ω meson from lattice QCD

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Based on:

1. **Yan *et al.***, **PRL** 133 (2024) 211906, **Editors' Suggestion**
2. **Yan *et al.***, **PRD** 111 (2025) 014503
3. **Yan *et al.***, arXiv 2504.xxxx on OpTion
4. **Yan *et al.***, arXiv 250x.xxxx on coupled channel $D_0^*(2300)$

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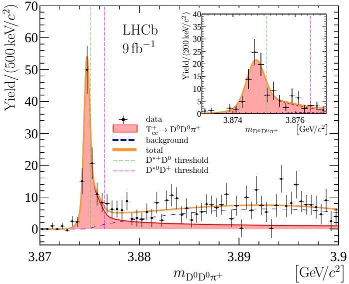
Introduction

- Many resonances decay into three-body states
- LHCb discovered a tetraquark candidate $T_{cc}(3875) \to DD^* \to DD\pi$ in 2022[1]

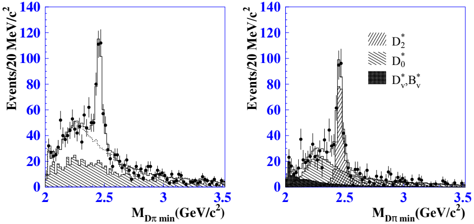


- Previous lattice studies lifted the pion mass and study the two-body DD^* scattering[2] and found the corresponding pole
- The real three-body scattering problem relies on the two-body $D\pi$ scattering amplitudes as an input
- Lattice QCD is the only way to calculate observables at low energy from the first principle
- We need to know how to solve 1. two-body and 2. three-body problems

[1] LHCb collaboration, NP 18 (2022) 751

[2] Padmanath *et al.*, PRL 129 (2022) 032002; Chen *et al.*, PLB 833 (2022) 137391; Lyu *et al.*, PRL 131 (2023) 161901; Collins *et al.*, PRD 109 (2024) 094509

- A broad resonance $D_0^* \to D\pi$ was found by Belle collboration[3] in 2004



- Puzzles on $D_0^*(2300)$
  - $m_{D_0^*} \approx m_{D_{s0}^*}$
  - $m_{D_0^*}^{\text{ChPT}} \ll m_{D_0^*}^{\text{exp.}}$

  could be explained by UChPT where the two-pole structure is proposed[4]
- Should be tested on the lattice at a series of m_π[5]

[3] Satpathy *et al.*, PRB 159 (2003) 553.
[4] For *e.g.*, Albaladejo *et al.*, PLB 767 (2017) 465.
[5] See previous lattice works in [Moir:2016srx, Gayer:2021xzv]

- Turn off the weak and electromagnetic interactions,

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\,(i\,(\gamma^{\mu} D_{\mu}) - m)\,\psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} \to \text{discretizing}$$

- Two-point function is calculated by

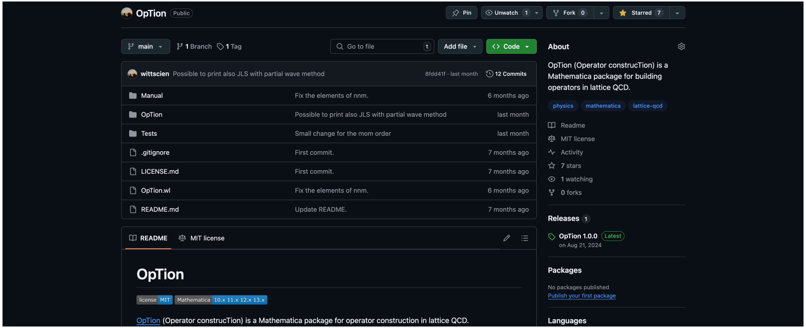
$$\langle\Omega|O(t)\,O^{\dagger}(0)|\Omega\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi}]\mathcal{D}[U]\mathrm{e}^{-S_F[\psi,\bar{\psi},U]-S_G[U]}(O(t)\,O^{\dagger}(0))[\psi,\bar{\psi},U]$$

- On the other hand,

$$\begin{aligned}\langle\Omega|O(t)\,O^{\dagger}(0)|\Omega\rangle &= \langle\Omega|O(t)\sum_{n}\frac{1}{2E_n}|E_n\rangle\langle E_n|O^{\dagger}(0)|\Omega\rangle \\ &= \frac{1}{2E_1}|\langle\Omega|O(t)|E_1\rangle|^2 e^{-E_1 t} + \text{exited states}\end{aligned}$$

- Towers of finite-volume energy levels E_n encode the interactions

- Now, how to build these operators O, so that we create $D\pi$ from the vacuum?
- Symmetry broken $O(3) \to O_h$ or even smaller group
- A package **OpTion**[6] (**Op**erator construc**Tion**) has been developed to construct general N-hadron operators. It is widely used and has become a routine tool in the CLQCD collaboration



- *e.g.* $D(+p_z)\pi(-p_z) - D(-p_z)\pi(+p_z)$ transforms as the T_1^- irrep

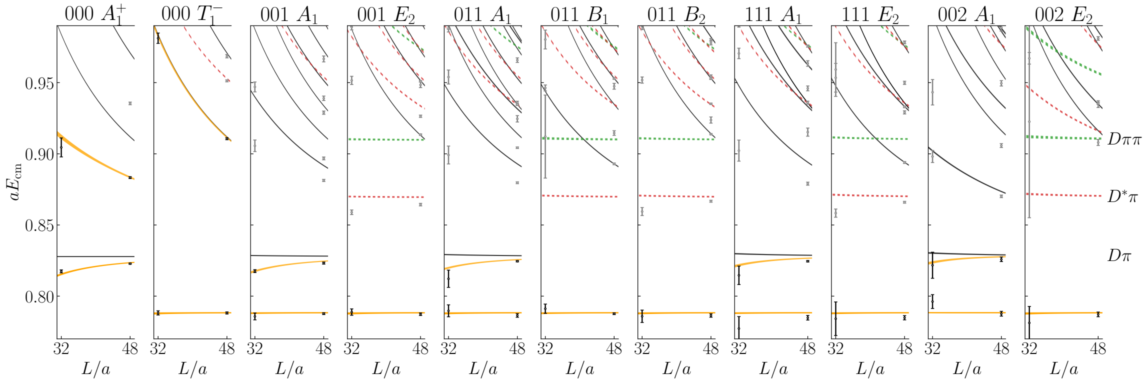
[6] `https://github.com/wittscien/OpTion`

Lattice setup

configuration	volume	a/fm	m_π/MeV	N_{cfgs}
C48P14	$48^3 \times 96$	$0.10530(18)$	$135.5(1.6)$	259
F32P21	$32^3 \times 64$	$0.07746(18)$	$210.9(2.2)$	459
F48P21	$48^3 \times 96$	$0.07746(18)$	$207.2(1.1)$	222
F32P30	$32^3 \times 96$	$0.07746(18)$	$303.2(1.3)$	567
F48P30	$48^3 \times 96$	$0.07746(18)$	$303.4(9)$	201
H48P32	$48^3 \times 144$	$0.05187(26)$	$317.2(0.9)$	274

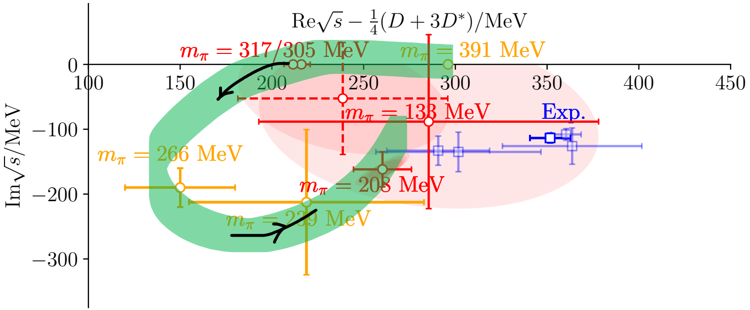
- CLQCD ensembles with $N_{\mathrm{f}} = 2 + 1$ Clover fermions [CLQCD, 2024]
- 4 different pion masses to track the chiral behavior
- 3 lattice spacings to estimate the discretization error

Finite-volume spectra at $m_\pi \approx 208$ MeV



- Strong attraction in S-wave
- The levels are interpreted by quantization conditions: f(energy levels, amplitude) $= 0$

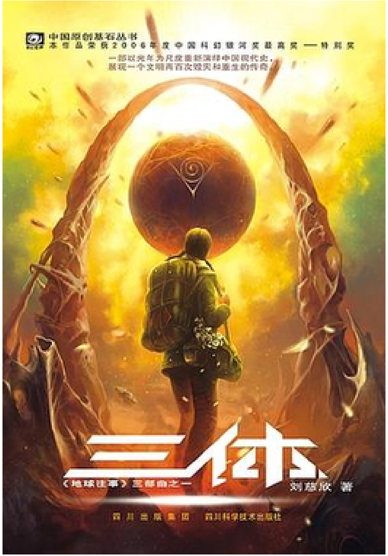
- Using the scattering amplitude, we perform an analytic continuation to find poles



- Only one pole is found → two-pole structure is to be found in coupled channel scattering
- An clear trend for the motion of one D_0^* pole is identified

bound state → virtual state → resonance

Introduction 2: Three-body: $\omega \to \pi\pi\pi$



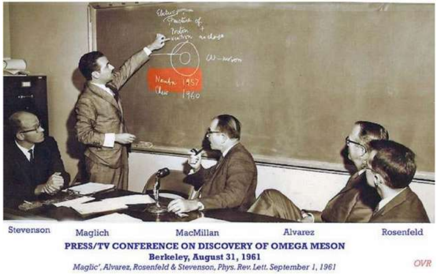
- Now we have the two-body input – we want to approach the three-body channel
- However, 3bd is too much more complicated (only one lattice calculation considered resonance in the history[a])
- We take a challenge on the ω meson
- crucial for T_{cc}[b], Roper, ...

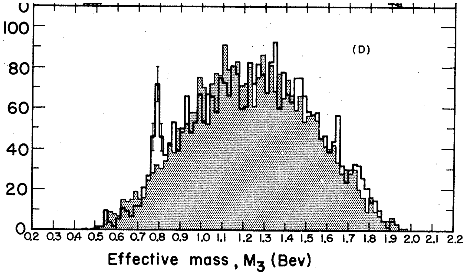
Why is this problem difficult?

- Three-body problem with resonances in two-body problem
  - $\pi\pi\pi \to \omega$
  - $\pi\pi \to \rho$
- Computationally most complicated among all isospin channel
- Complicated quantization conditions

[a] Mai:2021nul
[b] Hansen:2024ffk

ω: the first neutral vector meson (1961)[1]

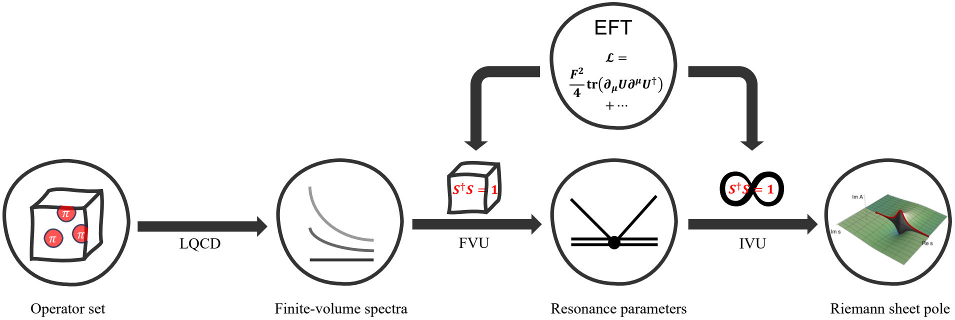




- The lightest hadron decaying into three particles: $\omega \to 3\pi$
- Phenomenologically interesting, for instance in the $N-N$ interaction, muon $g-2$, dark matter...[8]

[7] Maglic *et al.* (1961).
[8] Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov *et al.*, 1985; Connell *et al.* (1997); Bazavov *et al.* (2021).

The methodology

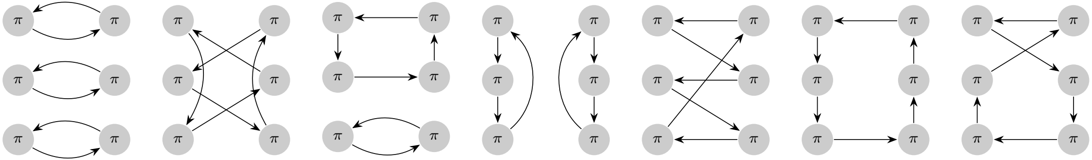


- $\omega, \rho\pi, \pi\pi\pi$ operator from OpTion[2]
- 2bd and 3bd spectra
- Develop FVU and the pertinent EFT to parametrize the three-body force
- Solve the integral equations and search for the ω poles

[2] `https://github.com/wittscien/OpTion`

Contraction & quantization conditions

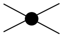
- Insanely many diagrams (202 for only $\pi\pi\pi \to \pi\pi\pi$, only 9 for the two-body problem)
- The topologies for $\pi\pi\pi \to \pi\pi\pi$

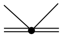


- Quantization conditions: **FVU** (Finite-Volume Unitarity)

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$

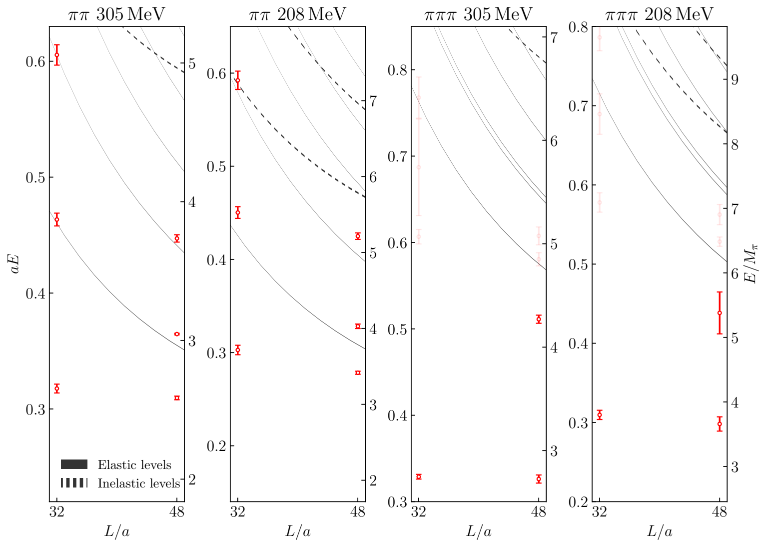
- **Combined fit** for $\pi\pi$ and $\pi\pi\pi$ spectra for all M_π's (**EFT4**)[3]

 $\sim \frac{\sigma - M_\rho^2}{g^2}$

 $\sim \frac{s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{g^2 f_\pi^6 (s - M_\omega^2)}$

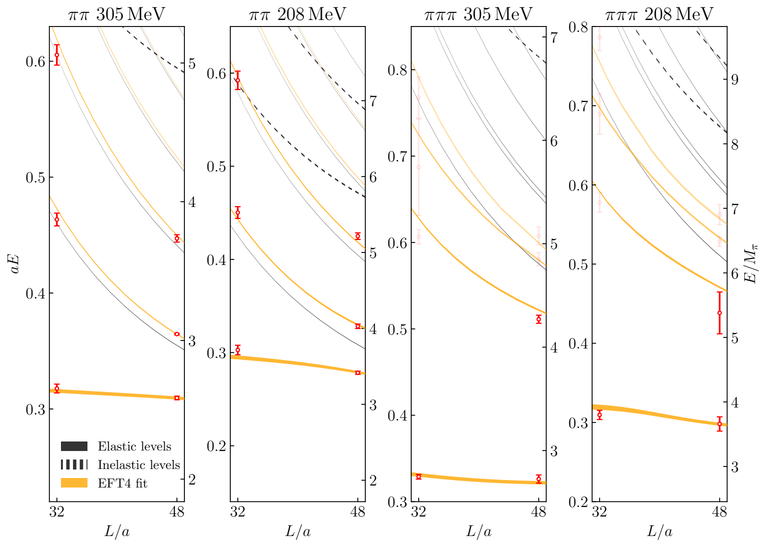
[3]For a review of the EFT form, see Meißner (1988).

Finite-volume spectra revisited



- Strong **attraction** in both the $\pi\pi$ and $\pi\pi\pi$ channels
- Restricted to be below the $\omega(1420)$ region, but high-lying energies are also well-predicted

Finite-volume spectra revisited



- Strong **attraction** in both the $\pi\pi$ and $\pi\pi\pi$ channels
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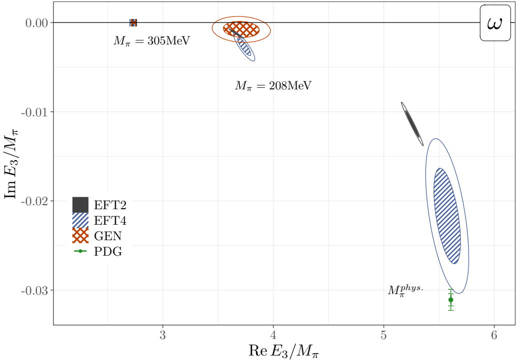
Pole positions of $\omega(782)$ as a function of m_π



- Solve the integral equation
- ω is a **bound state** at $M_\pi \approx 305\,\mathrm{MeV}$ and a **resonance** at $M_\pi \approx 208\,\mathrm{MeV}$
- Extrapolate to the **physical pion mass**, the poles agree well with the PDG values[4]

[4]See related discussions in Hoferichter (2023), Hoferichter (2019), Hoid (2020), Colangelo (2022), Colangelo (2018).

Summary

For $D_0^*(2300)$,

- An clear trend for the motion of one D_0^* pole is identified

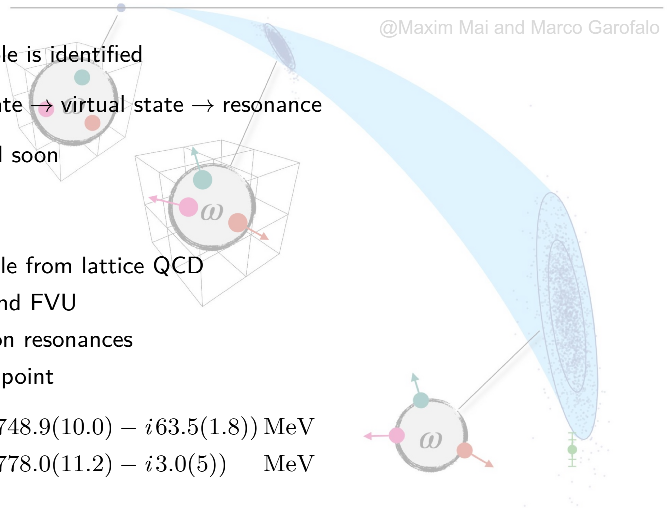
  bound state $\to$ virtual state $\to$ resonance

- Coupled $D\pi{-}D\eta{-}D_s\bar{K}$ analysis is expected soon
- The package OpTion is developed

For $\omega(782)$,

- First-ever determination of the ω-meson pole from lattice QCD
- Development of the FVU, matching EFT and FVU
- Paved the way to study heavier three-hadron resonances
- The ρ and ω pole positions at the physical point

$$\sqrt{s_\rho} = (748.9(10.0) - i63.5(1.8))\,\mathrm{MeV}$$
$$\sqrt{s_\omega} = (778.0(11.2) - i3.0(5))\quad\mathrm{MeV}$$



Thank you!