第十届XYZ物理研讨会，长沙，2025.04.11-14

在非厄密量子力学框架内研究 $K\bar{K}^*$和 $D\bar{D}^*$的共振态

孙宝玺

北京工业大学

2025.04.14

1. Content

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- 3. The $K\bar{K}^*$ interaction: f1(1285) and f1（1420）.
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1. Rectanglar potential-well

The rectanglar potential-well is defined by

$$V(x)=\begin{cases}0 & x\notin(0,L),\\ -V_0 & x\in(0,L),\end{cases}$$

with the depth of the well $V_0>0.$

The Schrodinger equation can be written as:

$$\begin{cases}\varphi''+k_0^2\varphi=0, & k_0=\sqrt{2ME}/\hbar & x\notin(0,L),\\ \varphi''+k^2\varphi=0, & k=\sqrt{2M(E+V_0)}/\hbar & x\in(0,L),\end{cases}$$

Moiseyev, Non-Hermitian Quantum Mechanics, 2011

1. Rectanglar potential-well

The general solution is

$$\begin{cases} \varphi = Ce^{ik_0x} + C'e^{-ik_0x} & (x<0), \\ \varphi = Ae^{ikx} + Be^{-ikx} & (0<x<L), \\ \varphi = De^{ik_0x} + D'e^{-ik_0x} & (x>L), \end{cases}$$

Incoming wave condition: $C' = D = 0$

Outgoing wave condition: $C = D' = 0$

The transcendental equation is

$$2\cot k2L = \frac{\pm i\left(k^2 + k_0^2\right)}{kk_0}.$$

Positive(negative) sign represents the outgoing(incoming) wave.

1. Rectanglar potential-well

The mechanism of transition from a bound state to a resonance state becomes evident when two variables are introduced.

$$\alpha = \sqrt{1 + \frac{E}{V_0}}, \quad \gamma = \sqrt{\frac{2MV_0L^2}{\hbar^2}}, \qquad V_0 > 0.$$

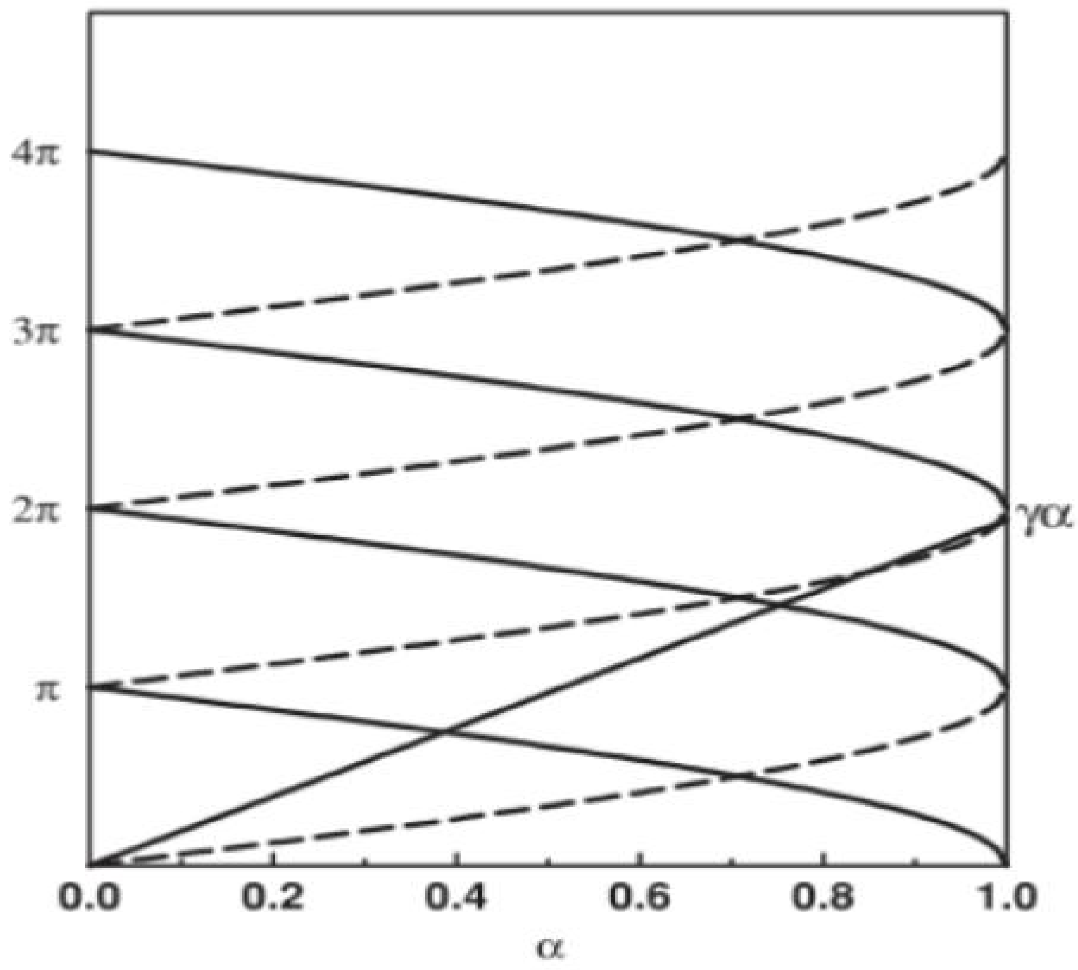
Bound states:

$$\gamma\alpha = (n-1)\pi + 2\cos^{-1}\alpha, \qquad n = 1,2,...$$

Virtual states:

$$\gamma\alpha = (n-1)\pi + 2\cos^{-1}\sqrt{1-\alpha^2}, \qquad n = 1,2,...$$

1. Rectanglar potential-well



2. One-Pion Exchange potential

The one-pion-exchange potential is:

$$V(x) = -g^2 \frac{e^{-mr}}{d},$$

where the distance in the denominator has been replaced with the range of force, $d = 1/m$.

Supposing the radial wave function $R(r) = \frac{u(r)}{r}$, the radial Schrodinger equation becomes

$$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2} + V(r)u(r) = Eu(r),$$

Sun et al., CTP, 76, 105301, 2024

2. One-Pion Exchange potential

With the variable substitution:

$x = \alpha e^{-\beta r}, \quad u(r) = J(x),$ with $\alpha = 2g\sqrt{2\mu d}, \quad \beta = \frac{1}{2d}, \quad \rho^2 = -8d^2\mu E,$

The Schrodinger equation becomes the Bessel eq.

$$\frac{d^2 J(x)}{dx^2} + \frac{1}{x}\frac{dJ(x)}{dx} + \left(1 - \frac{\rho^2}{x^2}\right)J(x) = 0,$$

$$r \to 0, \quad u(r) \to 0, \quad J_\rho(x) = J_\rho(\alpha) = 0,$$

If only one bound state exists, and the binding energy is given, the value of $\rho^2 = -8d^2\mu E$ can be determined, and the coupling $g^2 = \frac{\alpha^2}{8\mu d}$ is obtained with the first zero piont of $J_\rho(\alpha)$.

2. One-Pion Exchange potential

The Hankel functions are also solutions of the Bessel function.In a scattering process, the general solution of the Bessel equation is $u(r)=DH_{\rho'}^{(1)}(x)+D'H_{\rho'}^{(2)}(x),$

Incoming wave condition: $r\to 0,\quad u(r)\to 0,\quad H_{\rho'}^{(1)}(x)=H_{\rho'}^{(1)}(\alpha)=0,$

Outgoing wave condition: $r\to 0,\quad u(r)\to 0,\quad H_{\rho'}^{(2)}(x)=H_{\rho'}^{(2)}(\alpha)=0,$

With the same zero point(coupling constant),the order of $H_{\rho'}^{(2)}(\alpha)$ is determined, which is related to the energy of the resonance state,

$$E=\left(M - M_{Threshod}\right)-i\frac{\Gamma}{2}=-\frac{\rho'^2}{8d^2\mu}.$$

3. f1 (1285) or f1 (1420) ?

1. Roca, Oset, Singh, PRD, 72, 014002, 2005

f1(1285) is a $K\overline{K}^*$ bound state.

2. Wan, Zhao, Sun, 1808.08358[hep-ph]

f1(1420) is a $K\overline{K}^*$ resonance state, no other pole is detected.

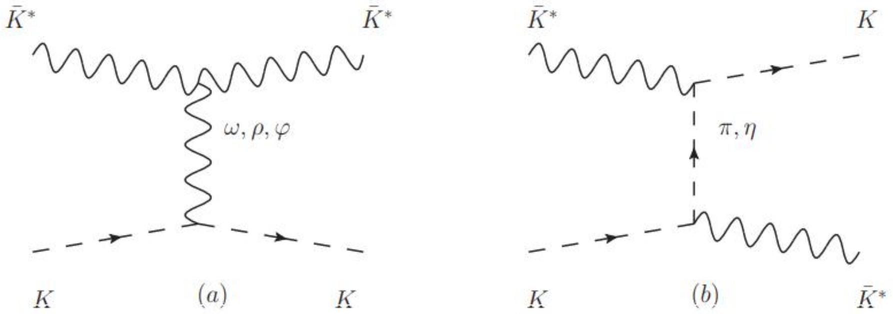
3. Debastiani, Aceti, Liang, Oset, PRD, 95, 034015, 2017

f1(1420) is related to a $K^*\overline{K}K$ triangle singularity.

3. KK* interaction

In the unitary coupled-channel approach, the vector meson transfer is dominant, while the pion exchange is neglected.

Wan, Zhao, Sun, 1808.08358[hep-ph]

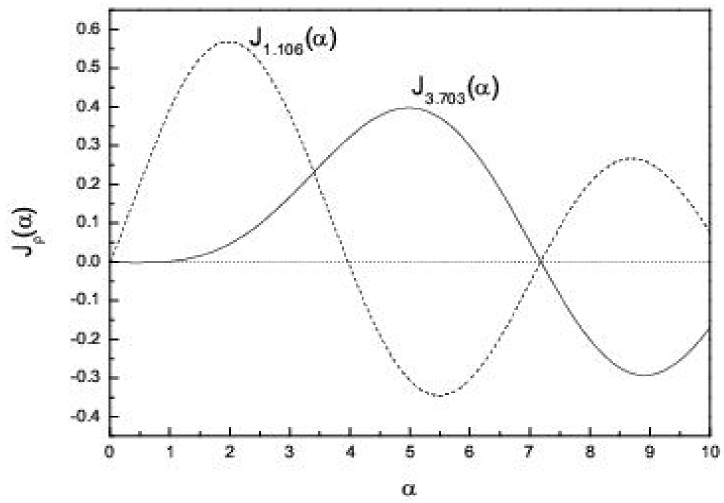


Actually, the OPEP is critical in the generation of resonance states.

3. KK* interaction

The f1(1285) particle is treated as a $K\bar{K}^*$ bound state with a binding energy of 105MeV, and the coupling constant is determined as g=1.682.

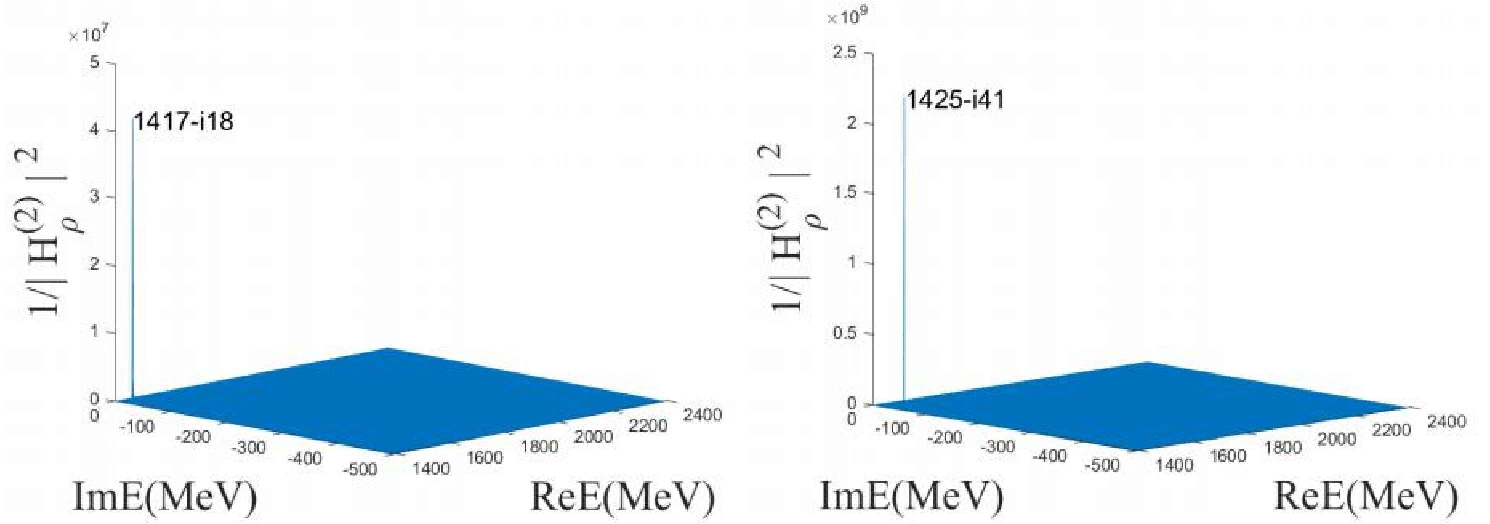
Another bound state about 9 MeV below the $K\bar{K}^*$ threshold.



3. Double-peak of f1(1420)?

- B. X. Sun, 2503.00478[hep-ph].

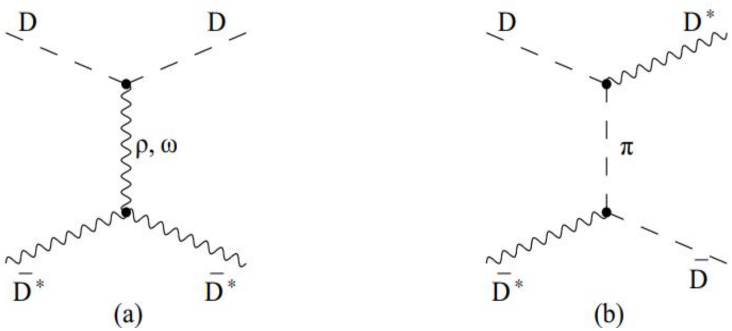
Energy	Name	$I^G\left(J^{PC}\right)$	Mass	Width
1417-i18	f1(1420)	$0^+\left(1^{++}\right)$	$1428.4^{+1.5}_{-1.3}$	56.7 ± 3.3
1425-i41	f1(1420)			



4. DD* interaction

In the unitary coupled-channel approach, the vector meson transfer is dominant, while the pion exchange is neglected.

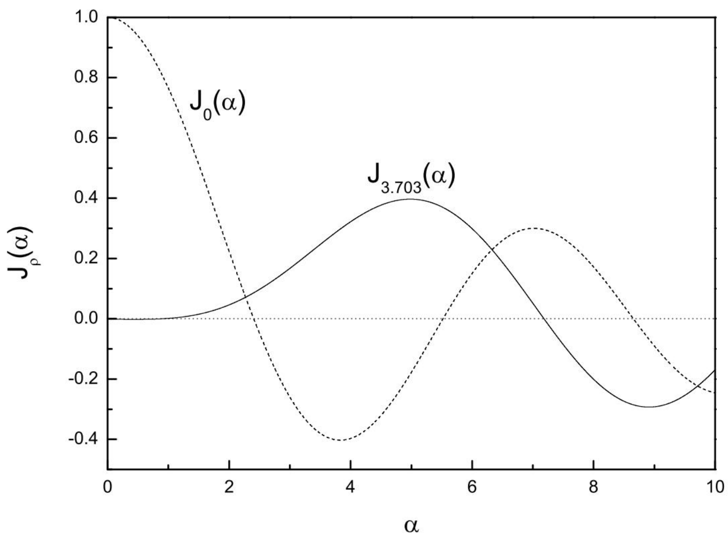
Sun,Wan, Zhao, CPC, 42, 053105, (2018)



Actually, the OPEP is critical in the generation of resonance states.

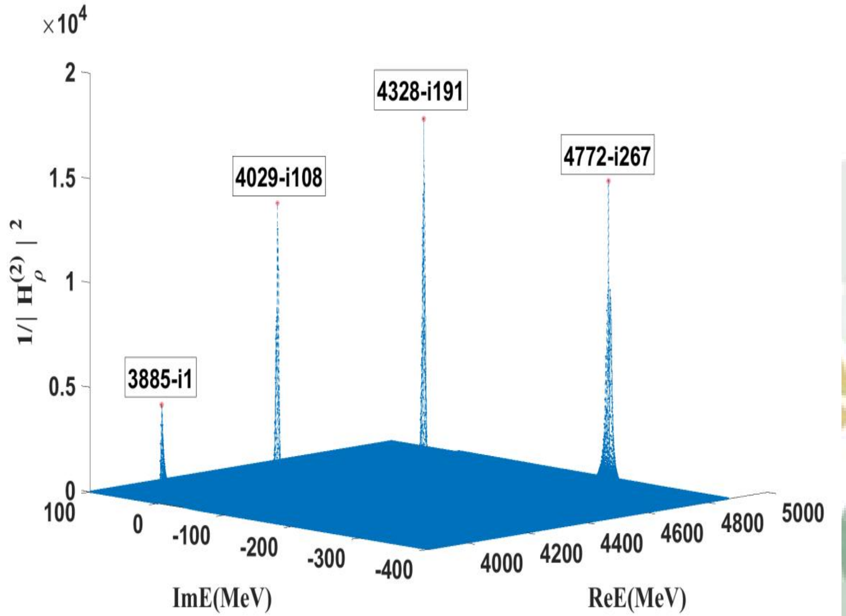
4. DD* interaction

The X(3872) particle is assumed to be a $D\overline{D}^*$ bound state with a zero binding energy since it lies at the $D^0\overline{D}^{*0}$ threshold. Therefore, the coupling constant is determined as g=0.323 according to the first zero point $\alpha=2.405$ of $J_0(x)$.



4. DD* interaction

If the X(3872) particle is a $D\overline{D}^*$ bound state, $Zc(3900)$ would be a $D\overline{D}^*$ resonance state. All states are isospin degenerate.



$D\overline{D}^*$	Energy	Name	$I^G(J^{PC})$	Mass	Width
1	3885-i1	$Zc(3900)$	$1^+(1^{+-})$	3887.1	28.4
2	4029-i108	$X(3940)$	$?^?(?^{??})$	3942	37
3	4328-i191	$\chi_{c1}(4274)$	$0^+(1^{++})$	4286	51
4	4772-i267	$\chi_{c1}(4685)$	$0^+(1^{++})$	4684	126

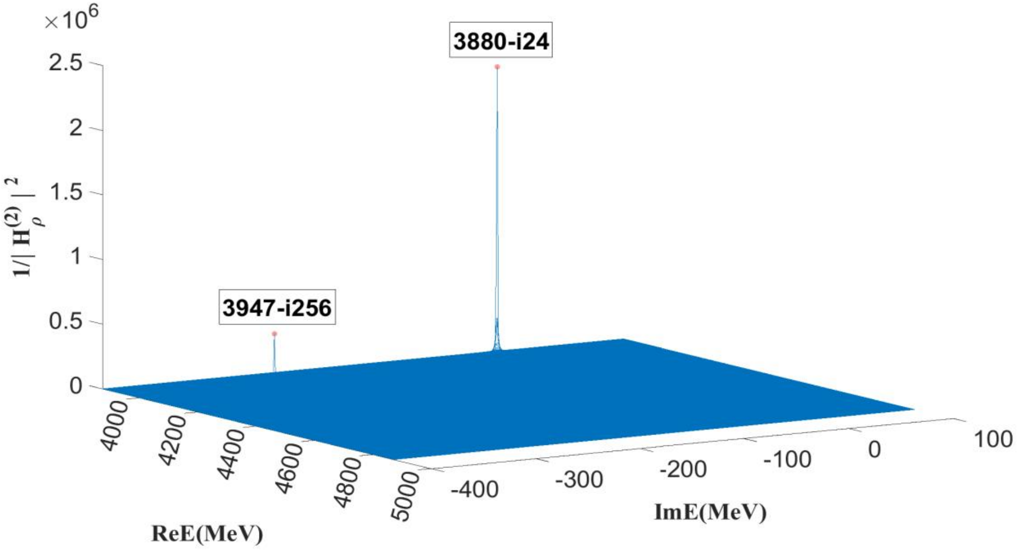
4. DD* interaction

The resonance state at 4029-i108MeV might correspond to the X(3940) particle, which is predicted as a partnenr state of X(3872) with $J^{PC}=1^{++}$.

Y. S. Kalashnikova, PRD, 72, 034010, 2005
P. G. Ortega, PRD, 81, 054023, 2010
Zhou and Xiao, PRD, 96, 054031, 2017
Q. Deng et al., 2312.10296[hep-ph].
H. Li et al., 2402.14541[heplat].
F. Giacosa et al., IJMPA, 34, 1950173, 2019.
G. J. Wang et al., 2306.12406[hep-ph].

4. DD* interaction

If the X(3872) particle is a $D^{+}D^{*-}/D^{-}D^{*+}$ bound state, the binding energy is about 8.11MeV, then the coupling constant $g=0.6520$. Two resonance states are detected.



5. Summary

B. X. Sun et al., Commun. Theor. Phys., 76, 105301, 2023

1. The interactions of hadrons are studied in the non-Hermitian quantum mechanics. By solving the Schrodinger equation， some resonance states are obtained when the outgoing wave condition is considered.
2. OPEP is critical in the formation of $K\bar{K}^*$ and $D\bar{D}^*$ bound and resonancd states.