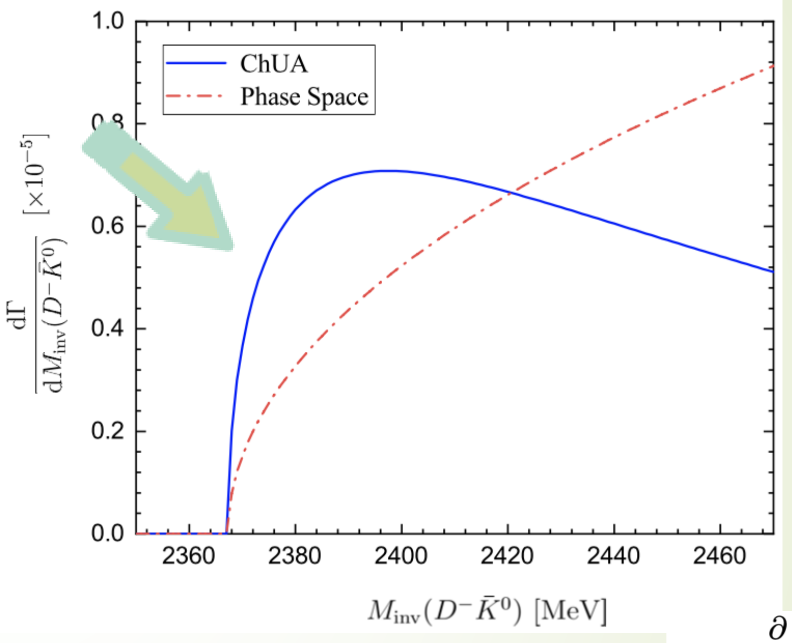
$$\tilde{p}_{\bar{K}} = \frac{\lambda^{1/2}\left(M_{\text{inv}}^2(\bar{K}\bar{D}), M_{\bar{K}}^2, M_{\bar{D}}^2\right)}{2M_{\text{inv}}(\bar{K}\bar{D})}$$

§3. Results

(1) Theoretical results

$q_{\max} = 706\ \mathrm{MeV}$ $A = 1$





$$g_1^2 = \lim_{s \to s_0} (s - s_0)\, T_{11}; \quad g_j = g_1 \lim_{s \to s_0} \frac{T_{1j}}{T_{11}}, \quad P_i = -g_i^2 \left. \frac{\partial G_i}{\partial s} \right|_{s=s_0}$$

$$-\frac{1}{a} = -8\pi \sqrt{s}\, T^{-1} \Big|_{s=s_{\rm th}}$$

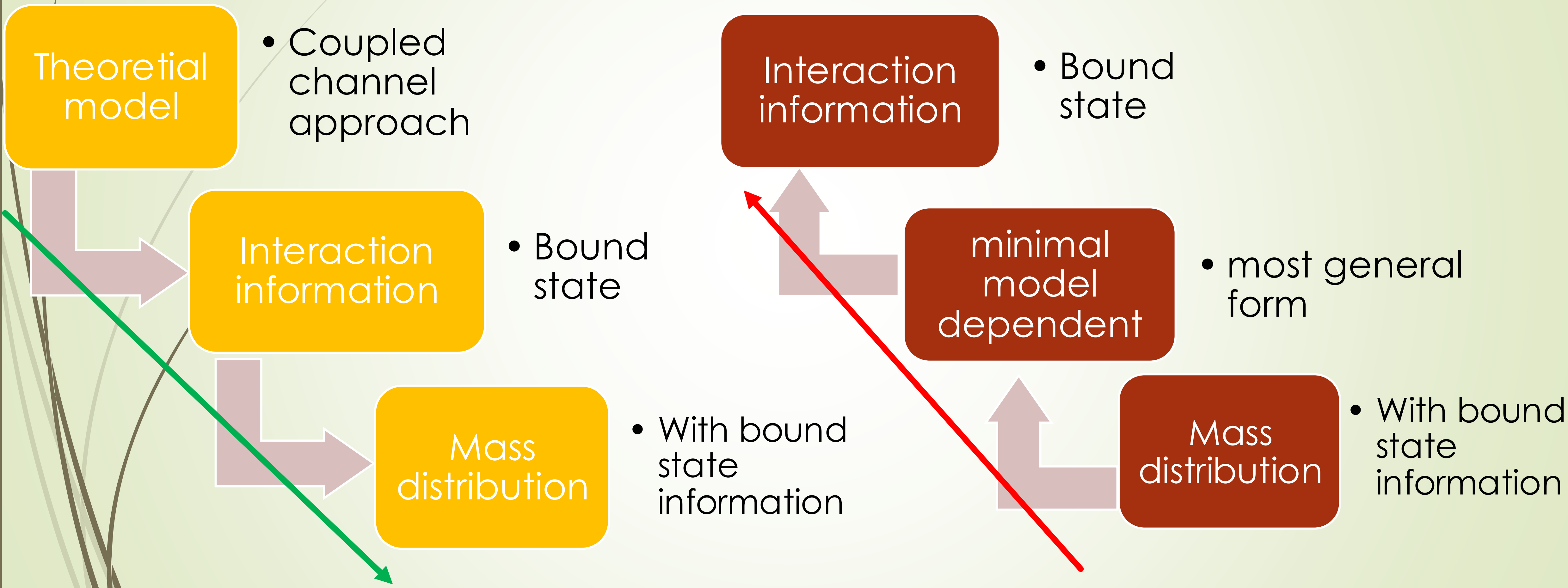
$$r_0 = \frac{\partial}{\partial k^2} 2\left(-8\pi \sqrt{s}\, T^{-1} + ik\right) = \frac{\sqrt{s}}{\mu} \frac{\partial}{\partial s} 2\left(-8\pi \sqrt{s}\, T^{-1} + ik\right)\Big|_{s=s_{\rm th}}$$

pole [MeV]	g_1	g_2	g_3
2317.85	8182.29	8144.59	-5571.38
πD_s	P_1	P_2	P_3
	0.34	0.29	0.04

a_1	a_2	a_3
0.70	$0.51 - i0.12$	$0.21 - i0.06$
$r_{0,1}$	$r_{0,2}$	$r_{0,3}$
-2.25	$0.14 - i2.41$	$0.12 + i0.11$

(2) What is the inverse problem?

Without experimental data of mass distribution ↛ **Fits with theoretical model**



Theoretical model

- Coupled channel approach

Interaction information

- Bound state

Mass distribution

- With bound state information

direct problem

Mass distribution

- With bound state information

minimal model dependent

- most general form

Interaction information

- Bound state

inverse problem

(3) How to do the inverse problem?

Assume an energy dependence interaction potential

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ & V_{11} & V_{13} \\ & & 0 \end{pmatrix}$$

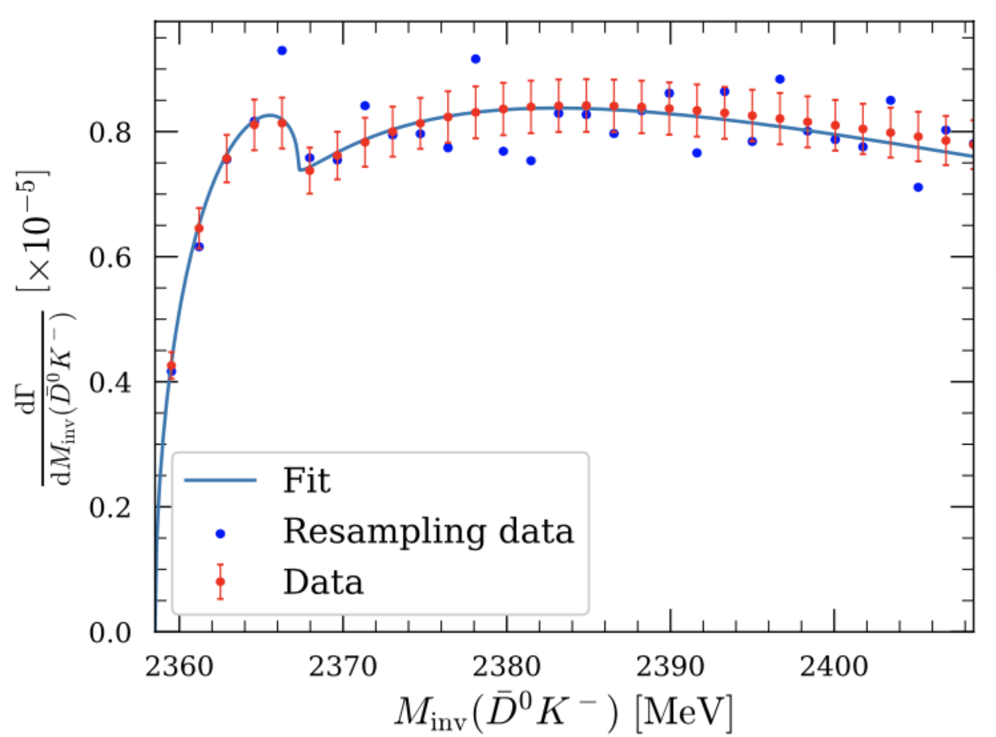
Isospin symmetric

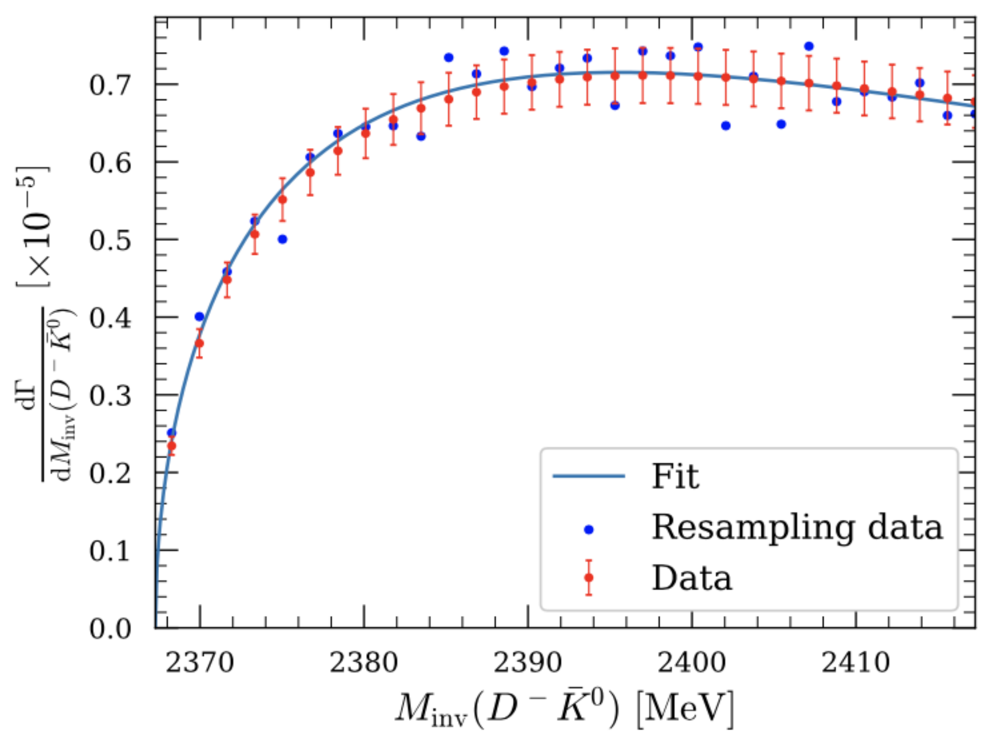
$$V_{11} = V'_{11} + \frac{\alpha}{M_V^2}(s - s_0) \qquad V_{12} = V'_{12} + \frac{\beta}{M_V^2}(s - s_0) \qquad V_{13} = V'_{13} + \frac{\gamma}{M_V^2}(s - s_0)$$

8 free parameters totally: $V'_{11},\ V'_{12},\ V'_{13},\ \alpha,\ \beta,\ \gamma,\ q_{\max}$ and A

Generating random centroids of the data from the theoretical results

Using the bootstrap or resampling method → **Doing 50 fits**





V'_{11}	V'_{12}	V'_{13}	α
-89.71 ± 31.50	-130.48 ± 27.73	98.15 ± 10.03	38.41 ± 128.72

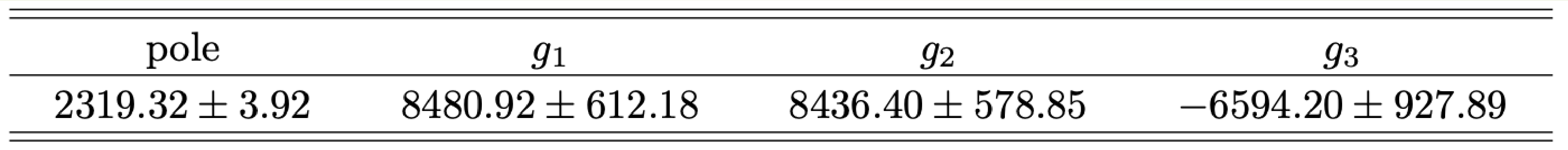
β	γ	$q_{\max}$ [MeV]	A
-103.88 ± 120.34	7.28 ± 33.75	688.38 ± 32.87	0.95 ± 0.09

$$|\bar{K}\bar{D}, I=0\rangle = \frac{1}{\sqrt{2}}\,|\bar{K}^0D^- + K^-\bar{D}^0\rangle$$

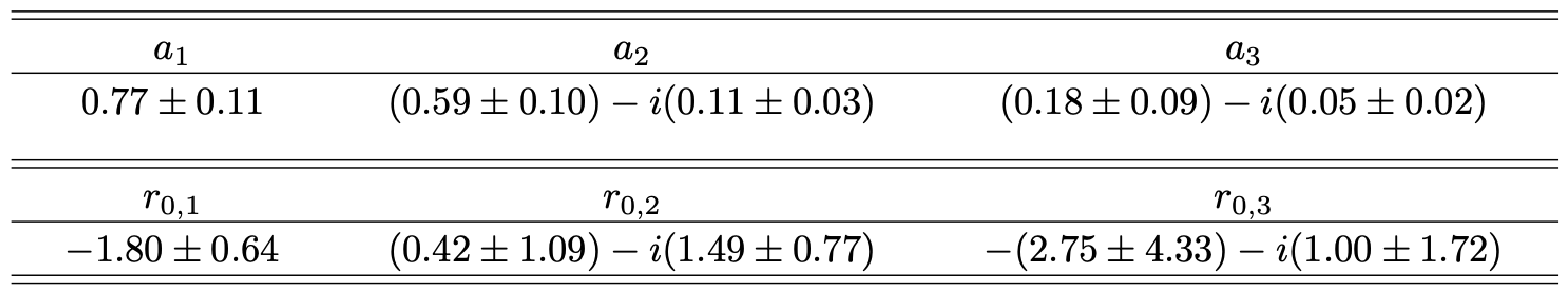
$$|\bar{K}\bar{D}, I=1\rangle = \frac{1}{\sqrt{2}}\,|\bar{K}^0D^- - K^-\bar{D}^0\rangle$$

$$V^{I=0} = \frac{1}{2}(V_{11}+V_{22}+2V_{12}) = V_{11}+V_{12},$$ attractive

$$V^{I=1} = \frac{1}{2}(V_{11}+V_{22}-2V_{12}) = V_{11}-V_{12}.$$ repulsive

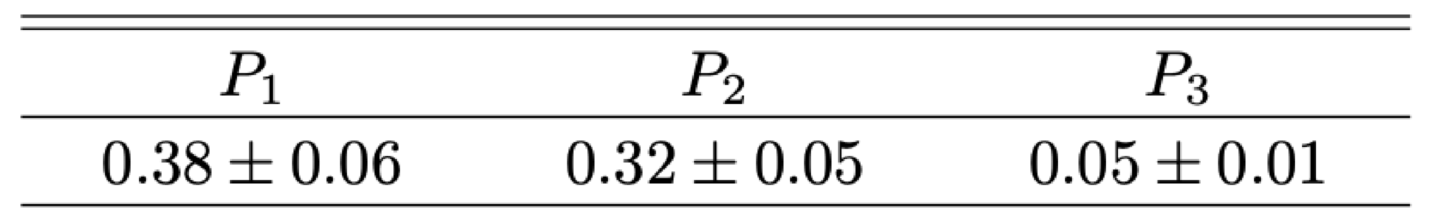


pole	g_1	g_2	g_3
2319.32 ± 3.92	8480.92 ± 612.18	8436.40 ± 578.85	-6594.20 ± 927.89



a_1	a_2	a_3
0.77 ± 0.11	$(0.59 \pm 0.10) - i(0.11 \pm 0.03)$	$(0.18 \pm 0.09) - i(0.05 \pm 0.02)$

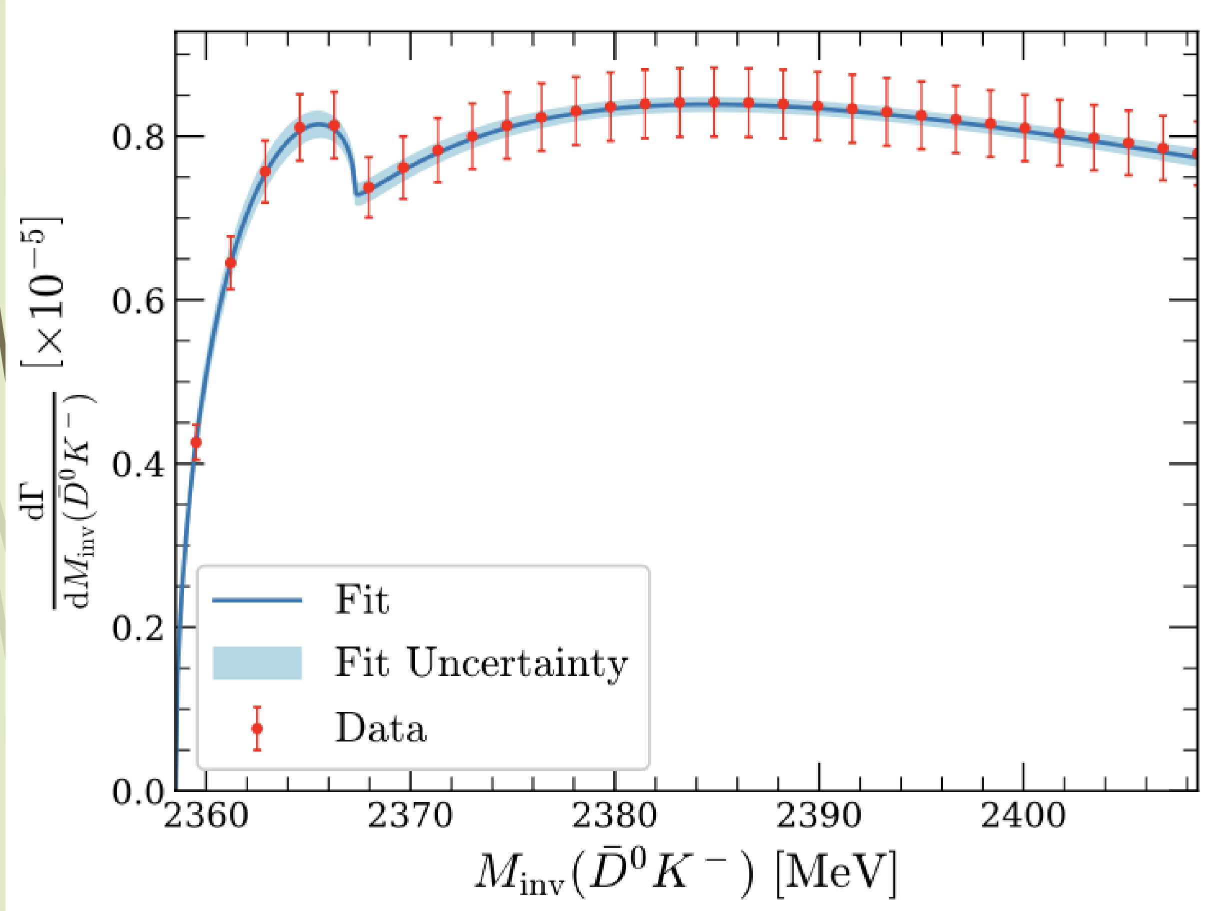
$r_{0,1}$	$r_{0,2}$	$r_{0,3}$
-1.80 ± 0.64	$(0.42 \pm 1.09) - i(1.49 \pm 0.77)$	$-(2.75 \pm 4.33) - i(1.00 \pm 1.72)$

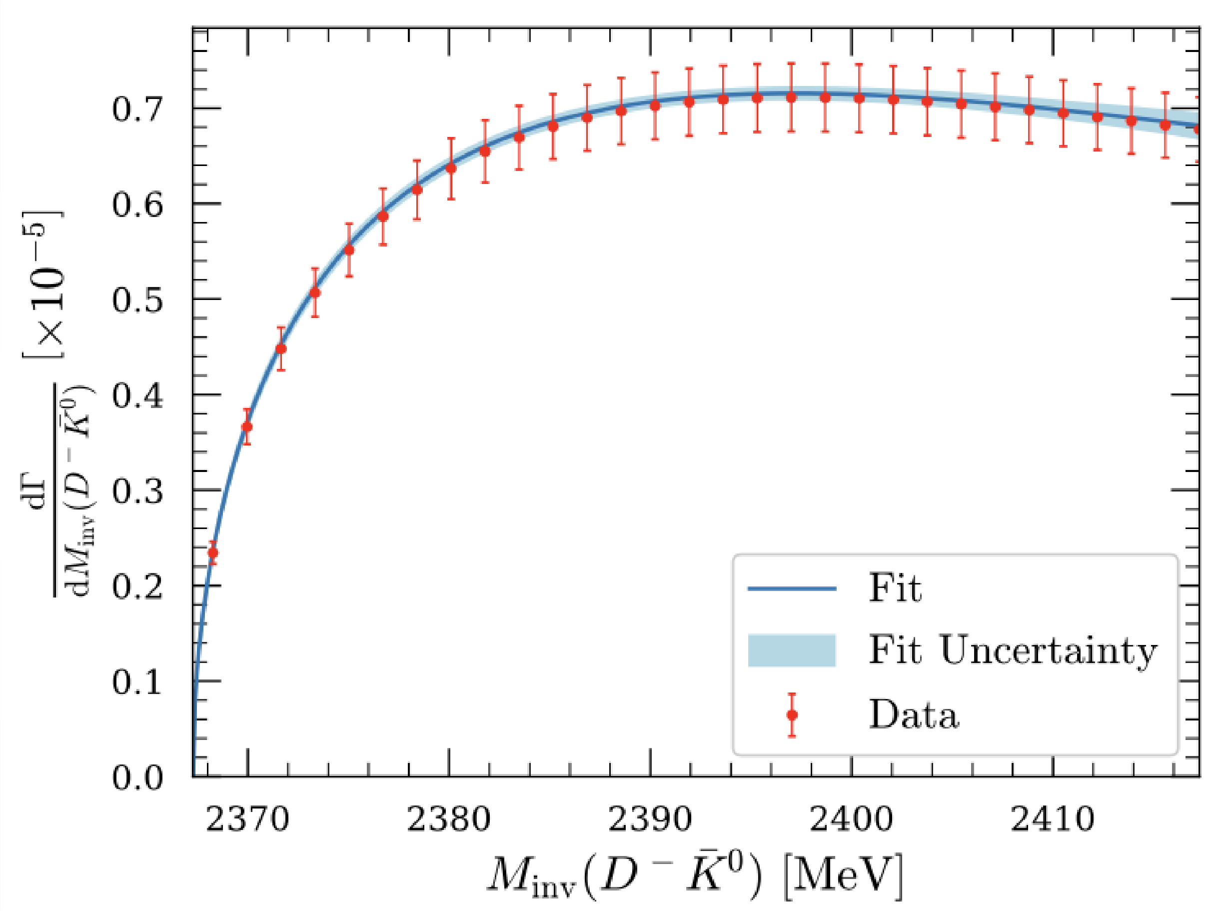


P_1	P_2	P_3
0.38 ± 0.06	0.32 ± 0.05	0.05 ± 0.01

Consistent with the theoretical results before

Results of the resampling with a narrow band for the uncertainty





§4. Summary

- We use the chiral unitary approach to dynamically generate the state $\boxed{D_{s0}^*(2317)}$
- Taking the pseudo data from theory, we use the resampling method for the inverse problem in the fitting of mass distribution, with a minimum model dependent method.
- The enhancement of this resonance at the *DK* threshold of the spectra can be tested by the information from the correlation functions.

Hope future experiments bring more clarifications on these issues…….

Thanks for your attention!

感谢大家的聆听！