

Unraveling charmonium mixing scheme for the $\psi(4220)$ and $\psi(4380)$ by a coupled-channel approach

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Based on

Zi-Long Man, Si-Qiang Luo, Zi-Yue Bai, and Xiang Liu, arXiv:2502.08072

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Outlines

1. Background

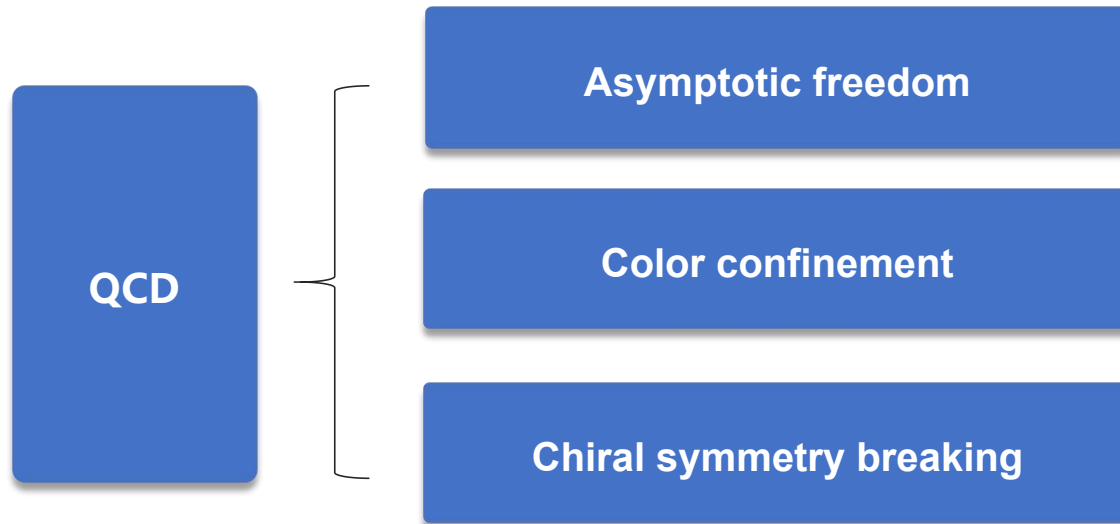
2. Formalism

3. Results for $\psi(4220)$ and $\psi(4380)$

4. Summary

Background

Hadron properties difficult to derive from QCD

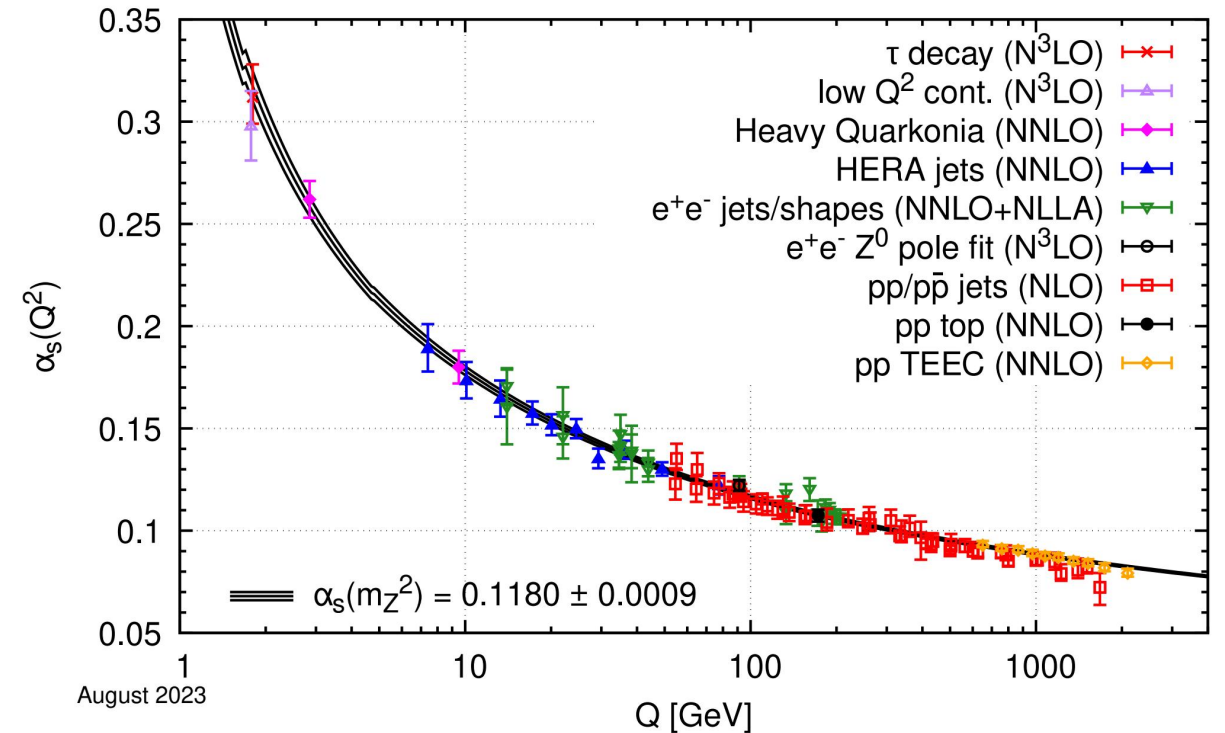


High energy: asymptotic freedom

Low energy: non-perturbative important

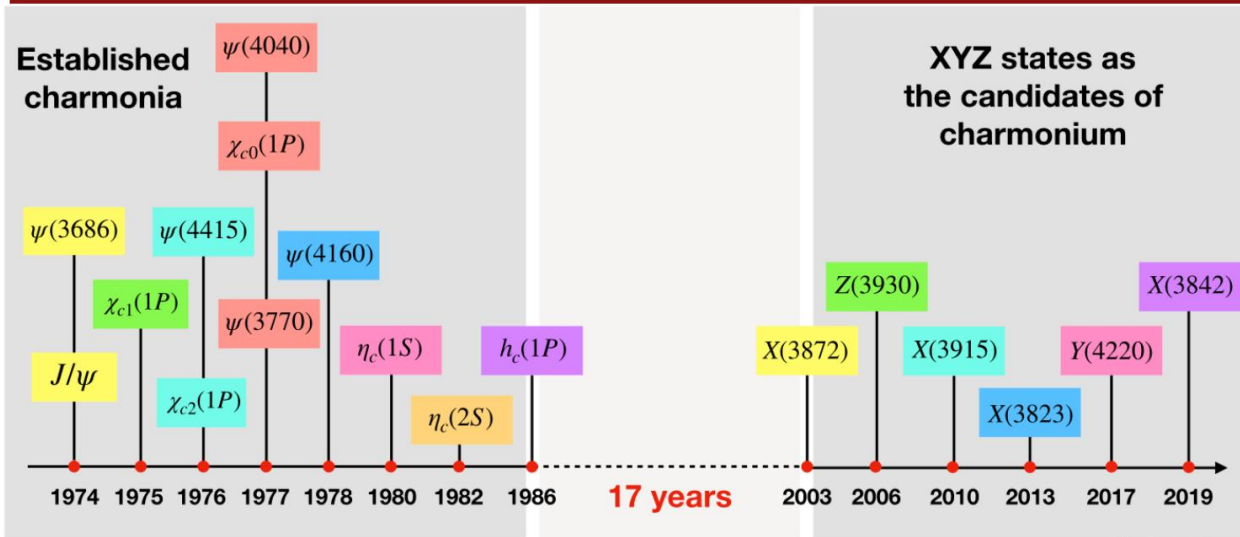
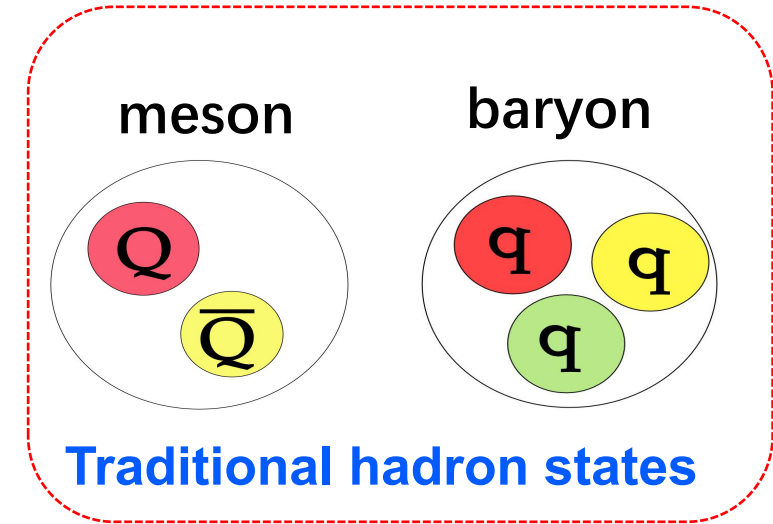
Hadron properties difficult to derive from QCD

Methods: Lattice QCD, Quark Model, Effective field theory et al.



Particle Data Group, Phys. Rev. D 110, 030001 (2024)

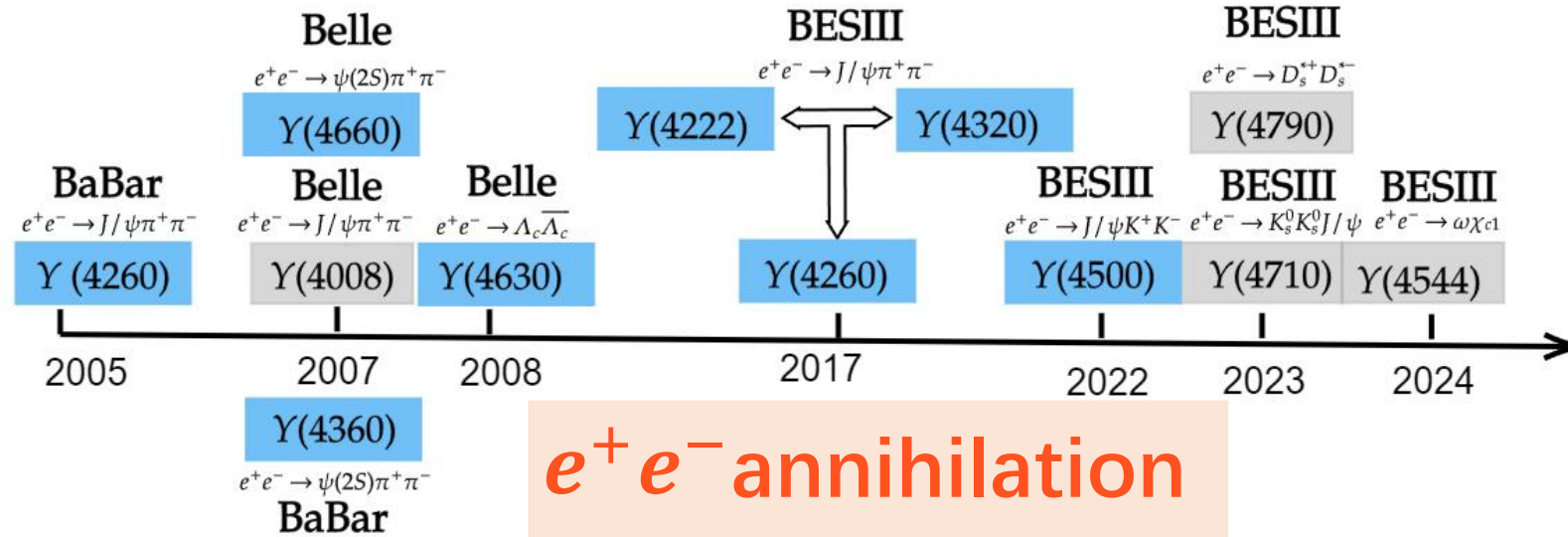
November Revolution



$$V(r) = -\frac{k}{r} + \frac{r}{a^2}$$

E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D 17, 30390 (1978)

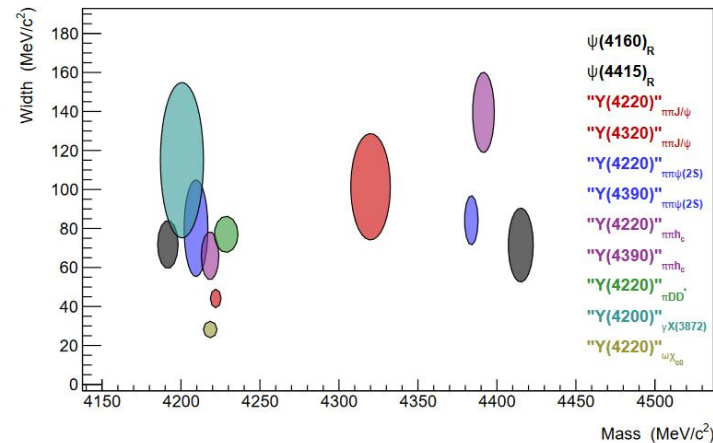
The “Y” problem



The Y Problem

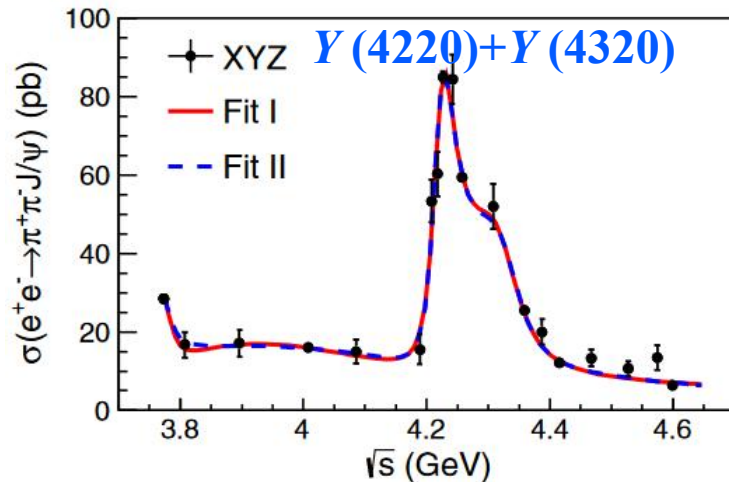
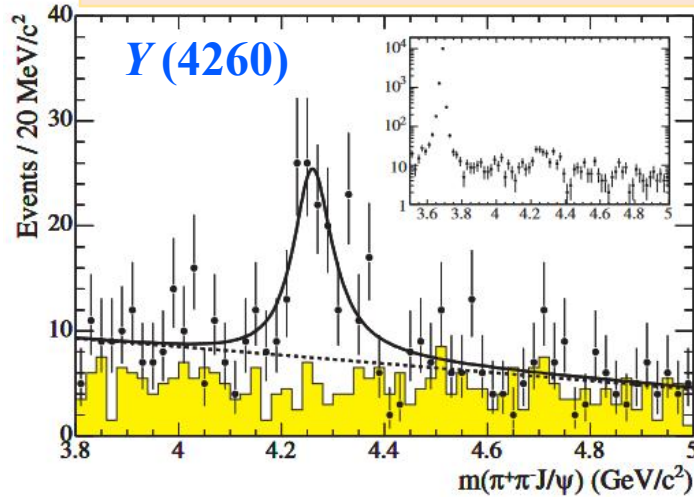
Exclusive e^+e^- cross sections have shown surprisingly complex behavior as a function of cms energy. The Y(4260) is more complex than a single ordinary resonance, as shown by the complicated lineshape in the $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross section in Fig. 3.10(e); the Y(4360) and Y(4660) are seen in $e^+e^- \rightarrow \pi^+\pi^-h_c$ in Fig. 3.10(f); the Y(4220) is seen in $e^+e^- \rightarrow \omega\chi_{c0}$ in Fig. 3.10(g) and so on. A summary of the masses and widths of resonances extracted from recent BESIII results is shown in Fig. 3.11. There is currently very little consistency between different reactions. Furthermore, none of these complicated features are apparently present in the inclusive e^+e^- cross section, which only shows evidence for the $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ [64]. This is the “Y” problem. Are the many peaks seen in e^+e^- cross sections really new states? Or are they the results of more subtle effects? With new data, will new patterns emerge? What are their exact line shapes? Will they match theoretical predictions, such as a very asymmetric line shape for the Y(4260) obtained within a molecular frame [69]? With our limited number of data points (cms energies), there is little hope in resolving the issue. We require (1) more data spread over a variety of cms energies, and (2) a global and simultaneous analysis of many final states. This latter

Parameters of the Peaks in e^+e^- Cross Sections

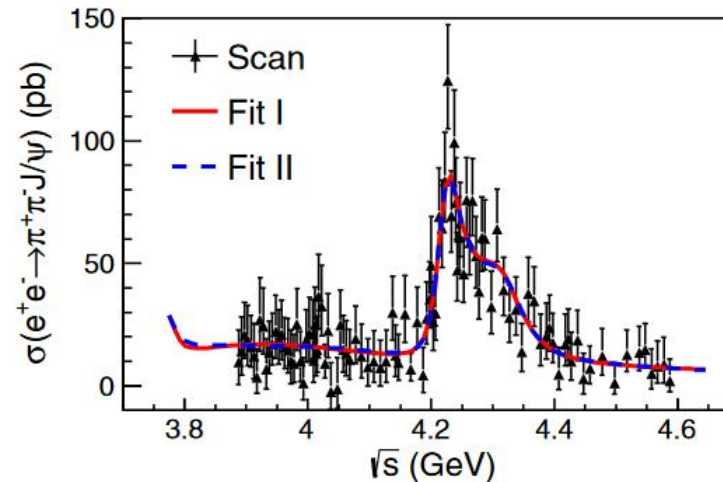
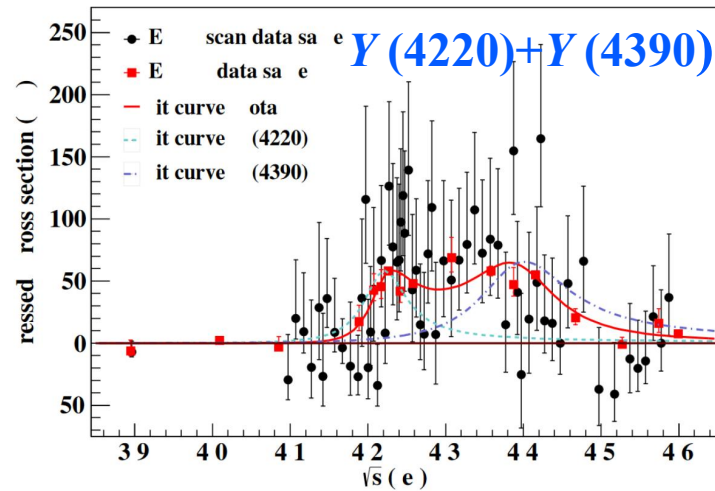


More precise experimental results

$$e^+e^- \rightarrow J/\psi\pi^+\pi^-$$

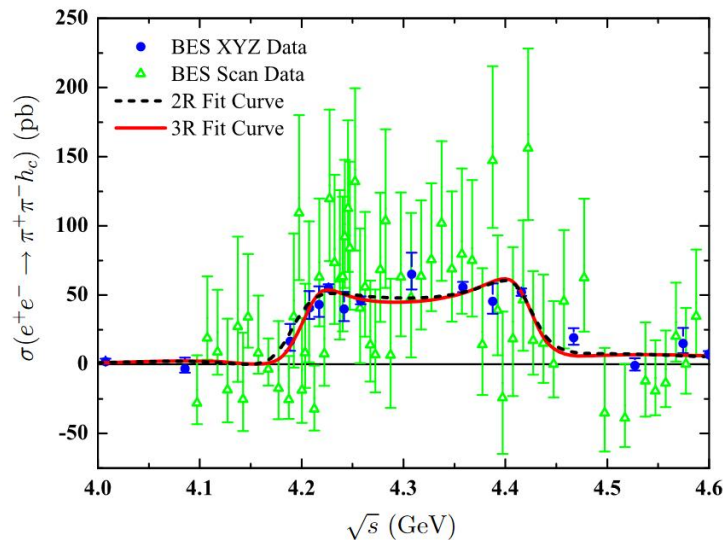
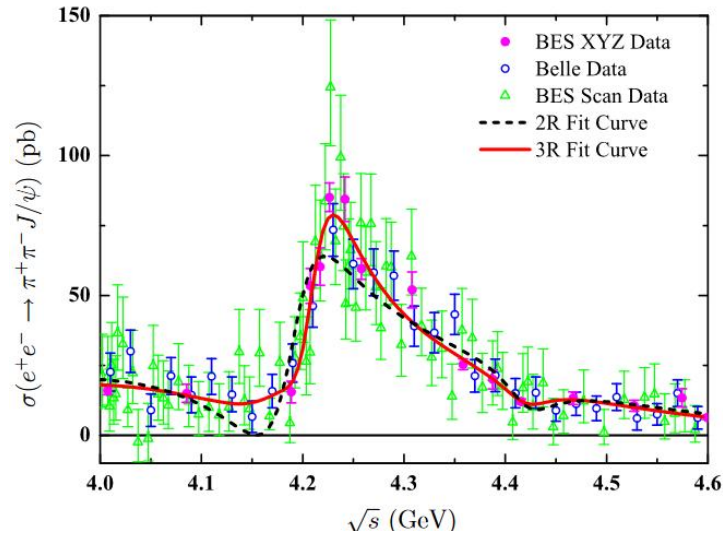


$$e^+e^- \rightarrow h_c\pi^+\pi^-$$



B. Aubert *et al.* [BaBar], Phys. Rev. Lett 95, 142001 (2005), M. Ablikim *et al.* [BESIII], Phys. Rev. Lett 118, 092001 (2017),
M. Ablikim *et al.* [BESIII], Phys. Rev. Lett 118, 092002 (2017)

Y (4220)



- **Y(4220) must be introduced.**
- **Y(4320) and Y(4390) can be killed by introducing interference effect between $\psi(4160)$ and $\psi(4415)$, and the continuum background**

D. Y. Chen, X. Liu, and T. Matsuki, Eur. Phys. J. C 78, 126 (2018)

Theoretical explanations to $Y(4220)$

➤ $D\bar{D}_1$ molecule

- T. Ji, X. K. Dong, F. K. Guo, and B. S. Zou, Phys. Rev. Lett. 129, 102002 (2022)
- L.V. Detten, V. Baru, C. Hanhart, Q. Wang, D. Winney, and Q. Zhao, Phys. Rev. D 109, 116002 (2024)
- M. Z. Liu and Q. Wu, Eur. Phys. J. C 78, 126 (2025)

➤ Tetraquark

- A. Ali, L. Maiani, A. V. Borisov, I. Ahmed, M. Jamil Aslam, A. Y. Parkhomenko, A. D. Polosa, and A. Rehman, Eur. Phys. J. C 78, 29 (2021)
- Z. G. Wang, Nucl. Phys. B 973, 115592 (2021)

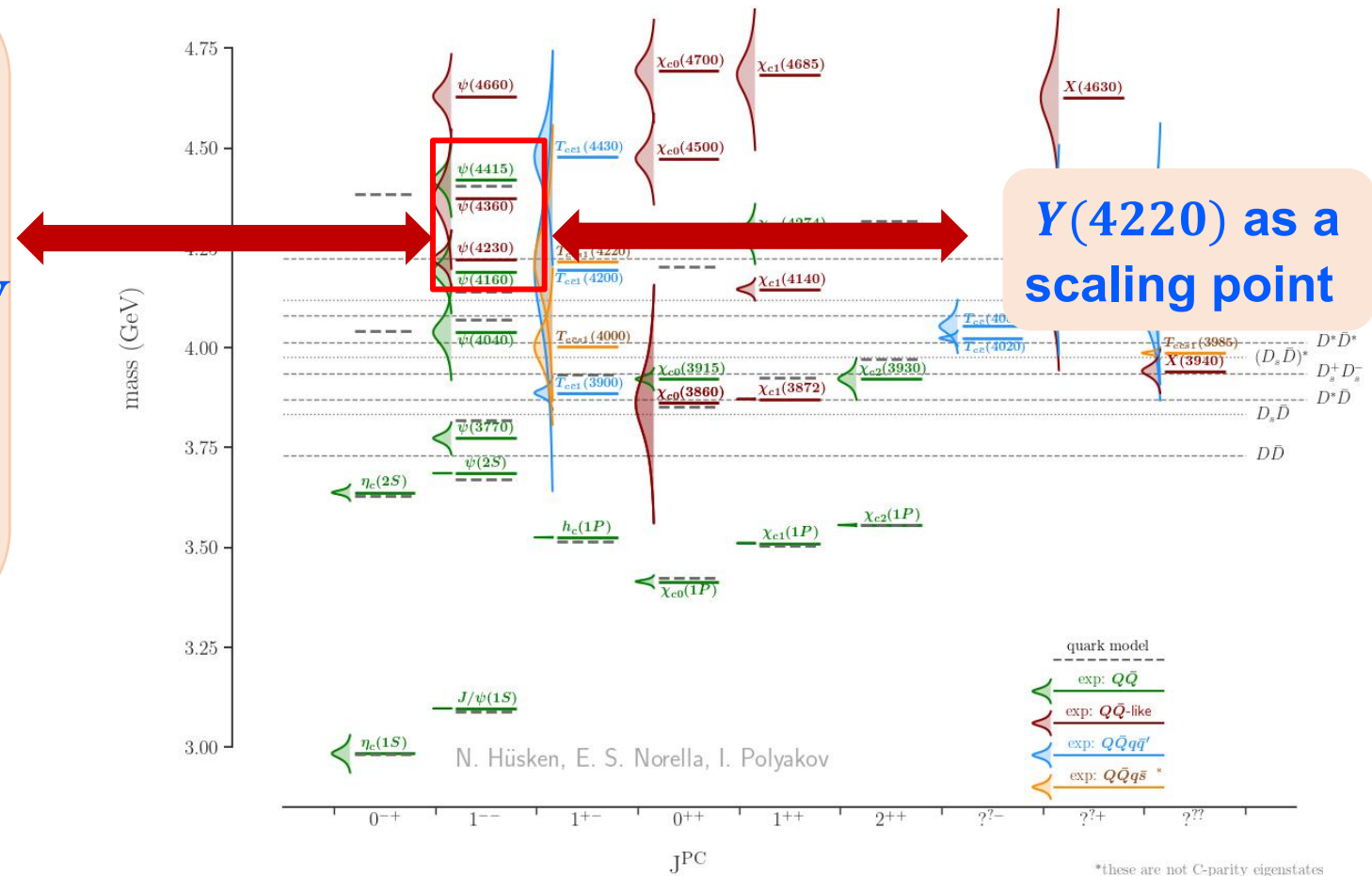
➤ Review

- H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016)
- H.X. Chen, W. Chen, X. Liu, Y.R. Liu, and S.L. Zhu, Rept. Prog. Phys. 80, 076201 (2017)
- F. K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
- Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019)
- N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, Phys. Rept. 873, 1 (2020)
- H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rept. Prog. Phys. 86, 026201 (2023)
- M. Z. Liu, Y. W. Pan, Z. W. Liu, T. W. Wu, J. X. Lu, and L. S. Geng, Phys. Rept. 1108,1 (2025)

Quenched potential model

$M_{\psi(4S)}$
 4433.0 MeV
 $\updownarrow \sim 200 \text{ MeV}$
 $M_{Y(4220)}$
 $4222.1 \pm 2.3 \text{ MeV}$

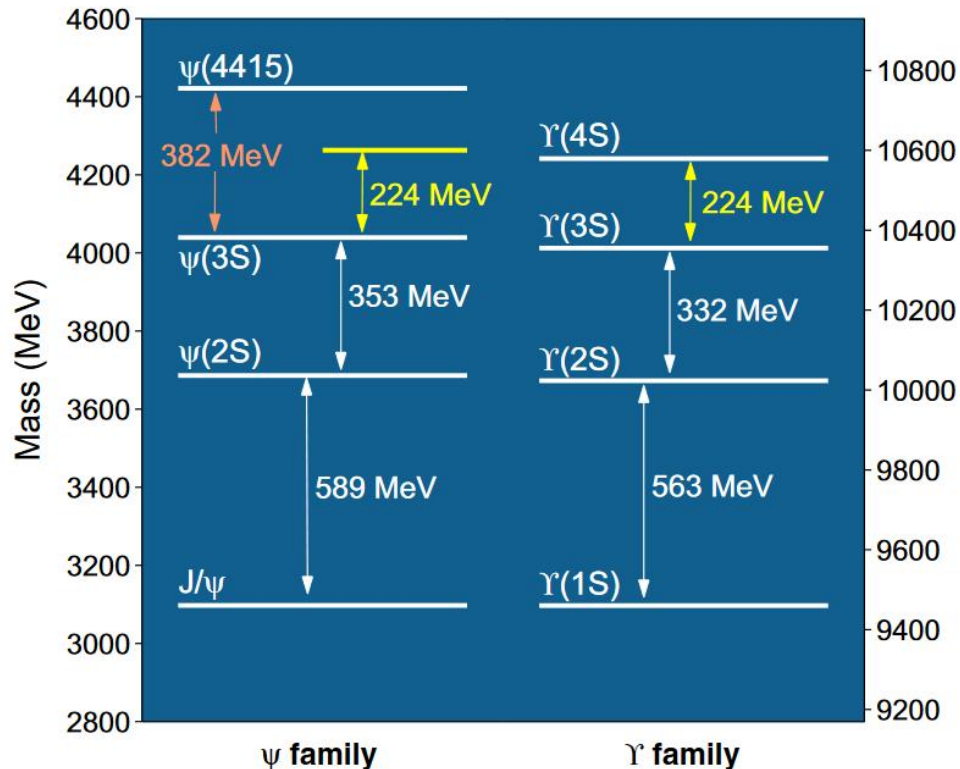
There is no room for the $Y(4220)$



N. Husken, E. S. Norella, and I. Polyakov, arXiv:2410.06923

Unquenched quark model

$$M_{\psi(4S)} \sim 4260 \text{ MeV}$$



L. P. He, D. Y. Chen, X. Liu, and T. Matsuki, Eur. Phys. J. C 74, 3208 (2014)

States	Mass (MeV)	Γ_{ee}^0 (keV)	Γ_{ee} (keV)	$\Gamma_{ee}^{\text{expt}}$ (keV)	Candidate
1S	3097	10.18	5.34	5.26 ± 0.37	$\psi(3097)$
2S	3686	4.13	2.17	2.14 ± 0.21	$\psi(3686)$
3S	4033	2.35	1.23	0.75 ± 0.15	$\psi(4040)$
4S	4262	1.46	0.77	0.77 ± 0.23	$\psi(4160)$
5S	4415	0.91	0.48	0.47 ± 0.10	$\psi(4415)$
1P	3526				$\chi(3526)_{\text{c.o.g.}}$
1D	3805				$\psi(3770)$
2D	4105				

Y. B. Dong, K. T. Chao, and D. H. Qin, Phys. Rev. D 51, 5064 (1995)

State		Expt.	Theor. of ours		Theor. of Ref. [5]	
			Mass	$\langle r^2 \rangle^{1/2}$	NR	GI
1S	$J/\psi(1^3S_1)$	3096.916 ± 0.011	3097	0.41	3090	3098
	$\eta_c(1^1S_0)$	2980.3 ± 1.2	2979		2982	2975
2S	$\psi'(2^3S_1)$	3686.093 ± 0.034	3673	0.91	3672	3676
	$\eta_c'(2^1S_0)$	3637 ± 4	3623		3630	3623
3S	$\psi(3^3S_1)$	4039 ± 1	4022	1.38	4072	4100
	$\eta_c(3^1S_0)$		3991		4043	4064
4S	$\psi(4^3S_1)$	4263_{-9}^{+8}	4273	1.87	4406	4450

B. Q. Li and K. T. Chao, Phys. Rev. D 79, 094004 (2009)

Coupled-channel model ~ Screened potential model

B. Q. Li, C. Meng, and K. T. Chao, Phys. Rev. D 80, 014012 (2009)

4S-3D mixing scheme

Screening potential

$$S^{scr}(r) = \frac{b(1 - e^{-\mu r})}{\mu} + c$$

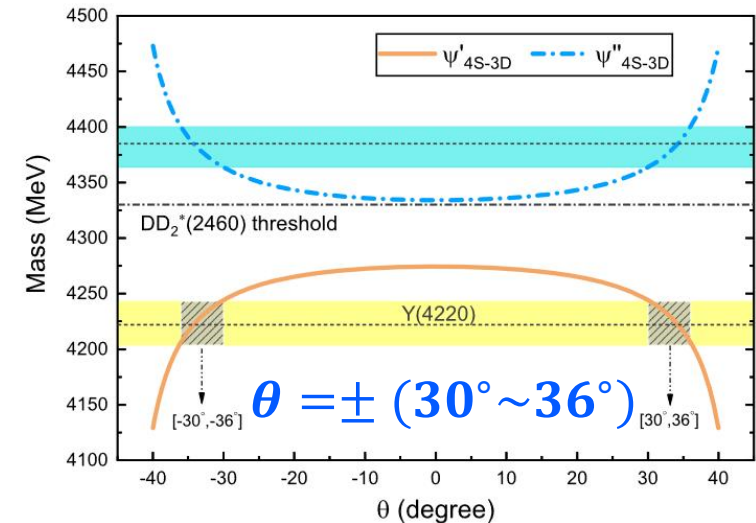
$$m_{\psi(4S)} \xleftarrow{\sim 50 \text{ MeV}} m_{Y(4220)} = 4222.1 \pm 2.3 \text{ MeV}$$

$$1^{--}(J^{PC}) = C_S |(n+1)^3S_1\rangle + C_D |(n)^3D_1\rangle$$

$$\begin{pmatrix} |\psi'_{4S-3D}\rangle \\ |\psi''_{4S-3D}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |4^3S_1\rangle \\ |3^3D_1\rangle \end{pmatrix}$$

$$m_{\psi'_{4S-3D}}^2 = \frac{1}{2} (m_{4S}^2 + m_{3D}^2 - \sqrt{(m_{3D}^2 - m_{4S}^2)^2 \sec^2 2\theta})$$

$$m_{\psi''_{4S-3D}}^2 = \frac{1}{2} (m_{4S}^2 + m_{3D}^2 + \sqrt{(m_{3D}^2 - m_{4S}^2)^2 \sec^2 2\theta})$$



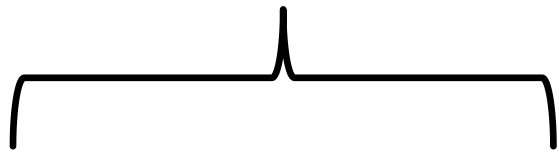
What dynamical mechanism constrains the mixing angle within this range?

Formalism

The mixing scheme induced by the tensor term

$$\begin{pmatrix} |\psi'_{4S-3D}\rangle \\ |\psi''_{4S-3D}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |4^3S_1\rangle \\ |3^3D_1\rangle \end{pmatrix}$$

4S-3D mixing dynamics



The tensor term

The coupled-channel effects

Godfrey-Isgur potential model

$$H_0 = (\vec{p}^2 + m_c^2)^{1/2} + (\vec{p}^2 + m_{\bar{c}}^2)^{1/2} + \tilde{V}_{eff}(\vec{p}, \vec{r})$$

$$H_T = \frac{4\alpha_s}{3m_c m_{\bar{c}} r^3} \left(\frac{3(\vec{S}_c \cdot \vec{r})(\vec{S}_{\bar{c}} \cdot \vec{r})}{r^2} - \vec{S}_c \cdot \vec{S}_{\bar{c}} \right)$$

$$\begin{pmatrix} M_S^0 & \langle \psi_S | H_T | \psi_D \rangle \\ \langle \psi_S | H_T | \psi_D \rangle & M_D^0 \end{pmatrix} \begin{pmatrix} C_S \\ C_D \end{pmatrix} = M \begin{pmatrix} C_S \\ C_D \end{pmatrix}$$

$$\theta = 0.5^\circ \ll \pm (30^\circ \sim 36^\circ)$$

We need to find an alternative source for the 4S-3D mixing scheme

The 4S-3D mixing scheme induced by a coupled-channel model

$$|A\rangle = C_S |\psi_S\rangle + C_D |\psi_D\rangle + \sum_{BC} \int C_{BC}(\vec{P}) |BC, \vec{P}\rangle$$

$$\begin{pmatrix} M_{4S}^0 & 0 \\ 0 & M_{3D}^0 \end{pmatrix} + \begin{pmatrix} 0 & \langle 4S | H_T | 3D \rangle \\ \langle 3D | H_T | 4S \rangle & 0 \end{pmatrix}$$

Bare Potential model

$$+ \sum_{BC} \begin{pmatrix} \langle 4S | \text{---} \overset{B}{\circ} \text{---} | 4S \rangle & \langle 4S | \text{---} \overset{B}{\circ} \text{---} | 3D \rangle \\ \langle 3D | \text{---} \overset{B}{\circ} \text{---} | 4S \rangle & \langle 3D | \text{---} \overset{B}{\circ} \text{---} | 3D \rangle \end{pmatrix}$$

Coupled-channel contribution

M. R. Pennington and D. J. Wilson,
Phys. Rev. D 76, 077502 (2007)
Z. Y. Zhou and Z. G. Xiao,
Phys. Rev. D 84, 034023 (2011)
Y. L. M. N. Anwar, and B. Z. Zou,
Phys. Rev. D 94, 034021 (2016)
M. X. Duan, S. Q. Luo, X. Liu, and T. Matsuki,
Phys. Rev. D 101, 054029 (2020)
R. H. Ni, Q. Deng, J. J. Wu, and X. H. Zhong,
arXiv:2501.15110

$$\begin{pmatrix} M_S^0 & \langle \psi_S | H_T | \psi_D \rangle & \sum_{BC} \int \langle \psi_S | H_I | BC, \vec{P} \rangle d^3 \vec{P} \\ \langle \psi_D | H_T | \psi_S \rangle & M_D^0 & \sum_{BC} \int \langle \psi_D | H_I | BC, \vec{P} \rangle d^3 \vec{P} \\ \sum_{BC} \int \langle BC, \vec{P} | H_I | \psi_S \rangle d^3 \vec{P} & \sum_{BC} \int \langle BC, \vec{P} | H_I | \psi_D \rangle d^3 \vec{P} & E_{BC} \end{pmatrix} \begin{pmatrix} C_S \\ C_D \\ C_{BC} \end{pmatrix} = M \begin{pmatrix} C_S \\ C_D \\ C_{BC} \end{pmatrix}$$



$$\begin{pmatrix} M_S^0 + \Delta M_S(M) & \langle \psi_S | H_T | \psi_D \rangle + \Delta M_{SD}(M) \\ \langle \psi_S | H_T | \psi_D \rangle + \Delta M_{SD}(M) & M_D^0 + \Delta M_D(M) \end{pmatrix} \begin{pmatrix} C_S \\ C_D \end{pmatrix} = M \begin{pmatrix} C_S \\ C_D \end{pmatrix}$$

Once subtracted method ($M_{J/\psi}$)

$$\Delta M_{SD}(M) = \text{Re} \sum_{BC} \int_0^\infty \frac{(M_{J/\psi} - M) \langle \psi_S^0 | H_I | BC, \vec{P} \rangle \langle BC, \vec{P} | H_I | \psi_D^0 \rangle P^2 dP}{(M - E_B - E_C)(M_{J/\psi} - E_B - E_C)}$$

$$\Delta M_{S(D)}(M) = \text{Re} \sum_{BC} \int_0^\infty \frac{(M_{J/\psi} - M) \langle \psi_{S(D)}^0 | H_I | BC, \vec{P} \rangle P^2 dP}{(M - E_B - E_C)(M_{J/\psi} - E_B - E_C)}$$

Coupled channel model

$$\langle \psi | H_I | BC \rangle = \mathcal{M}_{JL}(A \rightarrow B + C)$$

Quark pair
create model

$$H_I = -3\gamma \sum_m \langle 1m1 - m | 00 \rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \\ \times \mathcal{Y}_1^m \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1-m}^{34} \varphi_0^{34} \omega_0^{34} b_3^\dagger(\mathbf{p}_3) d_4^\dagger(\mathbf{p}_4),$$

Two-body strong
decay widths

$$\Gamma_{\text{total}} = \sum_{BC} \frac{2\pi P E_B E_C}{M} \sum_{JL} | \mathcal{M}_{JL}(A \rightarrow B + C) |^2$$

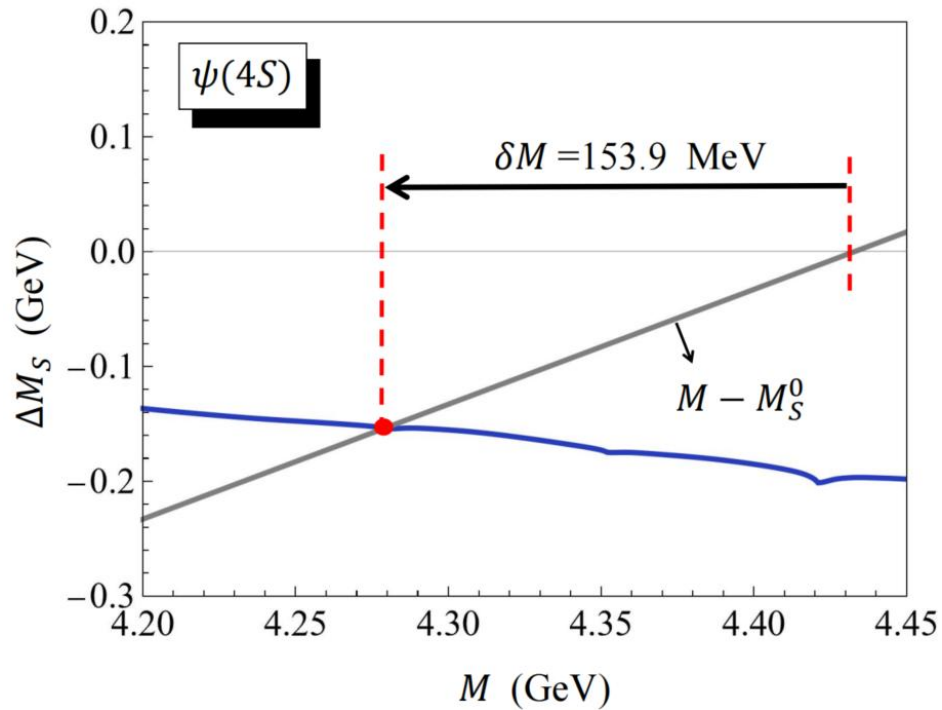
Numerical
wave function

$$\Psi_{nLM_L}(P) = \sum_{n1}^{n_{\max}} C_n R_{nL}(P) Y_{LM_L}(\Omega_P) \quad n_{\max} = 20$$

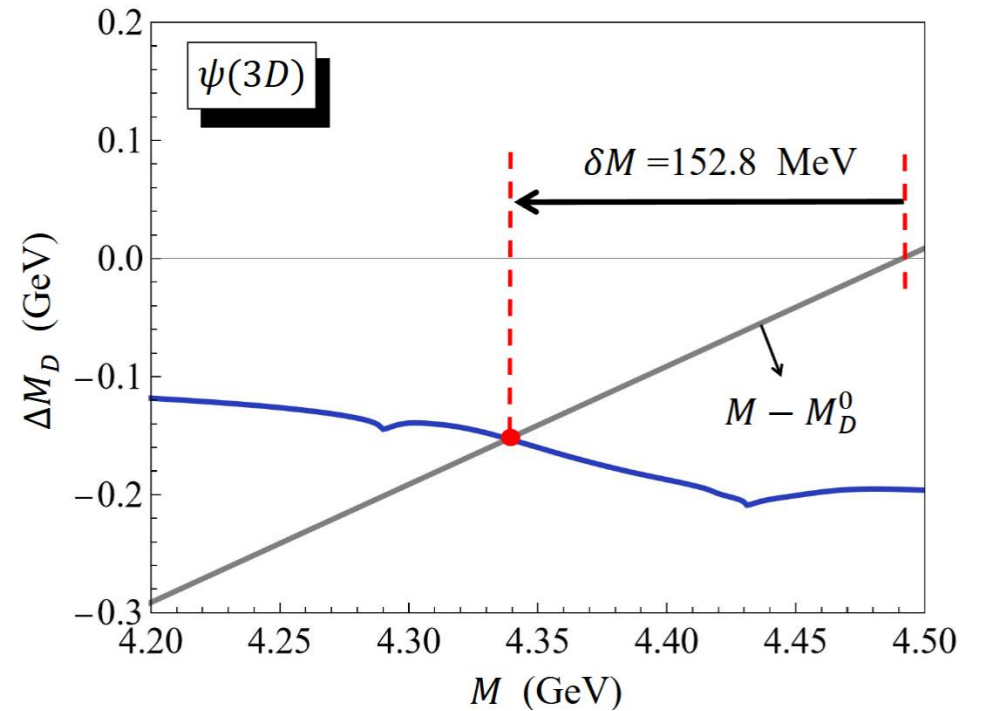
Results for $\psi(4220)$ and $\psi(4380)$

Results for $\psi(4S)$ and $\psi(3D)$

$$M - M_S^0 - \Delta M_S(M) = 0$$



$$M - M_D^0 - \Delta M_D(M) = 0$$



Results for $\psi(4S)$ and $\psi(3D)$

Channels	ΔM_i	$\Delta M_i / \sum_i \Delta M_i$	Γ_i	$\Gamma_i / \sum_i \Gamma_i$	ΔM_i	$\Delta M_i / \sum_i \Delta M_i$	Γ_i	$\Gamma_i / \sum_i \Gamma_i$
DD	-5.8	3.8%	0.3	0.6%	-10.7	7.0%	1.8	7.8%
DD^*	-12.3	8.0%	9.7	18.3%	-3.1	2.0%	3.4	14.4%
$D_s D_s$	-1.0	0.7%	0.2	0.3%	-1.1	0.7%	0.4	1.9%
$D^* D^*$	-35.1	22.8%	41.0	77.8%	-36.9	24.2%	8.0	34.0%
$D_s D_s^*$	-4.4	2.8%	0.4	0.8%	-1.3	0.9%	0.9	3.6%
$DD_0^*(2300)$	-	-	-	-	-	-	-	-
$D_s^* D_s^*$	-7.1	4.6%	1.1	2.1%	-10.1	6.6%	0.7	3.1%
$DD_1(2430)^0$	-16.1	10.4%	-	-	-16.4	10.7%	2.4	10.2%
$DD_1(2420)$	-8.7	5.7%	-	-	-14.5	9.5%	5.7	24.0%
$D_s D_{s0}(2317)$	-	-	-	-	-	-	-	-
$DD_2^*(2460)$	-8.5	5.5%	-	-	-5.1	3.3%	0.2	0.9%
$D^* D_0^*(2300)$	-12.1	7.8%	-	-	-8.4	5.5%	-	-
$DD_0(2550)^0$	-2.2	1.4%	-	-	-6.2	4.1%	-	-
$D^* D_1(2430)^0$	-22.8	14.8%	-	-	-21.4	14.0%	-	-
$D_s D_{s1}(2460)$	-2.3	1.5%	-	-	-3.3	2.2%	-	-
$D_s^* D_{s0}(2317)$	-2.2	1.4%	-	-	-1.1	0.7%	-	-
$D^* D_1(2420)$	-13.2	8.6%	-	-	-13.0	8.5%	-	-
Total	-153.9	100%	52.7	100%	-152.8	100%	23.6	100%
	$M = 4279.1$				$M = 4338.5$			
	$M_{\psi(4220)} = 4222.1 \pm 2.3$ [87]		$\Gamma_{\psi(4220)} = 49 \pm 7$ [87]					

$\psi(4S)$ strongly couples with:
 DD^* , $D^* D^*$, $DD_1(2430)$,
 $D^* D_1(2430)$, $D^* D_0(2300)$,
and $D^* D_1(2420)$

$\psi(3D)$ strongly couples with :
 $D^* D^*$, $DD_1(2430)^0$,
 $DD_1(2420)$, $D^* D_1(2430)^0$,
and $D^* D_1(2420)$

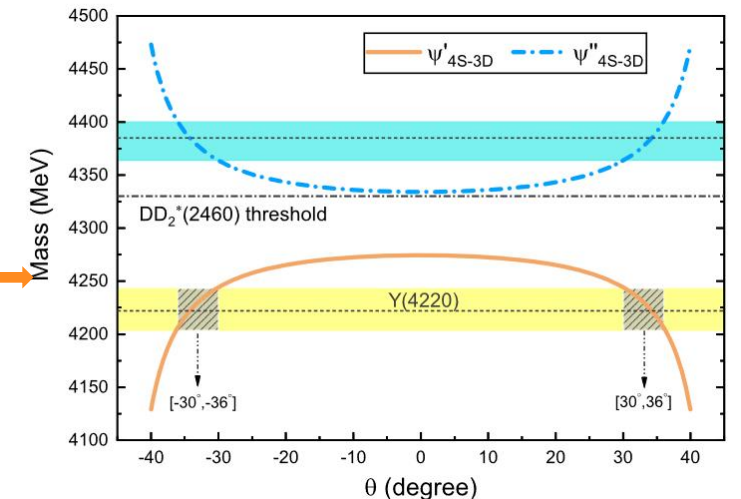
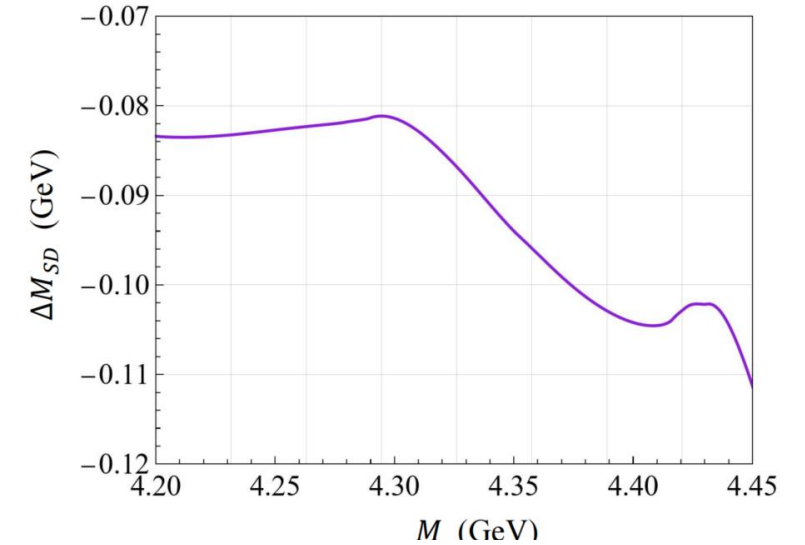
The mixing induced by the coupled -channel effects

$$\det \begin{vmatrix} M - M_S^0 - \Delta M_S(M) & \langle \psi_S | H_T | \psi_D \rangle + \Delta M_{SD}(M) \\ \langle \psi_D | H_T | \psi_S \rangle + \Delta M_{SD}(M) & M - M_D^0 - \Delta M_D(M) \end{vmatrix} = 0$$

$$M_{\psi'_{4S-3D}} = 4235.9 \text{ MeV} \quad M_{\psi''_{4S-3D}} = 4387.1 \text{ MeV}$$

$$\theta = 35^\circ$$

Consistent



Decay width for ψ'_{4S-3D}

$\theta = 35^\circ$		ψ'_{4S-3D} m=4235.9 MeV		ψ''_{4S-3D} m=4387.1 MeV	
Channels	Γ_i	$\Gamma_i / \sum_i \Gamma_i$	Γ_i	$\Gamma_i / \sum_i \Gamma_i$	
DD	0.2	0.5%	0.5	1.1%	
DD^*	1.7	5.4%	0.7	1.7%	
$D_s D_s$	0.6	1.9%	0.1	0.2%	
$D^* D^*$	29.2	91.4%	8.0	18.1%	
$D_s D_s^*$	0.0	0.0%	2.5	5.7%	
$DD_0^*(2300)$	—	—	—	—	
$D_s^* D_s^*$	0.3	0.8%	0.8	1.9%	
$DD_1(2430)^0$	—	—	5.7	13.0%	
$DD_1(2420)$	—	—	7.7	17.4%	
$D_s D_{s0}(2317)$	—	—	—	—	
$DD_2^*(2460)$	—	—	16.5	37.3%	
$D^* D_0^*(2300)$	—	—	1.6	3.7%	
Total	32.0	100%	44.1	100%	

$\Gamma_{\psi(4220)} = 49 \pm 7$ [87]

$\psi(4230)$ WIDTH		49 ± 7 MeV (S = 3.4)		
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
49 ± 7	OUR AVERAGE	Error includes scale factor of 3.4. See the ideogram below.		
$71.7 \pm 16.2 \pm 32.8$		¹ ABLIKIM	2023U BES3	$e^+ e^- \rightarrow K_S K_S J/\psi$
$81.6 \pm 17.8 \pm 9.0$		² ABLIKIM	2023X BES3	$e^+ e^- \rightarrow D^{*0} D^{*-} \pi^+$
$41.8 \pm 2.9 \pm 2.7$		³ ABLIKIM	2022AM BES3	$e^+ e^- \rightarrow \pi^+ \pi^- J/\psi$
$72.9 \pm 6.1 \pm 30.8$		⁴ ABLIKIM	2022AU BES3	$e^+ e^- \rightarrow K^+ K^- J/\psi$
$17.6 \pm 18.1 \pm 0.9$		⁵ ABLIKIM	2021AJ BES3	$e^+ e^- \rightarrow \pi^+ \pi^- \psi(2S)$
$47.2 \pm 22.8 \pm 10.5$		⁶ ABLIKIM	2020AG BES3	$e^+ e^- \rightarrow \mu^+ \mu^-$
$46.2 \pm 4.7 \pm 2.1$		⁷ ABLIKIM	2020N BES3	$e^+ e^- \rightarrow \pi^0 \pi^0 J/\psi$
$82.0 \pm 5.7 \pm 0.4$		⁷ ABLIKIM	2020O BES3	$e^+ e^- \rightarrow \eta J/\psi$
$28.2 \pm 3.9 \pm 1.6$		⁸ ABLIKIM	2019AJ BES3	$e^+ e^- \rightarrow \omega \chi_{c0}$
$77.0 \pm 6.8 \pm 6.3$		ABLIKIM	2019R BES3	$e^+ e^- \rightarrow \pi^+ D^0 D^{*-} + c.c.$
$115^{+38}_{-26} \pm 12$		⁹ ABLIKIM	2019V BES3	$e^+ e^- \rightarrow \gamma \chi_{c1}(3872)$
$66.0^{+12.3}_{-8.3} \pm 0.4$		ABLIKIM	2017G BES3	$e^+ e^- \rightarrow \pi^+ \pi^- h_c$
• • We do not use the following data for averages, fits, limits, etc. • •				
$41.2 \pm 16.0 \pm 16.4$		ABLIKIM	2020N BES3	$e^+ e^- \rightarrow \pi^0 T_{cc1}(3900)^0, T_{cc1}^0 \rightarrow \pi^0 J/\psi$
$44.1 \pm 4.3 \pm 2.0$		¹⁰ ABLIKIM	2017B BES3	$e^+ e^- \rightarrow \pi^+ \pi^- J/\psi$
$80.1 \pm 24.6 \pm 2.9$		¹¹ ABLIKIM	2017V BES3	$e^+ e^- \rightarrow \pi^+ \pi^- \psi(2S)$
$76.6 \pm 14.2 \pm 2.4$		¹² ZHANG	2017B RVUE	$e^+ e^- \rightarrow \pi^+ \pi^- \psi(2S)$
$54.2 \pm 2.6 \pm 1.0$		¹³ ZHANG	2017C RVUE	$e^+ e^- \rightarrow \pi^+ \pi^- J/\psi$ or $\psi(2S)$
$38 \pm 12 \pm 2$	180	¹⁴ ABLIKIM	2015C BES3	$e^+ e^- \rightarrow \omega \chi_{c0}$
$134.1 \pm 16.4 \pm 5.5$		¹⁵ LIU	2013B BELL	$e^+ e^- \rightarrow \gamma \pi^+ \pi^- J/\psi$
$114^{+16}_{-15} \pm 7$		¹⁶ LEES	2012AC BABR	$10.58 e^+ e^- \rightarrow \gamma \pi^+ \pi^- J/\psi$
$108 \pm 19 \pm 10$		^{17, 15} YUAN	2007 BELL	$10.58 e^+ e^- \rightarrow \gamma \pi^+ \pi^- J/\psi$

- Total width of ψ'_{4S-3D} is **32.0 MeV**
- $\psi(4220)$ can be regarded as the ψ'_{4S-3D}
- The dominant decay channel: **$D^* D^*$ (91.4%)**

Prediction

The properties of ψ''_{4S-3D}

Prediction

- The mass of ψ''_{4S-3D} is **4387.1 MeV**
- Total width of ψ''_{4S-3D} is **44.1 MeV**
- The dominant decay channels :

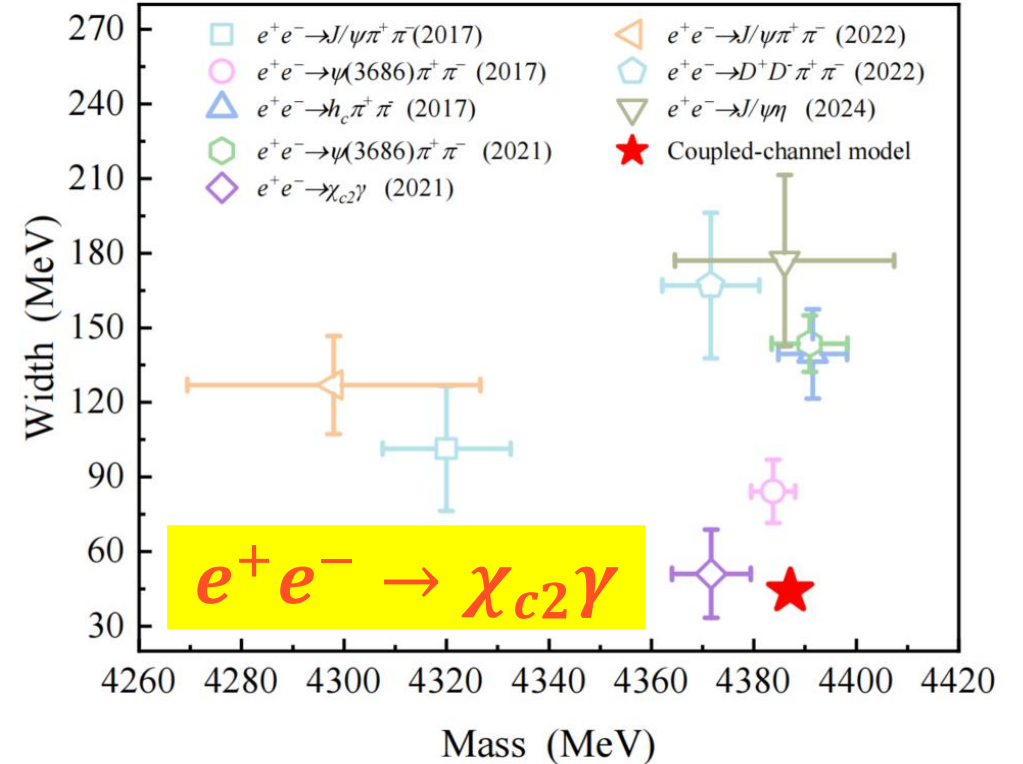
$$D^* D^* (18.1\%)$$

$$D^* D_1(2430) (13.0\%)$$

$$D^* D_1(2420) (17.4\%)$$

$$D^* D_2^* (37.3\%)$$

$$\Gamma(D^* D^*) : \Gamma(D^* D_1(2430)^0) : \Gamma(D^* D_1(2420)) : \Gamma(D^* D_2^*) = 1.4 : 1 : 1.4 : 2.9$$



$$M = 4371.7 \pm 7.5 \pm 1.8 \text{ MeV}$$

$$\Gamma = 51.1 \pm 17.6 \pm 1.9 \text{ MeV}$$

M. Ablikim *et al.* [BESIII],
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More precise experimental data are required to conclusively determine its properties

Summary

- $\psi(4220)$ can be assigned as the ψ'_{4S-3D} .
- The $\psi(4S) - \psi(3D)$ mixing angle is determined to be $\theta = 35^\circ$ by a dynamical analysis.
- $\psi(4380)$ can be searched for in D^*D^* , $D^*D_1(2430)$, $D^*D_1(2420)$, and $D^*D_2^*$ channels.

Thanks for your attention