Charmonia in an unquenched quark model

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Base on Phys. Rev. D 110 (2024) 5, 056034 ; Phys. Rev. D 109 (2024) 11, 116006

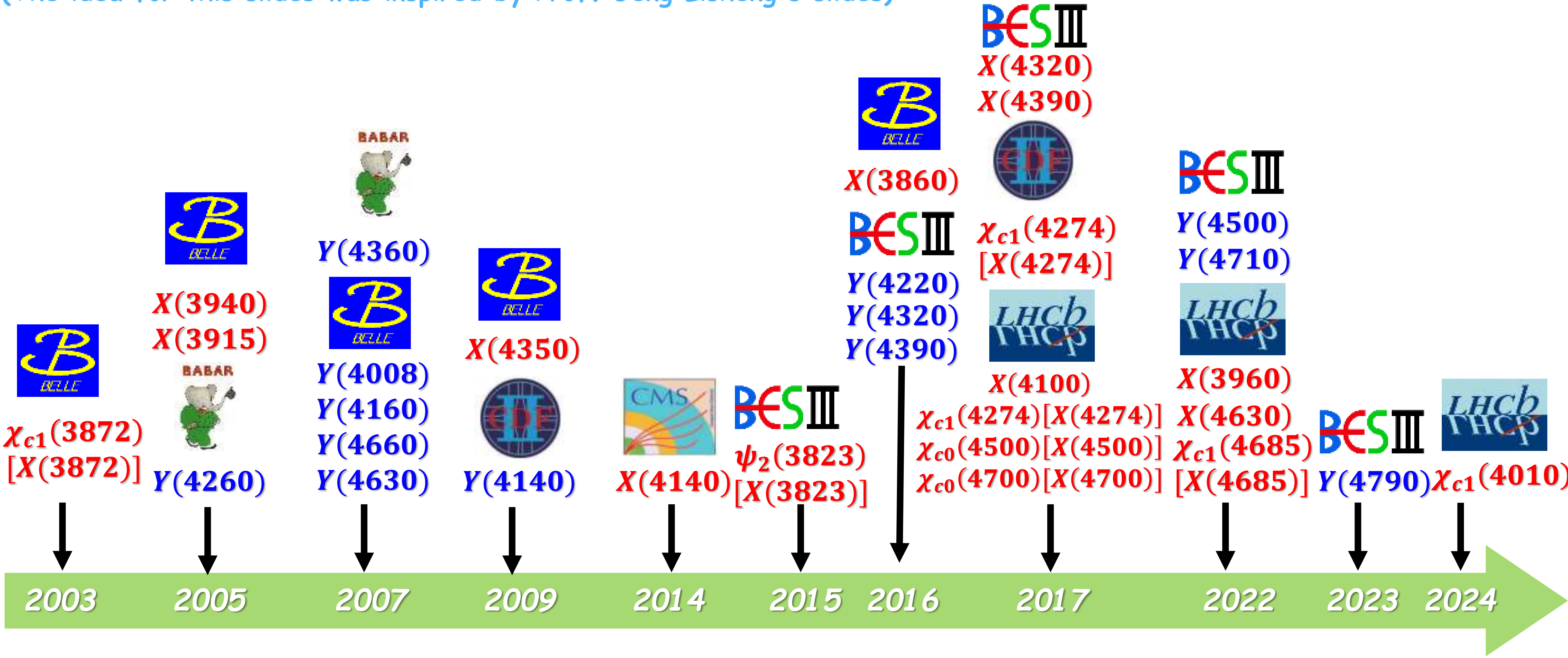
Together with 邓倩 (HUNNU), 李琦 (TCU), 吴佳俊 (UCAS), and 钟显辉 (HUNNU)

CONTENTS

- Background and theoretical framework
- Discussion
- Summary

A series of Charmonium(-like) states observed in experiments since 2003

(The idea for this slides was inspired by Prof. Geng Lisheng's slides)



2003: $\chi_{c1}(3872)$ [$X(3872)$]

2005: $X(3940)$, $X(3915)$; $Y(4260)$

2007: $Y(4360)$; $Y(4008)$, $Y(4160)$, $Y(4660)$, $Y(4630)$

2009: $X(4350)$; $Y(4140)$

2014: $X(4140)$

2015: $\psi_2(3823)$ [$X(3823)$]

2016: $X(3860)$; $Y(4220)$, $Y(4320)$, $Y(4390)$

2017: $X(4320)$, $X(4390)$; $\chi_{c1}(4274)$ [$X(4274)$]; $X(4100)$, $\chi_{c1}(4274)[X(4274)]$, $\chi_{c0}(4500)[X(4500)]$, $\chi_{c0}(4700)[X(4700)]$

2022: $Y(4500)$, $Y(4710)$; $X(3960)$, $X(4630)$, $\chi_{c1}(4685)$ [$X(4685)$]

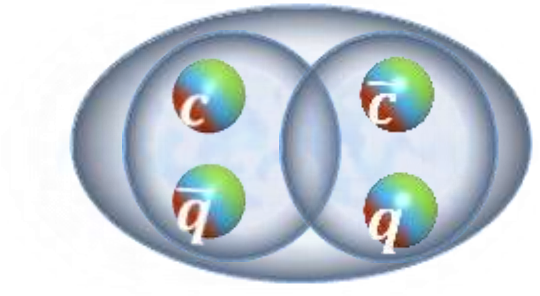
2023: $Y(4790)$

2024: $\chi_{c1}(4010)$

Understanding these charmonium(-like) states presents a significant challenge for conventional quenched quark models.

$\chi_{c1}(3872)$ Superstar!

➢ $D\bar{D}^*$ molecule ?



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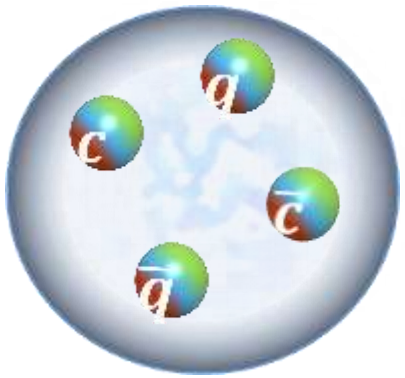
➢ Tetraquark ?



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➢ $c\bar{c} + D\bar{D}^*$?

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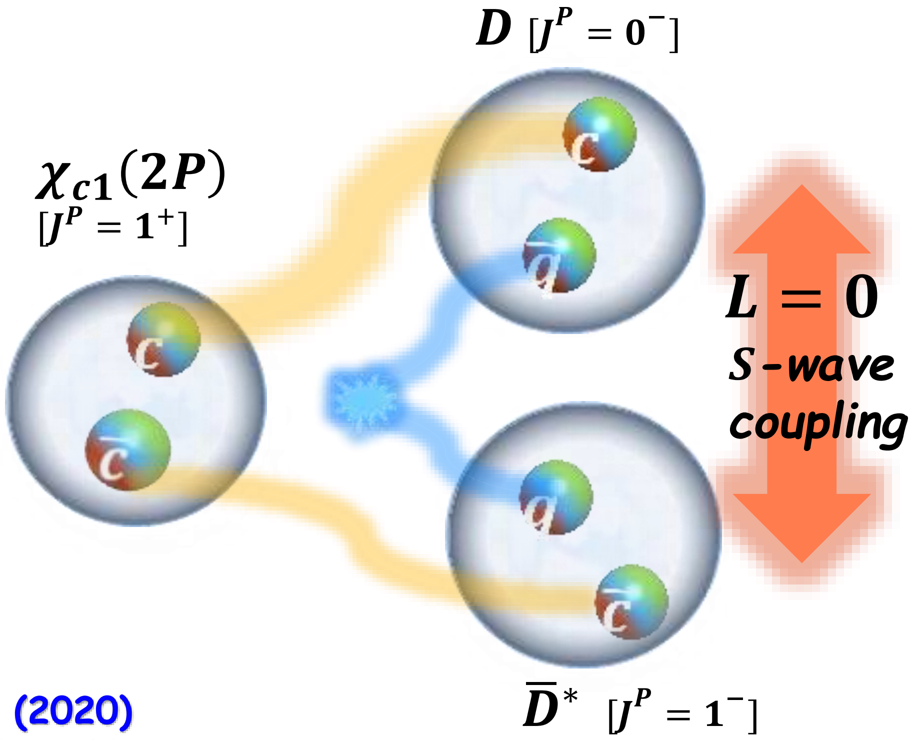
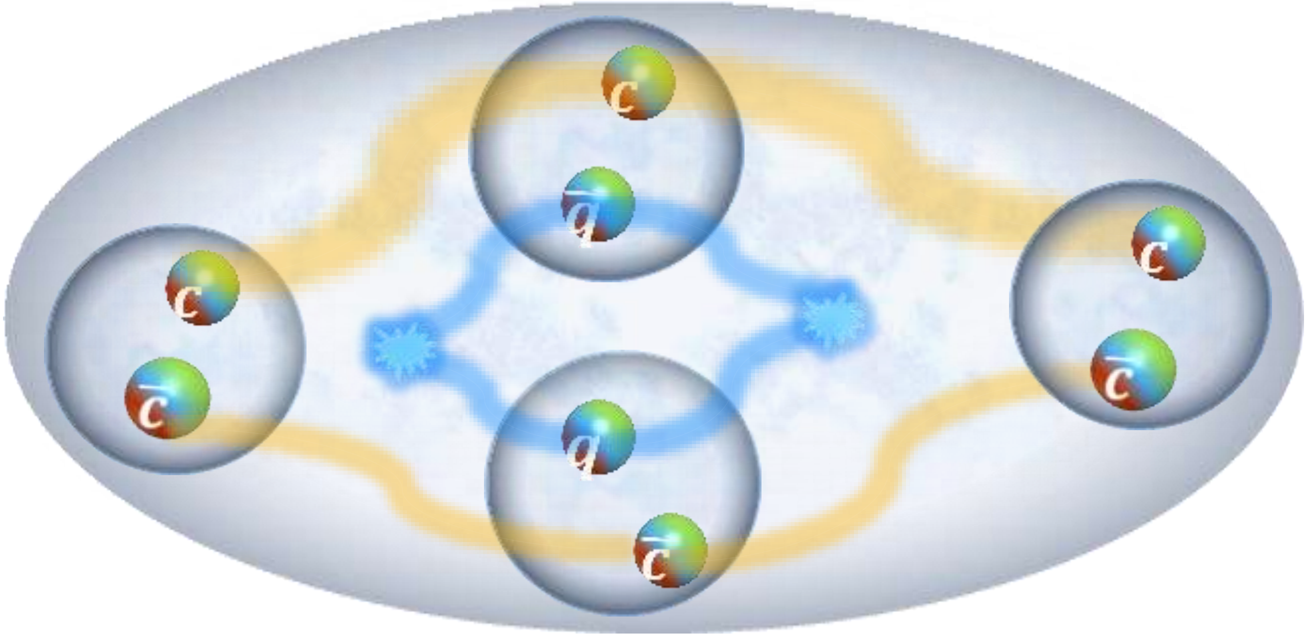
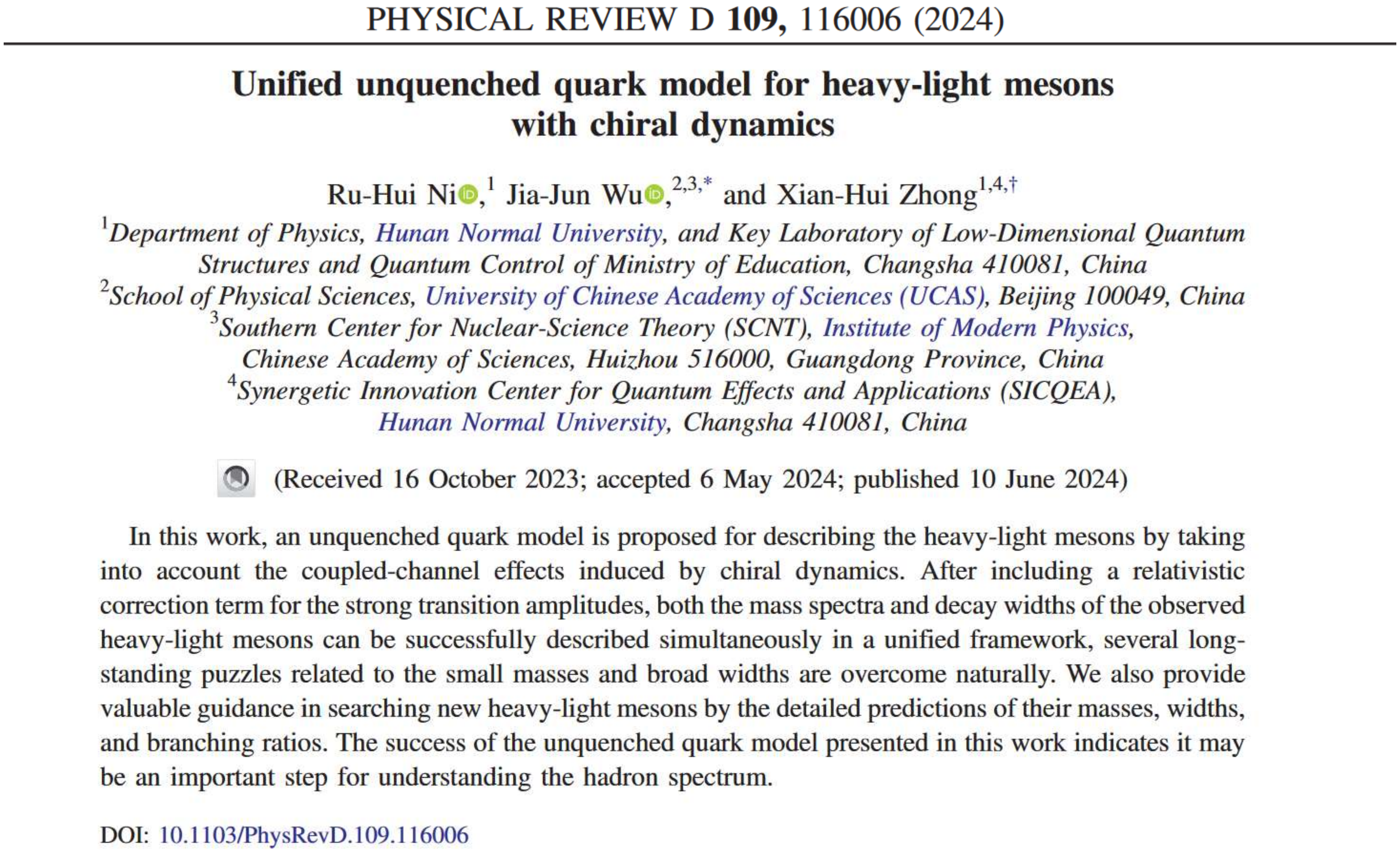




Unquenched Coupled-Channel Effects

— Two Key Aspects Overlooked by Most Studies:

- ➢ The mass shift contribution from the large-momentum region is unphysical.
- ➢ Criteria for selecting relevant coupling channels.



PHYSICAL REVIEW D **109,** 116006 (2024)

Unified unquenched quark model for heavy-light mesons with chiral dynamics

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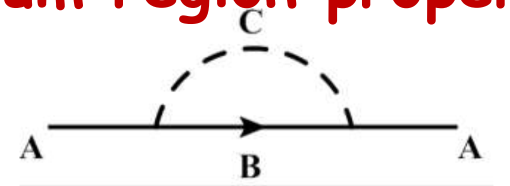
In this work, an unquenched quark model is proposed for describing the heavy-light mesons by taking into account the coupled-channel effects induced by chiral dynamics. After including a relativistic correction term for the strong transition amplitudes, both the mass spectra and decay widths of the observed heavy-light mesons can be successfully described simultaneously in a unified framework, several long-standing puzzles related to the small masses and broad widths are overcome naturally. We also provide valuable guidance in searching new heavy-light mesons by the detailed predictions of their masses, widths, and branching ratios. The success of the unquenched quark model presented in this work indicates it may be an important step for understanding the hadron spectrum.

DOI: 10.1103/PhysRevD.109.116006

A unified unquenched quark model provides a natural explanation for the low masses nature of $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ within the heavy-light meson spectrum (D, D_s, B, and B_s).

➢ Managed the unphysical mass shift in the large-momentum region properly.

$$\begin{pmatrix} \mathcal{H}_0 & \mathcal{H}_I \\ \mathcal{H}_I & \mathcal{H}_c \end{pmatrix} \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC}\int c_{BC}(\mathbf{q},M)d^3\mathbf{q}|BC,\mathbf{q}\rangle \end{pmatrix} = M \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC}\int c_{BC}(\mathbf{q},M)d^3\mathbf{q}|BC,\mathbf{q}\rangle \end{pmatrix}$$



$\mathcal{H}_0$ » OGE Potential

$$\mathcal{H}_0 = \sqrt{\boldsymbol{p}_1^2 + m_q^2} + \sqrt{\boldsymbol{p}_2^2 + m_{\bar{q}}^2} + V_0(r) + V_{sd}(r)$$

$$V_0(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + C_0$$

$$V_{sd}(r) = \mathcal{H}_{SS}(r) + \mathcal{H}_T(r) + \mathcal{H}_{LS}(r)$$

$\mathcal{H}_c$ » $\sqrt{m_B^2 + \boldsymbol{q}^2} + \sqrt{m_C^2 + \boldsymbol{q}^2}$

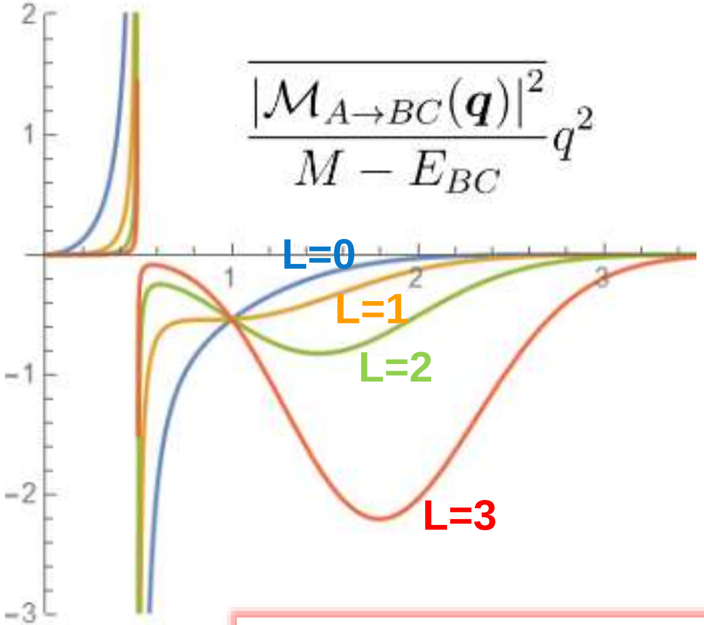
$\mathcal{H}_I$ » QPC model

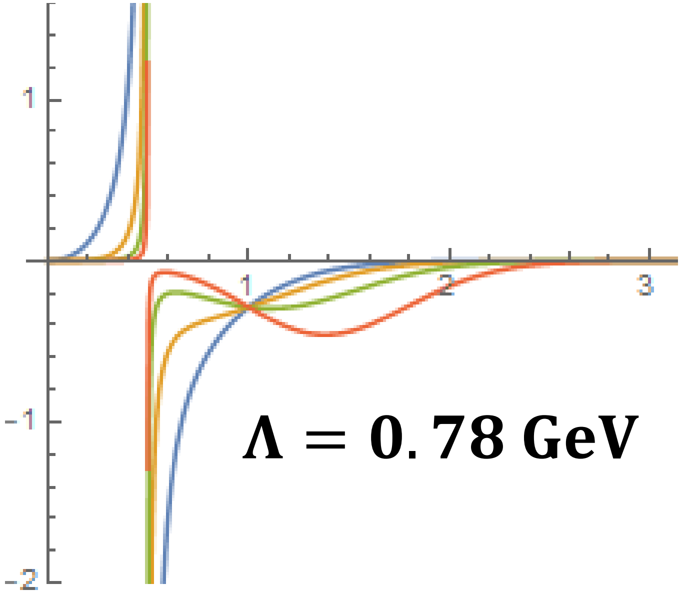
$$\mathcal{H}_I = -3\gamma\sum_m \langle 1m;1-m|00\rangle$$

$$\int d^3\mathbf{p}_3 d^3\mathbf{p}_4 \delta^3(\mathbf{p}_3+\mathbf{p}_4) y_{1m}\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4)$$

$$\boxed{M = M_A + \Delta M(M)} \quad \Delta M(M) = \sum_{BC}\mathcal{P}\int_0^\infty \frac{\overline{|\mathcal{M}_{A\to BC}(\mathbf{q})|^2}}{M - E_{BC}} q^2 dq$$

$$|\mathcal{M}(\mathbf{q})|^2 \propto e^{-a\mathbf{q}^2} f(\mathbf{q})\mathbf{q}^{2L}$$





$$\boxed{\langle BC,\mathbf{q}|\mathcal{H}_I|A\rangle \to \langle BC,\mathbf{q}|\mathcal{H}_I e^{-\mathbf{q}^2/(2\Lambda^2)}|A\rangle}$$

Suppressing Unphysical Mass Shift Contributions

— Softening the Hard Vertex in QPC model

➢ Criteria for Selecting Relevant Coupling Channels

It is unfeasible to calculate the self-energy function including an unlimited number of loops!!!

Once-subtracted method

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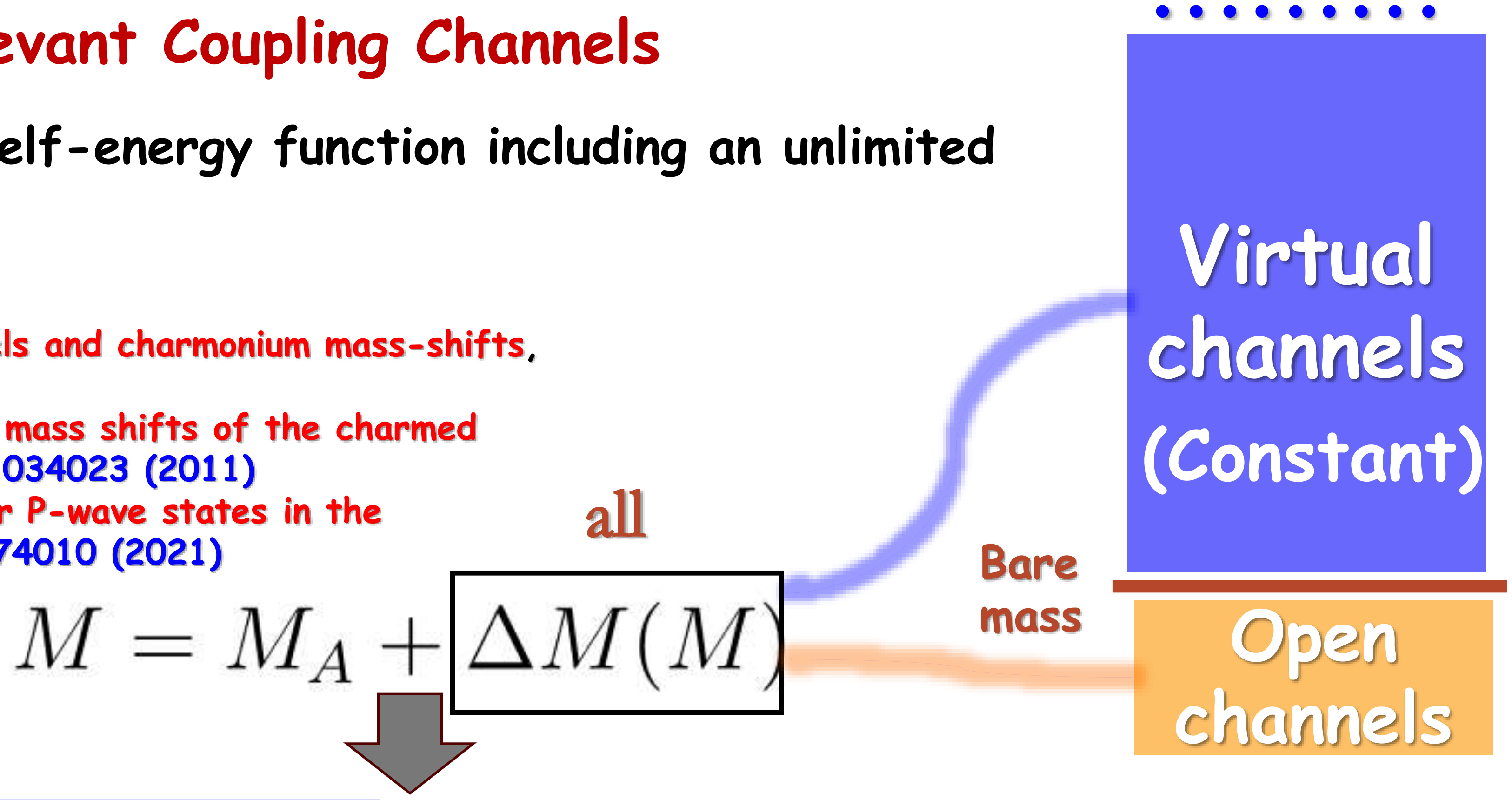
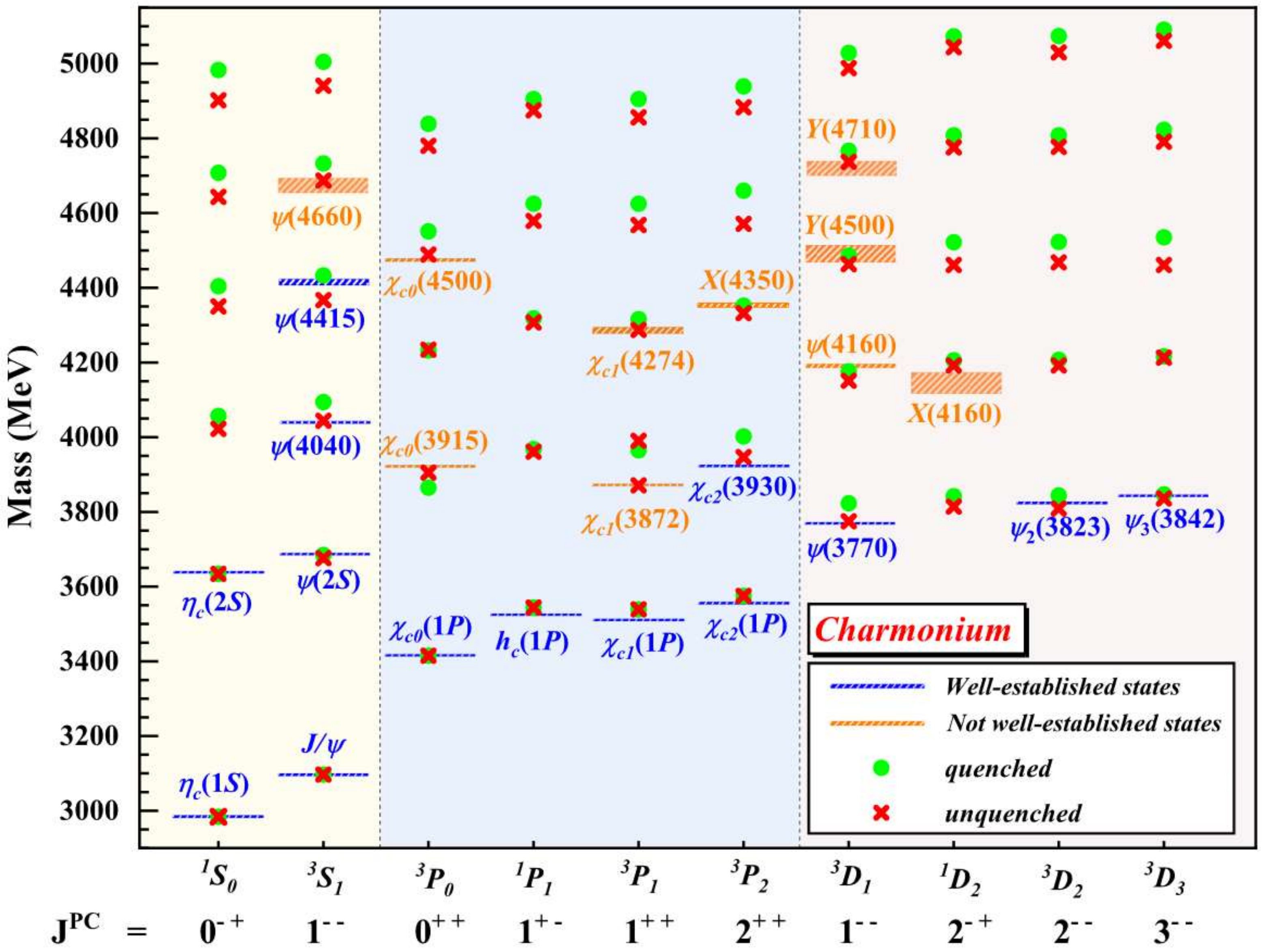


$$M = M_A + \boxed{\Delta M(M)}$$

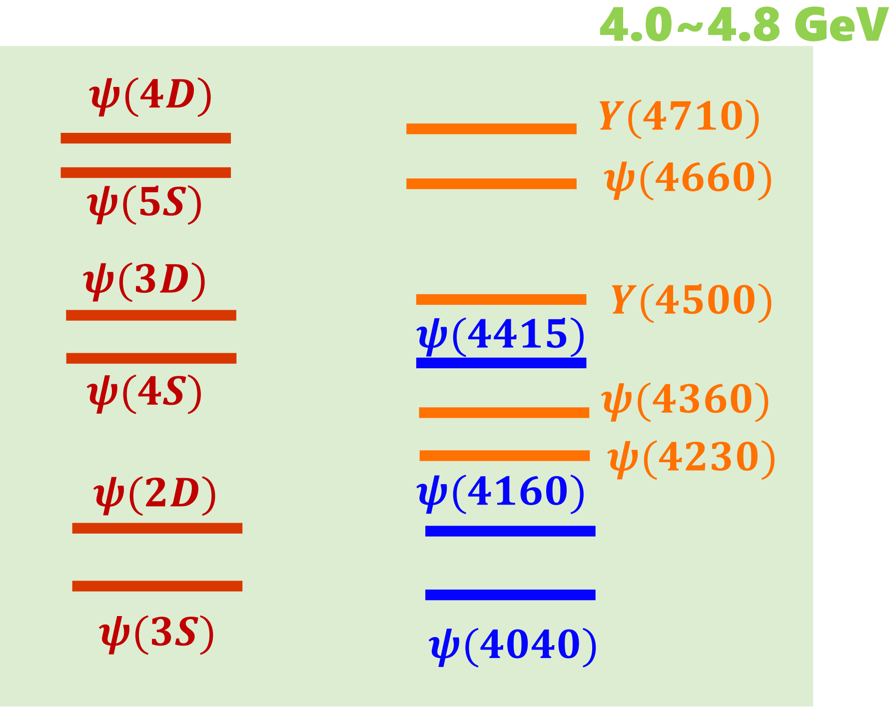
$$M = [M_A + \Delta M(M_0)] + [\Delta M(M) - \Delta M(M_0)]$$

$$\Delta M(M, M_0) = \mathrm{Re} \sum_{BC} \int_0^\infty \frac{(M_0 - M)\, \overline{|\mathcal{M}_{A\to BC}(\boldsymbol{q})|^2}}{(M - E_{BC})(M_0 - E_{BC})} q^2 dq$$

$M_0 = 3097$ **MeV for charmonia.**



- ***Vector states: "Y" problem***



4.0~4.8 GeV

$\psi(4D)$

$\psi(5S)$

$\psi(3D)$

$\psi(4S)$

$\psi(2D)$

$\psi(3S)$

$Y(4710)$

$\psi(4660)$

$Y(4500)$

$\psi(4415)$

$\psi(4360)$

$\psi(4230)$

$\psi(4160)$

$\psi(4040)$

- ***2P-wave states***

What is the true nature of $\chi_{c1}(3872)$?

Where is $\chi_{c0}(2P)$?

In potential model, there are only four free parameters: α, σ, C_0, r_c

In the QPC model, there is only one free parameter: the quark pair creation strength γ =0.422.

➢ "Y" problem

- » Y. S. Kalashnikova, Coupled-channel model for charmonium levels and an option for X(3872), Phys. Rev. D 72, 034010 (2005)
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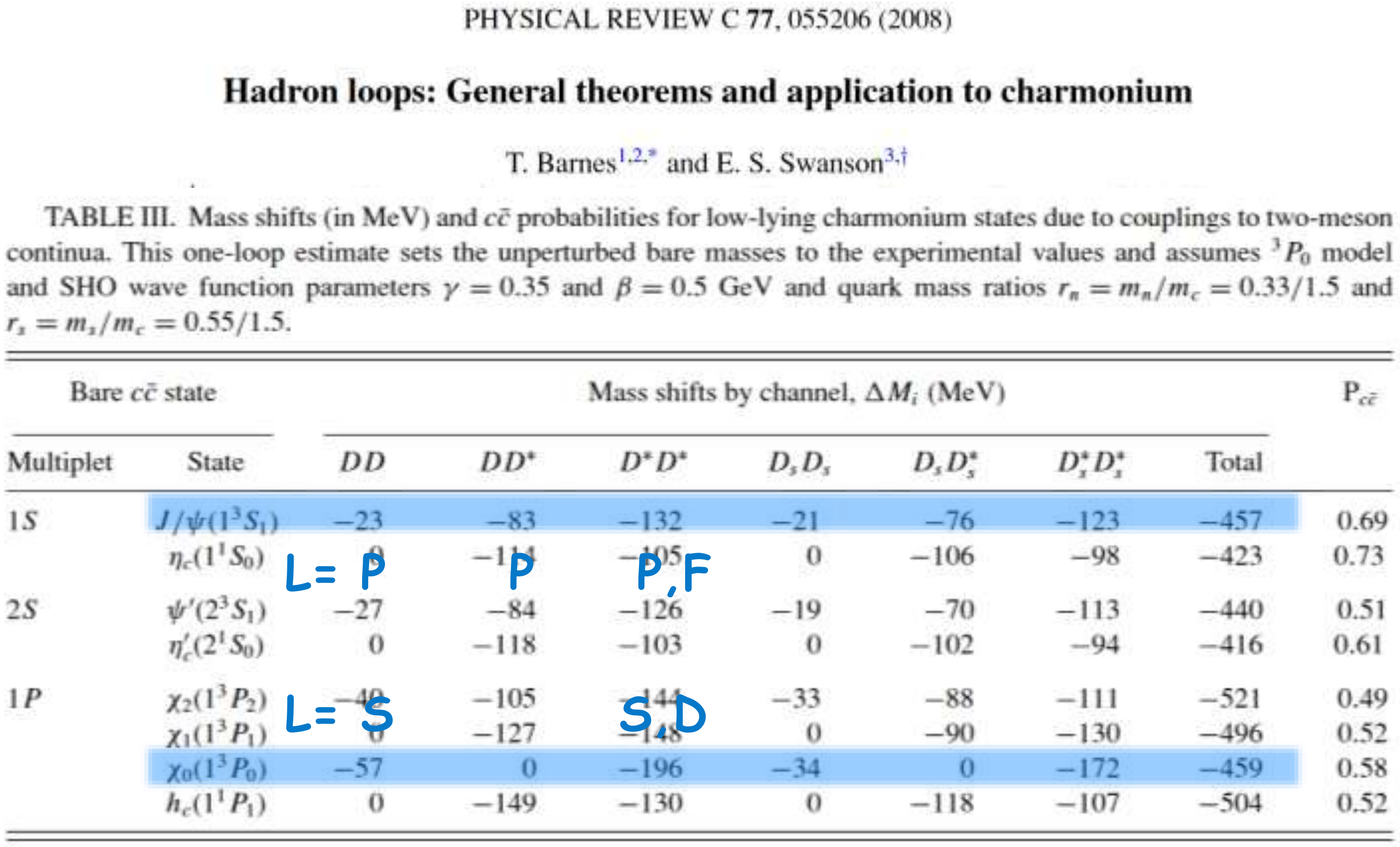


PHYSICAL REVIEW C **77**, 055206 (2008)

Hadron loops: General theorems and application to charmonium

T. Barnes[1,2,*] and E. S. Swanson[3,†]

TABLE III. Mass shifts (in MeV) and $c\bar{c}$ probabilities for low-lying charmonium states due to couplings to two-meson continua. This one-loop estimate sets the unperturbed bare masses to the experimental values and assumes ${}^3P_0$ model and SHO wave function parameters $\gamma = 0.35$ and $\beta = 0.5$ GeV and quark mass ratios $r_n = m_n/m_c = 0.33/1.5$ and $r_s = m_s/m_c = 0.55/1.5$.

Bare $c\bar{c}$ state		Mass shifts by channel, ΔM_i (MeV)							$P_{c\bar{c}}$
Multiplet	State	DD	DD^*	D^*D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	Total	
$1S$	$J/\psi(1^3S_1)$	−23	−83	−132	−21	−76	−123	−457	0.69
	$\eta_c(1^1S_0)$	0	−114	−105	0	−106	−98	−423	0.73
$2S$	$\psi'(2^3S_1)$	−27	−84	−126	−19	−70	−113	−440	0.51
	$\eta_c'(2^1S_0)$	0	−118	−103	0	−102	−94	−416	0.61
$1P$	$\chi_2(1^3P_2)$	−40	−105	−144	−33	−88	−111	−521	0.49
	$\chi_1(1^3P_1)$	0	−127	−148	0	−90	−130	−496	0.52
	$\chi_0(1^3P_0)$	−57	0	−196	−34	0	−172	−459	0.58
	$h_c(1^1P_1)$	0	−149	−130	0	−118	−107	−504	0.52

L= P   P   P,F

L= S   S,D

The significant mass shift induced by hadronic loops is largely unphysical.

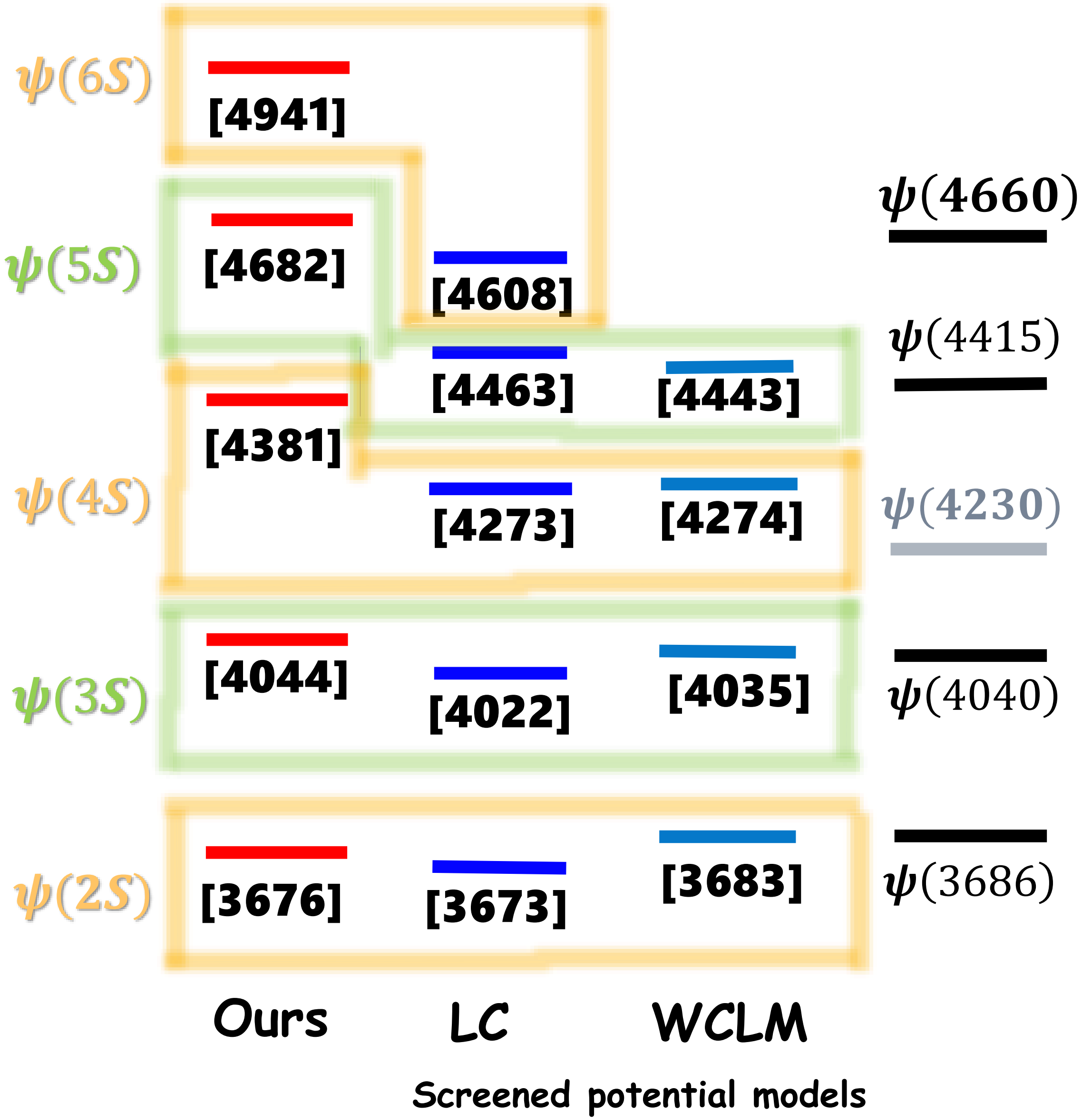
- » *B. Q. Li, C. Meng and K. T. Chao, Coupled-Channel and Screening Effects in Charmonium Spectrum, Phys. Rev. D 80, 014012 (2009)*

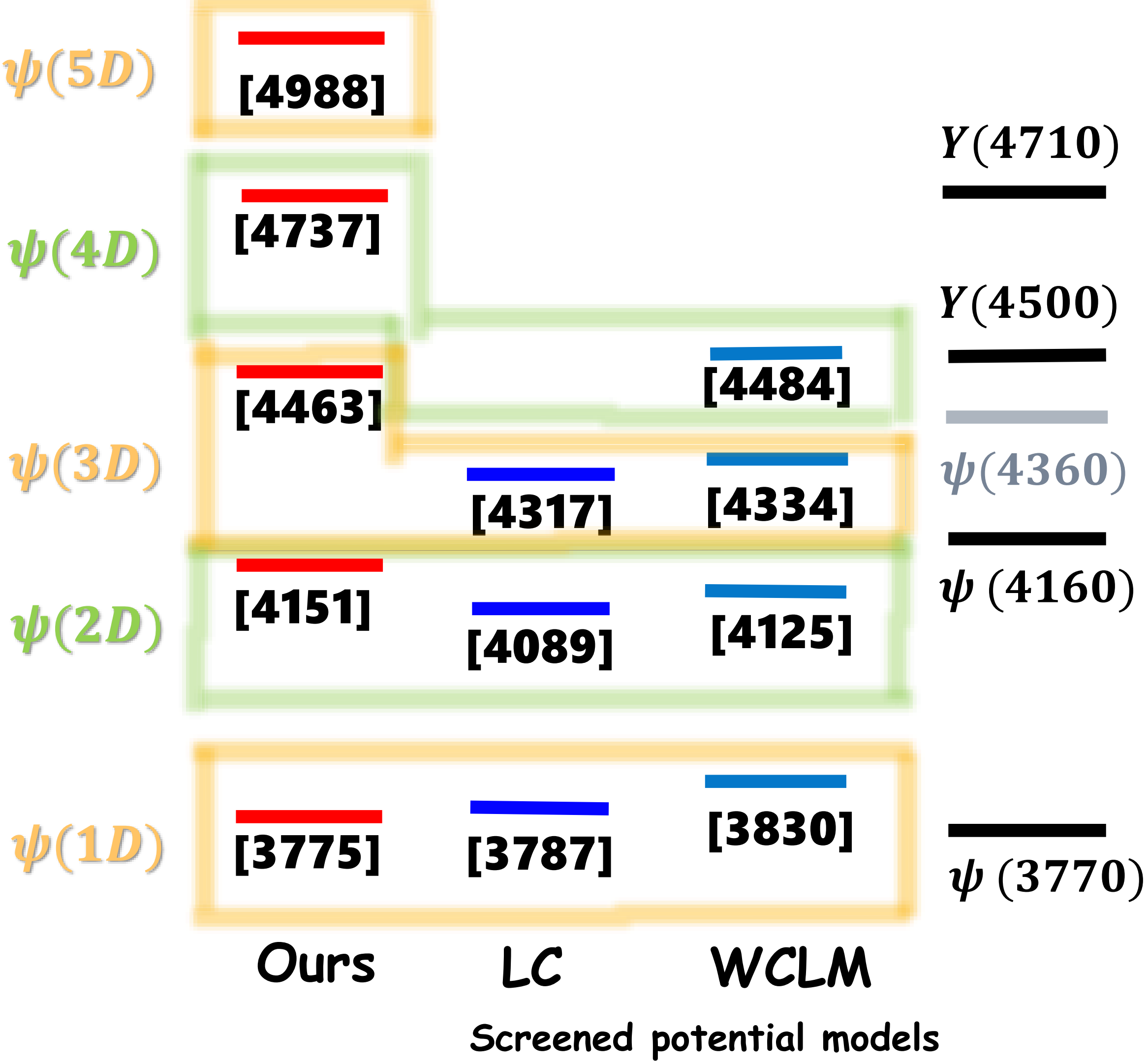
Screen potential model:

$\psi(4230)$—$\psi(4S)$ $\psi(4360)$—$\psi(3D)$

CCEFs ≃Screen potential effect ?

???

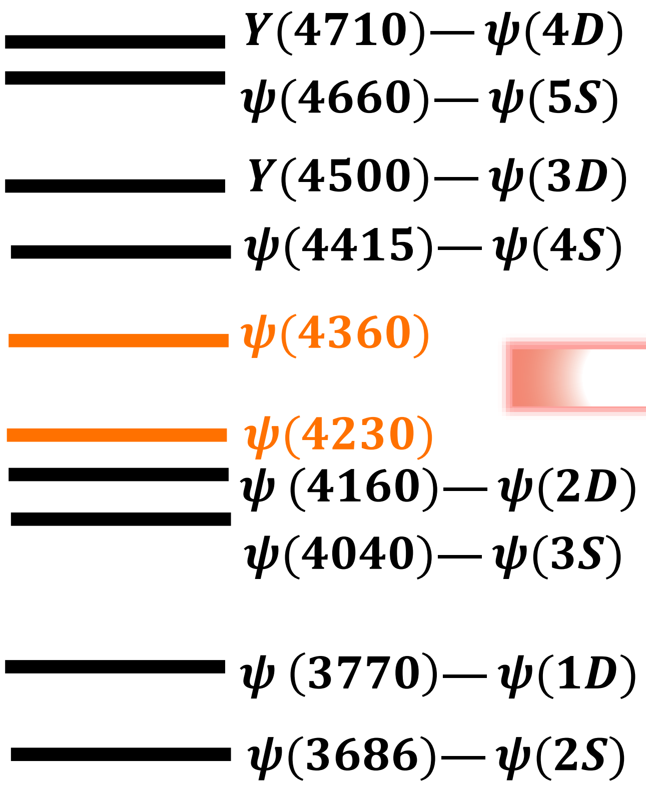




» LC: Bai-Qing Li, Kuang-Ta Chao.
Phys. Rev. D, 2009, 79:094004.

» WCLM: Jun-Zhang Wang, Dian-Yong Chen, Xiang Liu, et al.
Phys. Rev. D, 2019, 99(11):114003.

Our predicted mass shift amplitude due to the unquenched coupled-channel effect is on the order of 10s of MeV. Moreover, it does not exhibit the increasing trend from low to high energy levels predicted by the screened potential.



$Y(4710)$—$\psi(4D)$

$\psi(4660)$—$\psi(5S)$

$Y(4500)$—$\psi(3D)$

$\psi(4415)$—$\psi(4S)$

$\psi(4360)$

$\psi(4230)$

$\psi(4160)$—$\psi(2D)$

$\psi(4040)$—$\psi(3S)$

$\psi(3770)$—$\psi(1D)$

$\psi(3686)$—$\psi(2S)$

$\psi(4230)$: $D\,\bar{D}_1(2420)$ molecule?

aka $Y(4230)$; was $\psi(4260)$ $D\,\bar{D}_1(2420)$ threshold ~4280 MeV

$M_{exp} \simeq 4222.1 \pm 2.3$ MeV $\Gamma_{exp} \simeq 49 \pm 7$ MeV

» G. J. Ding, Phys. Rev. D 79, 014001 (2009) .

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» X. H. Liu and G. Li, Phys. Rev. D 88, 014013 (2013)

» Q. Wang, M. Cleven, F. K. Guo, C. Hanhart, U. G. Meissner, X. G. Wu and Q. Zhao, Phys. Rev. D 89, no.3, 034001 (2014)

» M. Cleven, Q. Wang, F. K. Guo, C. Hanhart, U. G. Meissner and Q. Zhao, Phys. Rev. D 90, no.7, 074039 (2014)

» W. Qin, S. R. Xue and Q. Zhao, Phys. Rev. D 94, no.5, 054035 (2016)

» M. Cleven and Q. Zhao, Phys. Lett. B 768, 52 (2017)

» L. von Detten, V. Baru, C. Hanhart, Q. Wang, D. Winney and Q. Zhao, Phys. Rev. D 109, no.11, 116002 (2024)

$\psi(4360)$: $D^*\,\bar{D}_1(2420)$ molecule?

aka $Y(4360)$; was $X(4360)$ $D^*\,\bar{D}_1(2420)$ threshold ~4430 MeV

$M_{exp} \simeq 4374 \pm 7$ MeV $\Gamma_{exp} \simeq 118 \pm 12$ MeV

» T. Ji, X. K. Dong, F. K. Guo and B. S. Zou, Phys. Rev. Lett. 129, no.10, 102002 (2022)

» F. Z. Peng, M. J. Yan, M. Sanchez and M. Pavon Valderrama, Phys. Rev. D 107, no.1, 016001 (2023)

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➢ $\chi_{c1}(2P)$: Two-pole structure? Our Key Prediction!

One bare state → CCEFs → Cusp → CCEq multiple solutions → case

How can multiple solutions in the CCEq be identified?

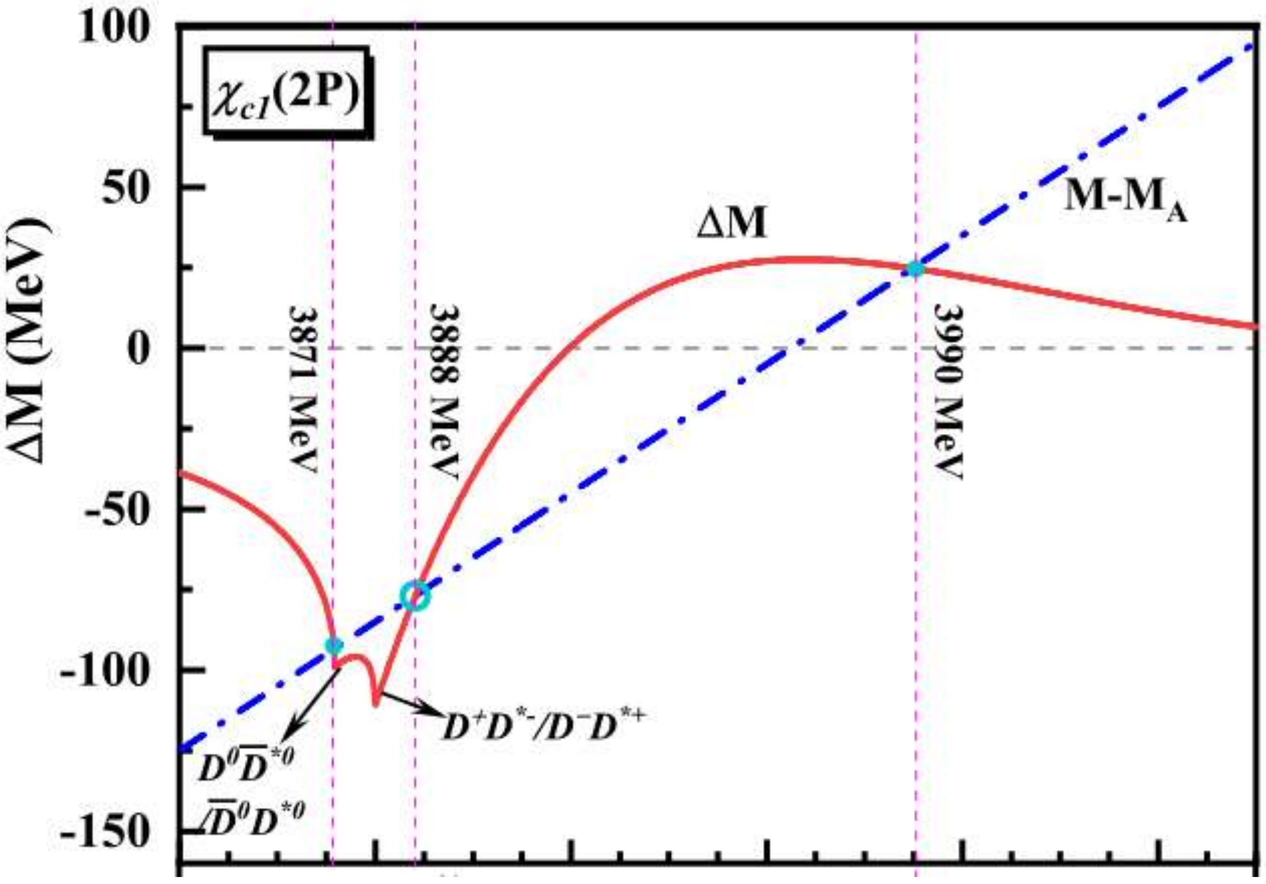
$$\mathrm{Re}\sum_{BC}\int_0^\infty \frac{(M_0-M)\boxed{|\mathcal{M}_{A\to BC}(\boldsymbol{q})|^2}}{(M-E_{BC})(M_0-E_{BC})}q^2dq$$

$$\boxed{|\mathcal{M}(\boldsymbol{q})|^2 \propto e^{-a\boldsymbol{q}^2}f(\boldsymbol{q})\boldsymbol{q}^{2L}}$$

L=0, $|\mathcal{M}(\boldsymbol{q})|^2 \to$ constant

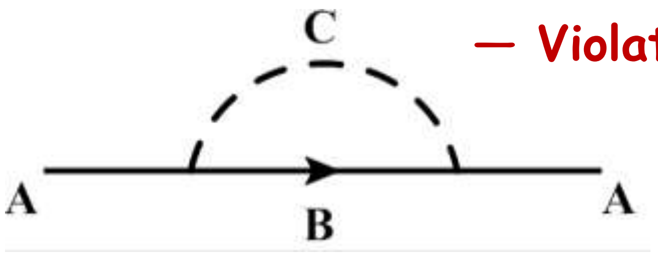
$\frac{\partial}{\partial M}g(M) \to$ **large** **Cusp**

Isgur early on pointed out the characteristic threshold behavior of this S-wave coupling.
[Phys.Rev.D 57,4041-4053(1998)]



The multiple solutions of the CCEq correspond to distinct peaks in the spectral function.

不稳定
束缚态



Unphysical Solutions
— Violation of the Wigner Condition

$$\frac{d\delta}{dk} \geqslant -\frac{1}{m_\pi}\frac{\sqrt{s}}{2k},$$

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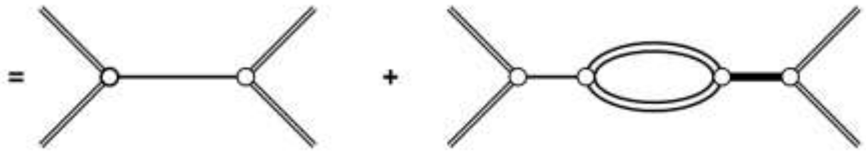
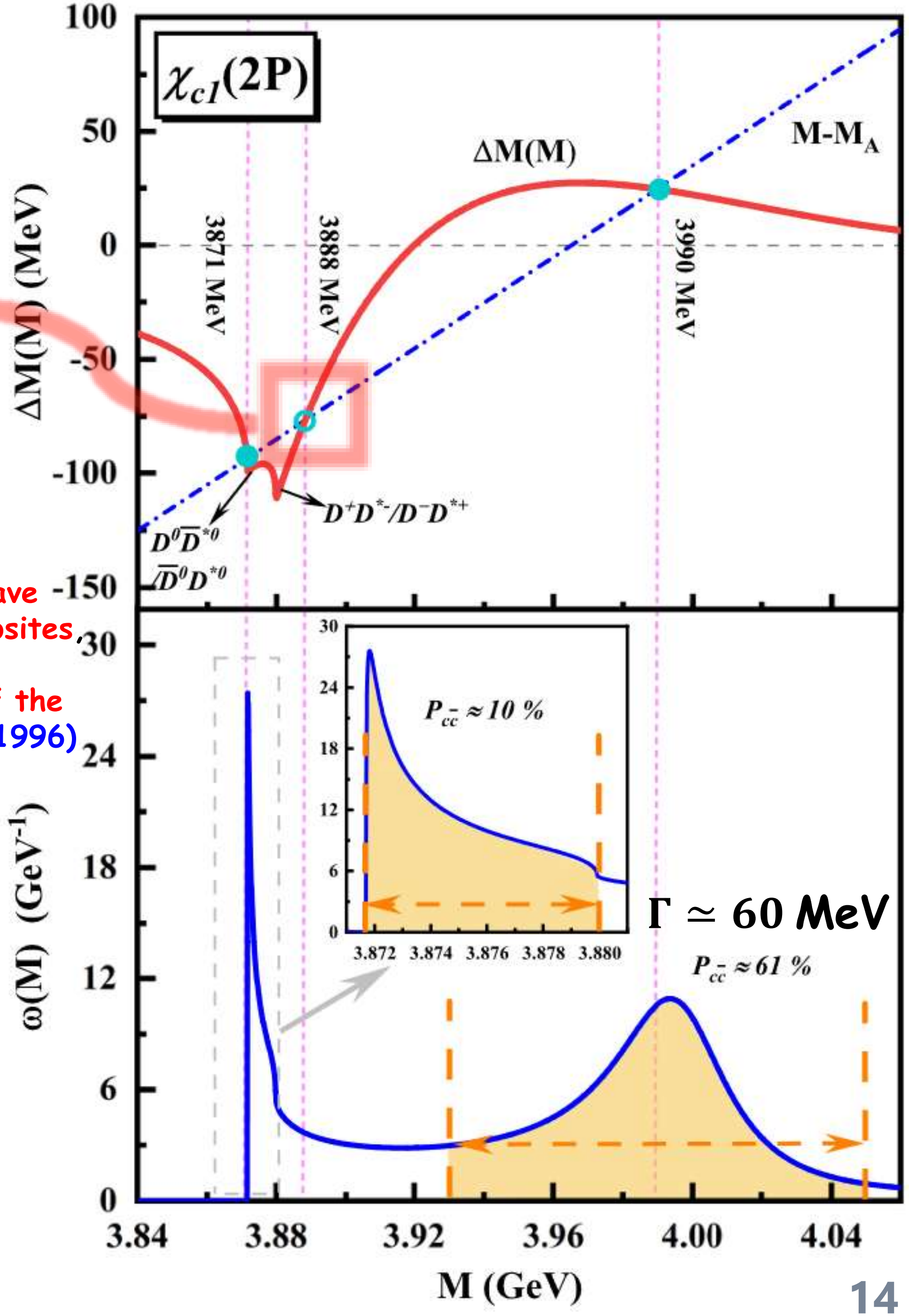
散射
共振态



T = V + V GT

$$\omega(M) = \frac{1}{2\pi}\frac{\Gamma_R}{(M-M_R)^2+\frac{\Gamma_R^2}{4}}.$$



Spectral density method: Bogdanova:1991zz ; Kalashnikova:2005ui ; Kalashnikova:2009gt ; Baru:2010ww ; Giacosa:2007bn ; Wang:2023snv

CCEq multiple solutions

	Lower-mass state [Molecular Dominance]	Higer-mass state [$q\bar{q}$ core Dominance]
$\chi_{c1}(2P)$	Mass (MeV)	Mass, Γ (MeV)
Kalashnikova	~3871	3990,27
DS	~3872	3950,35
OSEF	3871	3942,-
CRB	~3872	~4030-4190,~20-140
ZX	3871	3934,80
GPC	3872	3995,72
WYWOZ	~3872	3958,17
WLCMZ	~3872	~3980-4150,~10-140
LQCD	~3872	~4000-4070,30-70
Ours	**3871**	**3990,60**

$\chi_{c1}(3872)$ $\chi_{c1}(4010)$ 4013^{+8}_{-9}, 63 ± 13

Great agreement with our predictions!

» LHCb, Observation of New Charmonium or Charmoniumlike States in $B^+ \to D^{*\pm}D^{\mp}K^+$ Decays, Phys. Rev. Lett. 133, no.13, 131902 (2024)

» Kalashnikova:
Y. S. Kalashnikova, Phys. Rev. D 72, 034010 (2005)
» DS:
I. V. Danilkin and Y. A. Simonov,
Phys. Rev. Lett. 105, 102002 (2010)
» OSEF:
P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, Phys. Rev. D 81, 054023 (2010)
» CRB:
S. Coito, G. Rupp and E. van Beveren,
Eur. Phys. J. C 73, no.3, 2351 (2013)
» ZX:
Z. Y. Zhou and Z. Xiao,
Phys. Rev. D 96, 054031 (2017);96, 099905(E) (2017)
» GPC:
F. Giacosa, M. Piotrowska, and S. Coito,
Int. J. Mod. Phys. A 34, 1950173 (2019)
» WYWOZ:
G. J. Wang, Z. Yang, J. J. Wu, M. Oka and S. L. Zhu, Sci. Bull. 69, 3036-3041 (2024)
» WLCMZ:
J. Z. Wang, Z. Y. Lin, Y. K. Chen, L. Meng and S. L. Zhu,[arXiv:2404.16575 [hep-ph]]
» LQCD:
H. Li, C. Shi, Y. Chen, M. Gong, J. Liang, Z. Liu and W. Sun,
[arXiv:2402.14541 [hep-lat]].

Although different models provide overlapping interpretations of $\chi_{c1}(3872)$, there are significant differences in the details!

» LHCb, Observation of New Charmonium or Charmoniumlike States in $B^+ \to D^{*\pm}D^{\mp}K^+$ Decays, Phys. Rev. Lett. 133, no.13, 131902 (2024)

The other three new charmonium(-like) states

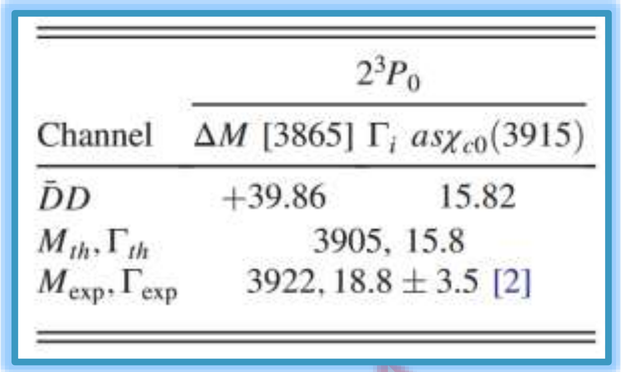
Resonance	J^{PC}	Exp. Mass, Γ (MeV)	States	Ours Mass, Γ (MeV)
$\eta_c(3945)$	0^{-+}	$3945^{+65}_{-45}, 130^{+193}_{-119}$	$\eta_c(3S)$	4022, 62
$h_c(4000)$	1^{+-}	$4000^{+46}_{-36}, 184^{+168}_{-106}$	$h_c(2P)$	3961, 66
$h_c(4300)$	1^{+-}	$4307^{+10}_{-11}, 58^{+56}_{-41}$	$h_c(3P)$	4307, 25

The results predicted by our unquenched quark model align with experimental observations within the error range.

➢ $\chi_{c0}(2P)$: *Where?*

$\chi_{c0}(3915)$ $M_{exp} \simeq 3922 \pm 2$ MeV

$\Gamma_{exp} \simeq 20 \pm 4$ MeV



Channel	2^3P_0	
	ΔM [3865]	Γ_i as $\chi_{c0}(3915)$
$\bar{D}D$	+39.86	15.82
M_{th}, Γ_{th}	3905, 15.8	
M_{exp}, Γ_{exp}	3922, 18.8 ± 3.5 [2]	

Quenched
~3840-3900
MeV

CCEFs play a crucial role in uncovering the puzzles surrounding $\chi_{c0}(2P)$!

$M_{exp} \simeq 3862^{+50}_{-35}$ MeV

$\Gamma_{exp} \simeq 200^{+180}_{-110}$ MeV

$\chi_{c0}(3860)$

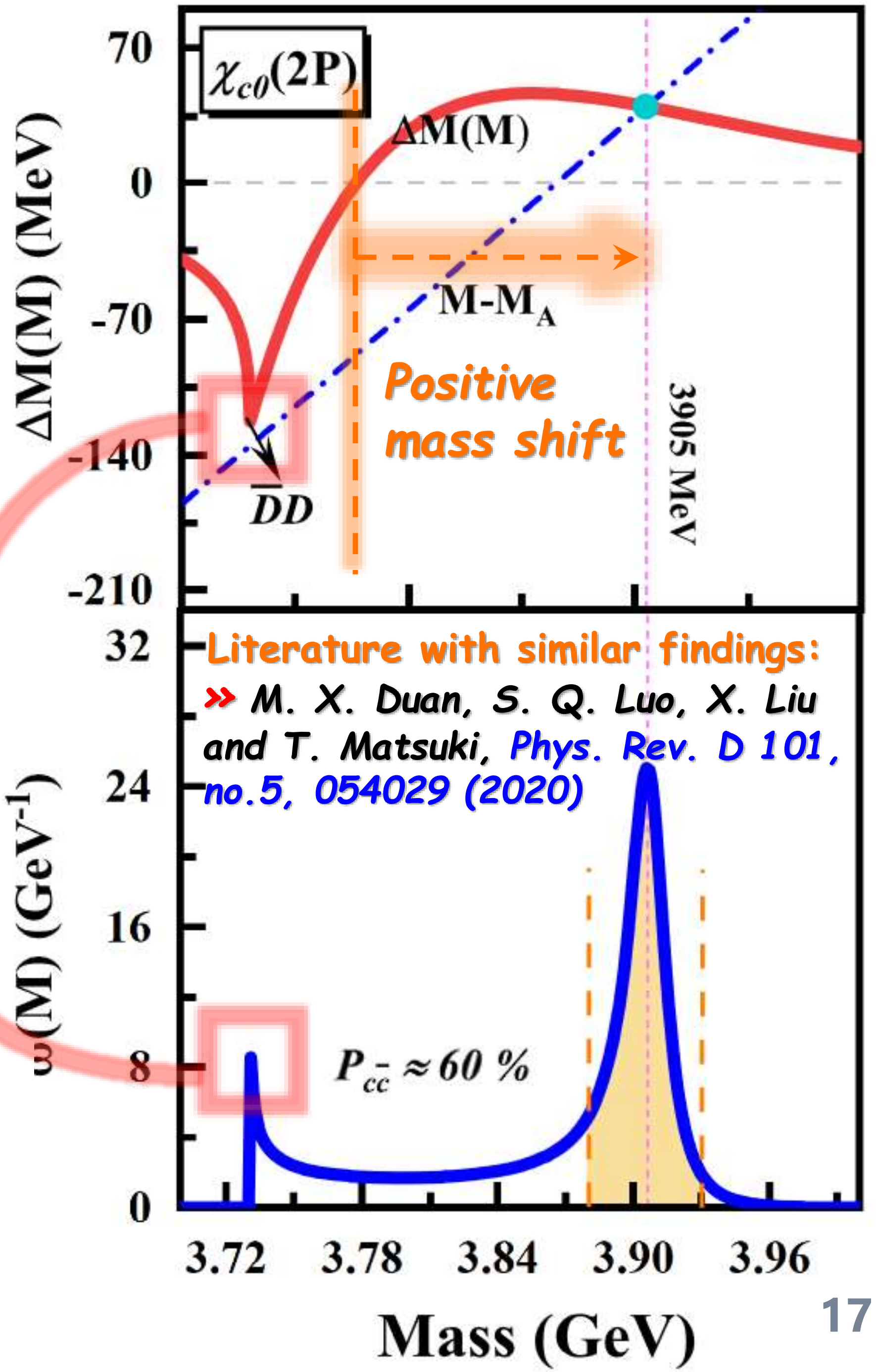
$\chi_{c0}(3730)$? Bound state? Virtual state?

Literature with similar findings:

P-wave charmonium contribution to hidden-charm states from lattice QCD

Pan-Pan Shi[1,2,3,*] Miguel Albaladejo[1,†] Meng-Lin Du[4,‡] Feng-Kun Guo[2,3,5,6,§] and Juan Nieves[1,¶]

arXiv:2410.19563v1 [hep-ph]



Literature with similar findings:
» M. X. Duan, S. Q. Luo, X. Liu and T. Matsuki, *Phys. Rev. D* 101, no.5, 054029 (2020)

SUMMARY

Two key points in CCEFs calculation [Phys.Rev.D 109(2024)11,116006]：

- Management of mass shifts
- Selection of coupling channels

Phys. Rev. D 110 (2024) 5, 056034 :

The mass correction induced by the unquenched coupled-channel effect seems to differ from the screened potential effect.

Unquenched charmonium spectrum

S-wave

$\psi(4040)-\psi(4S)$
$\psi(4415)-\psi(5S)$
$\psi(4660)-\psi(6S)$

P-wave

$\chi_{c0}(3915)-\chi_{c0}(2P)$, $\chi_{c2}(3930)-\chi_{c2}(2P)$
$\chi_{c1}(3872)-\chi_{c1}(2P)$, $\chi_{c1}(4010)-\chi_{c1}(2P)$
$\chi_{c1}(4274)-\chi_{c1}(3P)$, $X(4350)-\chi_{c2}(3P)$
$\chi_{c0}(4500)-\chi_{c0}(4P)$, $\chi_{c0}(4700)-\chi_{c0}(5P)$

D-wave

$\psi(3770)-\psi(1D)$, $\psi_2(3823)-\psi_2(1D)$
$\psi_3(3842)-\psi_3(1D)$, $\psi(4160)-\psi(2D)$
$X(4160)-\eta_{c2}(2D)$, $Y(4500)-\psi(3D)$
$Y(4710)-\psi(4D)$

$\chi_{c0}(3860)$? $\psi(4230)$?
$\chi_{c1}(4140)$? $Y(4360)$?
$\chi_{c1}(4685)$?

Thank you for your attention!