# Tri-meson state $\bar{B}\bar{B}^*\bar{B}^*$

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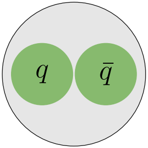
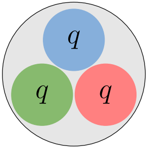
# Outline

- Conventional hadrons

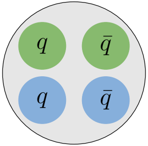


Meson

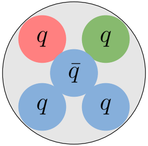


Baryon

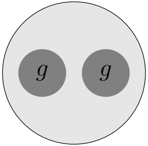
- Exotic hadrons



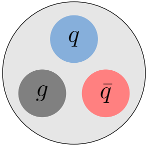
Four-quark



Five-quark



Glueball



Hybrid

# 1.2 New hadrons

- Discoveries in experiments
  - Staring from 2003, new hadronic states XYZ, $P_c$, $P_{cs}$ and $T_{cc}$
  - Exhibiting exotic properties.
- Theoretical descriptions
  - Excited charmonium
  - Compact multiquark states
  - $c\bar{c}$-gluon hybrid
  - Hadron molecular states
  - Threshold effect
  - ......
  - Still far away from a unified picture
    The most popular one is hadron molecular states,
    such as $X(3872)(D\bar{D}^*)$, $T_{cc}(3875)$ $(DD^*)$ and $P_c$ $(\Sigma\bar{D}^{(*)})$,......

# 1.3 Multi-hadron states

- In nuclear physics
  - Efimov effect, deuteron (np), deuteron+p(n)→ $^3H_e$ ($^3H$),......
- Similarly, in hadron physics

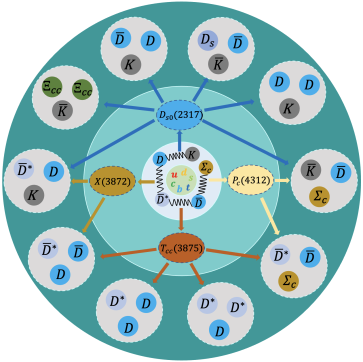



Figure 1: Multi-hadron states. Taken from Sci.Bull. 67, 1735 (2022).

- More theoretical predictions can be found in Phys. Rept. 1108, 1 (2025)

# 2.1 Model Hamiltonian

- Model Hamiltonian

$$H_n = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_c + \sum_{i>j}^{n}\left(V_{ij}^{oge} + V_{ij}^{con} + V_{ij}^{obe} + V_{ij}^{\sigma}\right)$$

- One-gluon-exchange and quark confinement

$$V_{ij}^{oge} = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j}\right), \quad V_{ij}^{con} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c r_{ij}^2$$

- One Goldstone boson exchange

$$V_{ij}^{obe} = V_{ij}^{\pi}\sum_{k=1}^{3}\mathbf{F}_i^k\mathbf{F}_j^k + V_{ij}^{K}\sum_{k=4}^{7}\mathbf{F}_i^k\mathbf{F}_j^k + V_{ij}^{\eta}(\mathbf{F}_i^8\mathbf{F}_j^8\cos\theta_P - \sin\theta_P)$$

$$V_{ij}^{\chi} = \frac{g_{ch}^2}{4\pi}\frac{m_\chi^3}{12m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\left(Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\right), \quad \chi = \pi,\ K,\ \eta$$

- $\sigma$-meson exchange

$$V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2}\left(Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right)$$

# 2.2 Model parameters

- Meson spectrum and adjustable parameters

Mass unit in MeV and root-mean-square unit in fm.

| State | $D$ | $D^*$ | $D_s$ | $D_s^*$ | $\bar{B}$ | $\bar{B}^*$ | $\bar{B}_s$ | $\bar{B}_s^*$ |
|---|---|---|---|---|---|---|---|---|
| Model prediction | 1867 | 2002 | 1972 | 2140 | 5259 | 5301 | 5377 | 5430 |
| PDG | 1869 | 2007 | 1968 | 2112 | 5280 | 5325 | 5366 | 5416 |
| $\langle r^2 \rangle^{\frac{1}{2}}$ | 0.68 | 0.82 | 0.52 | 0.69 | 0.73 | 0.77 | 0.57 | 0.62 |

Quark mass and $\Lambda_0$ unit in MeV, $a_c$ unit in MeV$\cdot$fm$^{-2}$, $r_0$ unit in MeV$\cdot$fm and $\alpha_0$ is dimensionless.

| Parameter | $m_{u,d}$ | $m_s$ | $m_c$ | $m_b$ | $a_c$ | $\alpha_0$ | $\Lambda_0$ | $r_0$ |
|---|---|---|---|---|---|---|---|---|
| Value | 280 | 512 | 1602 | 4936 | 40.78 | 4.55 | 9.17 | 35.06 |

- Applied to the $T_{cc}^+$, the model can match the experimental data well.

C.R. Deng and S.L. Zhu, $T_{cc}^+$ and its partners, Phys. Rev. D 105, 054015 (2022);

C.R. Deng and S.L. Zhu, Decoding the double heavy tetraquark state $T_{cc}^+$, Science Bulletin 67, 1522

# 3.1 WF of bottomed meson

- WF of bottomed meson

$$\Phi_{IJ}^{\bar{B}^{(*)}} = \chi_c \otimes \eta_i \otimes \psi_s \otimes \phi_{l_r m_r}(\mathbf{r})$$

  - Color part

$$\chi_c = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

  - Isospin part

$$\eta_i = b\bar{u}, \quad b\bar{d}$$

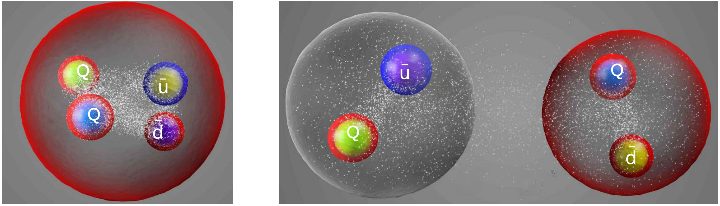
  - Spin part

$$S = 0: \quad \psi_s = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow); \quad S = 1: \quad \psi_s = \downarrow\downarrow, \quad \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad \uparrow\uparrow$$

  - Orbit part, Gaussian expansion method

$$\phi_{l_r m_r}(\mathbf{r}) = \sum_{n_r=1}^{n_{rmax}} c_{n_r} N_{n_r l_r} r^{l_r} e^{-\nu_{n_r} r^2} Y_{l_r m_r}(\hat{\mathbf{r}}), \quad \mathbf{r} = \mathbf{r}_b - \mathbf{r}_{\bar{q}}$$

# 3.2 Two configurations of $T_{bb}$




Diquark configuration    Meson-meson configuration

Figure 2: Two configurations

- Diquark configuration: compact, color force.
- Meson-meson configuration: relative loose, residual interactions.

# 3.3 WF of di-meson state $T_{bb}$

- WF of di-meson state $T_{bb}$ $(B^{(*)}\bar{B}^{*})$

$$\Psi^{T_{bb}}_{I_{12}J_{12}} = \sum_{\xi} c_{\xi}\mathcal{A}_{12}\left\{\left[\Phi^{\bar{B}^{(*)}}_{I_1 J_1}\Phi^{\bar{B}^{*}}_{I_2 J_2}\right]^{J_{12}}_{I_{12}}\phi_{l_\rho m_\rho}(\boldsymbol{\rho})\right\}.$$

  - $\xi = \{I_1, I_2, J_1, J_2, ......\}$ and $c_\xi$ can be determined by the model dynamics.
  - $\mathcal{A}_{12}$ serves as an antisymmetrization operator

$$\mathcal{A}_{12} = P_{b_1 b_2}P_{\bar{q}_1\bar{q}_2}, \qquad P_{b_1 b_2} = 1 - P_{b_1 b_2},\ P_{\bar{q}_1\bar{q}_2} = 1 - P_{\bar{q}_1\bar{q}_2}.$$

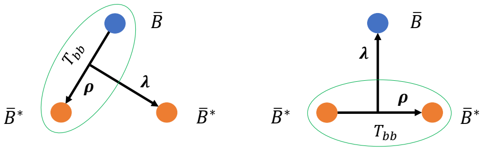
  - $\phi_{l_\rho m_\rho}(\boldsymbol{\rho})$ represents the relative motion WF between two mesons

$$\boldsymbol{\rho} = \frac{m_b\mathbf{r}_{b_1} + m_q\mathbf{r}_{\bar{q}_1}}{m_b + m_{\bar{q}}} - \frac{m_b\mathbf{r}_{b_2} + m_q\mathbf{r}_{\bar{q}_2}}{m_b + m_{\bar{q}}}.$$

Also, $\phi_{l_\rho m_\rho}(\boldsymbol{\rho})$ is expressed by Gaussian expansion method.

# 3.4 Wave functions of $H_{bbb}$

- Jaccobi coordinates for the trimeson state $\bar{B}\bar{B}^*\bar{B}^*$ ($H_{bbb}$).




- The total wave function of the tri-meson state $H_{bbb}$

$$\Psi_{IJ}^{H_{bbb}} = \sum_{\xi} c_\xi \mathcal{A}_{123} \left\{ \left[ \Psi_{I_{12}J_{12}}^{T_{bb}} \Phi_{I_3 J_3}^{\bar{B}^*} \right]_I^J \phi(\boldsymbol{\lambda}) \right\}.$$

  - $\mathcal{A}_{123}$ is antisymmetrization operator.

$$\mathcal{A}_{123} = P_{b_1 b_2 b_3} P_{\bar{q}_1 \bar{q}_2 \bar{q}_3}$$
$$P_{b_1 b_2 b_3} = 1 - P_{b_1 b_3} - P_{b_2 b_3}, \; P_{\bar{q}_1 \bar{q}_2 \bar{q}_3} = 1 - P_{\bar{q}_1 \bar{q}_3} - P_{\bar{q}_2 \bar{q}_3}.$$

  - $\phi_{l_\lambda m_\lambda}(\boldsymbol{\lambda})$ represents the relative motion WF between $T_{bb}$ and $\bar{B}$

$$\boldsymbol{\lambda} = \frac{m_b \mathbf{r}_{b_1} + m_q \mathbf{r}_{\bar{q}_1} + m_b \mathbf{r}_{b_2} + m_q \mathbf{r}_{\bar{q}_2}}{2(m_b + m_{\bar{q}})} - \frac{m_b \mathbf{r}_{b_3} + m_{\bar{q}} \mathbf{r}_{\bar{q}_3}}{m_b + m_{\bar{q}}}.$$

- Five possible isospin-spin configurations
  - Isospin antisymmetric: $[\bar{B}\bar{B}^*]_0^1$ and $[\bar{B}^*\bar{B}^*]_0^1$
  - Isospin symmetric: $[\bar{B}^*\bar{B}^*]_1^0$, $[\bar{B}\bar{B}^*]_1^1$, and $[\bar{B}^*\bar{B}^*]_1^2$
- Solving the four-body Schrödinger equation

$$(H_4 - E_4)\Psi_{I_{12}J_{12}}^{T_{bb}} = 0$$

- Binding energy

$$\Delta E = E_4 - M_{\bar{B}^{(*)}} - M_{\bar{B}^*}$$

- Contribution from each interaction

$$\Delta\langle V^\chi\rangle = \langle\Psi_{I_{12}J_{12}}^{T_{bb}}|V^\chi|\Psi_{I_{12}J_{12}}^{T_{bb}}\rangle - \langle\Phi_{I_1J_1}^{\bar{B}^{(*)}}|V^\chi|\Phi_{I_1J_1}^{\bar{B}^{(*)}}\rangle - \langle\Phi_{I_2J_2}^{\bar{B}^*}|V^\chi|\Phi_{I_2J_2}^{\bar{B}^*}\rangle.$$

# 4.2 Bound states of $T_{bb}$

- Bound states: $[\bar{B}\bar{B}^*]_0^1$ and $[\bar{B}^*\bar{B}^*]_0^1$

Binding energy $\Delta E_4$ and contribution from each part unit in MeV, and the size $\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}}$ in fm.

| $T_{bb}$ | $IJ^P$ | $\Delta E_4$ | $\Delta V^{\rm conf}$ | $\Delta V^{\rm coul}$ | $\Delta V^{\rm cm}$ | $\Delta T$ | $\Delta V^{\sigma}$ | $\Delta V^{\pi}$ | $\Delta V^{\eta}$ | $\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $[\bar{B}\bar{B}^*]_0^1$ | $01^+$ | $-10.0$ | $-6.3$ | $-8.9$ | $-14.3$ | 33.0 | $-9.3$ | $-4.4$ | 0.2 | 1.07 |
| $[\bar{B}^*\bar{B}^*]_0^1$ | $01^+$ | $-9.0$ | $-6.0$ | $-7.8$ | $-12.3$ | 29.4 | $-8.5$ | $-3.9$ | 0.2 | 1.11 |

  - Binding energy: $\Delta E_4 \simeq -10.0$ and $-9.0$ MeV.
  - Binding mechanisms: $V^{\rm conf}$, $V^{\rm coul}$, $V^{\rm cm}$, $V^{\sigma}$ and $V^{\pi}$.
  - Compact bound state: $\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}} \simeq 1.10$ fm while $\bar{B}^{(*)}$ is about 0.70 to 0.80 fm.
- Unbound states: $[\bar{B}^*\bar{B}^*]_1^0$, $[\bar{B}\bar{B}^*]_1^1$, and $[\bar{B}^*\bar{B}^*]_1^2$.

# 5.1 $H_{bbb}$ from $[\bar{B}\bar{B}^*]_0^1$

- $H_{bbb}$ from $[\bar{B}\bar{B}^*]_0^1$

Binding energy $\Delta E_6$ relative to $\bar{B}\bar{B}^*\bar{B}^*$ and contribution from each part unit in MeV, and the sizes $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}}$ and $\langle \boldsymbol{\lambda}^2\rangle^{\frac{1}{2}}$ in fm.

| $T_{bb}$ | $H_{bbb}$ | $IJ^P$ | $\Delta E_{6(4)}$ | $\Delta V^{\rm con}$ | $\Delta V^{\rm coul}$ | $\Delta V^{\rm cm}$ | $\Delta T$ | $\Delta V^{\sigma}$ | $\Delta V^{\pi}$ | $\Delta V^{\eta}$ | $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}}$ | $\langle \boldsymbol{\lambda}^2\rangle^{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[\bar{B}\bar{B}^*]_0^1$ | — | $01^+$ | −10.0 | −6.3 | −8.9 | −14.3 | 33.0 | −9.3 | −4.4 | 0.2 | 1.07 | — |
| | $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^0$ | $\frac{1}{2}0^-$ | −10.2 | −6.6 | −9.2 | −14.0 | 34.6 | −10.8 | −4.4 | 0.3 | 1.09 | 4.75 |
| $[\bar{B}\bar{B}^*]_0^1$ | $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^1$ | $\frac{1}{2}1^-$ | −10.0 | −6.3 | −8.9 | −14.3 | 33.0 | −9.3 | −4.4 | 0.2 | 1.07 | $\infty$ |
| | $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^2$ | $\frac{1}{2}2^-$ | −10.0 | −6.3 | −8.9 | −14.3 | 33.0 | −9.3 | −4.4 | 0.2 | 1.07 | $\infty$ |

- Bound state: $[[\bar{B}\bar{B}^*]_0^1\bar{B}^*]_{\frac{1}{2}}^0$.
  - Bingding energy: $\Delta E_6(\bar{B}\bar{B}^*\bar{B}^*) = -10.2$ MeV and $\Delta E_6'([\bar{B}\bar{B}^*]_0^1\bar{B}^*) = -0.2$ MeV.
  - Loose two-body bound state: $[\bar{B}\bar{B}^*]_0^1$ is almost unchanged, $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}} = 4.75$ fm.
  - Main binding mechanism: $\sigma$ meson exchange.
- Unbound states: $[[\bar{B}\bar{B}^*]_0^1\bar{B}^*]_{\frac{1}{2}}^1$ and $[[\bar{B}\bar{B}^*]_0^1\bar{B}^*]_{\frac{1}{2}}^2$.

# 5.2 $H_{bbb}$ from $[\bar{B}\bar{B}^*]^1_1$

- $H_{bbb}$ from $[\bar{B}\bar{B}^*]^1_1$

| $T_{bb}$ | $H_{bbb}$ | $IJ^P$ | $\Delta E_{6(4)}$ | $\Delta V^{\rm con}$ | $\Delta V^{\rm coul}$ | $\Delta V^{\rm cm}$ | $\Delta T$ | $\Delta V^{\sigma}$ | $\Delta V^{\pi}$ | $\Delta V^{\eta}$ | $\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}}$ | $\langle \boldsymbol{\lambda}^2 \rangle^{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[\bar{B}\bar{B}^*]^1_1$ | — | $01^+$ | unbound | | | | | | | | $\infty$ | — |
| | $\left[[\bar{B}\bar{B}^*]^1_1\bar{B}^*\right]^0_{\frac{1}{2}}$ | $\frac{1}{2}0^-$ | $-0.4$ | $-2.0$ | $-3.1$ | $-1.1$ | 15.2 | $-10.2$ | 0.3 | 0.5 | 2.19 | 1.49 |
| $[\bar{B}\bar{B}^*]^1_1$ | $\left[[\bar{B}\bar{B}^*]^1_1\bar{B}^*\right]^1_{\frac{1}{2}}$ | $\frac{1}{2}1^-$ | $-0.6$ | $-3.7$ | $-4.9$ | $-6.7$ | 24.2 | $-8.2$ | $-1.5$ | 0.1 | 2.64 | 1.28 |
| | $\left[[\bar{B}\bar{B}^*]^1_1\bar{B}^*\right]^2_{\frac{1}{2}}$ | $\frac{1}{2}2^-$ | unbound | | | | | | | | $\infty$ | $\infty$ |

- Bound states: $[[\bar{B}\bar{B}^*]^1_1\bar{B}^*]^0_{\frac{1}{2}}$ and $[[\bar{B}\bar{B}^*]^1_1\bar{B}^*]^1_{\frac{1}{2}}$.
  - Bingding energy: $\Delta E_6 = -0.4$ MeV and $-0.6$ MeV.
  - Loose three-body bound state: $\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}} = 2.19$ and 2.64 fm, $\langle \boldsymbol{\lambda}^2 \rangle^{\frac{1}{2}} = 1.49$ and 1.28 fm.
  - Main binding mechanism: $\sigma$ meson exchange
- Unbound state: $[[\bar{B}\bar{B}^*]^1_1\bar{B}^*]^2_{\frac{1}{2}}$.

# 5.3 Others

- $H_{bbb}$ from $[\bar{B}^*\bar{B}^*]_0^1$, $[\bar{B}^*\bar{B}^*]_1^0$, and $[\bar{B}^*\bar{B}^*]_1^2$

| $T_{bb}$ | $H_{bbb}$ | $IJ^P$ | $\Delta E_{6(4)}$ | $\Delta V^{\rm con}$ | $\Delta V^{\rm coul}$ | $\Delta V^{\rm cm}$ | $\Delta T$ | $\Delta V^{\sigma}$ | $\Delta V^{\pi}$ | $\Delta V^{\eta}$ | $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}}$ | $\langle \boldsymbol{\lambda}^2\rangle^{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[\bar{B}^*\bar{B}^*]_0^1$ | — | $01^+$ | −9.0 | −6.0 | −7.8 | −12.3 | 29.4 | −8.5 | −3.9 | 0.2 | 1.11 | — |
| $[\bar{B}^*\bar{B}^*]_0^1$ | $\left[\bar{B}[\bar{B}^*\bar{B}^*]_0^1\right]_{\frac{1}{2}}^1$ | $\frac{1}{2}1^-$ | −9.0 | −6.0 | −7.8 | −12.3 | 29.4 | −8.5 | −3.9 | 0.2 | 1.11 | $\infty$ |
| $[\bar{B}^*\bar{B}^*]_1^2$ | — | $12^+$ | unbound | | | | | | | | $\infty$ | — |
| $[\bar{B}^*\bar{B}^*]_1^2$ | $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^2\right]_{\frac{1}{2}}^2$ | $\frac{1}{2}2^-$ | unbound | | | | | | | | $\infty$ | $\infty$ |
| $[\bar{B}^*\bar{B}^*]_1^0$ | — | $10^+$ | unbound | | | | | | | | $\infty$ | — |
| $[\bar{B}^*\bar{B}^*]_1^0$ | $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^0\right]_{\frac{1}{2}}^0$ | $\frac{1}{2}0^-$ | −0.7 | −3.1 | −2.2 | −4.4 | 22.5 | −11.7 | −1.9 | 0.2 | 2.09 | 1.43 |

- Unbound state: $\left[\bar{B}[\bar{B}^*\bar{B}^*]_0^1\right]_{\frac{1}{2}}^1$ and $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^2\right]_{\frac{1}{2}}^2$.
- Bound states: $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^0\right]_{\frac{1}{2}}^0$
  - Bingding energy: $\Delta E_6 = -0.7$ MeV.
  - Loose three-body bound state: $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}} = 2.09$ fm and $\langle \boldsymbol{\lambda}^2\rangle^{\frac{1}{2}} = 1.43$ fm.
  - Main binding mechanism: $\sigma$ meson exchange

- Spatial configuration

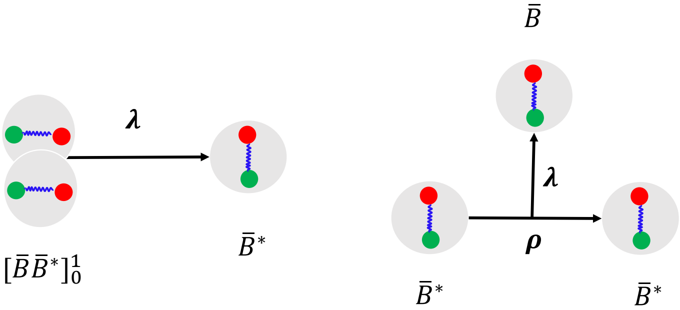
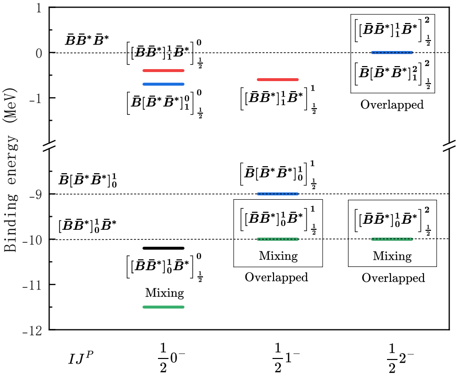



Figure 3: Two-body molecule (left) and three-body molecule (right)

- Tow-body bound state: $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^0$
- Three-body bound states: $\left[[\bar{B}\bar{B}^*]_1^1\bar{B}^*\right]_{\frac{1}{2}}^0$, $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^0\right]_{\frac{1}{2}}^0$, $\left[[\bar{B}\bar{B}^*]_1^1\bar{B}^*\right]_{\frac{1}{2}}^1$

# 5.5 Spectrum and coupled channel effects

- Binding energy spectrum




- Coupled channel effects: $IJ^P = \frac{1}{2}0^-$, $\Delta E_6 = -11.5$ MeV, $\langle \boldsymbol{\lambda}^2 \rangle^{\frac{1}{2}} = 2.20$ fm.
  $[[\bar{B}\bar{B}^*]^1_0\bar{B}^*]^0_{\frac{1}{2}}$ (80%), $[\bar{B}[\bar{B}^*\bar{B}^*]^0_1]^0_{\frac{1}{2}}$ (14%), $[[\bar{B}\bar{B}^*]^1_1\bar{B}^*]^0_{\frac{1}{2}}$ (6%).

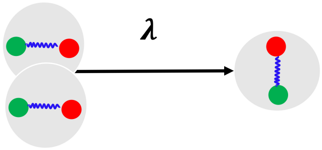



Figure 4: np correlation in the $^3H_e$ or $^3H$ and $[\bar{B}\bar{B}^*]_0^1$ in the tri-meson.

- np short-rang correlation, Jefferson Lab, Nature 609, 41 (2022).
  - $IJ = 01^+$, np$\gg$nn or pp, partly overlapped, quark and gluon structures
- $[\bar{B}\bar{B}^*]_0^1$ short-rang correlation
  - $IJ = 01^+$, $[\bar{B}\bar{B}^*]_0^1(80\%) \gg [\bar{B}\bar{B}^*]_1^1$ (6%) and $[\bar{B}^*\bar{B}^*]_1^0$ (14%)
  - Partly overlapped, quark and gluon structures, such as quark delocalization effect.

# 5.6 Correlations of meson pairs

- Two-body bound state of $[\bar{B}_1\bar{B}_2^*]_0^1$ and $\bar{B}_3^*$

$$\left[[\bar{B}_1\bar{B}_2^*]_0^1\bar{B}_3^*\right]_{\frac{1}{2}}^0=\left[[\bar{B}_1\bar{B}_3^*]_0^1\bar{B}_2^*\right]_{\frac{1}{2}}^0+\left[\bar{B}_1[\bar{B}_2^*\bar{B}_3^*]_0^{0,2}\right]_{\frac{1}{2}}^0,\ \Delta E{=}-0.2\text{ MeV}$$

- Three-body bound states of $\bar{B}_1\bar{B}_2^*\bar{B}_3^*$, $[\bar{B}_1\bar{B}_2^*]_1^1$ and $[\bar{B}_2^*\bar{B}_3^*]_1^0$ are unbound.

$$\left[[\bar{B}_1\bar{B}_2^*]_1^1\bar{B}_3^*\right]_{\frac{1}{2}}^0=\frac{1}{\sqrt{3}}\left(\left[[\bar{B}_1\bar{B}_3^*]_0^1\bar{B}_2^*\right]_{\frac{1}{2}}^0+\left[\bar{B}_1[\bar{B}_2^*\bar{B}_3^*]_0^{0,2}\right]_{\frac{1}{2}}^0\right),\ \Delta E{=}-0.4\text{ MeV}$$

$$\left[\bar{B}_1[\bar{B}_2^*\bar{B}_3^*]_1^0\right]_{\frac{1}{2}}^0=\frac{1}{\sqrt{3}}\left(\left[[\bar{B}_1\bar{B}_2^*]_0^1\bar{B}_3^*\right]_{\frac{1}{2}}^0+\left[[\bar{B}_1\bar{B}_3^*]_0^1\bar{B}_2^*\right]_{\frac{1}{2}}^0\right),\ \Delta E{=}-0.7\text{ MeV}$$

$$\left[[\bar{B}_1\bar{B}_2^*]_1^1\bar{B}_3^*\right]_{\frac{1}{2}}^1=\frac{1}{\sqrt{3}}\left(\left[[\bar{B}_1\bar{B}_2^*]_0^1\bar{B}_3^*\right]_{\frac{1}{2}}^1+\left[\bar{B}_1[\bar{B}_2^*\bar{B}_3^*]_0^1\right]_{\frac{1}{2}}^1\right),\ \Delta E{=}-0.6\text{ MeV}.$$

- Binding mechanism: $[\bar{B}\bar{B}^*]_0^1$, the orbital components of the pair $[\bar{B}\bar{B}^*]_0^1$ encompasses not only the ground state but also angular excitations

- Summary
  - Four bound isospin-spin configurations, $\Delta E \in (-1.0, 0)$ MeV.
    $^3H$ or $^3H_e$-like two-body state: $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^0$
    Loose three-body state: $\left[[\bar{B}\bar{B}^*]_1^1\bar{B}^*\right]_{\frac{1}{2}}^0$, $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^0\right]_{\frac{1}{2}}^0$, and $\left[[\bar{B}\bar{B}^*]_1^1\bar{B}^*\right]_{\frac{1}{2}}^1$
  - After coupling of the configurations with $\frac{1}{2}0^-$, $\Delta E = -1.5$ MeV
    $\left[[\bar{B}\bar{B}^*]_0^1\bar{B}^*\right]_{\frac{1}{2}}^0$ (80%), $\left[\bar{B}[\bar{B}^*\bar{B}^*]_1^0\right]_{\frac{1}{2}}^0$ (14%), $\left[[\bar{B}\bar{B}^*]_1^1\bar{B}^*\right]_{\frac{1}{2}}^0$ (6%).
  - Strong correlation meson pair $[\bar{B}\bar{B}^*]_0^1$, responsible for the binding mechanism of trimeson state $\bar{B}\bar{B}^*\bar{B}^*$.

Thank you for your attention!