Doubly heavy tetraquark states with various configurations

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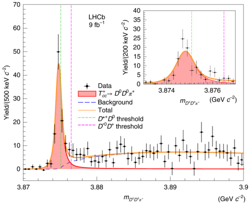
Introduction

Experimentally,

- The first doubly charmed tetraquark $T_{cc}(3875)^+$ discovered in the $D^0D^0\pi^+$ channel [LHCb:2021vvq]

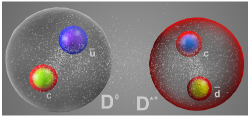
$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) \sim -300\text{keV}$$

- $X(3872) \to$ XYZ states
  $T_{cc}(3875)^+ \to$ doubly heavy exotic states?



Theoretically,

- $T_{cc}^+(3875)$: natural interpretation as the $D^{*+}D^0$ molecular state
- Other doubly heavy tetraquarks: compact tetraquark / hadronic molecule



Quark Model + Effective Few-Body Methods:

A possible framework for unified descriptions

Constituent Quark Model

- Success in conventional hadrons → nontrivial extension to multiquark
  - Richer color structure

$$\mathbf{3}_q \otimes \mathbf{3}_q \otimes \bar{\mathbf{3}}_{\bar{q}} \otimes \bar{\mathbf{3}}_{\bar{q}} \to \mathbf{1}_{[qq]_{\bar{3}}[\bar{q}\bar{q}]_3} \oplus \mathbf{1}_{[qq]_6[\bar{q}\bar{q}]_{\bar{6}}}$$

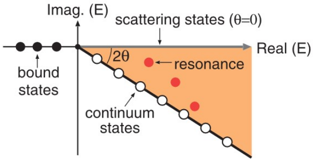
  - Complicated few-body calculations
- Do not make *a priori* assumptions about clustering of quarks → various types of multiquark states in a unified framework
- AL1 model: one-gluon-exchange + linear confinement [Silvestre-Brac:1996myf]

$$V_{ij} = -\frac{3}{16}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left(-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda + \frac{8\pi\kappa'}{3m_i m_j} \frac{\exp\left(-r_{ij}^2/r_0^2\right)}{\pi^{3/2} r_0^3} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \right)$$

- Theoretical uncertainties $\sim$ tens of MeV

Complex Scaling Method

- Resonant states: poles at $E = m - i\frac{\Gamma}{2}$, not square-integrable
- Previous works: states above thresholds with real energies — possible false states
- Complex scaling method: transform the asymptotic behavior of wave functions by analytical continuation [Aguilar:1971ve, Balslev:1971vb]



$$U(\theta)\boldsymbol{r} = \boldsymbol{r}e^{i\theta} \quad U(\theta)\boldsymbol{p} = \boldsymbol{p}e^{-i\theta}$$

$$H(\theta) = \sum_{i=1}^{4}\left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i}\right) + \sum_{i<j=1}^{4} V_{ij}(r_{ij}e^{i\theta})$$

$$H(\theta)\Psi(\theta) = E(\theta)\Psi(\theta)$$

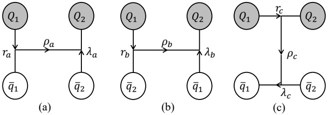
Solve bound, resonant and continuum states simultaneously!

Wave Function

- Color-spin wave functions: complete set

$$\chi^{s_1,s_2,S}_{\bar{3}_c\otimes 3_c} = \left[(Q_1Q_2)^{s_1}_{\bar{3}_c}\,(\bar{q}_1\bar{q}_2)^{s_2}_{3_c}\right]^{S}_{1_c} \qquad \chi^{s_1,s_2,S}_{6_c\otimes\bar{6}_c} = \left[(Q_1Q_2)^{s_1}_{6_c}\,(\bar{q}_1\bar{q}_2)^{s_2}_{\bar{6}_c}\right]^{S}_{1_c}$$

- Spatial wave functions: Gaussian expansion method [Hiyama:2003cu]



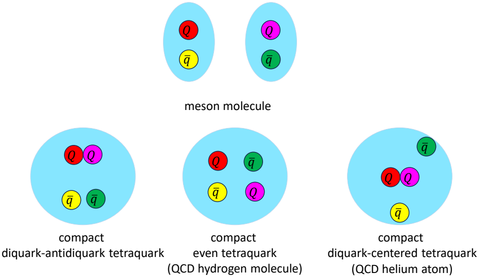
$$\Phi^{(\mathrm{jac})}_{n_1,n_2,n_3} = \phi_{n_1}(r_{jac})\phi_{n_2}(\lambda_{jac})\phi_{n_3}(\rho_{jac}) \qquad \phi_{n_i}(r) = N_{n_i} e^{-\nu_{n_i} r^2}$$

  - dimeson (a,b) and diquark-antidiquark (c) configurations
  - consistently describe molecular, compact states and their couplings

- $\sim 10^4$ bases to solve four-body Schrödinger equation with strong correlations in color, spin and spatial d.o.f.

Spatial Structures

- Meson molecules & compact tetraquarks: different internal structures and binding mechanisms
- 3 typical types of compact tetraquarks expected in $QQ\bar{q}\bar{q}$
- Identification by rms radius



$$r_{ij}^{\rm rms} \equiv {\rm Re}\left[\sqrt{\frac{\langle \Psi_{\rm nA}(\theta)|r_{ij}^2 e^{2i\theta}|\Psi_{\rm nA}(\theta)\rangle}{\langle \Psi_{\rm nA}(\theta)|\Psi_{\rm nA}(\theta)\rangle}}\right]$$

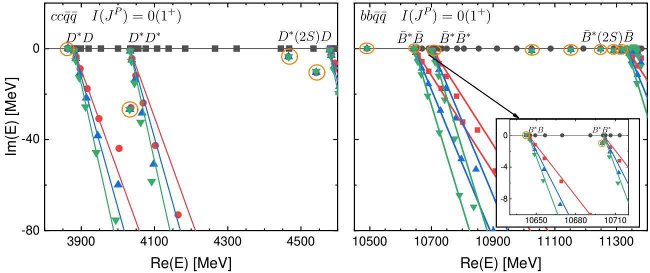
A novel definition to avoid ambiguities from antisymmertrization

Bound states

System	$I(J^P)$	M	ΔE	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Configuration
$cc\bar{q}\bar{q}$	$0(1^+)$	3864	-14	58%	42%	0.71	0.64	1.13	1.16	1.02	1.22	M. (D^*D)
$bb\bar{q}\bar{q}$	$0(1^+)$	10642	-1	33%	67%	0.66	0.63	2.06	2.07	1.98	2.15	M. ($\bar{B}^*\bar{B}$)
$bc\bar{q}\bar{q}$	$0(2^+)$	7363	-3	27%	73%	0.66	0.70	1.95	1.97	1.86	2.05	M. ($\bar{B}^*D^*$)
$bc\bar{q}\bar{q}$	$0(0^+)$	7129	-26	48%	52%	0.64	0.64	0.91	0.95	0.76	1.03	C.E.
	$0(1^+)$	7185	-27	60%	40%	0.67	0.66	0.88	0.93	0.71	1.00	C.E.
$bb\bar{q}\bar{q}$	$0(1^+)$	10491	-153	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	10647	-64	91%	9%	0.56	0.67	0.71	0.61	0.36	0.76	C.DC.

- D^*D molecule with $\Delta E = -14$ MeV as candidate for $T_{cc}(3875)^+$
- Bound states with various configurations
  - Molecular (M.) shallow bound state
  - Compact even tetraquark (C.E.)
  - Compact diquark-centered tetraquark (C.DC.) deeply bound state
- No isovector bound state → importance of “good” antidiquark $(\bar{q}\bar{q})^{S=0,I=0}_{3_c}$

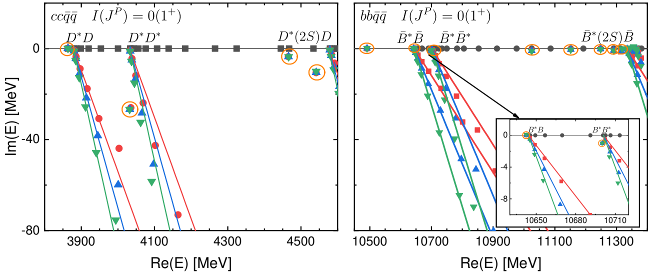
Resonant states: isoscalar $QQ\bar{q}\bar{q}$



- Lowest resonant states near the D^*D^* and $\bar{B}^*\bar{B}^*$ thresholds
- First T_{bb} resonance as $\bar{B}^*\bar{B}^*$ molecule

$M-i\Gamma/2$	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Config.
10491	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
10642	33%	67%	0.66	0.63	2.06	2.07	1.98	2.15	M. ($\bar{B}^*\bar{B}$)
$10700-1i$	44%	56%	0.67	0.67	1.96	1.96	1.88	2.02	M. ($\bar{B}^*\bar{B}^*$)
$11025-1i$	98%	2%	1.08	1.07	1.08	1.08	0.33	0.83	C.DC.

Resonant states: isoscalar $QQ\bar{q}\bar{q}$



- Second T_{bb} resonance as radial excitation of the deeply bound T_{bb} state
- More resonant states in higher energy region and $bc\bar{q}\bar{q}$ system

$M-i\Gamma/2$	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Config.
10491	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
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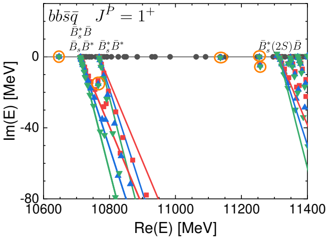
$10700-1i$	44%	56%	0.67	0.67	1.96	1.96	1.88	2.02	M. ($\bar{B}^*\bar{B}^*$)
$11025-1i$	98%	2%	1.08	1.07	1.08	1.08	0.33	0.83	C.DC.

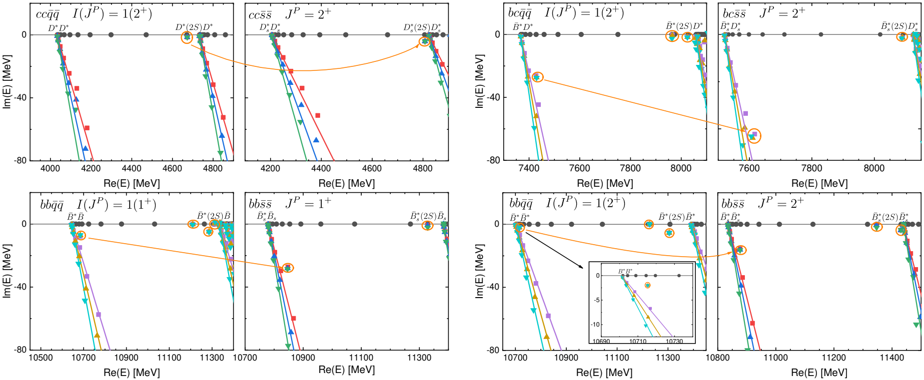
Resonant states: $QQ\bar{s}\bar{q}$



$M - i\Gamma/2$	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{s}}$	$r^{\rm rms}_{Q_2\bar{q}}$	$r^{\rm rms}_{Q_1\bar{q}}$	$r^{\rm rms}_{Q_2\bar{s}}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{s}\bar{q}}$	Config.
10647	91%	9%	0.56	0.67	0.71	0.61	0.36	0.76	C.DC.
$10766 - 16i$	92%	8%	0.58	0.71	0.74	0.63	0.36	0.85	C.DC.

- Lowest $T_{bb\bar{s}}$ resonant state near $\bar{B}_s^*\bar{B}^*$ threshold
- Similar to $bb\bar{q}\bar{q}$ system: the resonant state is the radial excitation in the light d.o.f of the deeply bound state

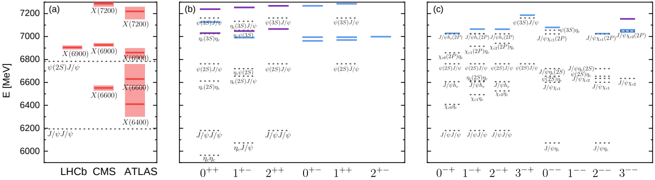
Resonant states: isovector $QQ\bar{q}\bar{q}$ and $QQ\bar{s}\bar{s}$



- A series of compact tetraquark resonant states
- Resemblance between energy spectra of isovector $QQ\bar{q}\bar{q}$ and $QQ\bar{s}\bar{s}$ systems $\rightarrow$ same internal symmetries
- Mass differences between $QQ\bar{q}\bar{q}$ resonances and their strange partners $\sim$ 150 MeV

Other systems

- Successfully describe $T^*_{cs0}(2870)^0$ [Chen:2023syh] , $X(6900)$ and $X(7200)$ [Wu:2024euj,Wu:2024hrv]
- Both 0^{++} and 2^{++} candidates for $X(6900)$ and $X(7200)$ are found
- All $cc\bar{c}\bar{c}$ resonances are compact tetraquark states



		BW_1	BW_2	BW_3
Interference	m (MeV)	$6593\,^{+15}_{-14} \pm 25$	$6847\,^{+10}_{-10} \pm 15$	$7173\,^{+9}_{-10} \pm 13$
(Run 2+Run 3)	Γ (MeV)	$446\,^{+66}_{-54} \pm 87$	$135\,^{+16}_{-14} \pm 14$	$73\,^{+18}_{-15} \pm 10$
Interference	m (MeV)	$6638\,^{+43+16}_{-38-31}$	$6847\,^{+44+48}_{-28-20}$	$7134\,^{+48+41}_{-25-15}$
(Run 2 [12])	Γ (MeV)	$440\,^{+230+110}_{-200-240}$	$191\,^{+66+25}_{-49-17}$	$97\,^{+40+29}_{-29-26}$

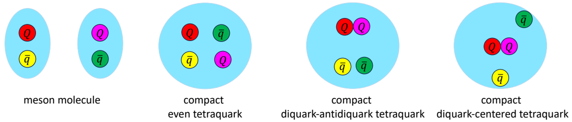
	J^{PC}	0^{++}	0^{++}	2^{++}	2^{++}
Quark Model	m (MeV)	6978	7167	7017	7204
	Γ (MeV)	72	38	78	58

Open questions for $cc\bar{c}\bar{c}$:

- If experimental states are determined to be 2^{++}, where are 0^{++} states?
- Where is $X(6600)$? It's too wide to be calculated.

Summary

- Framework: QM + CSM + GEM
- Various types of tetraquark states in a unfied framework



- A series of doubly heavy tetraquark bound and resonant states
  - D^*D molecular shallow bound state as candidate for $T_{cc}^+(3875)$
  - Bound states in $bb\bar{q}\bar{q}, bc\bar{q}\bar{q}, bb\bar{s}\bar{q}$ systems
  - Resonant states near D^*D^* and $\bar{B}^*\bar{B}^*$ thresholds, $\cdots$
- The capability and limit of QM are to be explored with comprehensive theoretical calculations.

Thanks for your listening

Backup: RMS Radii

- Molecular or compact states: rms radii
- Ambiguities from antisymmertrization:

$$\Psi = \mathcal{A}\Psi_{\rm nA} = \mathcal{A}\sum_{s_1\geq s_2}[(Q_1\bar{q}_1)^{s_1}_{1_c}(Q_2\bar{q}_2)^{s_2}_{1_c}]^S_{1_c}\otimes|\psi_1^{s_1s_2}\rangle$$

$$= \sum_{s_1\geq s_2}\Big([(Q_1\bar{q}_1)^{s_1}_{1_c}(Q_2\bar{q}_2)^{s_2}_{1_c}]^S_{1_c}\otimes|\psi_1^{s_1s_2}\rangle + [(Q_1\bar{q}_1)^{s_2}_{1_c}(Q_2\bar{q}_2)^{s_1}_{1_c}]^S_{1_c}\otimes|\psi_2^{s_1s_2}\rangle$$

$$+[(Q_1\bar{q}_2)^{s_1}_{1_c}(Q_2\bar{q}_1)^{s_2}_{1_c}]^S_{1_c}\otimes|\psi_3^{s_1s_2}\rangle + [(Q_1\bar{q}_2)^{s_2}_{1_c}(Q_2\bar{q}_1)^{s_1}_{1_c}]^S_{1_c}\otimes|\psi_4^{s_1s_2}\rangle\Big)$$

The quarks belong to both constituent mesons, $\langle\Psi|r^2_{Q\bar{q}}|\Psi\rangle$ cannot reflect the size of the constituent meson.

- Define rms radii using $\Psi_{\rm nA}$

$$r_{ij}^{\rm rms} \equiv {\rm Re}\left[\sqrt{\frac{\langle\Psi_{\rm nA}(\theta)|r_{ij}^2 e^{2i\theta}|\Psi_{\rm nA}(\theta)\rangle}{\langle\Psi_{\rm nA}(\theta)|\Psi_{\rm nA}(\theta)\rangle}}\right]$$