Theoretical study of hadronic molecular states with quark composition $bc\bar{s}\bar{q}$

刘文颖 东南大学

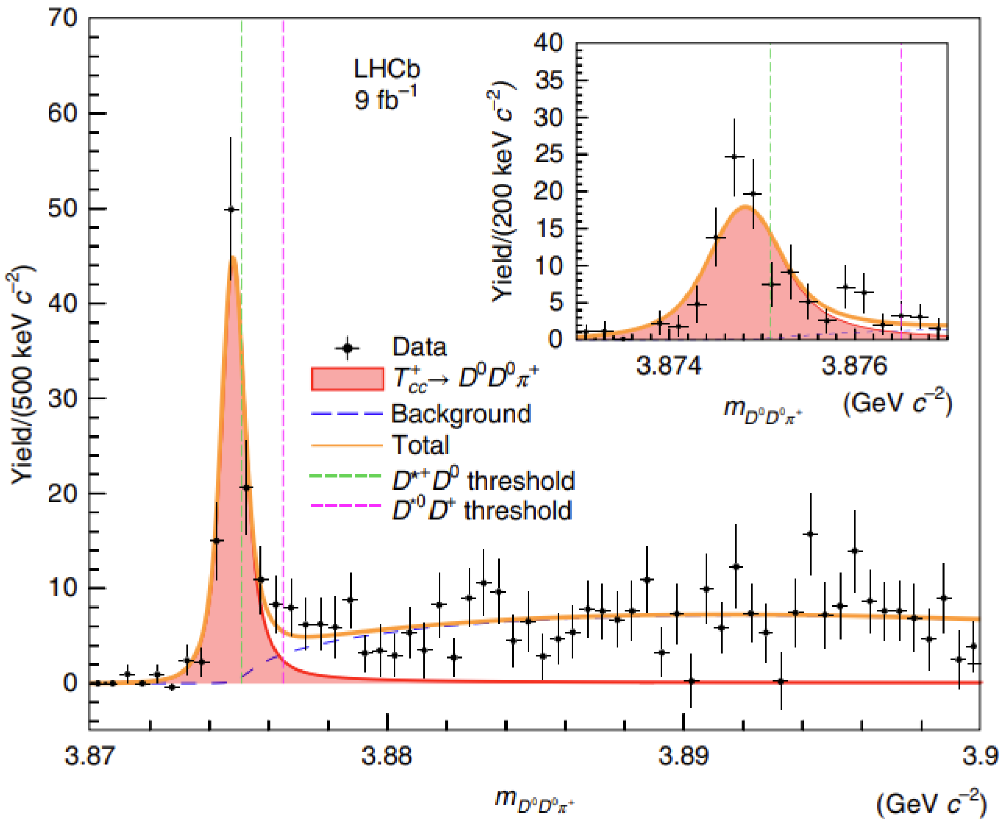
合作：陈华星，王恩，Eulogio Oset

Contents

- Motivation
- Coupled Channel Bethe-Salpeter Equation
- Numerical Results
- Summary

Contents

The double charmed tetraquark state T_{cc}



The distribution of the $D_0 D_0 \pi^+$ mass

- The BW mass and width of T_{cc}

$$M_{\rm BW} = M_{D^{*+}} + M_{D^0} - (273 \pm 61 \pm 5^{+11}_{-14})\ \text{keV},$$
$$\Gamma_{\rm BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}.$$

- Considering the experimental resolution produces the resonance

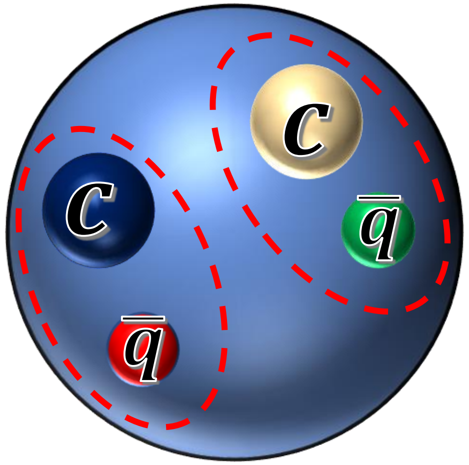
$$m_{\rm pole} = M_{D^{*+}} + M_{D^0} - (360 \pm 40^{+0}_{-4})\ \text{keV},$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\ \text{keV}.$$

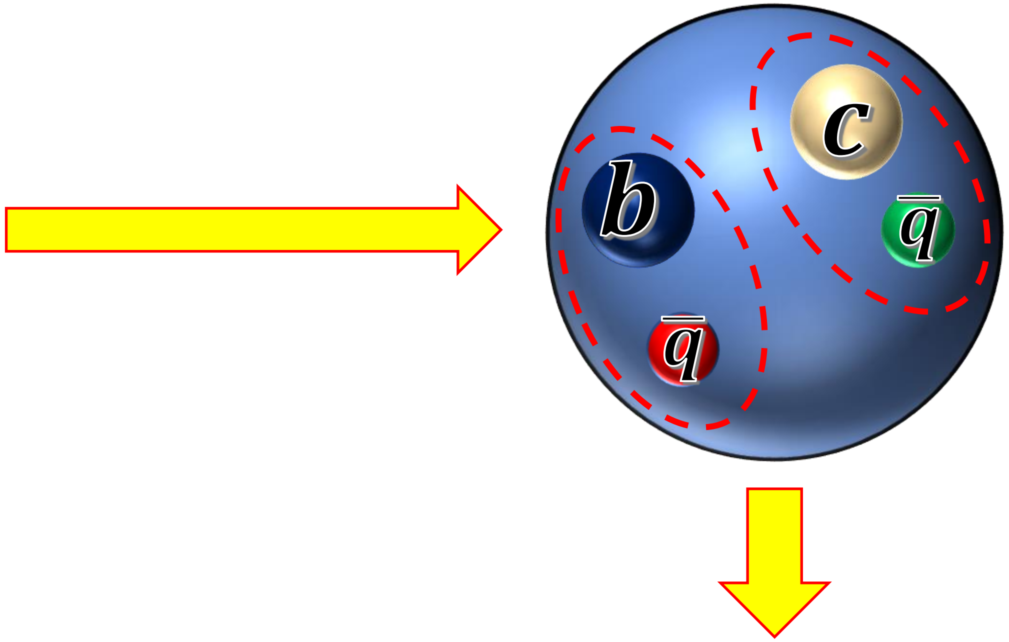
- quantum numbers, $(I)J^P = (0)1^+$

LHCb, Nat. Phys. 18, 751 (2022).

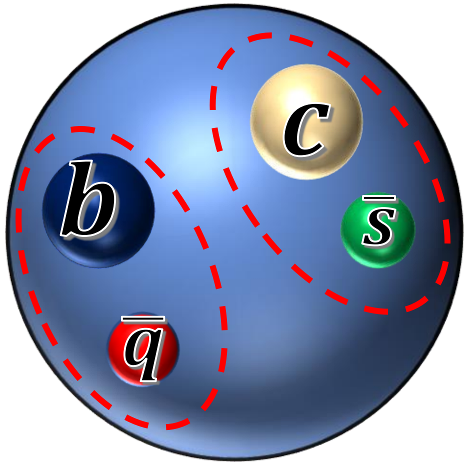
The possible heavy quark partner of Z_{cs}/T_{cc} states

T_{cc} :$cc\bar{q}\bar{q}$





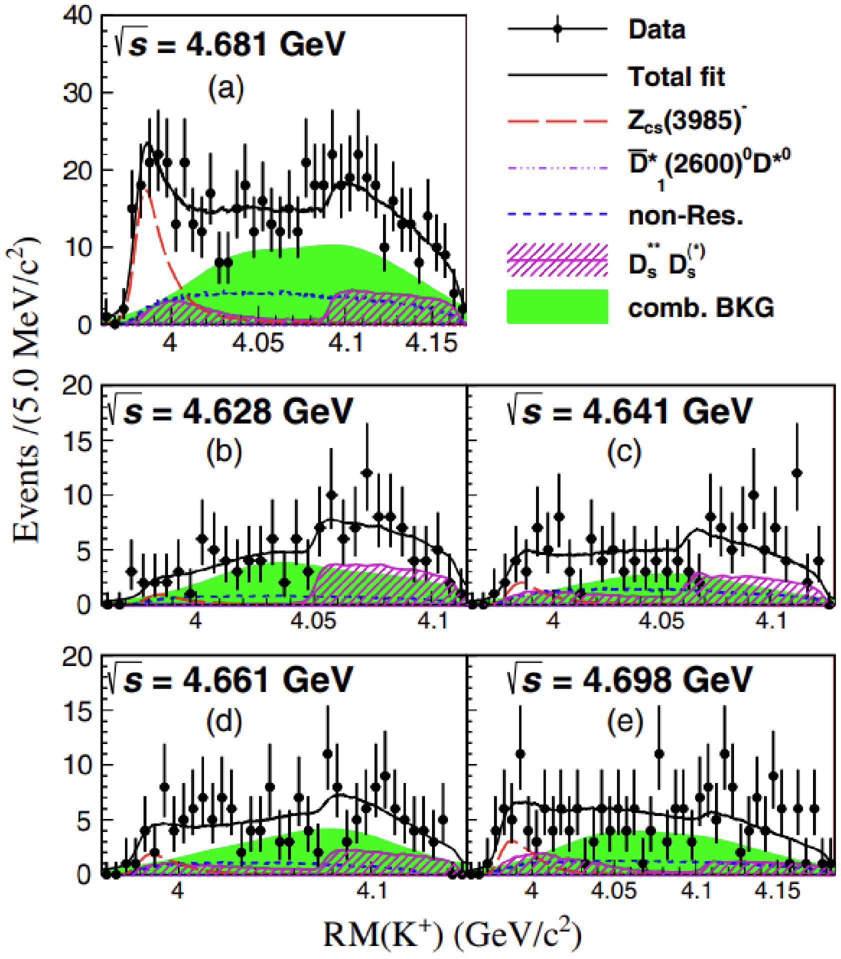
T_{bc} :$bc\bar{q}\bar{q}$



$T_{bc\bar{s}}$:$bc\bar{s}\bar{q}$

- In the heavy quark limit, the b/c quark could be considered as a rest color resource, so the two different quark components could have similar properties.

The hidden-charm tetraquark state $Z_{cs}(3985)$



K^+ recoil-mass spectra in $e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$

BESIII, Phys. Rev. Lett. 126, 102001 (2021).

- In 2021, $Z_{cs}(3985)$, the possible strange partner of $Z_c(3900)$ was reported by BESIII Collaboration.

$$M_{Z_{cs}} = 3982.5^{+1.8}_{-2.6} \pm 2.1$$

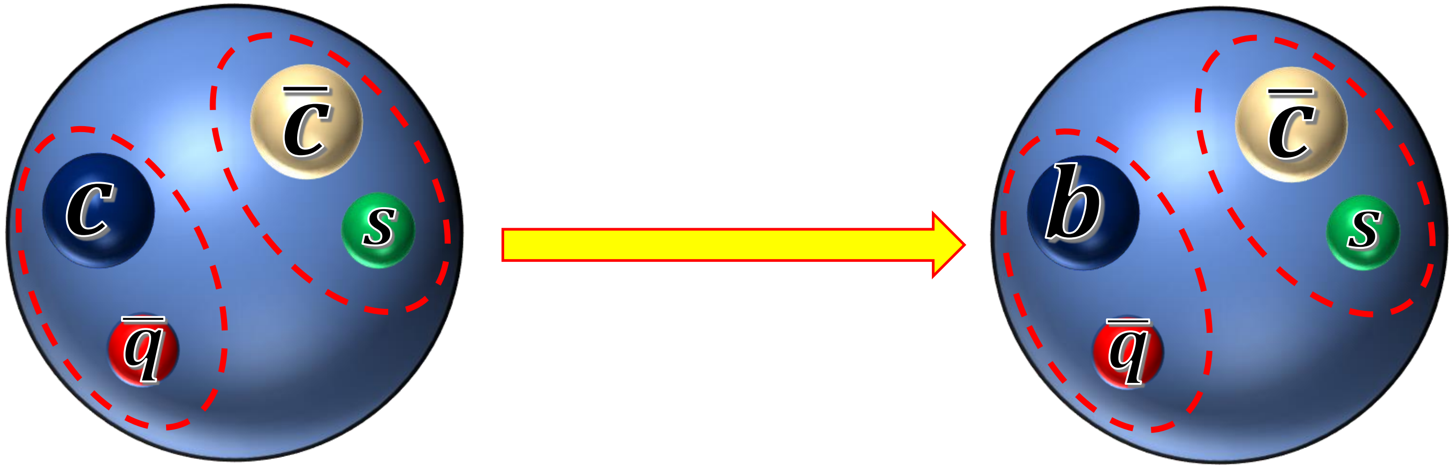
$$\Gamma_{Z_{cs}} = 12.8^{+5.3}_{-4.4} \pm 3.0$$

- $M_{Z_{cs}} - M_{D_s^*} - M_{\bar{D}} \sim 2$ MeV

Is $Z_{cs}(3985)$ a molecular state ?

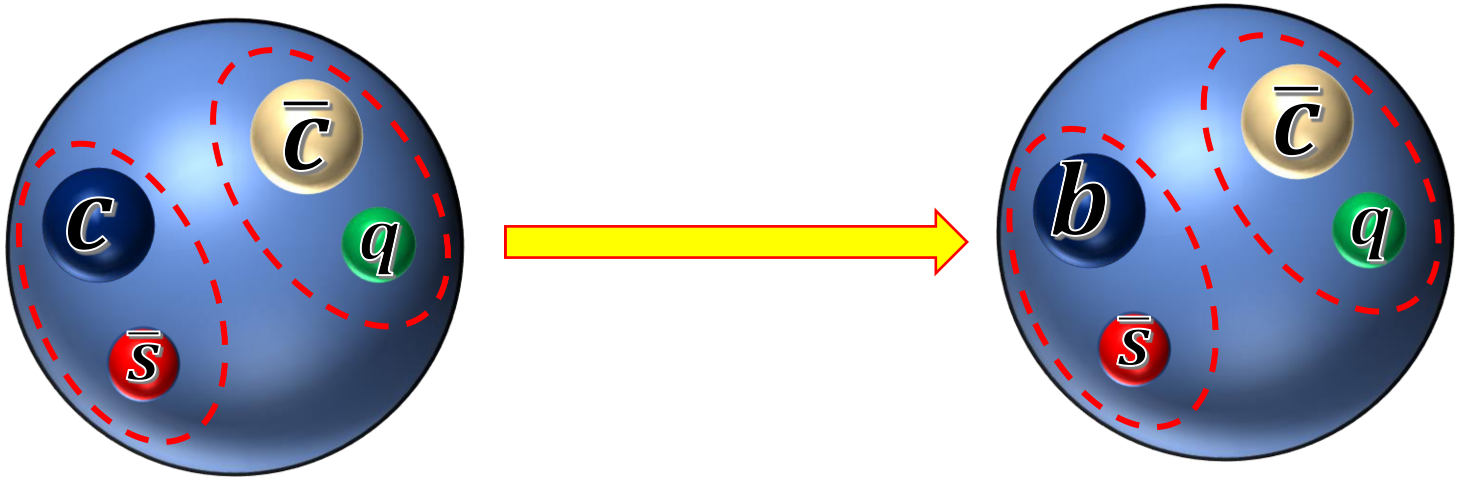
The possible heavy quark partner of Z_{cs}/T_{cc} states

Z_{cs}:$c\bar{c}s\bar{q}$



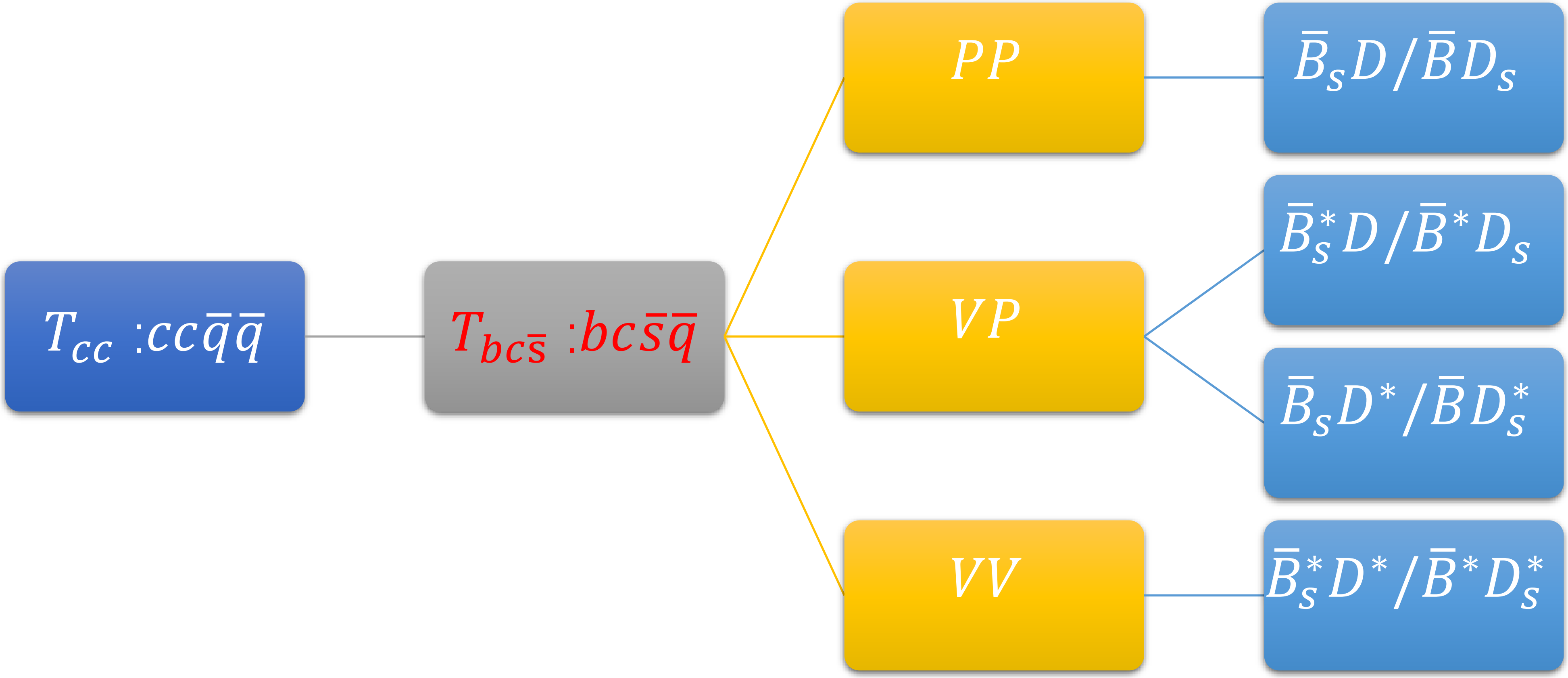
$T_{b\bar{c}s}$:$b\bar{c}s\bar{q}$

$\bar{Z}_{cs}$:$c\bar{c}q\bar{s}$



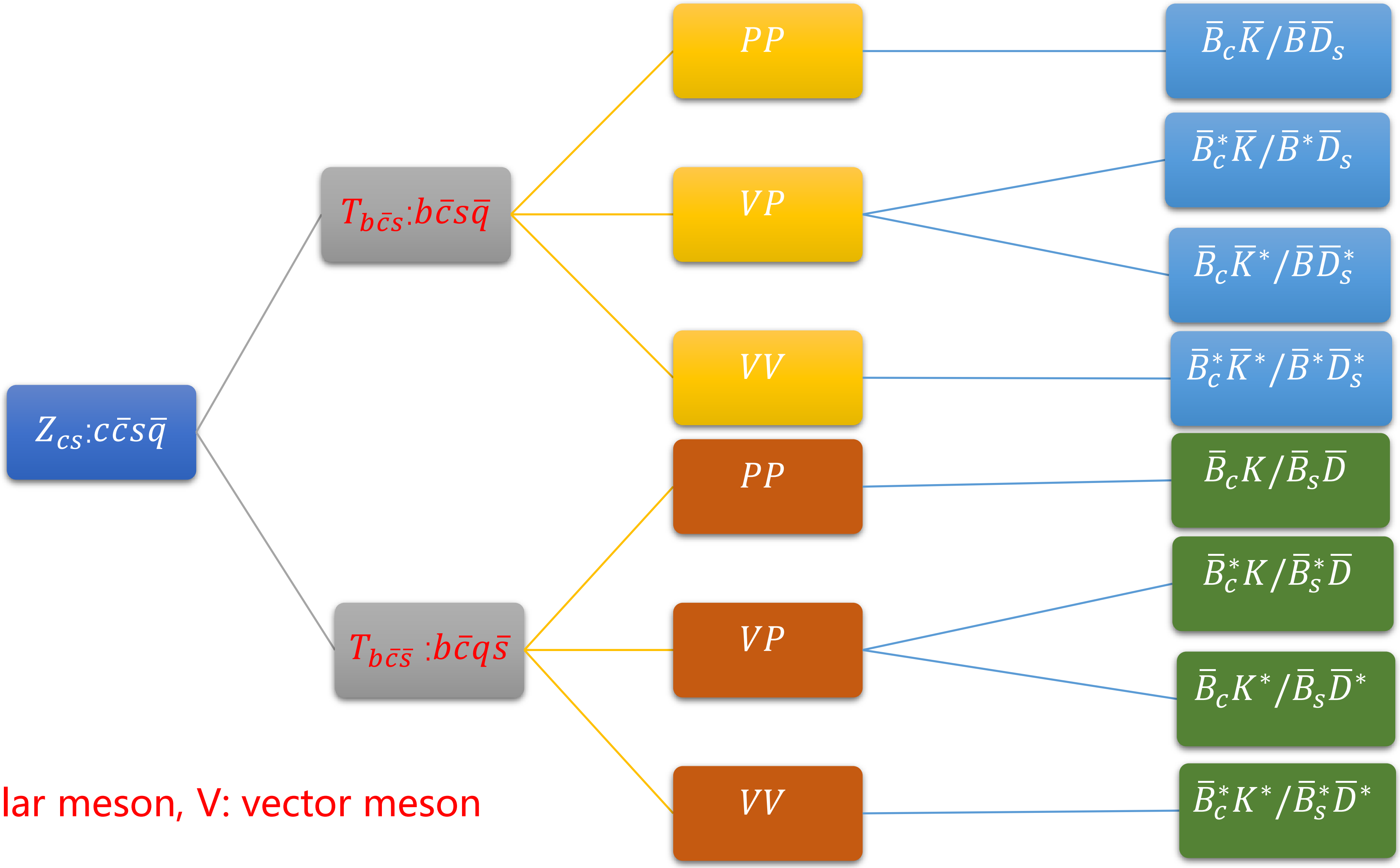
$T_{b\bar{c}\bar{s}}$:$b\bar{c}q\bar{s}$

The possible heavy quark partner of T_{cc} states



P: pseudoscalar meson, V: vector meson

The possible heavy quark partner of Z_{cs} states



P: pseudoscalar meson, V: vector meson

Contents

Workflow

output

Unitary scattering amplitude

T

Bethe-Salpeter equation

input

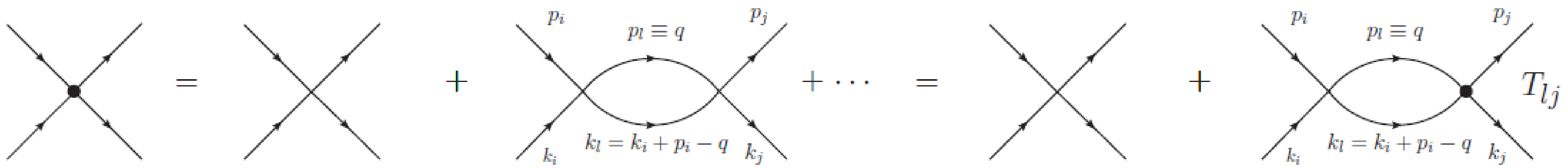
Tree level amplitude

V

Meson-meson loop function

G

BS-eq



$$\boxed{T_{ij} \;=\; V_{ij} \;+\; \sum_l V_{il} G_{ll} V_{lj} \;+\cdots\;=\; V_{ij} \;+\; V_{il} G_{ll} T_{lj}}$$

Bethe-Salpeter equation in coupled channels

$$T = V + VGT$$

$$\Downarrow$$

$$T = \frac{V}{1 - VG}$$

Loop function

- Bethe-Salpeter equation:

$$T_{PP/VP/VV}(s) = \frac{V_{PP/VP/VV}(s)}{1 - V_{PP/VP/VV}(s)G(s)}$$

- Loop function:

$$G_{ii}(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(p-q)^2 - m_2^2 + i\epsilon}.$$

Cutoff regularization

$$G_{ii}(s) = \int_0^{q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

q_{max} is generally taken to be 400-600 MeV, when dealing with hidden-charm molecular states.

Loop function

- Looking for poles s_p of T on complex plane,

Mass

$$s_p = a + b\, i$$

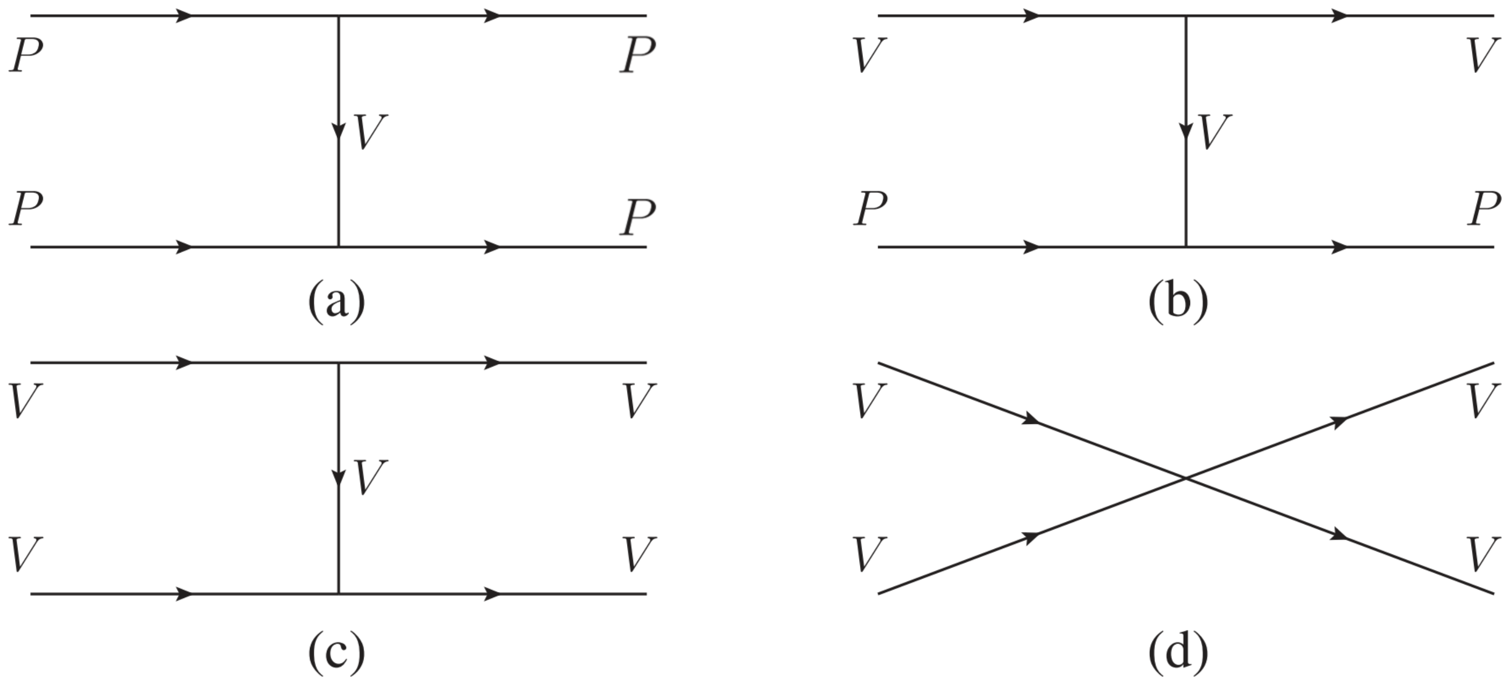
Width

$$G_{ii}^{II}(s) = G_{ii}(s) + i\frac{k}{4\pi\sqrt{s}}, \qquad k(s) = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}/(2\sqrt{s})$$

- Coupling constants are defined as the residue of the amplitude at the poles:

$$T_{ij}(s) = \frac{g_i g_j}{s - s_p^2} \qquad g_i^2 = \lim_{\sqrt{s}\to s_p}(s - s_p^2)T_{ii}(s)$$

Vector meson exchange formalism



Feynman diagrams for the interactions between charmed (-strange) mesons and bottom(-strange) mesons

Vertices involving P and V within the Local Hidden Gauge

$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\mu P] V^\mu \rangle,$$

$$\mathcal{L}_{VVV} = ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V^\mu V_\mu) V^\nu \rangle,$$

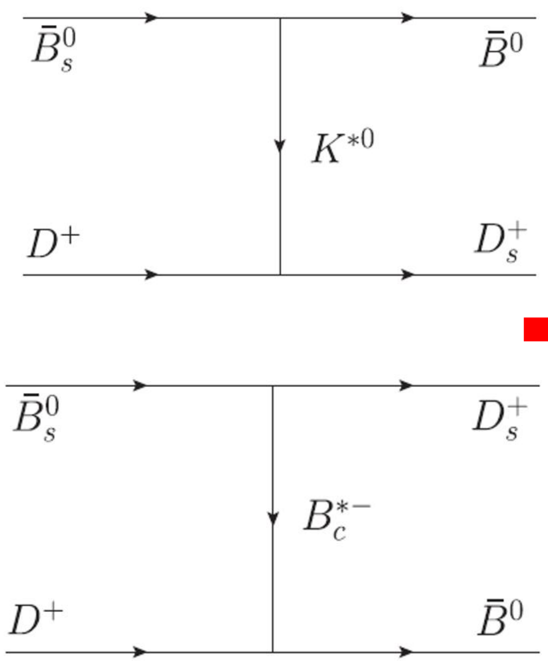
$$\mathcal{L}_{VVVV} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle.$$

P: pseudoscalar meson, V: vector meson

P-P interaction in $bc\bar{s}\bar{q}$ system

- Two coupled channels in $bc\bar{s}\bar{q}$ system,

$$\bar{B}_s^0 D^+, \qquad \bar{B}^0 D_s^+ .$$



Heavy vector exchange negligible

Interaction potential:

$$\boxed{V_{PP}(s) = C_{PP} \times g^2 (p_1 + p_3)(p_2 + p_4),}$$

Coefficient matrix:

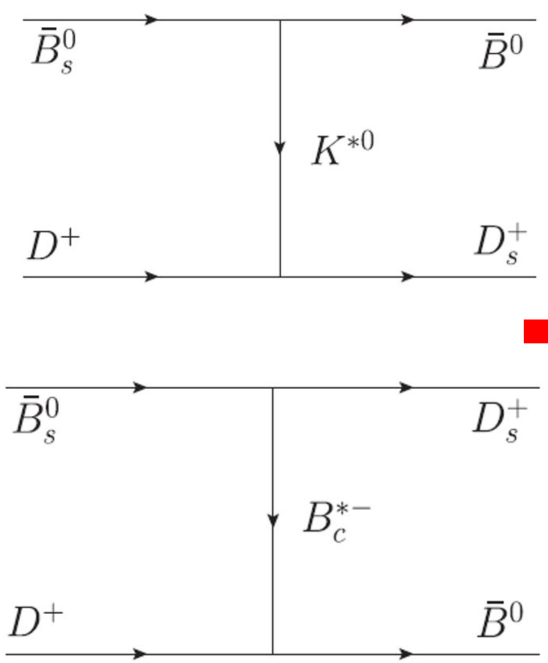
$$C_{PP} = \left(\begin{array}{c|cc} J=0 & \bar{B}_s^0 D^+ & \bar{B}^0 D_s^+ \\ \hline \bar{B}_s^0 D^+ & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^+ & \frac{1}{m_{K^*}^2} & 0 \end{array} \right),$$

C_{PP}: + repulsive, - attractive

P-P interaction in $bc\bar{s}\bar{q}$ system

- Two coupled channels in $bc\bar{s}\bar{q}$ system,

$$\bar{B}_s^0 D^+, \qquad \bar{B}^0 D_s^+ .$$



Heavy vector exchange negligible

Interaction potential:

$$\boxed{V_{PP}(s) = C_{PP} \times g^2 (p_1 + p_3)(p_2 + p_4),}$$

Coefficient matrix:

$$C_{PP} = \left(\begin{array}{c|cc} J=0 & \bar{B}_s^0 D^+ & \bar{B}^0 D_s^+ \\ \hline \bar{B}_s^0 D^+ & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^+ & \frac{1}{m_{K^*}^2} & 0 \end{array} \right),$$

repulsive　　vanished

The formation of molecular states generally requires a diagonal attractive potential

P-P interaction in $bc\bar{s}\bar{q}$ system

Mass of coupled channels in $bc\bar{s}\bar{q}$ system

$\bar{B}_s^0 D^+$	$\bar{B}^0 D_s^+$
7236.6	7248.0

Similar to forming an isospin channel !

- mixing of the two channels

$$\boxed{\begin{aligned} |(\bar{B}D)_s^+; J=0\rangle &= \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^+\rangle_{J=0} + |\bar{B}^0 D_s^+\rangle_{J=0}), \\ |(\bar{B}D)_s^-; J=0\rangle &= \frac{1}{\sqrt{2}}(|\bar{B}_s^0 D^+\rangle_{J=0} - |\bar{B}^0 D_s^+\rangle_{J=0}), \end{aligned}}$$

$$C_{PP} = \left(\begin{array}{c|cc} J=0 & \bar{B}_s^0 D^+ & \bar{B}^0 D_s^+ \\ \hline \bar{B}_s^0 D^+ & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^+ & \frac{1}{m_{K^*}^2} & 0 \end{array}\right) \longrightarrow C'_{PP} = \left(\begin{array}{c|cc} J=0 & (\bar{B}D)_s^+ & (\bar{B}D)_s^- \\ \hline (\bar{B}D)_s^+ & \frac{1}{m_{K^*}^2} & 0 \\ (\bar{B}D)_s^- & 0 & -\frac{1}{m_{K^*}^2} \end{array}\right)$$

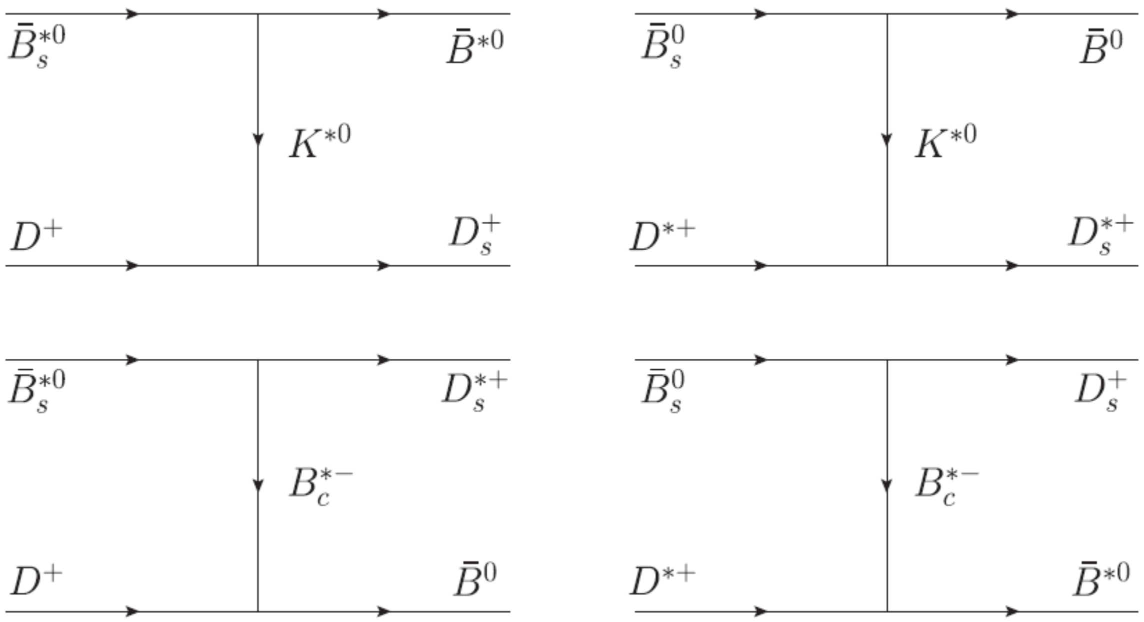
Repulsive

Attractive

V-P interaction in $bc\bar{s}\bar{q}$ system

- Four coupled channels in $bc\bar{s}\bar{q}$ system,

$$\boxed{\bar{B}_s^{*0}D^+, \qquad \bar{B}^{*0}D_s^+, \qquad \bar{B}_s^0 D^{*+}, \qquad \bar{B}^{*0}D_s^{*+}}$$



Vector meson exchange mechanism in VP sector

$$|(\bar{B}^*D)_s^+; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^{*0}D^+\rangle_{J=1} + |\bar{B}^{*0}D_s^+\rangle_{J=1})$$

$$|(\bar{B}^*D)_s^-; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^{*0}D^+\rangle_{J=1} - |\bar{B}^{*0}D_s^+\rangle_{J=1})$$

$$|(\bar{B}D^*)_s^+; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0D^{*+}\rangle_{J=1} + |\bar{B}^0D_s^{*+}\rangle_{J=1})$$

$$|(\bar{B}D^*)_s^-; J=1\rangle = \frac{1}{\sqrt{2}}(|\bar{B}_s^0D^{*+}\rangle_{J=1} - |\bar{B}^0D_s^{*+}\rangle_{J=1})$$

Four orthogonal bases in VP sector

V-P interaction in $bc\bar{s}\bar{q}$ system

$$C_{VP} = \begin{array}{c|cccc} J=1 & \bar{B}_s^{*0}D^+ & \bar{B}^{*0}D_s^+ & \bar{B}_s^0 D^{*+} & \bar{B}^0 D_s^{*+} \\ \hline \bar{B}_s^{*0}D^+ & 0 & \frac{1}{m_{K^*}^2} & 0 & 0 \\ \bar{B}^{*0}D_s^+ & \frac{1}{m_{K^*}^2} & 0 & 0 & 0 \\ \bar{B}_s^0 D^{*+} & 0 & 0 & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^0 D_s^{*+} & 0 & 0 & \frac{1}{m_{K^*}^2} & 0 \end{array}$$

$\Longrightarrow$

$$C'_{VP} = \begin{array}{c|cccc} J=1 & (\bar{B}^*D)_s^+ & (\bar{B}^*D)_s^- & (\bar{B}D^*)_s^+ & (\bar{B}D^*)_s^- \\ \hline (\bar{B}^*D)_s^+ & \frac{1}{m_{K^*}^2} & 0 & 0 & 0 \\ (\bar{B}^*D)_s^- & 0 & -\frac{1}{m_{K^*}^2} & 0 & 0 \\ (\bar{B}D^*)_s^+ & 0 & 0 & \frac{1}{m_{K^*}^2} & 0 \\ (\bar{B}D^*)_s^- & 0 & 0 & 0 & -\frac{1}{m_{K^*}^2} \end{array}$$

No diagonal attraction potential !

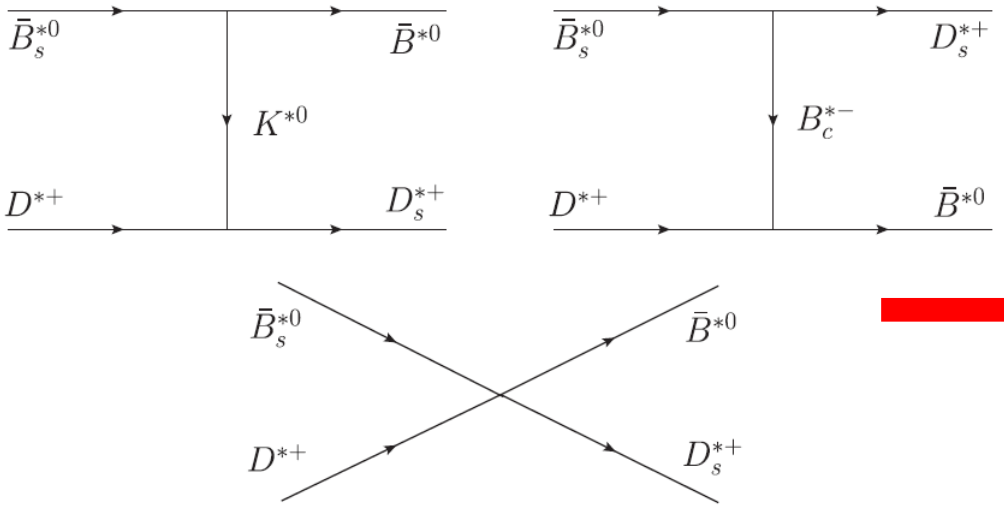
Attractive

Repulsive

$\Downarrow$

Possible molecular states

V-V interaction in $bc\bar{s}\bar{q}$ system



Vector meson exchange mechanism in VV sector

Interaction potential:

$$V_{VV}(s) = V_{VV}(s)^{ex} + V_{VV}(s)^{co}.$$

$$V_{VV}(s)^{ex} = C_{VV} \times g^2 (p_1 + p_3)(p_2 + p_4)\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4,$$

$$C'_{VV} = \left(\begin{array}{c|cc} J = 0,1,2 & (\bar{B}^* D^*)_s^+ & (\bar{B}^* D^*)_s^- \\ \hline (\bar{B}^* D^*)_s^+ & \frac{1}{m_{K^*}^2} & 0 \\ (\bar{B}^* D^*)_s^- & 0 & -\frac{1}{m_{K^*}^2} \end{array}\right).$$

$$V_{VV}(s)^{co} = m_{K^*}^2 \cdot C_{VV} \times \begin{cases} -4g^2 & \text{for } J = 0, \\ 0 & \text{for } J = 1, \\ 2g^2 & \text{for } J = 2. \end{cases}$$

Contents

Pole position and coupling

The binding energies E_B and the couplings g_i of the bound states on the physical (first Riemann) sheet for the $bc\bar{s}\bar{q}$ systems with $q_{max} = 600$ MeV.

Content: $bc\bar{s}\bar{d}$	$I(J^P)$	E_B (MeV)	Channel	$\lvert g_i \rvert$ (GeV)
$\lvert(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	15.7	$\bar{B}_s^0 D^+$	19
			$\bar{B}^0 D_s^+$	21
$\lvert(\bar{B}^* D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	17.3	$\bar{B}_s^{*0} D^+$	20
			$\bar{B}^{*0} D_s^+$	21
$\lvert(\bar{B}D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	16.4	$\bar{B}_s^0 D^{*+}$	20
			$\bar{B}^0 D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	13.6	$\bar{B}_s^{*0} D^{*+}$	19
			$\bar{B}^{*0} D_s^{*+}$	21
$\lvert(\bar{B}^* D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	18.2	$\bar{B}_s^{*0} D^{*+}$	21
			$\bar{B}^{*0} D_s^{*+}$	23
$\lvert(\bar{B}^* D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	20.5	$\bar{B}_s^{*0} D^{*+}$	22
			$\bar{B}^{*0} D_s^{*+}$	24

- Six bound state in $bc\bar{s}\bar{q}$ system with cutoff parameter $q_{max} = 600$ MeV.
- 1 pole generated from the P-P interaction, 2 poles generated from the V-P interaction and 3 poles generated from the V-V interaction.
- No width.
  - Not consider the width of the initial and final states.
  - Not consider the box diagrams with Kaon exchange.

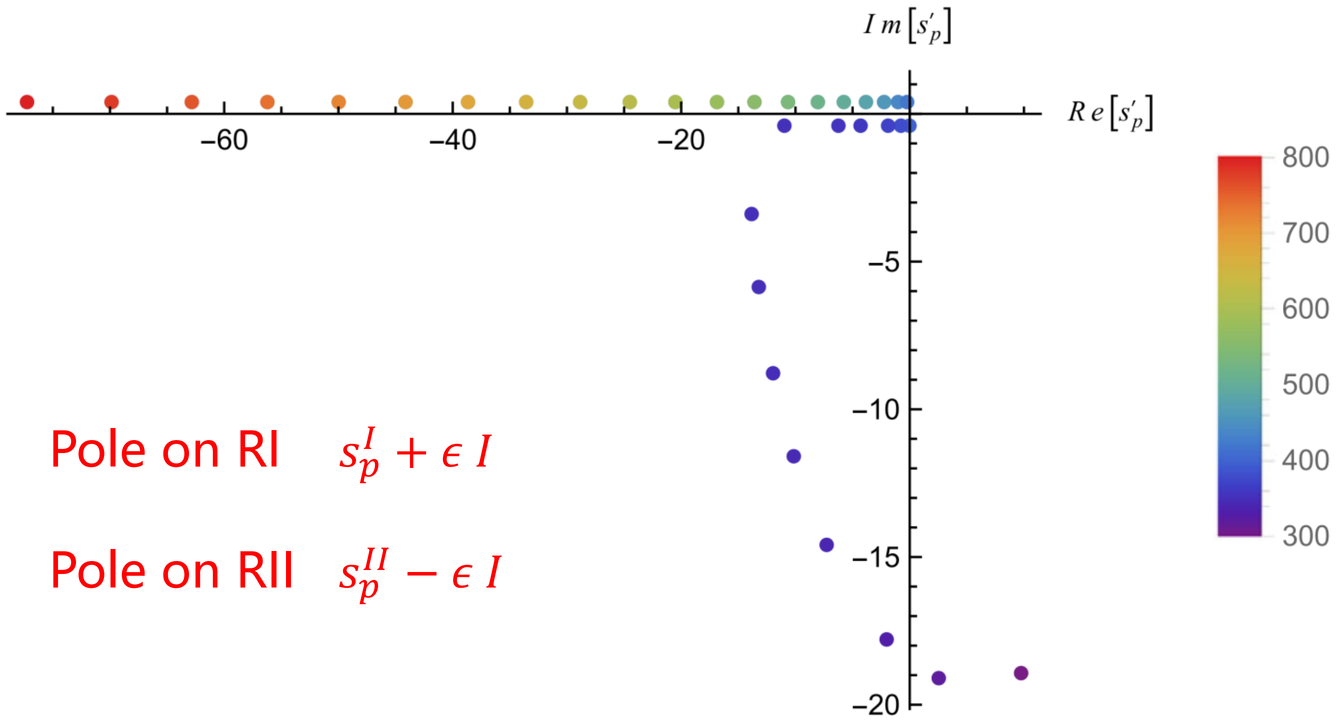
Pole position and coupling

The pole position s_p of the virtual states on the second Riemann sheet for the $bc\bar{s}\bar{q}$ systems with $q_{max} = 400$ MeV.

Content: $bc\bar{s}\,\bar{d}$	$I(J^P)$	Pole	Channel	Threshold
$\lvert(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	$7235.2 + i0$	$\bar{B}_s^0 D^+$	7236.6
			$\bar{B}^0 D_s^+$	7248.0
$\lvert(\bar{B}^*D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7284.9 + i0$	$\bar{B}_s^{*0} D^+$	7285.1
			$\bar{B}^{*0} D_s^+$	7293.1
$\lvert(\bar{B}D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7375.7 + i0$	$\bar{B}_s^0 D^{*+}$	7377.2
			$\bar{B}^0 D_s^{*+}$	7391.9
$\lvert(\bar{B}^*D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	$7423.0 + i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9
$\lvert(\bar{B}^*D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	$7425.4 + i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9
$\lvert(\bar{B}^*D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	$7425.6 + i0$	$\bar{B}_s^{*0} D^{*+}$	7425.7
			$\bar{B}^{*0} D_s^{*+}$	7436.9

- No bound state pole on the first Riemann sheet.
- six virtual state near threshold on the second Riemann sheet (++) in $bc\bar{s}\bar{q}$ system has been found with cutoff parameter $q_{max} = 400$ MeV.

Pole position and coupling



Pole on RI $s_p^I + \epsilon\, I$

Pole on RII $s_p^{II} - \epsilon\, I$

- When $q_{max} > 410$ MeV ,

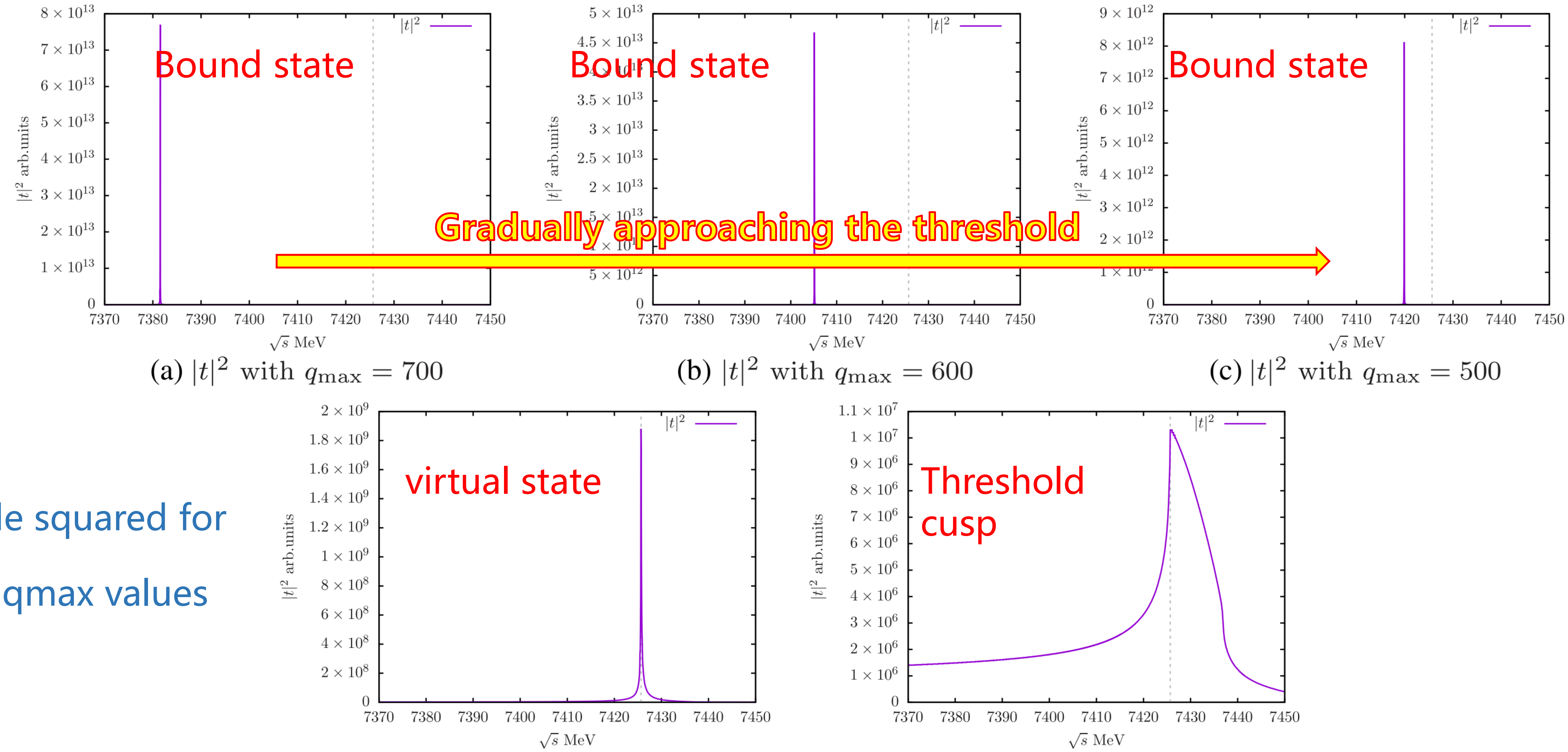
  a bound state pole appears,

  while it becomes a virtual state

  when $q_{max} < 410$ MeV.

The pole position $s'_p = s_p - m_{thr}$ of the combination $|(\bar{B}^* D^*)_s^-; J = 2 >$ as a function of the cutoff momentum $q_{max} = 300 - 800$ MeV.

Different behaviors of poles



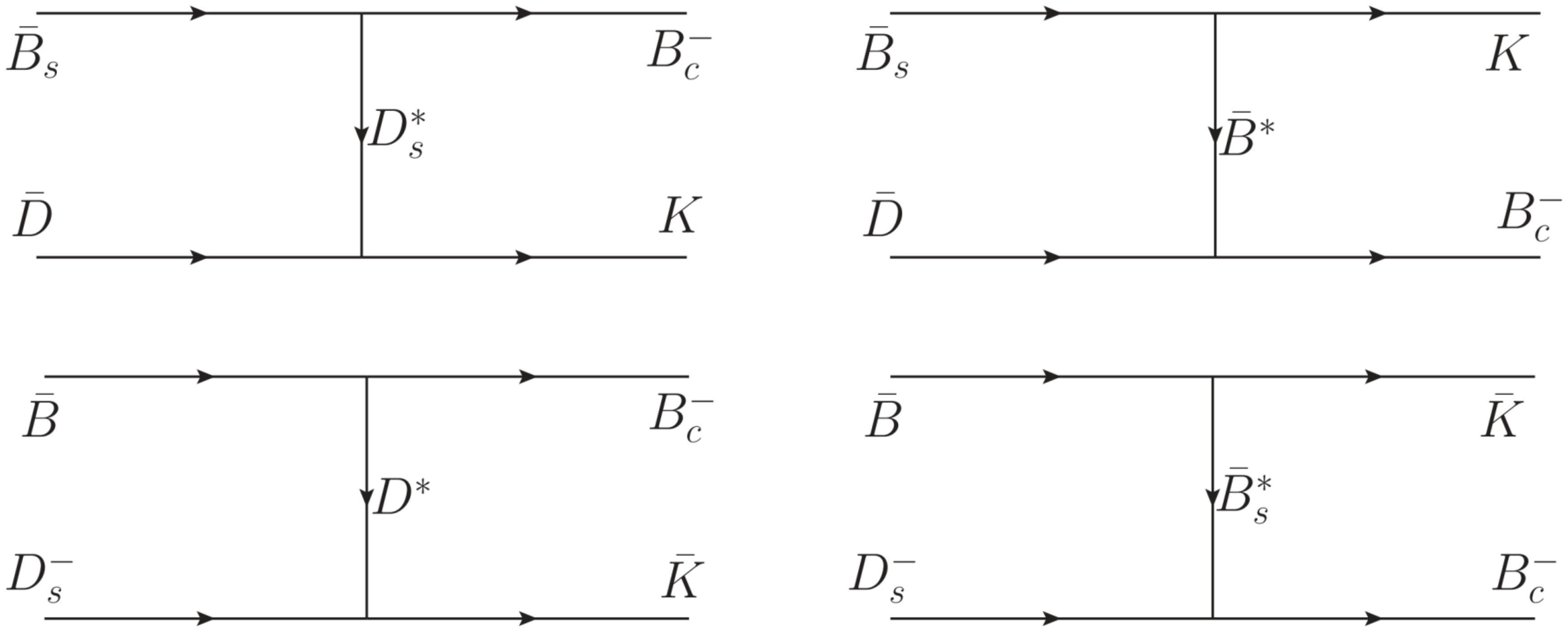
(a) $|t|^2$ with $q_{\max} = 700$ (b) $|t|^2$ with $q_{\max} = 600$ (c) $|t|^2$ with $q_{\max} = 500$

(d) $|t|^2$ with $q_{\max} = 400$ (e) $|t|^2$ with $q_{\max} = 300$

Amplitude squared for different qmax values

$b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$ system

Interactions in the $b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$ systems



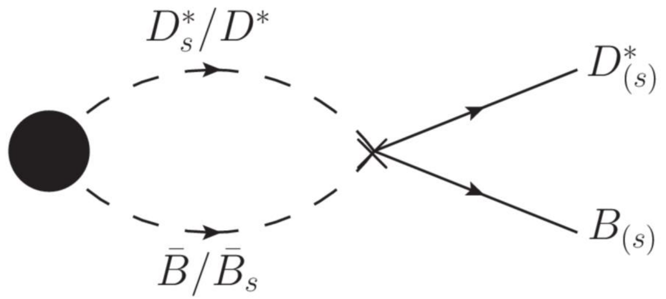
- No light vector meson exchange

- No deep bound pole has been found

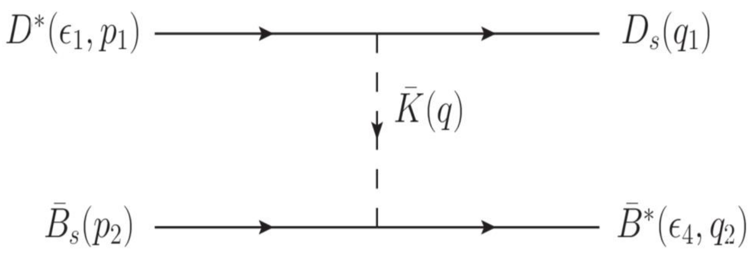
Does this mean that $Z_{cs}(3985)$ is not a molecular state, at least not a bound state?

Evaluate width

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}D^*)_s;\ 1^+\rangle$

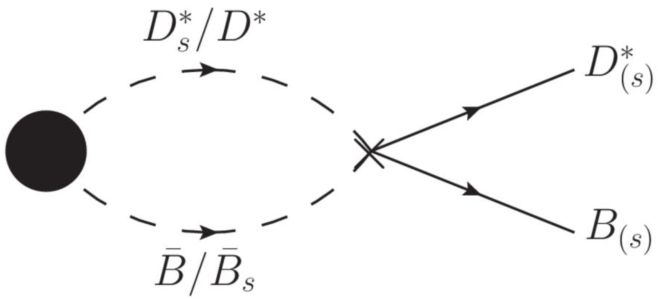


Kaon exchange mechanism

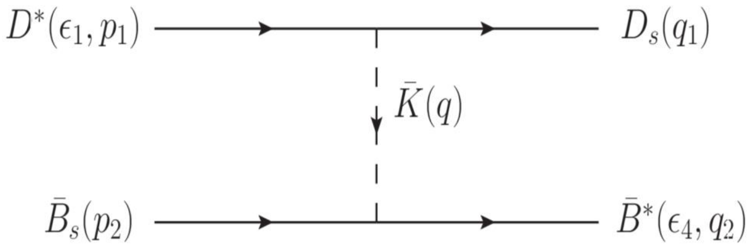
Content: $bc\bar{s}\bar{q}$	$I(J^P)$	Width (MeV)	Channel
$\|(\bar{B}D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	0	$\bar{B}_s^0 D^+$
			$\bar{B}^0 D_s^+$
$\|(\bar{B}^*D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	0	$\bar{B}_s^{*0} D^+$
			$\bar{B}^{*0} D_s^+$
$\|(\bar{B}D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	25^{+8}_{-11}	$\bar{B}_s^0 D^{*+}$
			$\bar{B}^0 D_s^{*+}$
$\|(\bar{B}^*D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	43^{+22}_{-23}	$\bar{B}_s^{*0} D^{*+}$
			$\bar{B}^{*0} D_s^{*+}$
$\|(\bar{B}^*D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	36^{+15}_{-17}	$\bar{B}_s^{*0} D^{*+}$
			$\bar{B}^{*0} D_s^{*+}$
$\|(\bar{B}^*D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	44^{+20}_{-22}	$\bar{B}_s^{*0} D^{*+}$
			$\bar{B}^{*0} D_s^{*+}$

Evaluate width

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}D^*)_s;\, 1^+\rangle$



Kaon exchange mechanism

$|(\bar{B}D^*)_s;\, 1^+\rangle$

$$i\mathcal{M}_2 = ig_D\, ig_B\, \epsilon_1 \cdot (q_1 - q)\, \epsilon_4^* \cdot (q - p_2)\, \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}$$

$|(\bar{B}^*D^*)_s;\, 0^+/2^+\rangle$

$$i\mathcal{M}_3 = ig_D\, ig_B\, \epsilon_1 \cdot (q_1 - q)\, \epsilon_2 \cdot (q_2 + q)\, \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}\, \mathcal{P}_{0/2}$$

$|(\bar{B}^*D^*)_s; 1^+\rangle$

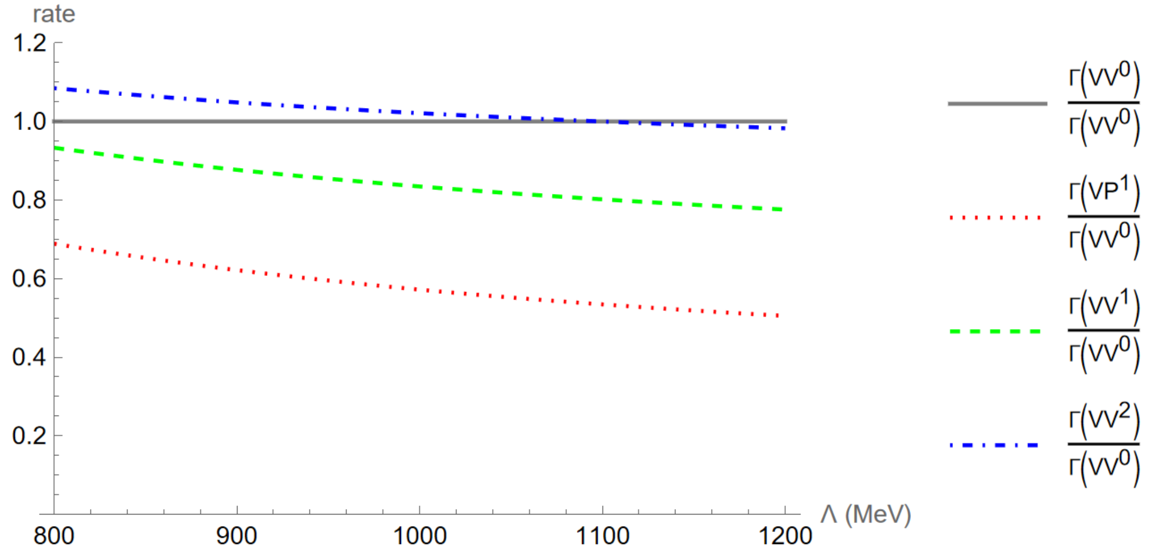
$$i\mathcal{M}_4 = \frac{-G_D'}{\sqrt{2}}\, ig_B\, p_1^\mu \epsilon_1^\nu q_1^\alpha \epsilon_3^{*\beta} \epsilon_{\mu\nu\alpha\beta}\, \epsilon_2 \cdot (q_2 + q)\, \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}\, \mathcal{P}_1$$

$$\Lambda = 800 - 1200 \text{ MeV}$$

Evaluate width

Unpublished manuscript

Content: $bc\bar s\bar q$	$I(J^P)$	Width (MeV)	Channel
$\vert(\bar B D)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	0	$\bar B_s^0 D^+$ $\bar B^0 D_s^+$
$\vert(\bar B^* D)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	0	$\bar B_s^{*0} D^+$ $\bar B^{*0} D_s^+$
$\vert(\bar B D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	25^{+8}_{-11}	$\bar B_s^0 D^{*+}$ $\bar B^0 D_s^{*+}$
$\vert(\bar B^* D^*)_s^-; J=0\rangle$	$\frac{1}{2}(0^+)$	43^{+22}_{-23}	$\bar B_s^{*0} D^{*+}$ $\bar B^{*0} D_s^{*+}$
$\vert(\bar B^* D^*)_s^-; J=1\rangle$	$\frac{1}{2}(1^+)$	36^{+15}_{-17}	$\bar B_s^{*0} D^{*+}$ $\bar B^{*0} D_s^{*+}$
$\vert(\bar B^* D^*)_s^-; J=2\rangle$	$\frac{1}{2}(2^+)$	44^{+20}_{-22}	$\bar B_s^{*0} D^{*+}$ $\bar B^{*0} D_s^{*+}$



$$\frac{\Gamma_{VP^1}}{\Gamma_{VV^0}} = 0.57^{+0.12}_{-0.06}$$

$$\frac{\Gamma_{VV^1}}{\Gamma_{VV^0}} = 0.83^{+0.10}_{-0.06}$$

$$\frac{\Gamma_{VV^2}}{\Gamma_{VV^0}} = 1.02^{+0.06}_{-0.04}$$

The ratio of the widths remains relatively stable !

Contents

Summary and Outlook

- Six bound states in $bc\bar{s}\bar{q}$ system with the binding energies about 10-20 MeV has been found when cutoff parameter $q_{max} = 600$ MeV, those bound states change to virtual states when cutoff parameter $q_{max} = 400$ MeV.

- No deeply bound pole has been found in the $b\bar{c}s\bar{q}$ and $b\bar{c}\bar{s}q$ system, for there is no light vector exchange.

- The widths of molecule states in $bc\bar{s}\bar{q}$ system are estimated:

$$\Gamma_{|(\bar{B}D^*)_s;1^+\rangle} \simeq 25^{+8}_{-11}\ \mathrm{MeV}$$
$$\Gamma_{|(\bar{B}^*D^*)_s;0^+\rangle} \simeq 43^{+22}_{-23}\ \mathrm{MeV}$$
$$\Gamma_{|(\bar{B}^*D^*)_s;1^+\rangle} \simeq 36^{+15}_{-17}\ \mathrm{MeV}$$
$$\Gamma_{|(\bar{B}^*D^*)_s;2^+\rangle} \simeq 44^{+20}_{-22}\ \mathrm{MeV}$$

Thanks for your attention!

报告人: 刘文颖

2025/04/13

Back-up

BS-eq within LHG

$$P=\begin{pmatrix} \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}+\frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 & B^+ \\ \pi^- & \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}-\frac{\pi^0}{\sqrt{2}} & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}}+\sqrt{\frac{2}{3}}\eta' & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & \eta_c & B_c^+ \\ B^- & \bar{B}^0 & \bar{B}_s^0 & B_c^- & \eta_b \end{pmatrix}, \qquad V=\begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} & B^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi & B_c^{*+} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & B_c^{*-} & \Upsilon \end{pmatrix}.$$

A flavor SU(5) symmetry is assumed

BS-eq within LHG

$$V_{VV}(s)^{co} = m_{K^*}^2 \cdot C_{VV} g^2 (-2\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu).$$

$$C_{VV} = \left(\begin{array}{c|cc} J=0,1,2 & \bar{B}_s^{*0} D^{*+} & \bar{B}^{*0} D_s^{*+} \\ \hline \bar{B}_s^{*0} D^{*+} & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^{*0} D_s^{*+} & \frac{1}{m_{K^*}^2} & 0 \end{array}\right),$$

$$\begin{aligned} \mathcal{P}^{(0)} &= \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu \\ \mathcal{P}^{(1)} &= \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) \\ \mathcal{P}^{(2)} &= \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu, \end{aligned}$$

BS-eq within LHG

$$V_{VV}(s)^{co} = m_{K^*}^2 \cdot C_{VV} \times \begin{cases} -4g^2 & \text{for } J = 0, \\ 0 & \text{for } J = 1, \\ 2g^2 & \text{for } J = 2. \end{cases}$$

$$C_{VV} = \left(\begin{array}{c|cc} J = 0, 1, 2 & \bar{B}_s^{*0} D^{*+} & \bar{B}^{*0} D_s^{*+} \\ \hline \bar{B}_s^{*0} D^{*+} & 0 & \frac{1}{m_{K^*}^2} \\ \bar{B}^{*0} D_s^{*+} & \frac{1}{m_{K^*}^2} & 0 \end{array} \right),$$

Loop function

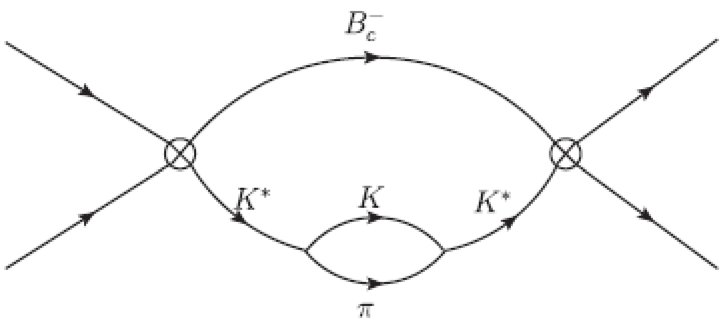
Loop function:

$$G_{ii}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(p-q)^2 - m_2^2 + i\epsilon},$$

dimensional regularization

$$\begin{aligned}
G_{ii}(s) =& \frac{1}{16\pi^2} \Bigg\{ a_{ii}(\mu) + \ln \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \ln \frac{m_2^2}{m_1^2} \\
&+ \frac{q_{cmi}(s)}{\sqrt{s}} \left[\ln\left(s - (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \right. \\
&+ \ln\left(s + (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \\
&- \ln\left(-s - (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \\
&\left. - \ln\left(-s + (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \right] \Bigg\},
\end{aligned}$$

Results



$$G(s) = \int_0^{q_{\max}} \frac{q^2 dq}{4\pi^2} \frac{\omega_{B_c^{(*)}} + \omega_{K^*}}{\omega_{B_c^{(*)}} \omega_{K^*}} \frac{1}{\sqrt{s} + \omega_{B_c^{(*)}} + \omega_{K^*}} \times \frac{1}{\sqrt{s} - \omega_{K^*} - \omega_{B_c^{(*)}} + i \frac{\sqrt{s'}}{2\omega_{K^*}} \Gamma_{K^*}(s')},$$

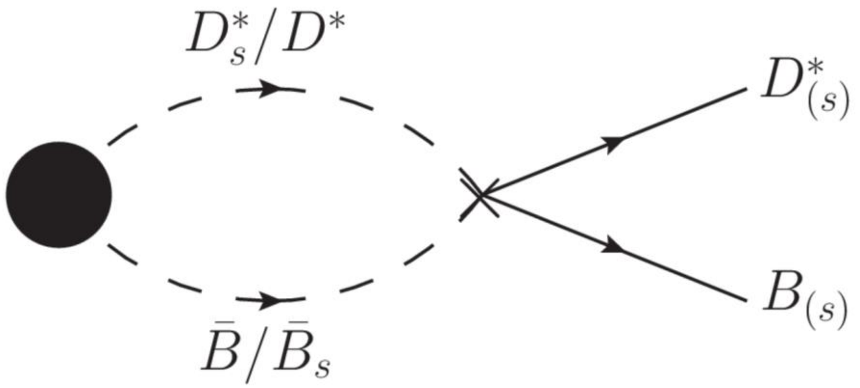
where $s' = (\sqrt{s} - \omega_{B_c^{(*)}})^2 - \vec{q}^2$ and

$$\Gamma_{K^*}(s') = \Gamma_{K^*}(m_{K^*}^2) \frac{m_{K^*}^2}{s'} \left(\frac{p_\pi(s')}{p_\pi(m_{K^*}^2)} \right)^3 \times \Theta(\sqrt{s'} - m_K - m_\pi),$$

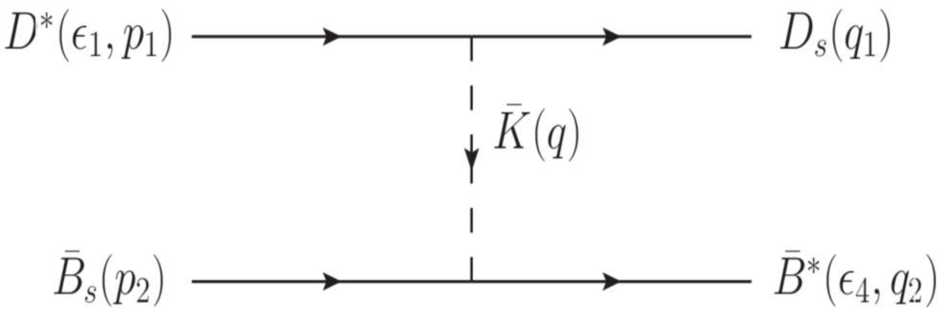
A. Feijoo, W. H. Liang and E. Oset, Phys. Rev. D 104, no.11, 114015 (2021)

Decay property of $|(\bar{B}D^*)_s;\ 1^+\rangle$

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}D^*)_s;\ 1^+\rangle$



Kaon exchange mechanism

$$i\mathcal{M} = G_{MM} \cdot g_{MMX} \cdot i\mathcal{M}_2$$

$$d\Gamma = \frac{1}{32\pi^2} \left|\mathcal{M}\right|^2 \frac{|\boldsymbol{p}_1|}{M^2}\, d\Omega$$

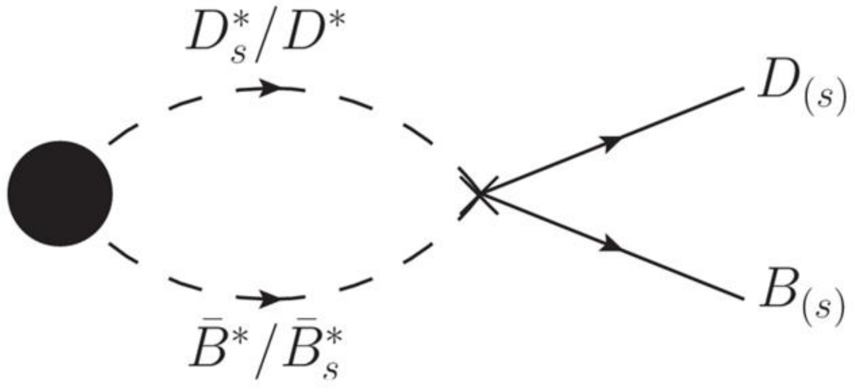
$$i\mathcal{M}_2 = i g_D\, i g_B\, \epsilon_1 \cdot (q_1 - q)\, \epsilon_4^* \cdot (q - p_2)\, \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}$$

$$\Lambda = 800 - 1200 \text{ MeV}$$

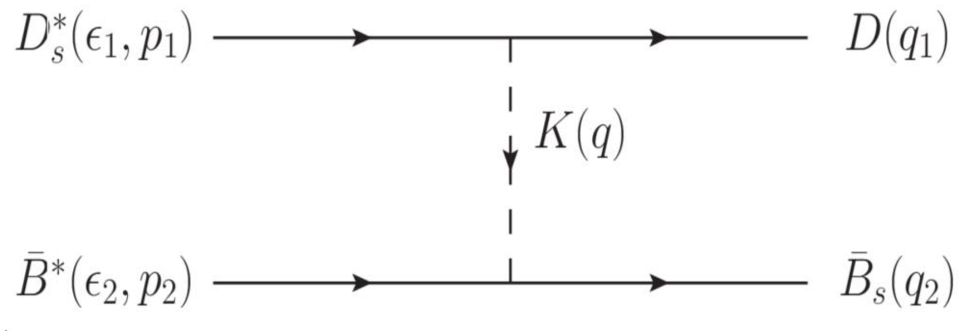
G_{MM}: meson-meson loop function
g_{MMX}: Coupling constant

Decay property of $|(\bar{B}^* D^*)_s;\ 0^+/2^+\rangle$

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}^* D^*)_s;\ 0^+/2^+\rangle$



Kaon exchange mechanism

$$i\mathcal{M} = G_{MM} \cdot g_{MMX} \cdot i\mathcal{M}_3$$

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\boldsymbol{p}_1|}{M^2} d\Omega$$

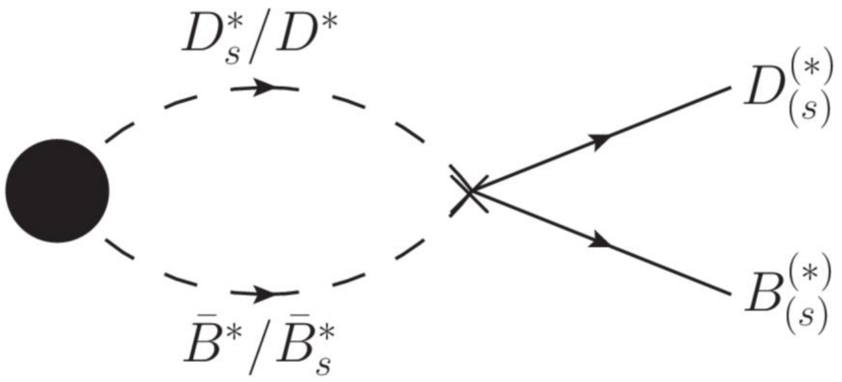
$$i\mathcal{M}_3 = ig_D\, ig_B\, \epsilon_1 \cdot (q_1 - q)\, \epsilon_2 \cdot (q_2 + q)\, \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2}\, \mathcal{P}_{0/2}$$

$$\Lambda = 800 - 1200 \text{ MeV}$$

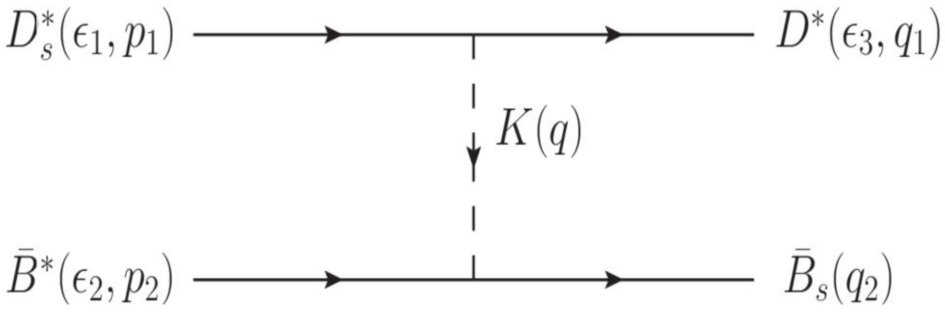
G_{MM}: meson-meson loop function
g_{MMX}: Coupling constant

Decay property of $|(\bar{B}^* D^*)_s; 1^+\rangle$

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}^* D^*)_s;\ 1^+\rangle$



Kaon exchange mechanism

$$i\mathcal{M} = G_{MM} \cdot g_{MMX} \cdot i\mathcal{M}_4$$

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\boldsymbol{p}_1|}{M^2} d\Omega$$

G_{MM}: meson-meson loop function
g_{MMX}: Coupling constant

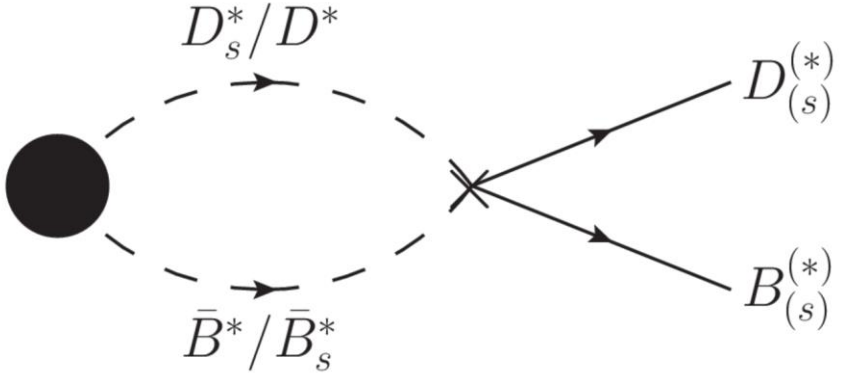
$$i\mathcal{M}_4 = \frac{-G'_D}{\sqrt{2}} i g_B\, p_1^\mu \epsilon_1^\nu q_1^\alpha \epsilon_3^{*\beta} \epsilon_{\mu\nu\alpha\beta}\, \epsilon_2 \cdot (q_2 + q) \frac{i}{q^2 - m_K^2} \times \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2} \mathcal{P}_1$$

$$\Lambda = 800 - 1200 \text{ MeV}$$

$$\mathcal{L} = \frac{iG'}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \delta_\mu V_\nu \delta_\alpha V_\beta P \rangle$$

Spin projection operator

Unpublished manuscript



Strong decay Feynman diagram for $|(\bar{B}^* D^*)_s;\ 1^+\rangle$

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu$$

$$\mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu)$$

$$\mathcal{P}^{(2)} = \frac{1}{2}(\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3}\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu,$$

$$\begin{aligned}\mathcal{L} = {} & g_X^0 \frac{1}{\sqrt{3}} X^\dagger V_1 \cdot V_2 \\ & + g_X^1 \frac{1}{\sqrt{2}} \epsilon^{ijk} X^{i\dagger} V_1^j V_2^k \\ & + g_X^2 X^{ij} V_1^i V_2^j \\ & + h.c.\end{aligned}$$

Decay property

Unpublished manuscript

$|(\bar{B}D^*)_s;1^+\rangle$ *PV* sector

$$\Gamma_{X'_{VP}\to\bar{B}^*D_s}\simeq 8.9^{+3.0}_{-3.9}\,\mathrm{MeV}$$
$$\Gamma_{X'_{VP}\to\bar{B}^*_sD}\simeq 15.9^{+5.2}_{-6.9}\,\mathrm{MeV}$$
$$\Gamma_{X'_{VP}}\simeq 24.8^{+8.2}_{-10.8}\,\mathrm{MeV}$$

$|(\bar{B}^*D^*)_s;0^+\rangle$ *VV*, *J* = 0 sector

$$\Gamma_{X^0_{VV}\to\bar{B}_sD}\simeq 24.0^{+12.3}_{-12.8}\,\mathrm{MeV}$$
$$\Gamma_{X^0_{VV}\to\bar{B}D_s}\simeq 19.3^{+9.7}_{-10.2}\,\mathrm{MeV}$$
$$\Gamma_{X^0_{VV}}\simeq 43.3^{+22.0}_{-23.0}\,\mathrm{MeV}$$

$|(\bar{B}^*D^*)_s;1^+\rangle$ *VV*, *J* = 1 sector

$$\Gamma_{X^1_{VV}\to\bar{B}^*_sD}\simeq 15.1^{+6.6}_{-7.5}\,\mathrm{MeV}$$
$$\Gamma_{X^1_{VV}\to\bar{B}_sD^*}\simeq 4.6^{+1.3}_{-1.8}\,\mathrm{MeV}$$
$$\Gamma_{X^1_{VV}\to\bar{B}^*D_s}\simeq 14.2^{+6.1}_{-7.0}\,\mathrm{MeV}$$
$$\Gamma_{X^1_{VV}\to\bar{B}D^*_s}\simeq 2.2^{+0.5}_{-0.8}\,\mathrm{MeV}$$
$$\Gamma_{X^1_{VV}}\simeq 36.1^{+14.5}_{-17.1}\,\mathrm{MeV}$$

$|(\bar{B}^*D^*)_s;2^+\rangle$ *VV*, *J* = 2 sector

$$\Gamma_{X^2_{VV}\to\bar{B}_sD}\simeq 12.4^{+6.2}_{-6.6}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}\to\bar{B}D_s}\simeq 10.2^{+5.0}_{-5.4}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}\to\bar{B}^*_sD}\simeq 9.1^{+4.0}_{-4.5}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}\to\bar{B}_sD^*}\simeq 2.8^{+0.8}_{-1.1}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}\to\bar{B}^*D_s}\simeq 8.5^{+3.7}_{-4.2}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}\to\bar{B}D^*_s}\simeq 1.3^{+0.3}_{-0.5}\,\mathrm{MeV}$$
$$\Gamma_{X^2_{VV}}\simeq 44.3^{+20.0}_{-22.3}\,\mathrm{MeV}$$

Production

$$S^{\mathrm{mac}}_{K^*} = 1 - i t_{K^*} \frac{1}{\sqrt{2\omega_{K^*}}} \frac{1}{\sqrt{2\omega_K}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\mathcal{V}^{3/2}} (2\pi)^4 \delta(P_{\mathrm{in}} - P_{\mathrm{out}}), \tag{15}$$

$$S^{\mathrm{mac}}_{B^*} = 1 - i t_{B^*} \frac{1}{\sqrt{2\omega_{B^*}}} \frac{1}{\sqrt{2\omega_B}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\mathcal{V}^{3/2}} (2\pi)^4 \delta(P_{\mathrm{in}} - P_{\mathrm{out}}). \tag{16}$$

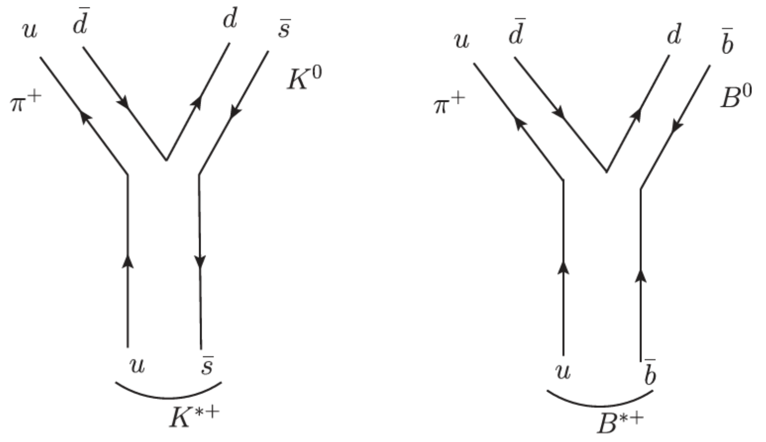


FIG. 5. Diagram of the transition $K^{*+} \to K^0\pi^+$ (left) and $B^{*+} \to B^0\pi^+$ (right).

Let us then compare the $K^{*+} \to K^0\pi^+$ and $B^{*+} \to B^0\pi^+$ transitions as shown in Fig. 5. As we can see in the figure, the transitions are identical and governed by the light quarks, with the $\bar{s}$ quark in K^{*+} and $\bar{b}$ quark in B^{*+} playing the role of a spectator. The transition amplitudes are thus identical at the quark microscopic level, but we must take into account that when used at the macroscopic level of the K^{*+} or B^{*+} there are normalization factors $(2\omega)^{-1/2}$ which are different for the K^{*+}, K^0 or B^{*+}, B^0 fields. This is taken easily into account by constructing the S matrix at the macroscopic level. At the microscopic level we have (we follow Mandl + Shaw normalization of the fields [59])

$$S^{\mathrm{mic}} = 1 - i t \sqrt{\frac{2m_L}{2E_L}} \sqrt{\frac{2m'_L}{2E'_L}} \sqrt{\frac{1}{2\omega_\pi}} \frac{1}{\mathcal{V}^{3/2}} (2\pi)^4 \delta(P_{\mathrm{in}} - P_{\mathrm{out}}), \tag{14}$$

with m_L, E_L, m'_L, and E'_L the masses (constituent) of the incoming and outgoing light quarks, $\mathcal{V}$ the volume of the box where states are normalized to unity, and ω_π the pion energy. At the macroscopic level we have for the K^{*+} and B^{*+}

Production

$$\frac{t_{B^*}}{t_{K^*}} \equiv \frac{\sqrt{m_{B^*} m_B}}{\sqrt{m_{K^*} m_K}} \simeq \frac{m_{B^*}}{m_{K^*}}. \quad (17)$$

For a B^* at rest, as we shall assume in our evaluations, t is proportional to $\vec{\epsilon} \cdot \vec{q}$, with $\vec{q}$ the pion momentum and $\vec{\epsilon}$ the polarization vector of the vector meson (corrections of the order of $|\vec{p}_{B^*}|/m_{B^*}$ coming next can be safety neglected). It is interesting to compare what we get in our approach to the results of Ref. [58]. In Ref. [58], the width for $B^{*+} \to B^0 \pi^+$ (or $D^{*+} \to D^0 \pi^+$) is given by

$$\Gamma = \frac{g_H^2}{6\pi \tilde{f}_\pi^2} |\vec{p}_\pi|^3, \quad (18)$$

with g_H the coupling appearing in the heavy hadron Lagrangian and $\tilde{f}_\pi = \sqrt{2} f_\pi$. For the same amplitude, our approach, considering Eq. (17), is given by

$$\Gamma = \frac{1}{6\pi} \frac{1}{m_{B^*}^2} g^2 \left(\frac{m_{B^*}}{m_{K^*}} \right)^2 |\vec{p}_\pi|^3. \quad (19)$$

By taking $g^2/m_{K^*}^2 = (m_V/2f_\pi m_{K^*})^2 \equiv \frac{1}{4f_\pi^2}$, we have the relationship

$$\frac{g_H^2}{2} \equiv \frac{1}{4}; \qquad g_H = \frac{1}{\sqrt{2}}. \quad (20)$$

The same result would appear if we use another heavy vector decay like D^*. Our approach, with the consideration of the field normalizations, leads to a g_H independent of flavor and furthermore provides a value for it of $(\sqrt{2})^{-1}$. This value is in good agreement with the latest lattice QCD result [62] for the $B^* \to B\pi$ decay

$$g_H = 0.57 \pm 0.1. \quad (21)$$

The heavy quark plays the role of a spectator at the quark level.

$$\Gamma_{D^* \to D\pi} = \frac{1}{6\pi} \frac{q^3}{m_{D^*}^2} g_D^2$$

$$g_D = g \cdot \frac{m_{D^*}}{m_{K^*}}$$

$$g_B = g \cdot \frac{m_{B^*}}{m_{K^*}}$$

$$g_D^{exp} = 8.41$$

$$g_B^{lattice} = 21.4$$

$$g_D^{theo} = 9.7$$

$$g_B^{theo} = 25.9$$