

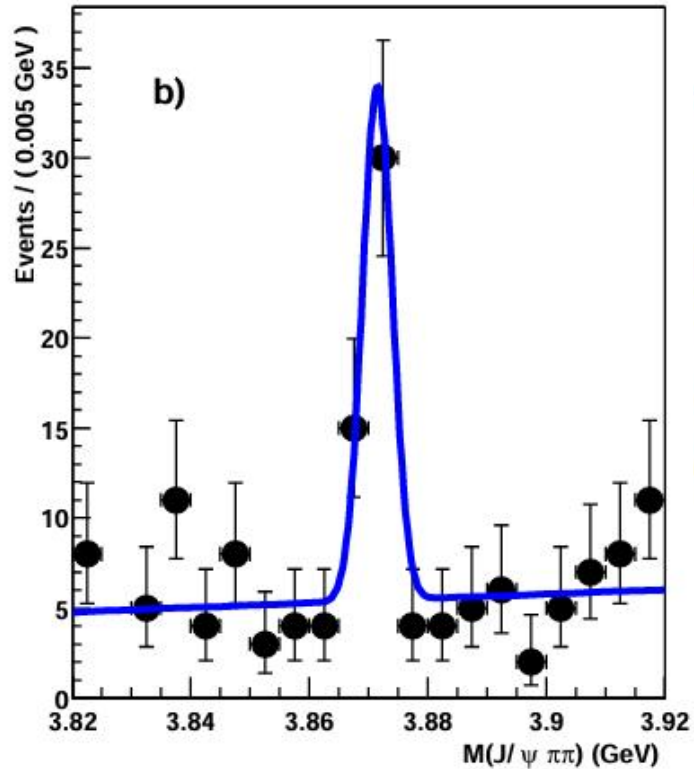
# **Zc(3900) (and X(3872)) on the phenomenological and lattice side**

*Based on: PhysRevD.110.114029, 2502.05789*

**Kang Yu**  
**University of Chinese Academy Science**  
**Coauthors: Guang-Juan Wang, Jia-Jun Wu, Zhi Yang**

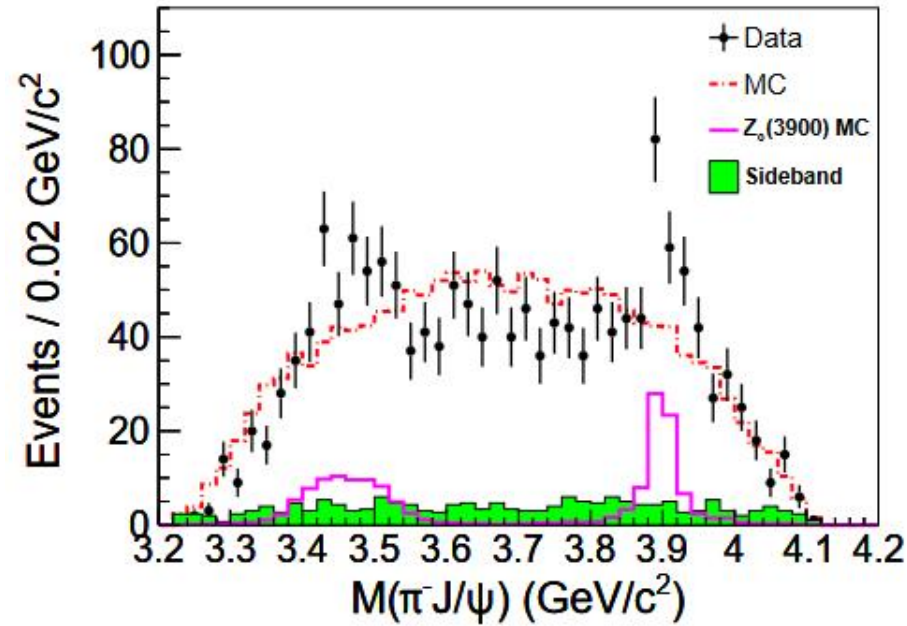
# Exp line shapes of the three exotic states

2003  
X(3872)



*Belle PRL 91 (26): 262001*

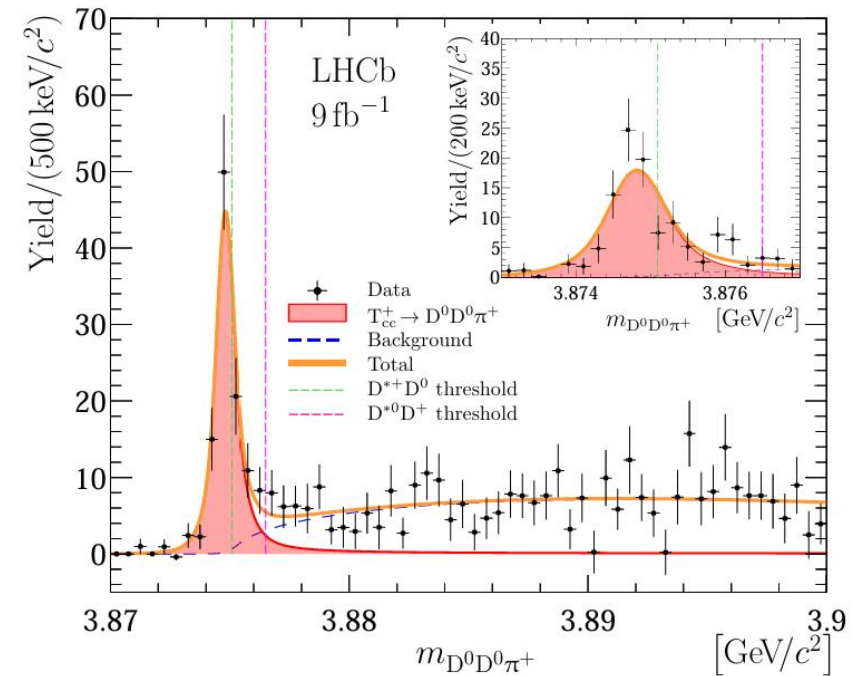
2013  
Z<sub>c</sub>(3900)



*BES PRL 110 (25): 252001*

.....

2022  
T<sub>cc</sub>



*LHCb Nature Physics Vol.18(2022)*

## ph relationship between the three states

**X(3872) :  $c\bar{c}$  core ( $\chi_{c1}(2P)$ ) +  $D\bar{D}^*(I=0)$  FSI**

supported by lattice study

*S. Prelovsek et.al 1503.03257*  
*G.S. Bali et.al 1110.2381.*

OBE interaction

$V_{1\pi}, V_{\rho}, V_{\omega}$

**Tcc :  $DD^*(I=0)$  interaction**

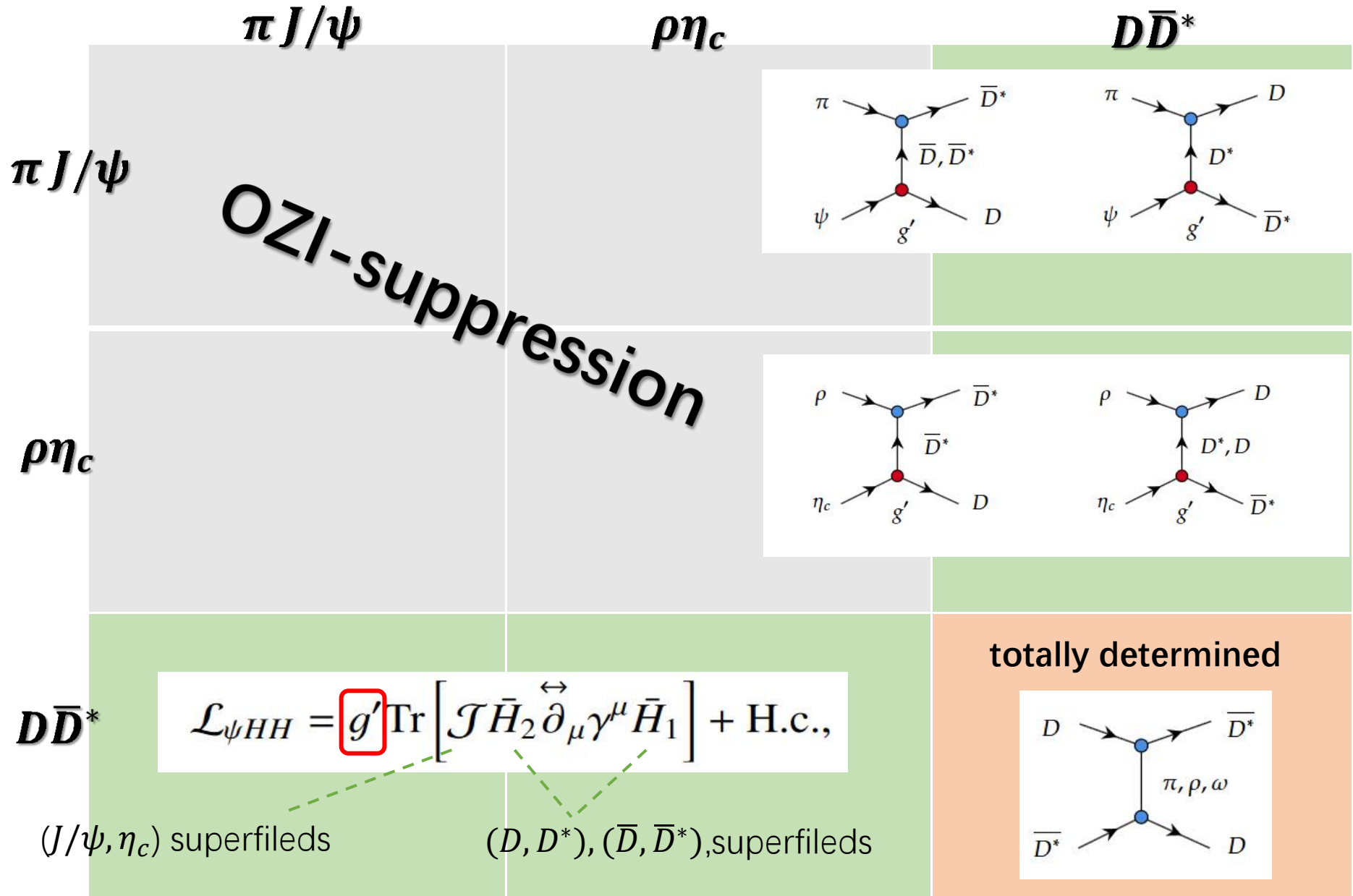
**Zc(3900) :  $D\bar{D}^*(I=1)$  interaction  
(also couple with  $\pi J/\psi, \rho\eta_c$ )**

established in *Jia-Jun Wu et.al 2306.12406*  
(binding energy of X3872  $\surd$  line shape of Tcc  $\surd$ )

The inclusion of  $\rho\eta_c$  is implied by lattice study  
and recent experiment result

*HALQCD 10.1088/1361-6471/aa9afd*

# hadron-hadron interactions for $Z_c(3900)$

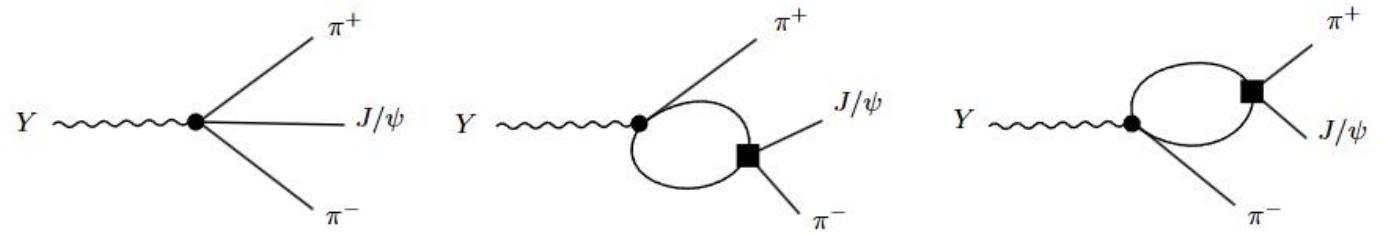


$$\mathcal{L}_{\psi HH} = g' \text{Tr} \left[ \mathcal{J} \bar{H}_2 \overset{\leftrightarrow}{\partial}_\mu \gamma^\mu \bar{H}_1 \right] + \text{H.c.},$$

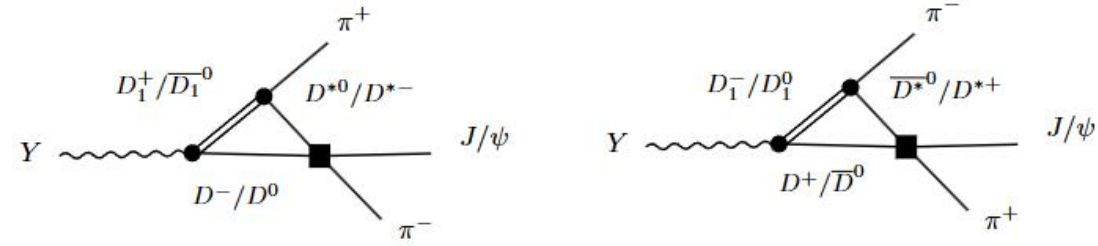
$(J/\psi, \eta_c)$  superfields

$(D, D^*), (\bar{D}, \bar{D}^*),$  superfields

# Production Amp for $Y(4230)$ decay

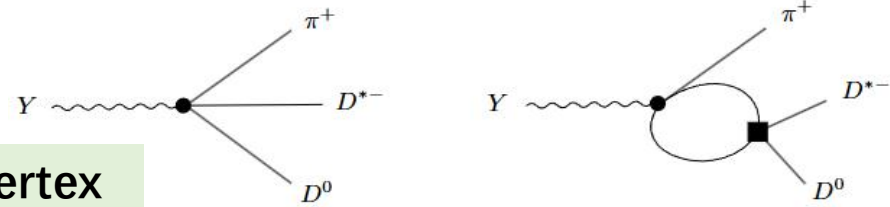


$$e^+e^- \rightarrow \pi^+\pi^- J/\psi$$

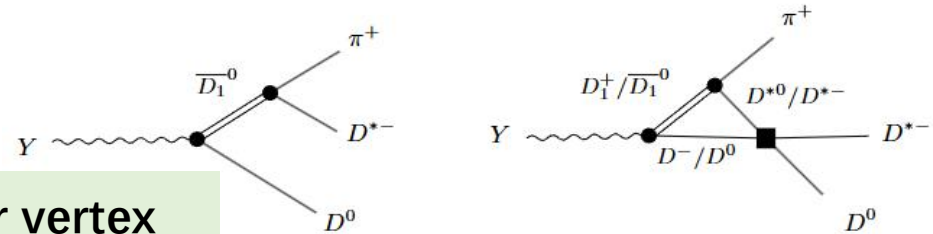


$$e^+e^- \rightarrow \pi D \bar{D}^*$$

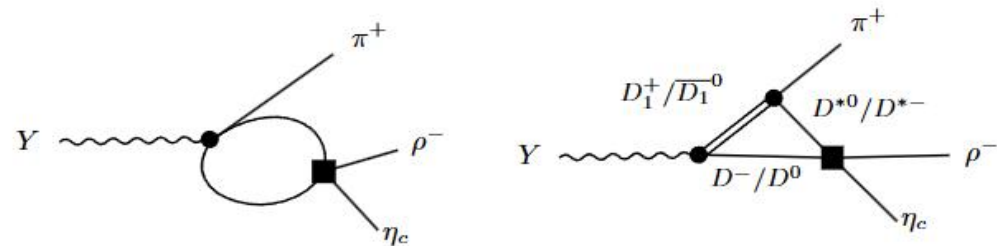
four-point vertex



$D_1 \bar{D}$  molecular vertex



$$e^+e^- \rightarrow \pi \rho \eta_c$$



## Fitting formulas for line shape

$$dN \sim N \int |M|^2 d\Phi + P$$

scale factor      amplitude squared      phase space      incoherent polynomials

$$b_0 \left( \sqrt{s_{\pi^- J/\psi}} - m_\pi - m_{J/\psi} \right)^{b_1} \left( \sqrt{s} - m_\pi - \sqrt{s_{\pi^- J/\psi}} \right)^{b_1} \Rightarrow \text{mimic } \pi\pi \text{ FSI}$$

*BES PhysRevLett.112.022001*

$$c_0 \left( \sqrt{s_{D^0 D^{*-}}} - m_D - m_{D^{*-}} \right)^{c_1} \left( \sqrt{s} - m_\pi - \sqrt{s_{D^0 D^{*-}}} \right)^{c_2} \Rightarrow \text{mimic possible background contribution}$$

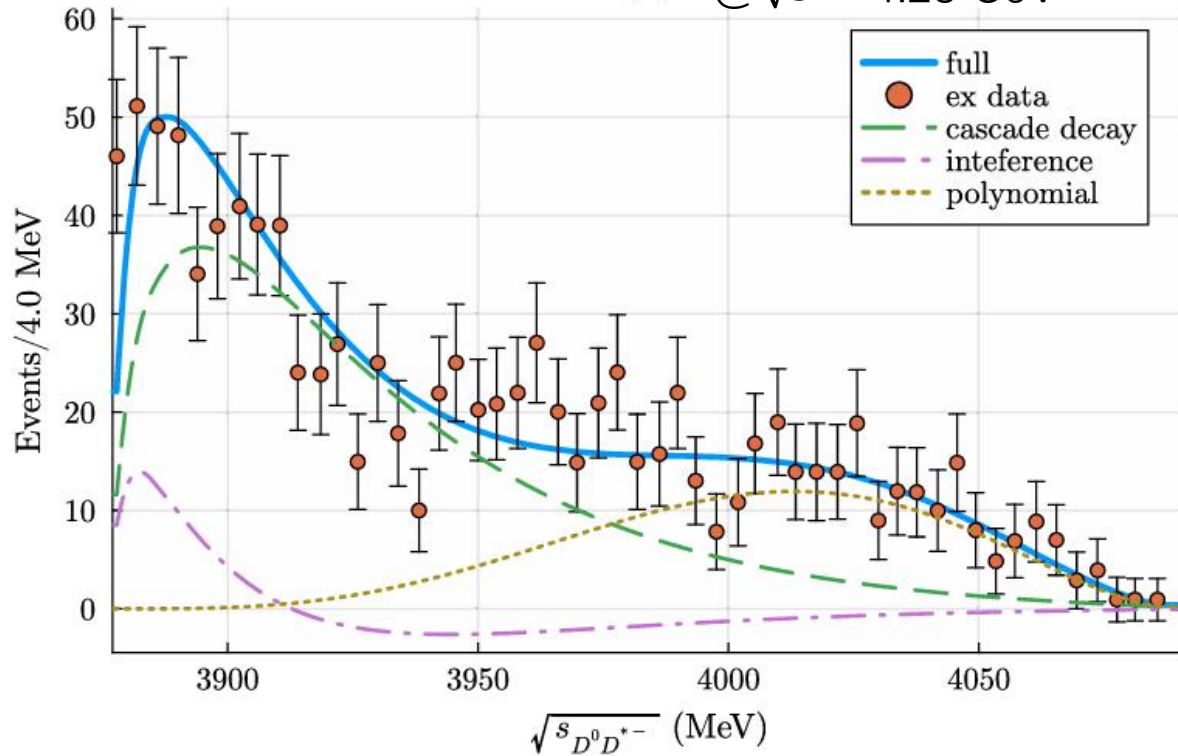
$$\# \text{. par} = 2 \cdot 11 + 1 = 23$$

#. ex data  $\sim 250$  @  $\sqrt{s} = 4.23, 4.26 \text{ GeV}$

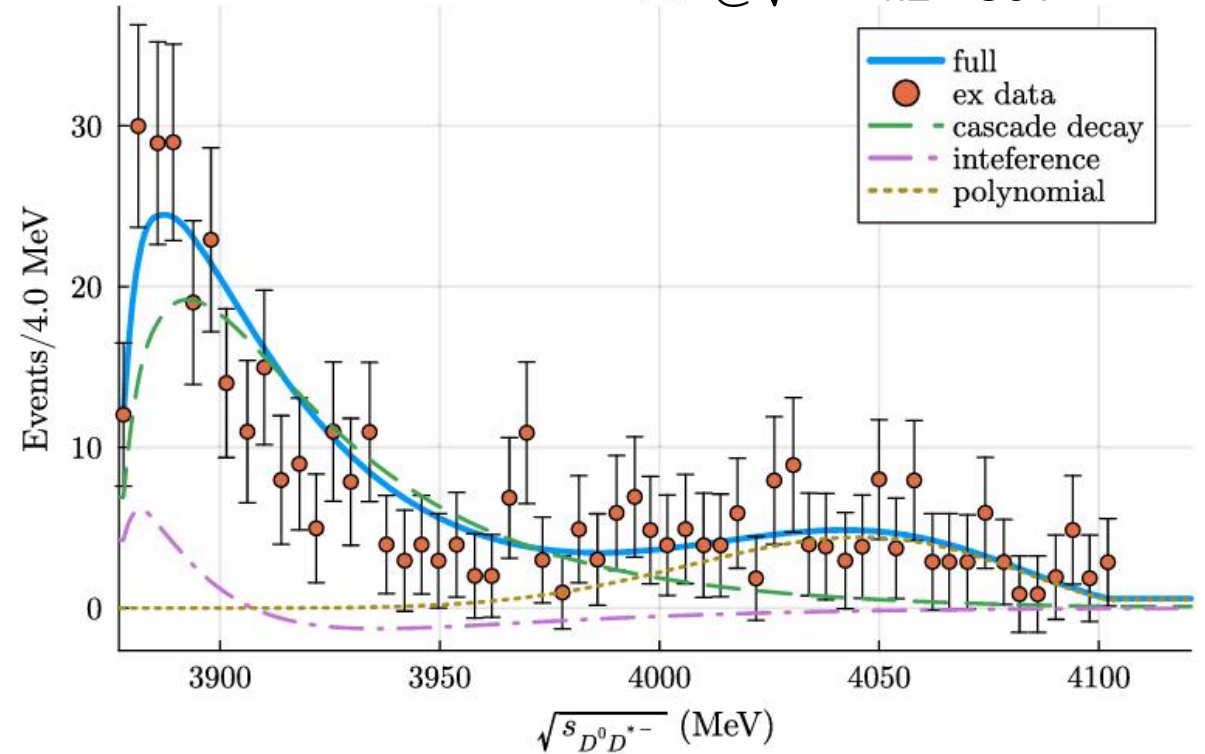
we get a  $\hat{\chi}^2 \sim 1.6$  fitting results

## Fitted line shape of $D^0 D^{*-}$ distribution

(a) @  $\sqrt{s} = 4.23$  GeV



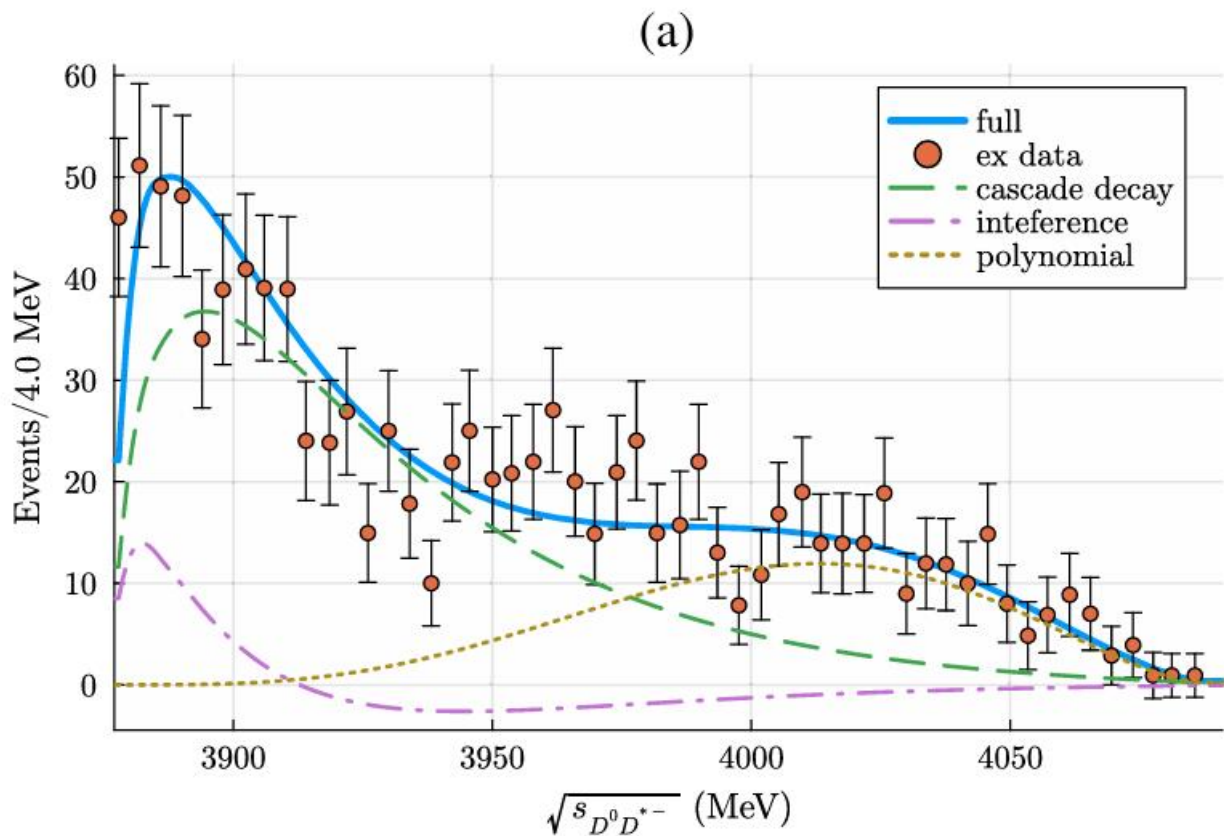
(b) @  $\sqrt{s} = 4.26$  GeV



- Tree-level cascade decay  $Y \rightarrow D_1 \bar{D} \rightarrow D \bar{D}^* \pi$  contributes most.
- interference between tree-level cascade decay and triangle diagram enhances.
- polynomial only contributes at the tail.



# Fitted line shape of $D^0 D^{*-}$ distribution

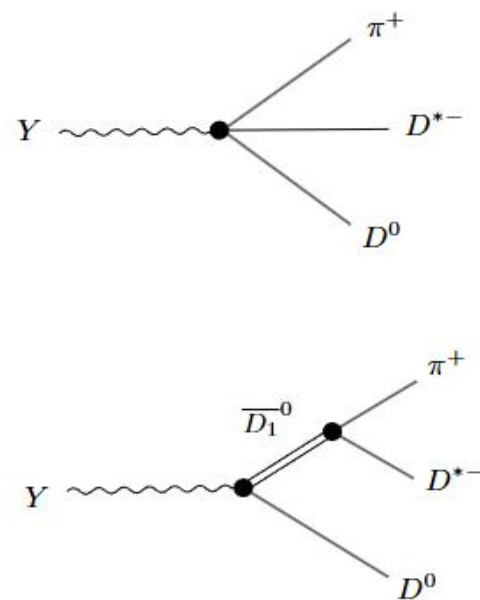


$I=1$  OBE is relatively weak

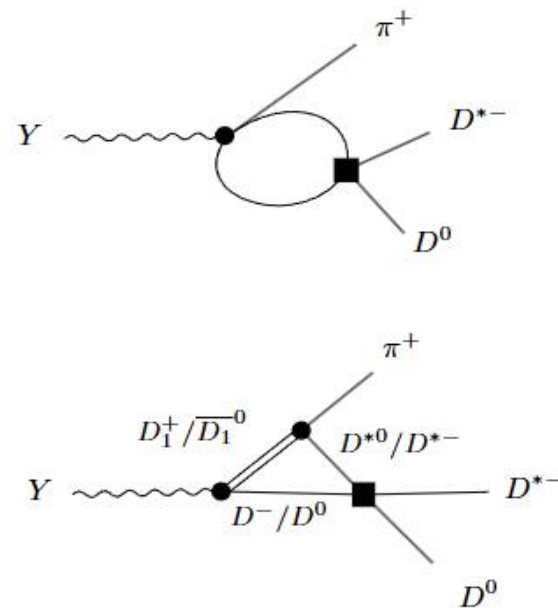
→ tree level dominates

→ cascade decay contributes to peak most

tree-level



re-scattering

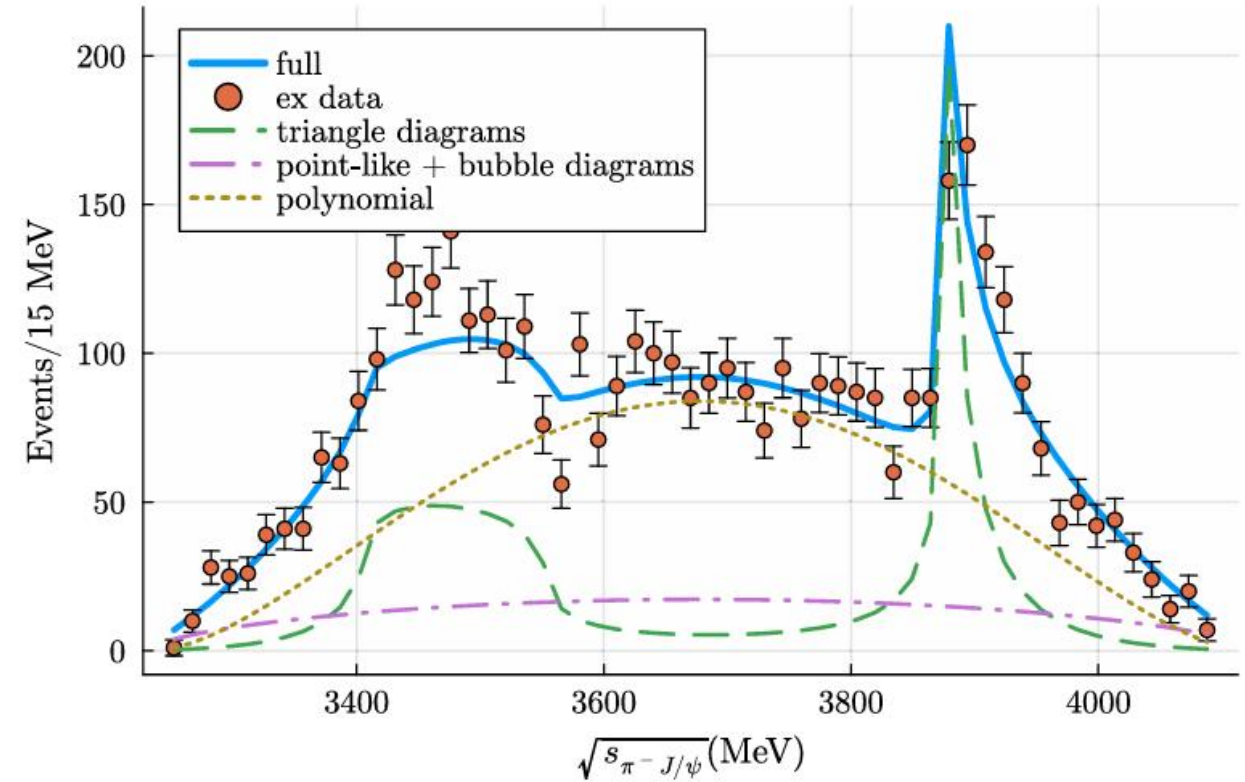
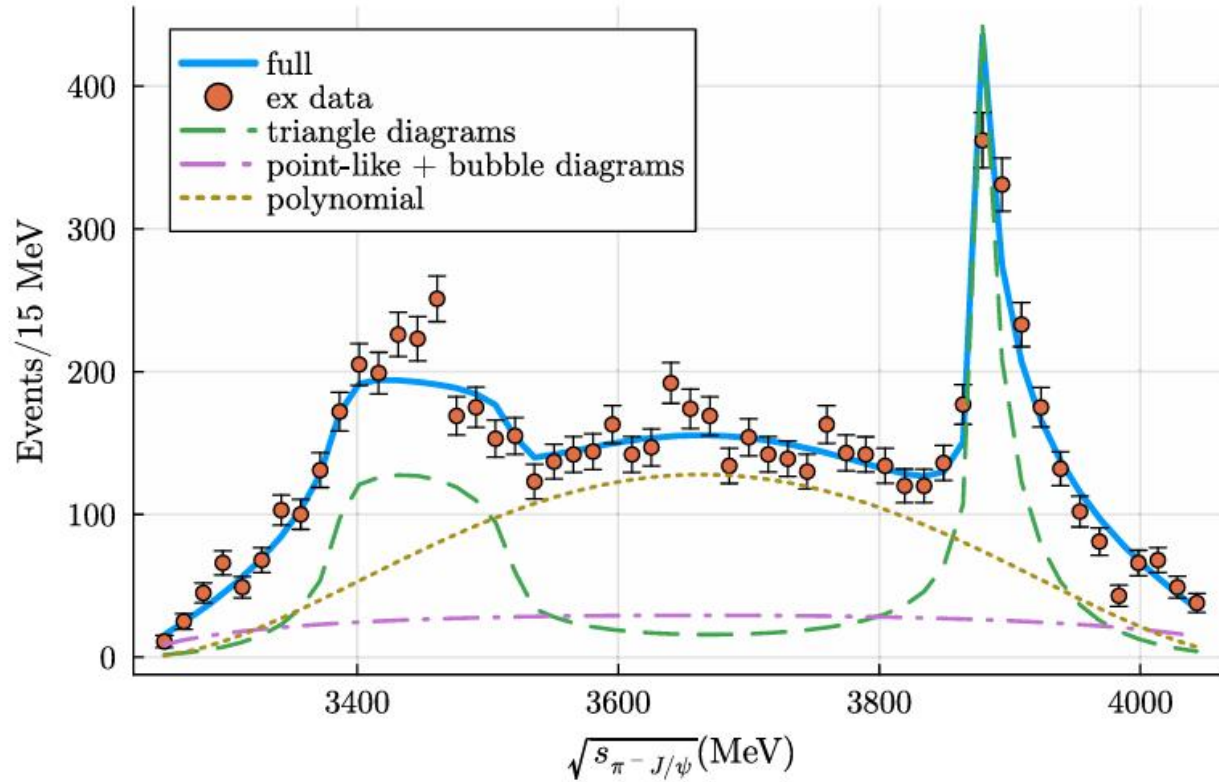




## Fitted line shape of $\pi J/\psi$ distribution

(a) @  $\sqrt{s} = 4.23$  GeV

(b) @  $\sqrt{s} = 4.26$  GeV

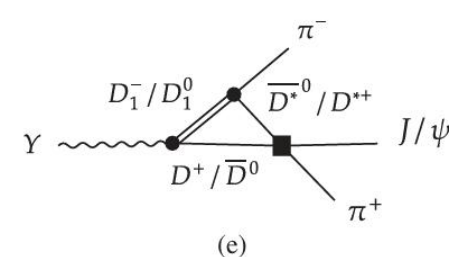
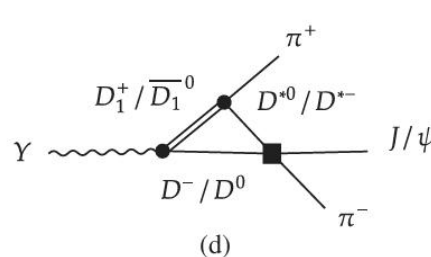
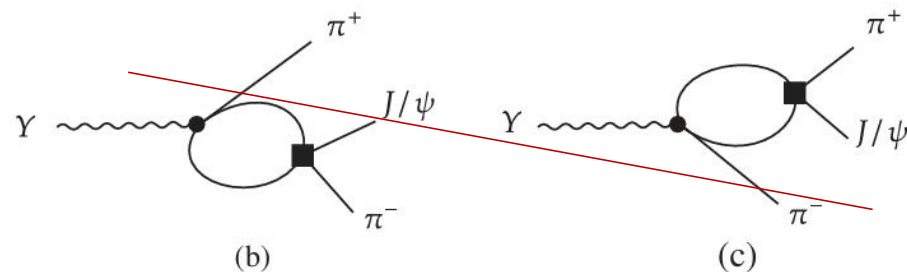
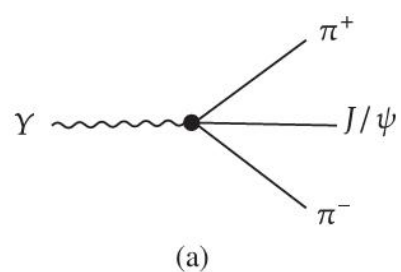
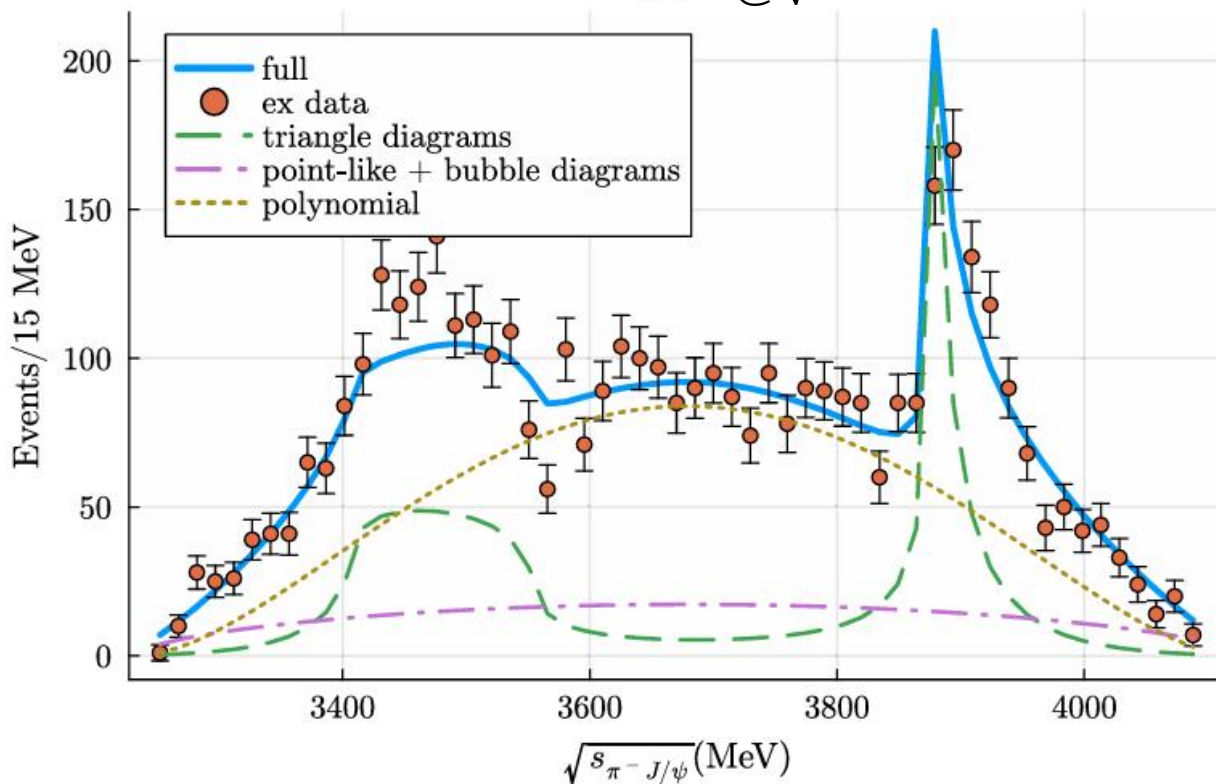
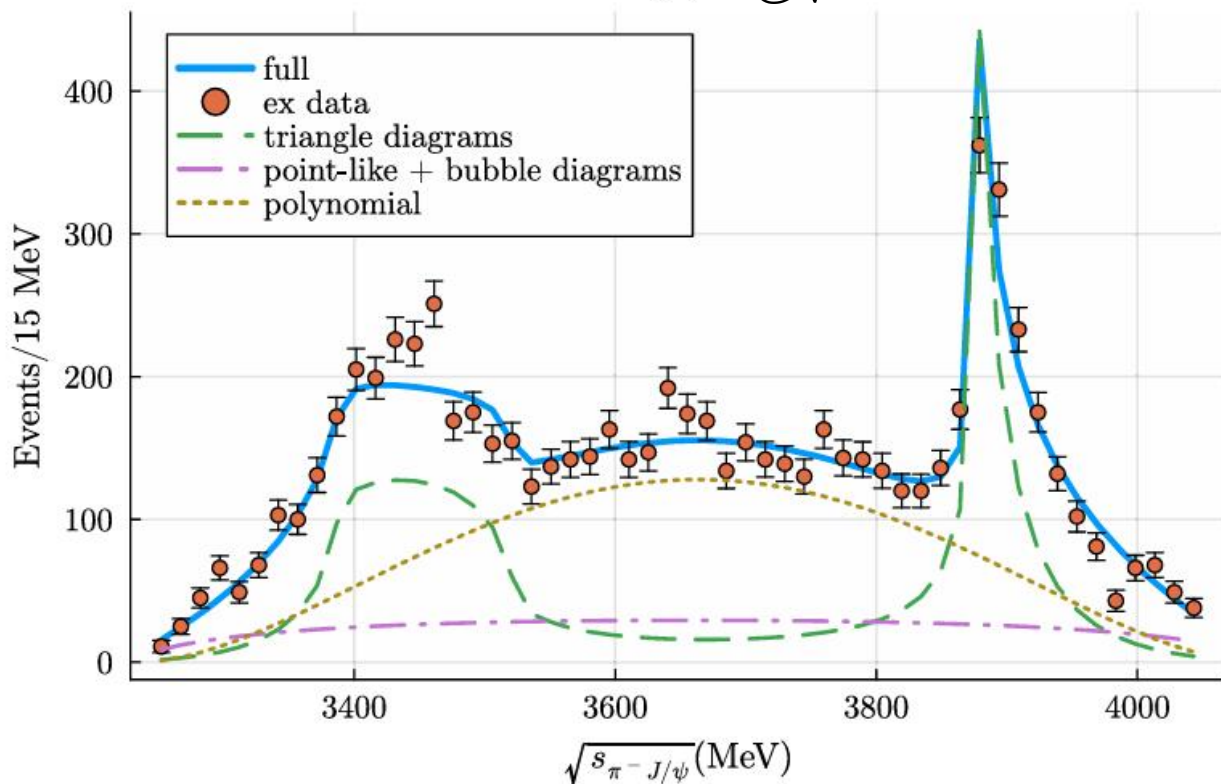


- triangle diagram  $Y \rightarrow \bar{D}_1 D (\rightarrow D \bar{D}^* \pi) \rightarrow \pi \pi J/\psi$  produce the peak
- polynomial produce the similar line shape of  $\pi\pi$  FSI as in *BES PhysRevLett.112.022001*

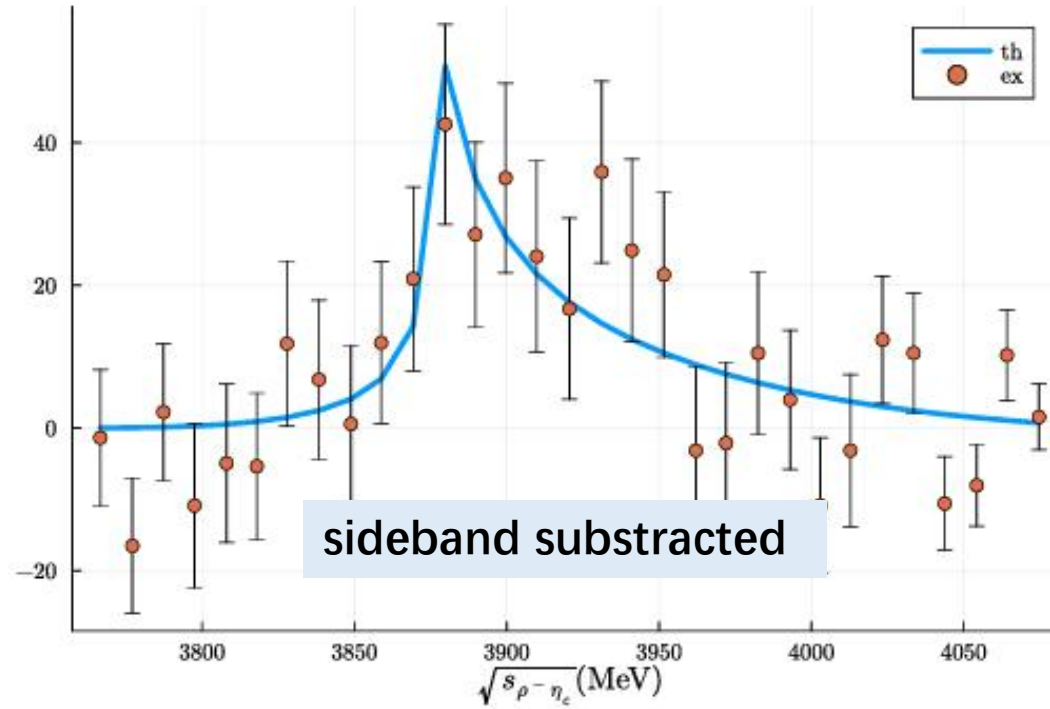
# Fitted line shape of piJ/psi distribution

(a) @  $\sqrt{s} = 4.23$  GeV

(b) @  $\sqrt{s} = 4.26$  GeV



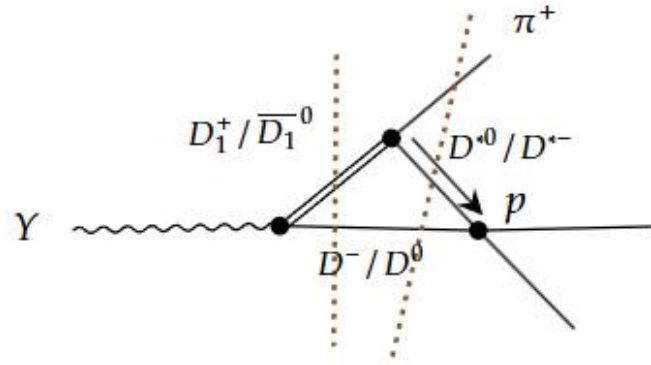
## Fitted line shape of rhoEta\_c distribution



peak contributed by triangle diagram

exp data from: [C.Z. Yuan 10.1142/S0217751X18300181](https://arxiv.org/abs/1011.42/S0217751X18300181)

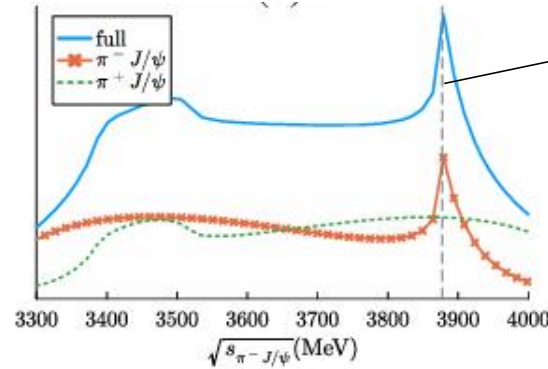
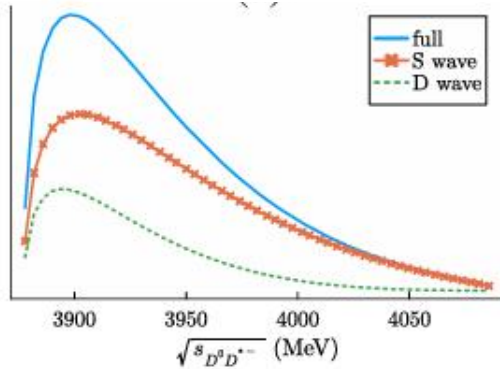
# line shape by pure triangle loop



triangle diagram = pure triangle loop + T-matrix (FSI)

$$\sim V_{D_1 D^* \pi} \int \frac{d^3 p}{\sqrt{s} - E_D - E_{D_1} + \frac{i\Gamma_{D_1}}{2}} \frac{1}{\sqrt{s} - E_\pi - E_{D^*} - E_D + i0^+}$$

width of  $D_1(2420)$  is taken into account



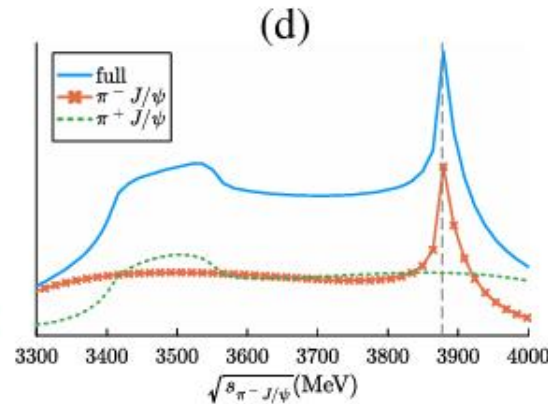
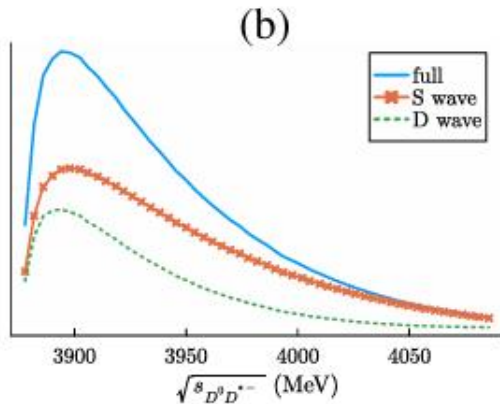
Indeed a cusp Exactly at  $D\bar{D}^*$  threshold

$$\sqrt{s} - E_\pi - E_{D^*} - E_D = 0 \text{ can hold above the } D\bar{D}^* \text{ threshold}$$

However...

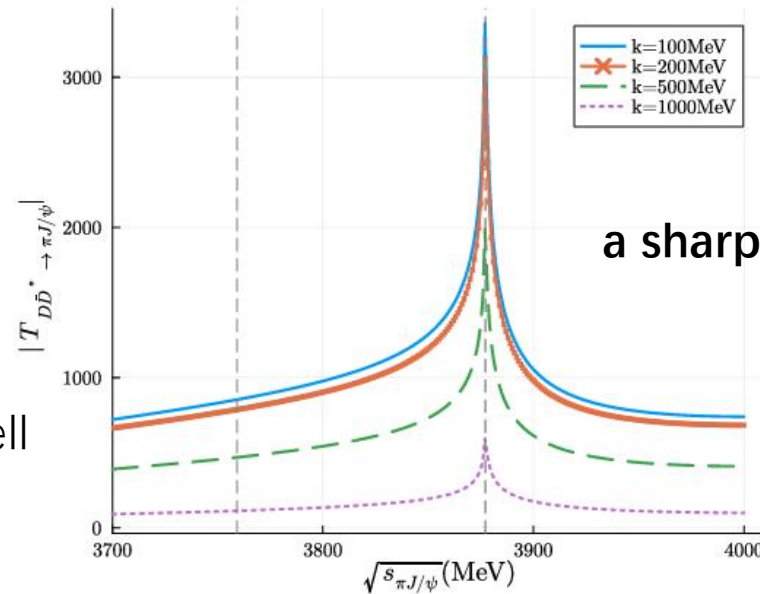
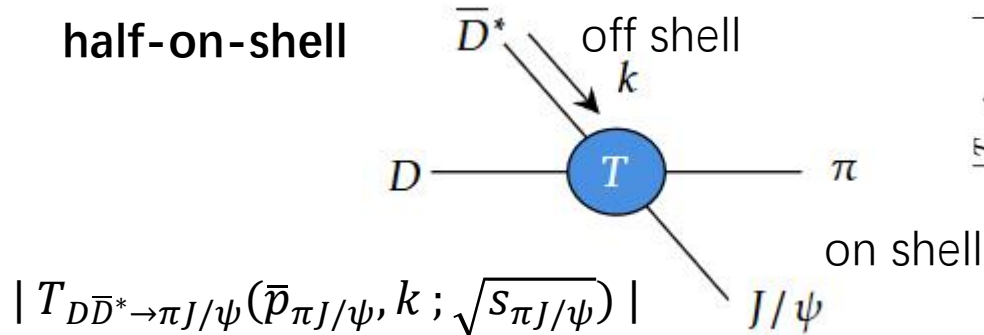
- for  $D\bar{D}^*$ , peak not close enough
- for  $\pi J/\psi$ , peak not high enough

pure triangle loop is not all of the story



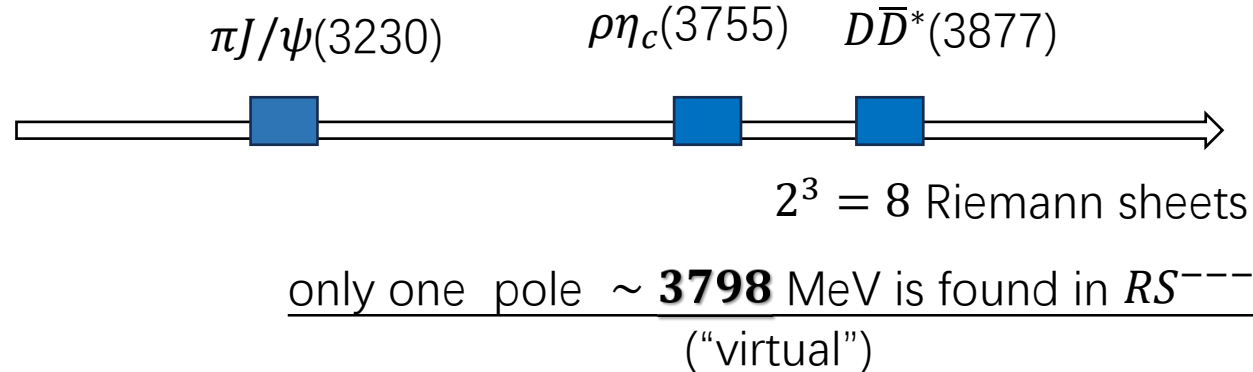


## cusps from T-matrix



a sharp cusp EXACTLY at  $D\bar{D}^*$  threshold  
(thanks to OPE)

## pole of T-matrix



Reference [25]	$3798_{-31}^{+25}$	Virtual
	$3902(6) - 38(9)i$	Resonance
Reference [19]	$3831_{-38}^{+27}$	Virtual
	$3894(6) - 30(13)i$	Resonance
Reference [21]	3870	Virtual
Reference [20]	3879	Virtual
Reference [22]	3872	Virtual
Reference [26]	$3880(3) - 13(1)i$	Resonance
Reference [32]	$3884 - 22i$	Resonance
Reference [27]	3840	Virtual
Reference [62]	3839(11)	Virtual

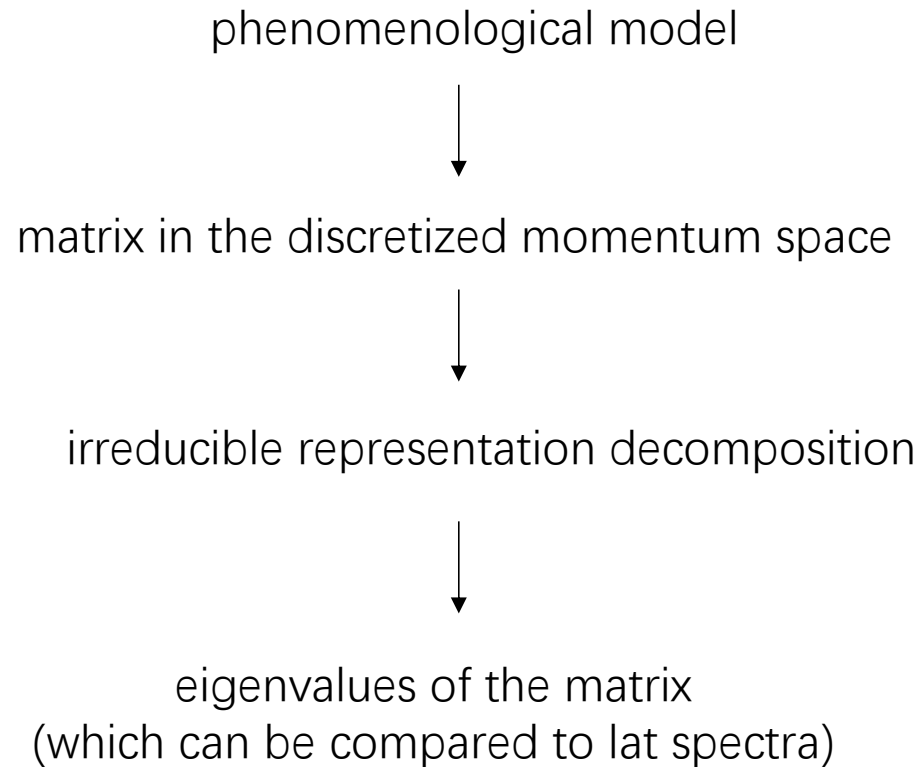
👉 peak from triangle diagram is threshold cusp  
mainly from T-matrix and enhanced by pure triangle loop

a virtual pole far below  $D\bar{D}^*$  threshold in LQCD at larger  $m_\pi$

*PhysRevLett.117.242001*

# Finite Volume Hamiltonian method: from ph to lat

## workflow of FVH method



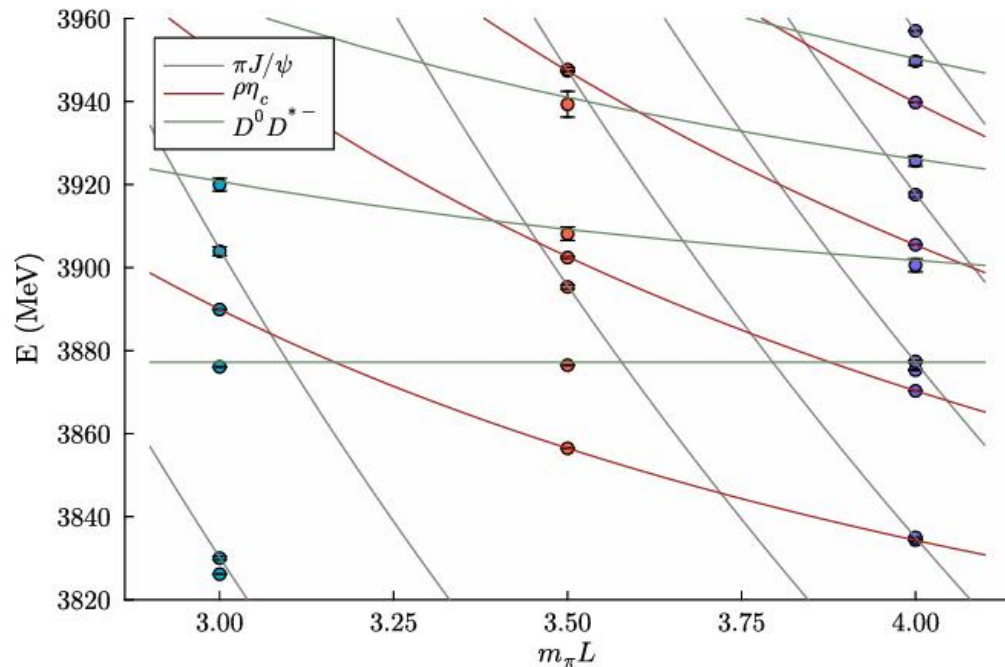
*2502.05789*

**provide a systematic calculation procedure**

(also address left-hand cut issue)

(generalization to three-body system is in progress)

## Finite Volume energy levels: $Z_c(3900)$



all levels are close to the non-interacting levels  
( $I=1$  OBE is weak)

*Prelovsek et.al PhysRevD.91.014504*

*"The levels appear near the noninteracting energies of the two-particle states"*

*HSC 10.1007/JHEP11(2017)033*

*"the majority of energies lie close to the non-interacting levels."*

*MILC, <http://arxiv.org/abs/1411.1389>*

*"The mixing is evidently too weak to produce a state distinct from the noninteracting scattering states"*

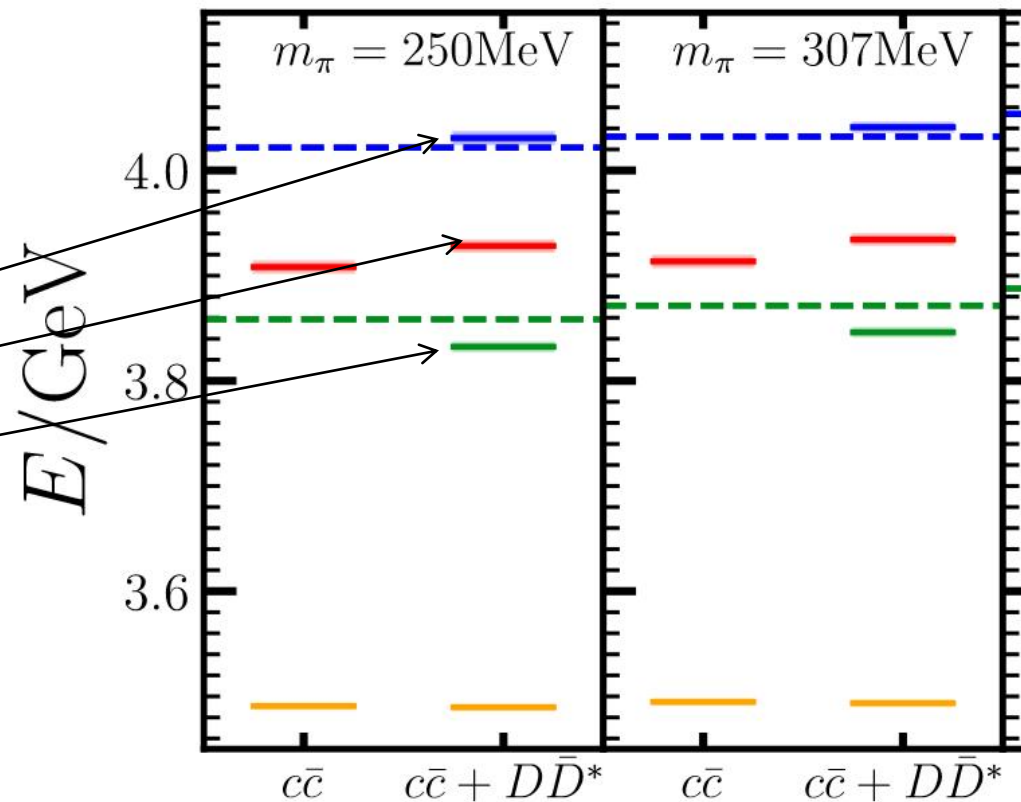
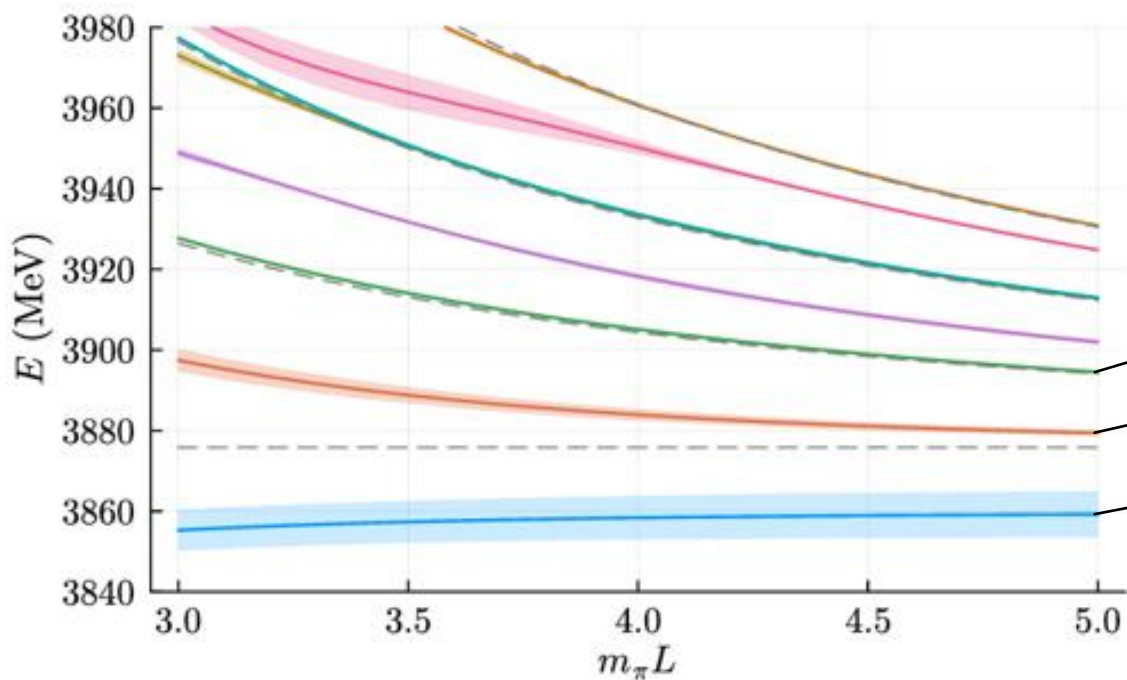


## Finite Volume energy levels: X(3872)

Signal of X(3872) on the lattice is reported by several lattice studies

MILC, <http://arxiv.org/abs/1411.1389>  
Prelovsek et.al *PhysRevLett.111.192001*  
HaoZheng Li 2402.14541  
...

$c\bar{c}$  core ( $\chi_{c1}(2P)$ ) +  $D\bar{D}^*(I=0)$



2402.14541

## Summary

- A OBE model reproducing: the binding energy of X(3872), line shape of Zc(3900) and Tcc
- Based on the current experiment data, Zc(3900) is more likely to be a cusp in our model.
- The finite volume spectra is calculated. The results are qualitatively same as the lattice study



