Spectra of low-lying 1^{-+} light hybrid and their two-body hadronic decays

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Based on arXiV: 2504.05818 [hep-ph]

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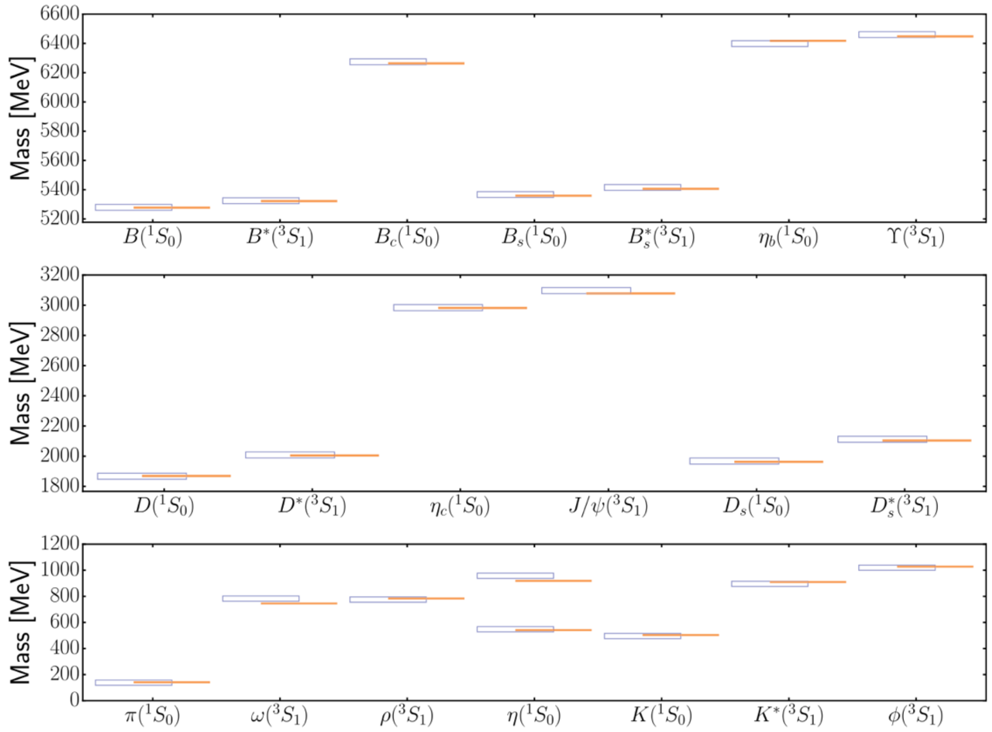
Outline

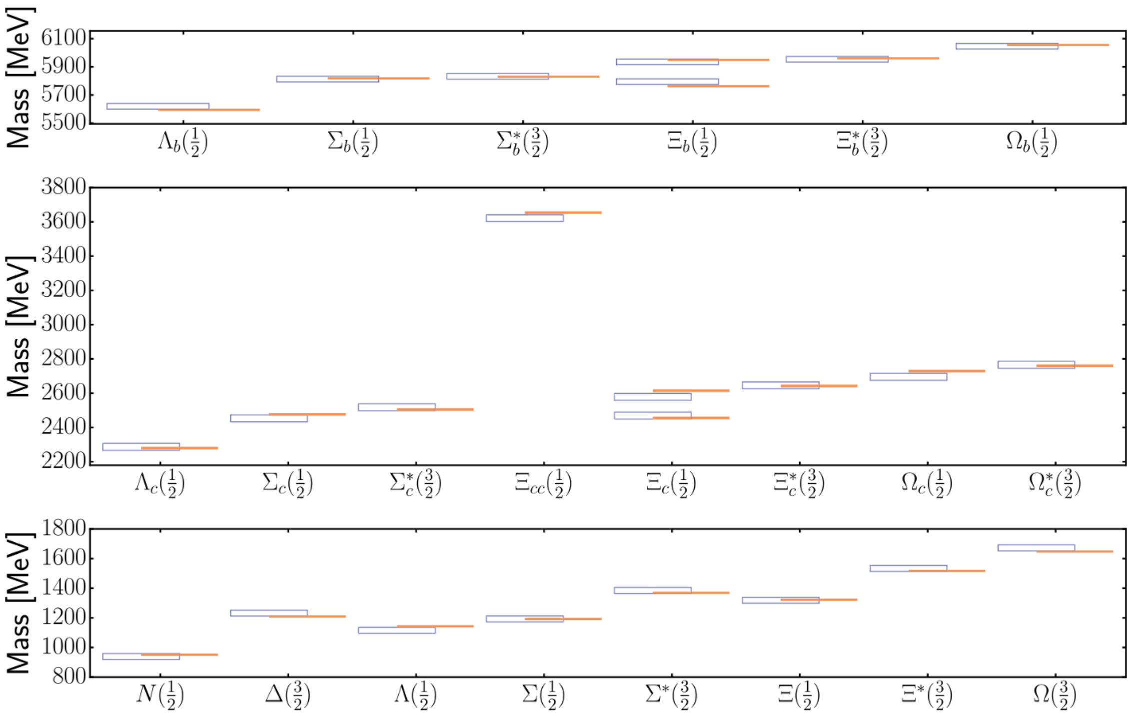
- Motivation
- Method
- Results
- Summary

Motivation

Success of potential model in meson, baryon, and multi-quark states explanation

one gluon exchange, goldstone boson exchange, hidden local symmetry, etc.





Eur. Phys. J., C 83 (2023) 12, 1159

Motivation

$$L_{QCD} = \bar{\psi}_i\left(i\gamma^{\mu}D_{\mu} - m_i\right)\psi_i - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} \Leftarrow \begin{cases} D_{\mu} = \partial_{\mu} - ig_s G^{a}_{\mu}T^{a} \\ G^{a}_{\mu\nu} = \partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu} + g_s f^{abc}G^{b}_{\mu}G^{c}_{\nu} \end{cases}$$

⇒ one same g_s in D_{μ} and $G^{a}_{\mu\nu}$, $g_s^2 = 4\pi\alpha_s$ is of $\log^{-1}$ form

quark → constituent quark ⇒ meson, baryon

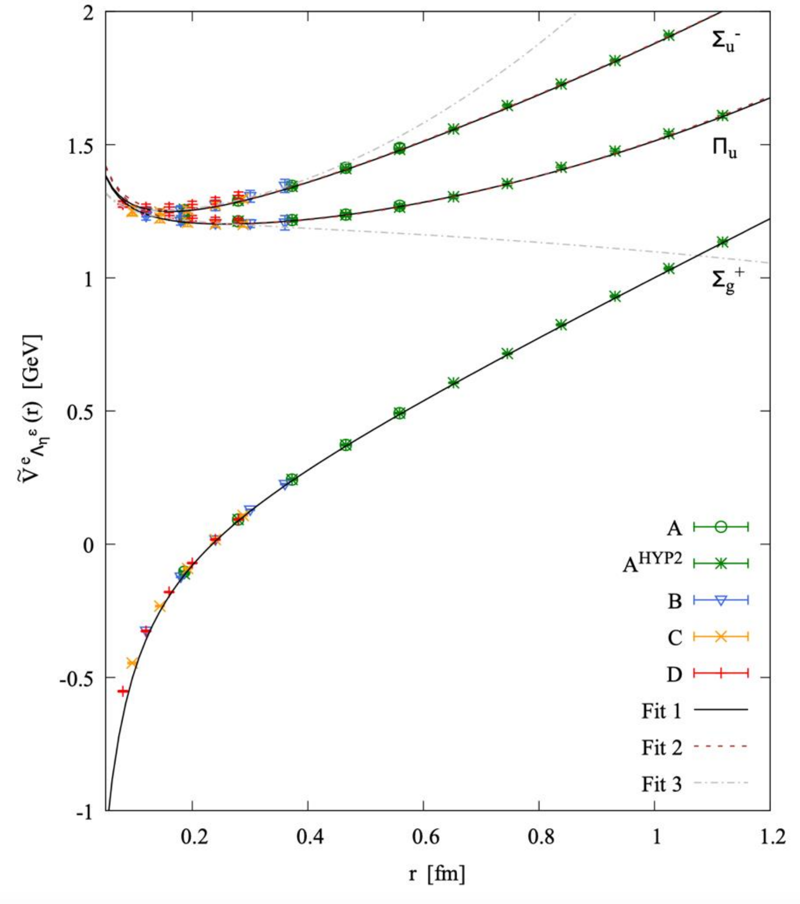
chiral constituent quark model: $\alpha_s = \frac{\alpha_0}{\log\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)}$ ⇒ behavior of g_s ?

⇒ naive: if the behavior of g_s can be used to the quark-gluon interaction directly ?

⇒ if properties of hybrid can be inferred directly ?

Motivation

Most simple hybrid: $q\bar{q}g$ $\Rightarrow$ How we treat g in $q\bar{q}g$?



Phys. Rev., D 105 (2022) 5, 054503

lattice QCD: modeling effective potentials for excited gluon fields, which facilitate interactions between quarks

$$V_{q\bar{q}g}(r) = \frac{A_1}{r} + A_2 r^2 + V_0 + \xi_2 \sqrt{\frac{b}{\xi_1}}, \quad (1)$$

where q and $\bar{q}$ denote the light (u, d) and strange (s) quarks. The parameters are set as $A_1 = 0.0958$ GeV, $A_2 = 0.01035$ GeV^3, $b = 0.165$ GeV^2, $\xi_1 = 0.04749$, and $\xi_2 = 0.5385$. The constant V_0 varies with flavor: $V_0^{(q\bar{q})} = -0.48$ GeV, $V_0^{(q\bar{s})} = -0.40$ GeV, and $V_0^{(s\bar{s})} = -0.33$ GeV.

We solve the spinless Salpeter equation:

$$H = \sqrt{p_q^2 + m_q^2} + \sqrt{p_{\bar{q}}^2 + m_{\bar{q}}^2} + V_{q\bar{q}g}(r), \quad (2)$$

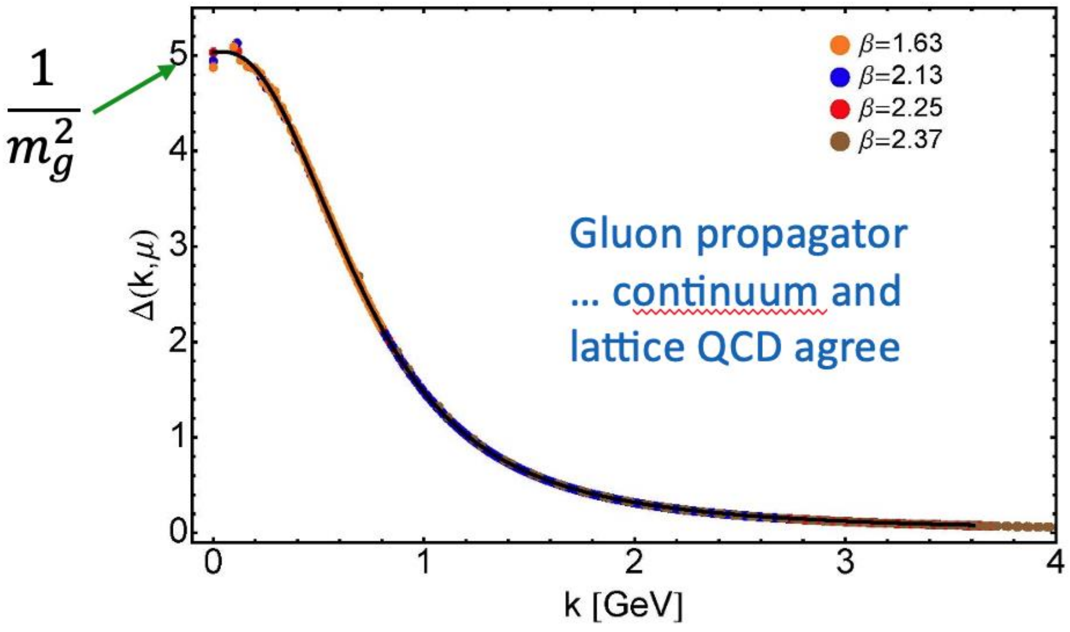
arXiV: 2503.01443

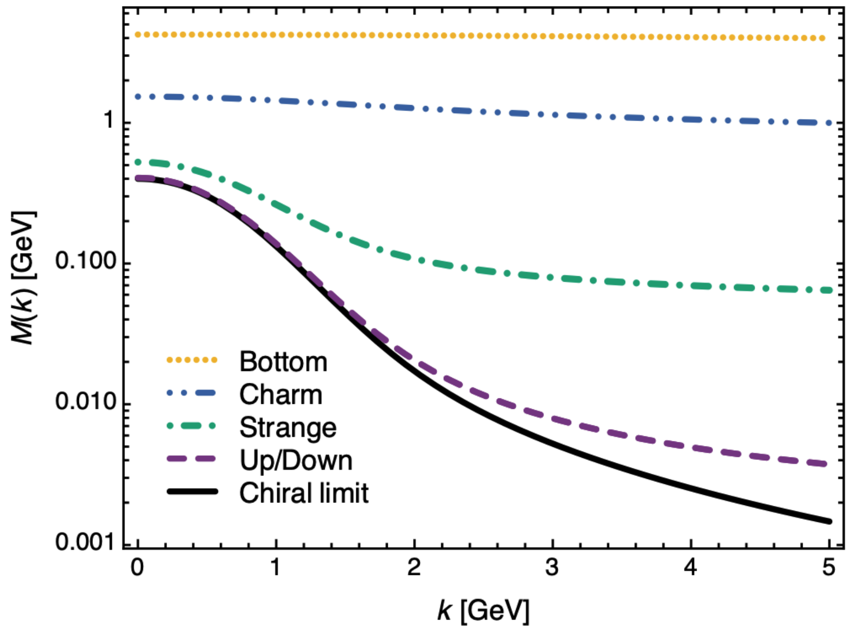
None business with g_s

Motivation

If this thing can be more interesting ?

gluon becomes a constituent as quark ? if yes, how much is its mass ?





Schwinger mechanism:

Few Body Syst. 63 (2022) 2, 42

$$m_g \approx \frac{1}{2} m_p, m_{u/d} \approx \frac{1}{3} m_p, m_s \approx 526\ MeV \Rightarrow familar\ values$$

Motivation

Treating hybrid as a three-body problem

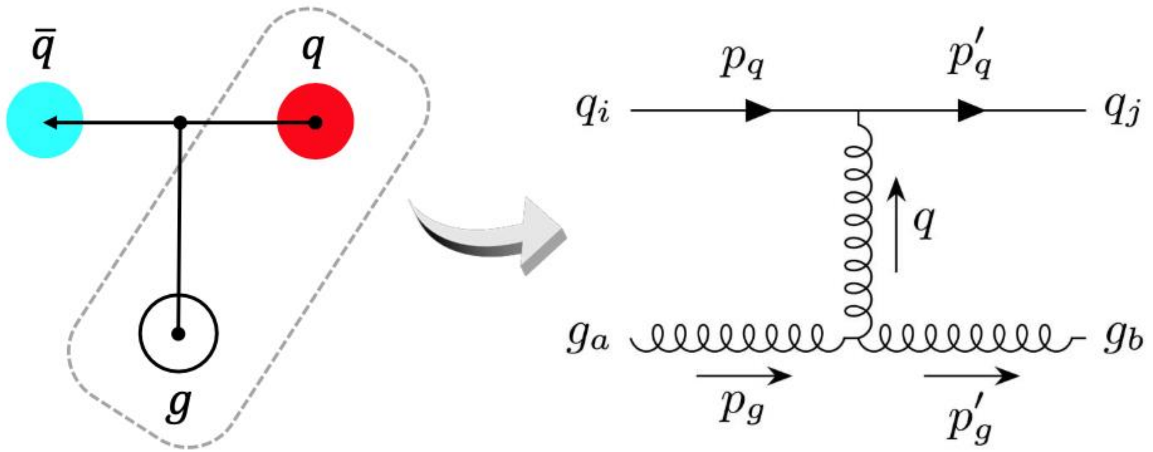
Keep the same model parameters as previous meson spectra calculations

Just adding only one parameter $\boldsymbol{m_g}$: constituent gluon mass

Research object: 1^{-+} light hybrid $\begin{cases} exotic\ quantum\ number \\ theoretical\ results\ for\ crosscheck \\ \pi_1(1400/1600), \eta_1(1855), \pi_1(2015)\ as\ candidates \end{cases}$

Method

Three-body problem: GEM method



Derived from QCD Lagrangian

$$\hat{H} = \frac{\hat{p}^2_{q,\bar{q}}}{2\mu_{q,\bar{q}}} + \frac{\hat{p}^2_{q\bar{q},g}}{2\mu_{q\bar{q},g}} + \hat{V}_{q\bar{q}} + \hat{V}_{qg} + \hat{V}_{\bar{q}g},$$

$$\alpha_s(\mu) = \frac{\alpha_0}{\log\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)},$$

f: *structure constants*

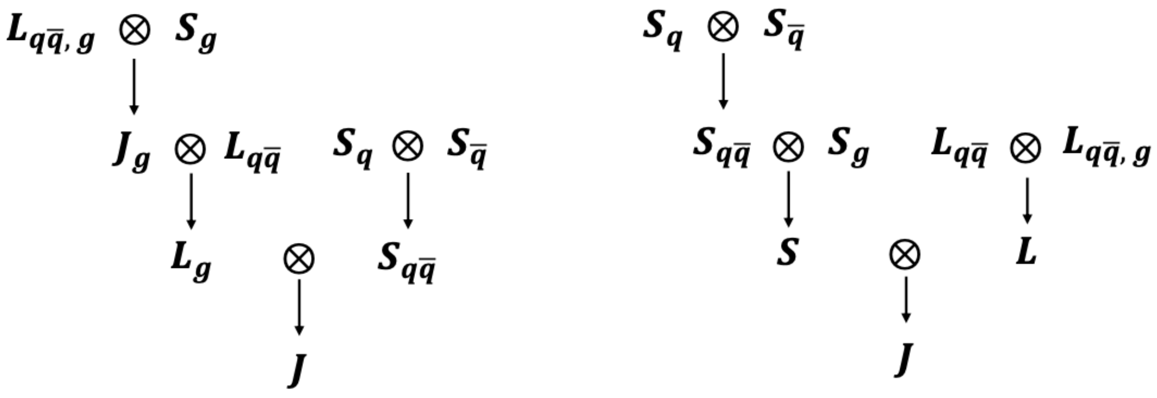
$$V_{q\bar{q}}(\boldsymbol{r}) = V^{CON}_{q\bar{q}}(\boldsymbol{r}) + V^{OGE}_{q\bar{q}}(\boldsymbol{r}) + V^{GBE}_{q\bar{q}}(\boldsymbol{r}),$$

$$\begin{aligned} V_{qg}(\boldsymbol{r}) &= V^{CON}_{qg}(\boldsymbol{r}) + V^{OGE}_{qg}(\boldsymbol{r}), \\ V^{CON}_{qg}(\boldsymbol{r}) &= V^{CON}_{q\bar{q}}(\boldsymbol{r}) \text{ with } \boldsymbol{\lambda}_c \cdot \boldsymbol{\lambda}^*_c \to i\boldsymbol{\lambda}^d \cdot \boldsymbol{f}^d, \\ V_{\bar{q}g}(\boldsymbol{r}) &= V_{qg}(\boldsymbol{r}) \text{ with } \boldsymbol{\lambda}_c \to -\boldsymbol{\lambda}^*_c, \end{aligned}$$

$$\begin{aligned} V_{qg}(\boldsymbol{r}) = & \frac{\alpha_s}{2}\boldsymbol{\lambda}_c \cdot \boldsymbol{f}_c \left[\frac{1}{r} - \left(\frac{2\pi}{3m_g^2} + \frac{\pi}{2m_q^2}\right)\delta(\boldsymbol{r}) - \frac{\boldsymbol{S}_q \cdot \boldsymbol{L}_q}{2m_q^2 r^3}\right. \\ & + \frac{\boldsymbol{S}_g \cdot \boldsymbol{L}_g}{2m_g^2 r^3} - \frac{1}{m_g m_q r^3}[\boldsymbol{S}_g \cdot \boldsymbol{L}_q - \boldsymbol{S}_q \cdot \boldsymbol{L}_g] \\ & + \frac{1}{2m_g^2 r^3}\left(\boldsymbol{S}_g \cdot \boldsymbol{S}_g - 3\frac{(\boldsymbol{S}_g \cdot \boldsymbol{r})(\boldsymbol{S}_g \cdot \boldsymbol{r})}{r^2}\right) \\ & \left. - \frac{8\pi}{3m_g m_q}\boldsymbol{S}_g \cdot \boldsymbol{S}_q \delta(\boldsymbol{r})\right], \end{aligned}$$

Method

Wave function: Two coupling schemes



(*a*) (*b*)

$$
\begin{aligned}
\psi^{J,g}_{q\bar{q}g} &= \sum_{J_{q\bar{q}},S,L} \sqrt{(2J_{q\bar{q}}+1)(2L_g+1)}(-1)^{S_{q\bar{q}}+L_{q\bar{q}}+J_g+J} \\
&\times \begin{Bmatrix} S_{q\bar{q}} & L_{q\bar{q}} & J_{q\bar{q}} \\ J_g & J & L_g \end{Bmatrix} \sqrt{(2S+1)(2L+1)} \\
&\times \sqrt{(2J_{q\bar{q}}+1)(2J_g+1)} \begin{Bmatrix} S_{q\bar{q}} & L_{q\bar{q}} & J_{q\bar{q}} \\ 1 & L_{q\bar{q},g} & J_g \\ S & L & J \end{Bmatrix} \\
&\times \psi^{J,LS}_{q\bar{q}g}.
\end{aligned}
$$

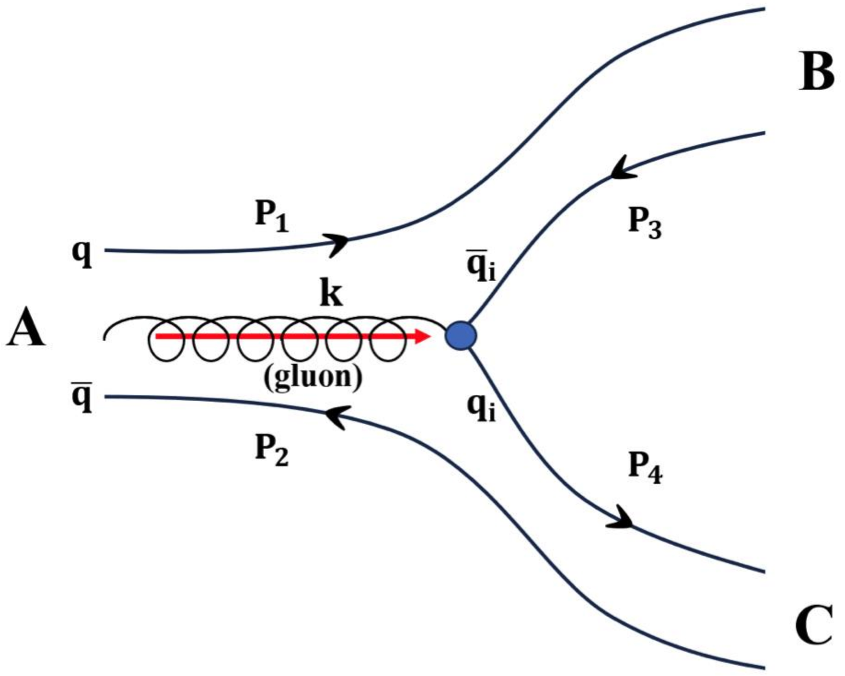
(a) Excited gluon mode, commonly used
(b) S-L coupling, easier to construct spin

Color wave function:

$$
|\psi^c_{q\bar{q}g}\rangle = \frac{\delta_{ad}}{\sqrt{8}} \frac{(\lambda^a)_{bc}}{\sqrt{2}} f^{dbc}\ |q^b\rangle \otimes |\bar{q}^c\rangle \otimes |g^d\rangle.
$$

Method

Decay mechanism:



$$\hat{H}_I = i\sqrt{4\pi\alpha_s}\frac{(\lambda^a)_{bc}}{2}\int d^3\vec{x}\ \bar{q}^c(\vec{x})\gamma^\mu q^b(\vec{x})A^a_\mu(\vec{x}),$$

$$\hat{T} = 3i\sqrt{\pi\alpha_s}(\lambda^a)_{bc}\sum_{s,s',m}\int\frac{d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{k}}{\sqrt{2m_g}(2\pi)^6}\delta(\vec{p}_1+\vec{p}_2-\vec{k})$$
$$\times\langle 1,m;1,-m|0,0\rangle\langle 1,-m;\frac{1}{2},s'|\frac{1}{2},s\rangle$$
$$\times d^{c\dagger}_{s'}(\vec{p}_1)b^{b\dagger}_s(\vec{p}_2)a^a_m(\vec{k}),$$

Amplitude: Similar with *Phys. Lett.*, B 650, 390-400

Parameter

$GEM: r_{min} = 0.1\ fm, r_{max} = 2.0\ fm, n_{max} = 8$

Confinement: screened, linear, square

action parameters are consistent. Masses of π, η, K adopt experimental values, while other parameters — $m_\sigma = 3.42$ fm^{-1}, $\Lambda_\pi = \Lambda_\sigma = 4.2$ fm^{-1}, $\Lambda_\eta = \Lambda_K = 5.2$ fm^{-1}, $\theta_p = -15°$, $g^2_{ch}/(4\pi) = 0.54$ — are adopted from Ref. [1]

		[Scr. , Lin. , Squ.]
Gluon mass	m_g (MeV)	450
Quark masses	$m_{u,d}$ (MeV)	313
	m_s (MeV)	[555 , 525 , 536]
	m_c (MeV)	[1752 , 1731 , 1728]
	m_b (MeV)	[5100 , 5100 , 5112]
Confinement	a_c (MeV fm^{-n})	[430 , 160 , 101]
	Δ (MeV)	[181.1, -131.1, -78.3]
	μ_c (fm^{-1})	[0.7 , - , -]
	a_s	0.777
OGE	α_0	[2.12 , 2.65 , 3.67]
	Λ_0 (fm^{-1})	[0.113, 0.075, 0.033]
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (MeV fm)	28.17
	$\hat{r}_g$ (MeV fm)	34.5

TABLE III: Meson spectrum calculated by using three sets of parameters and confinement potentials (unit: MeV).

States	[Scr. , Lin. , Squ.]	Exp.
π	[132.18 , 140.08 , 134.87]	139.57
η	[684.74 , 680.19 , 669.21]	547.86
ρ	[773.92 , 775.33 , 772.26]	775.26
ω	[697.89 , 703.70 , 701.59]	782.66
K	[472.58 , 496.21 , 489.37]	493.68
K^*	[908.39 , 917.90 , 913.55]	891.67
η'	[824.19 , 832.80 , 821.47]	957.78
$h_1(1170)$	[1247.02 , 1271.19 , 1314.82]	1166.00
$b_1(1235)$	[1234.79 , 1250.19 , 1281.93]	1229.50
$a_1(1260)$	[1204.76 , 1213.91 , 1238.72]	1230.00
$f_1(1285)$	[1149.08 , 1212.92 , 1283.19]	1281.80
$\pi(1300)$	[1286.99 , 1345.44 , 1453.58]	1300.00
$K_1(1270)$	[1342.73 , 1293.69 , 1287.15]	1253.00
$K_1(1400)$	[1416.25 , 1425.47 , 1453.16]	1403.00
$K(1460)$	[1465.49 , 1490.60 , 1571.79]	1482.40

Isospin-0 states are not so good mainly due to mixing

No change on the previous parameters

Spectra

arXiV: 2503.01443: constituent gluon in 1^{-+} light hybrid should be electric if wants lowest mass

$\Rightarrow Translating: S_{q\bar{q}} = 1, L_{q\bar{q}} = 0, L_{q\bar{q},g} = 1, J_g = 1, L_g = 1, J = 1, \quad constituent\ gluon: 1^{+-}$

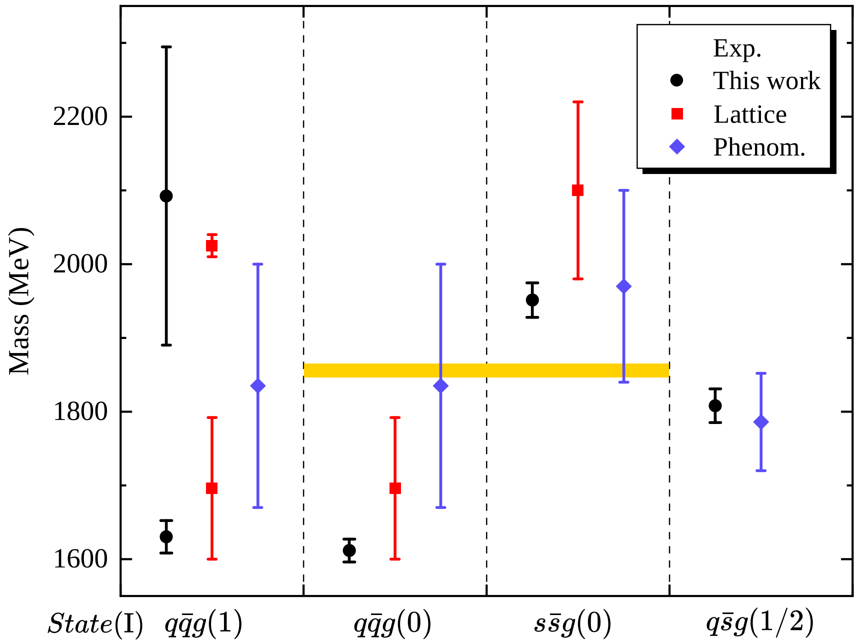


TABLE IV: Predicted masses of 1^{-+} hybrid mesons under three sets of confinement potentials (unit: MeV).

States(I)	$M_{Scr.}$	$M_{Lin.}$	$M_{Squ.}$
$q\bar{q}g(1)$	1608.4	1631.3	1652.4
	1890.2	1987.1	2130.1
	2087.0	2294.5	2600.5
$q\bar{q}g(0)$	1596.2	1614.7	1627.2
$s\bar{s}g(0)$	1974.6	1927.9	1957.8
$q\bar{s}g(\frac{1}{2})$	1793.9	1785.2	1830.9

1. Ground states are consistent
2. First excited states have two modes ?
3. There's another $\eta_1(1640)$?

$$\begin{bmatrix} \eta_1^{low} \\ \eta_1^{high} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} q\bar{q}g \\ s\bar{s}g \end{bmatrix} \Rightarrow \theta \approx 28.8° \Rightarrow$$

Decays

States	Channels	[Scr. , Lin. , Squ.]	[[46] , [50]]
$\pi_1(1600)$	$b_1(1235)\pi$	[53.3 , 69.7 , 88.0]	[244 , 56.6]
	$f_1(1285)\pi$	[9.0 , 11.5 , 13.6]	[15 , 8.4]
	$\rho\pi$	[1.0 , 1.3 , 1.4]	[2 , -]
	Total	[63.3 , 82.5 , 103.0]	[261 , 65]
$\eta_1(1640)$	$a_1(1260)\pi$	[35.5 , 45.8 , 58.3]	[55 , 29.3]
	$\pi(1300)\pi$	[2.9 , 1.5 , 0.1]	[5 , 0.4]
	Total	[38.4 , 47.3 , 58.4]	[60 , 29.7]
$\eta_1'(1855)$	$a_1(1260)\pi$	[11.0 , 16.6 , 24.5]	[- , 18.1]
	$f_1(1285)\eta$	[6.4 , 6.7 , 8.6]	[- , 5.6]
	$\pi(1300)\pi$	[4.3 , 3.5 , 2.4]	[- , 1.1]
	$K_1(1270)\bar{K}$	[176.0 , 160.8 , 228.0]	[157 ,162.5]
	$K^*\bar{K}$	[1.0 , 1.4 , 1.6]	[2 , -]
	Total	[198.7 , 189.0 , 265.1]	[159 ,187.3]
$K_1(1^{-+})$	$K\pi$	[0.6 , 0.8 , 1.3]	[1 , -]
	$K^*\pi$	[1.3 , 1.6 , 2.3]	[3 , -]
	$K^*\eta$	[0.2 , 0.2 , 0.4]	[1 , -]
	$K_1(1270)\pi$	[36.5 , 46.1 , 46.5]	[106 , -]
	$K_1(1400)\pi$	[0.0 , 0.0 , 2.2]	[146 , -]
	$h_1(1170)K$	[6.8 , 3.6 , 5.8]	[16 , -]
	$K(1460)\pi$	[1.4 , 1.0 , 0.5]	[2 , -]
	Total	[46.8 , 53.3 , 59.0]	[275 , -]

1. Most of the decay widths are consistent
2. Width of $\pi_1(1600) \to b_1(1235)\pi$ are totally different
3. Mixing of $K_1(1^1P_1)$ and $K_1(1^3P_1)$ for K_1 states
4. No $\eta\eta'$ decay channel for $\eta_1(1855)$, OMG😱
5. Narrow width of ground π_1 hybrid ?
6. Find $\eta_1(1855)$ on $K_1(1270)\bar{K}$ channel ?
7. Find ground K_1 hybrid on $K_1(1270)\pi$ channel ?
8. Verify $\eta_1(1640)$ on $a_1(1260)\pi$ channel ?

Summary

- Proper constituent gluon mass as the last piece to construct hybrid ?
- Nearly everything are constituent well with the same meson parameters.
- $\pi_1(1600)$ and $\eta_1(1855)$, or decay mechanism, need to be studied further.
- Verify $\eta_1(1640)$ on $a_1(1260)\pi$ channel and K_1 hybrid on $K_1(1270)\pi$ channel.
- Studies on other quantum numbers should be done to see if it's coincidence.

Thank you ~