The propagator for S-wave threshold states and Study on $X(3872)$

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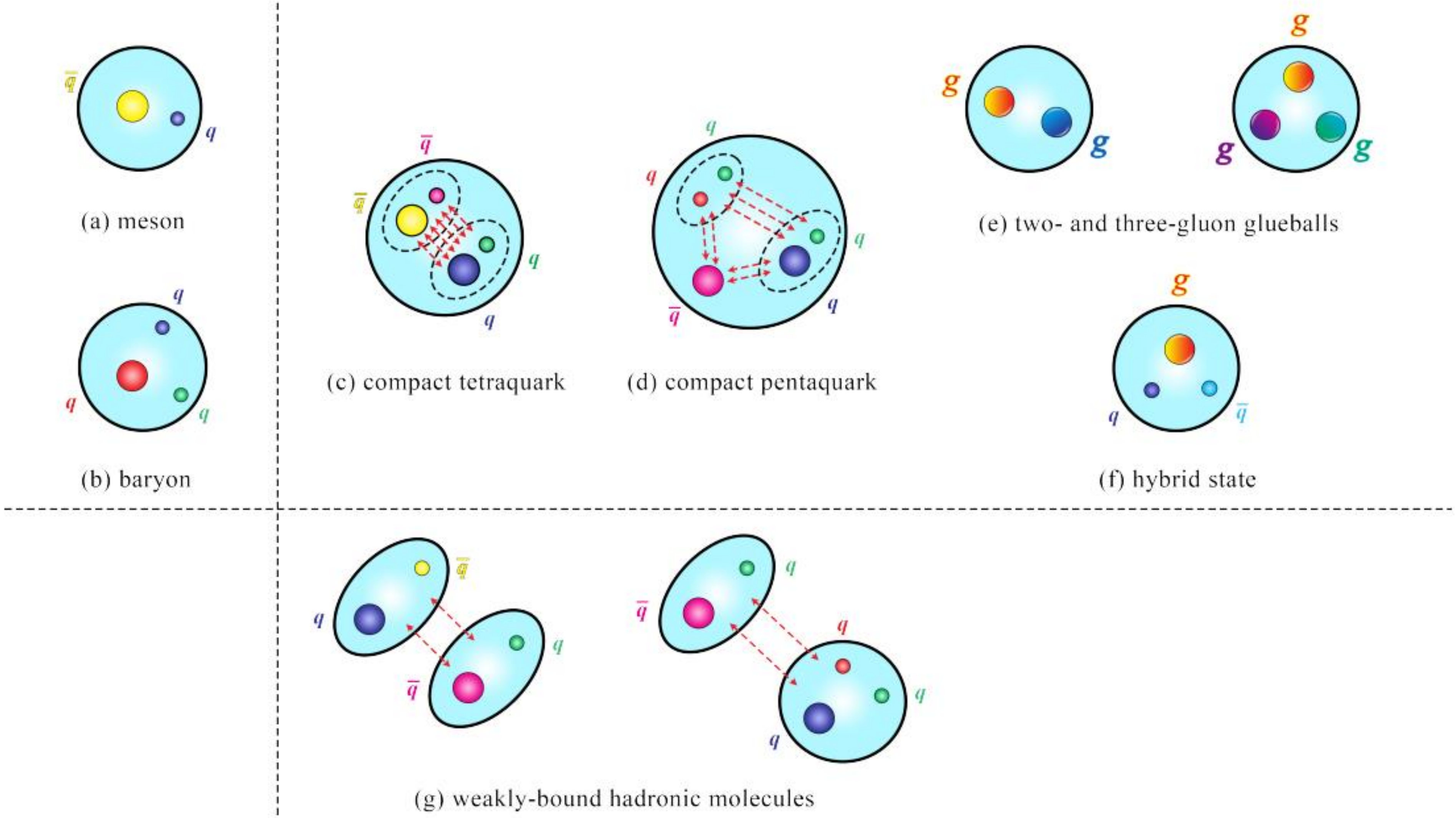
Outline

- Introduction
- The propagator in EFT incorporating Weinberg's compositeness theorem
- Study on $X(3872)$ using the propagator
- Summary


(a) meson
(b) baryon
(c) compact tetraquark
(d) compact pentaquark
(e) two- and three-gluon glueballs
(f) hybrid state
(g) weakly-bound hadronic molecules

Weinberg's compositeness theorem

$$a = [2(1-Z)/(2-Z)]/\sqrt{2\mu B} + \mathcal{O}(m_\pi^{-1})$$

$$r = -[Z/(1-Z)]/\sqrt{2\mu B} + \mathcal{O}(m_\pi^{-1})$$

The a is scattering length, r is effective range. The Z is the wave function renormalization constant Z, presenting the probability of finding a compact component in the state, the hadron structure information encoded in Z .

Weinberg S. Phys. Rev., 1963, 130: 776
Weinberg S. Phys. Rev. B, 1965, 137: 672

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Relations: $g^2 = \frac{2\pi\sqrt{2\mu B}}{\mu^2}(1-Z),\ g_0^2 = g^2/Z\ ,\ B_0 = \frac{2-Z}{Z}B$

The propagator for the S-wave near-threshold state is written as

$$\mathrm{G_X}(E) = \frac{iZ}{D_{EFT}(E)}, \qquad D_{EFT}(E) = E + B + \tilde{\Sigma}'(E) + i\,\Gamma/2\,,$$

$$\tilde{\Sigma}'(E) = -g^2[\frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} + \frac{\mu\sqrt{2\mu B}}{4\pi B}(E-B)].$$

For a two-body channel, denoted as DD, with a threshold M_{th} and a near-threshold state X with mass M and width Γ.

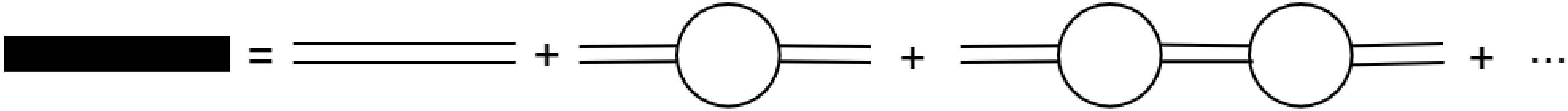


Figure. Full propagator for the near-threshold state.
The double line denotes the bare state.

The full propagator can be rewritten as

$$i\Delta = \frac{iZ}{2E + (2-Z)B - g^2 \frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} + iZ\,\Gamma_0/2}$$

$$= \frac{iZ}{E + B - g^2 \frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} - (1-Z)(E-B) + iZ\,\Gamma_0/2}$$

$$= \frac{iZ}{E + B - g^2 \frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} - g^2 \frac{\mu\sqrt{2\mu B}}{4\pi B}(E-B)] + iZ\,\Gamma_0/2}$$

We can find $\boldsymbol{\Gamma} = \boldsymbol{Z\Gamma_0}$

For $X(3872)$, we may also consider the charged DD channel. The full propagator, which include the charged DD channel, can be written as

$$G_{X(3872)} = \frac{iZ}{E + B + \tilde{\Sigma}'(E) + i\,\Gamma/2},$$

$$\tilde{\Sigma}'(E) = -g^2\left[\frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} + \frac{\mu\sqrt{2\mu B}}{4\pi B}(E - B)\right] -$$

$$g_c^2\left[\frac{\mu_c}{2\pi}\sqrt{-2\mu_c(E-\delta) - i\epsilon} + \frac{\mu_c\sqrt{2\mu_c(B+\delta)}}{4\pi(B+\delta)}(E - B - 2\delta)\right].$$

- Breit-Wigner amplitude:

$$f(E)=\frac{1}{D_{BW}(E)},D_{BW}(E)=E+B+i\,\Gamma/2$$

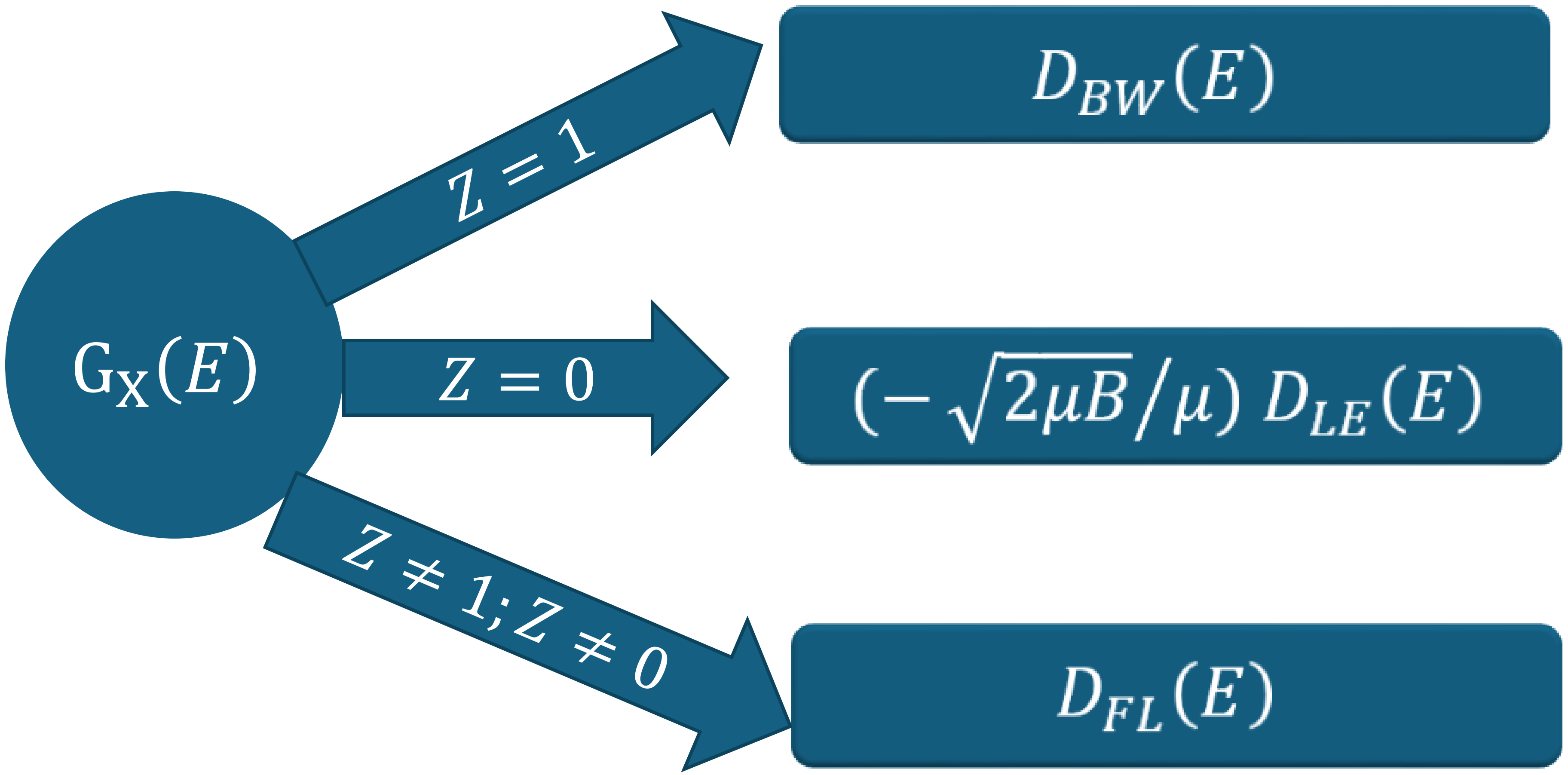
- Flatté amplitude:

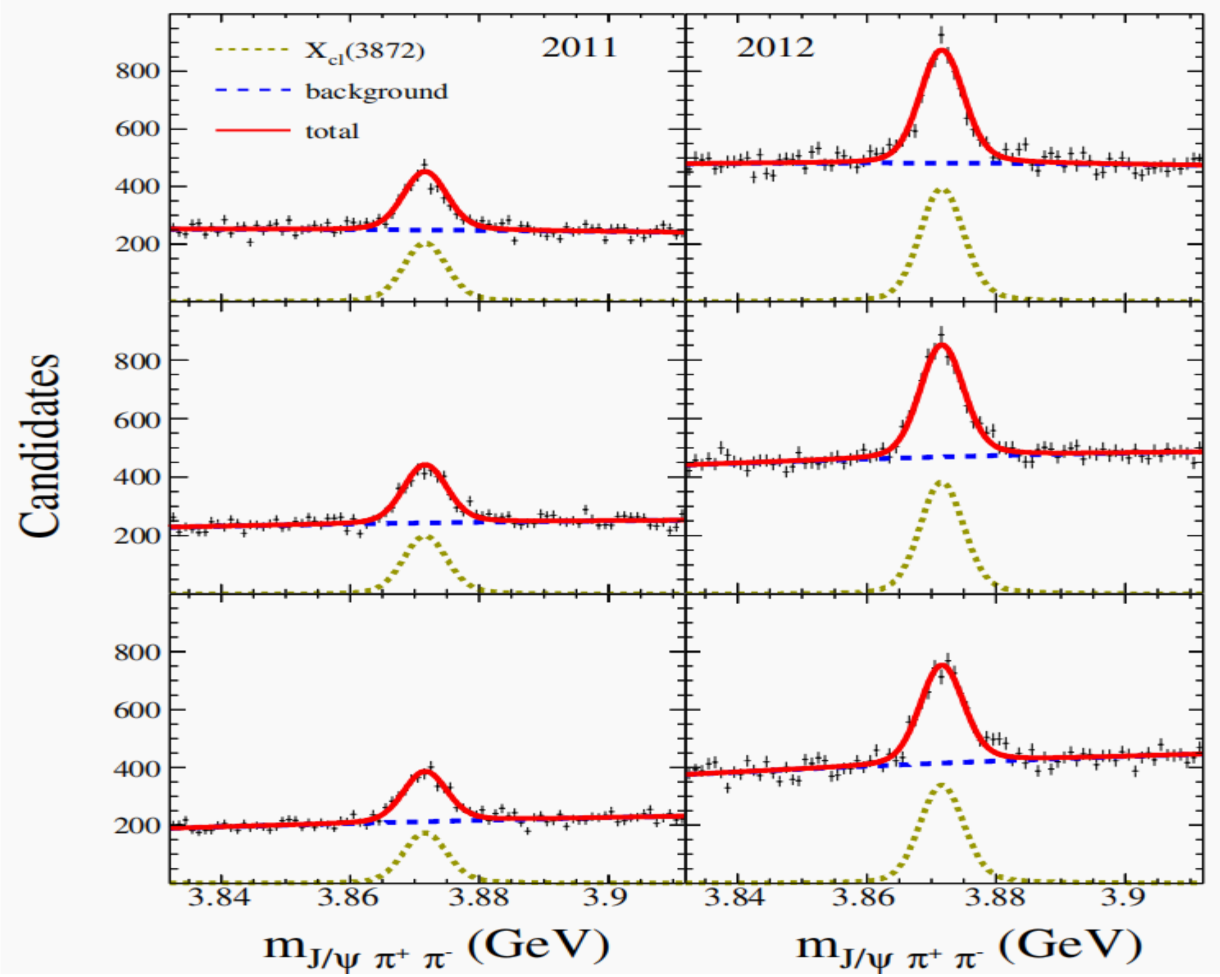
$$f(E)=\frac{1}{D_{FL}(E)},D_{FL}(E)=E-E_f-\frac{1}{2}g_1\sqrt{-2\mu E}+i\frac{1}{2}\Gamma_f$$

- Low-energy scattering amplitude:

$$f(E)=\frac{1}{D_{LE}(E)},D_{LE}(E)=-1/a\ +\sqrt{-2\mu E-i\epsilon}$$

S. M. Flatte, Phys. Lett. B 63 (1976) 224–227.
E. Braaten and M. Lu, Phys. Rev. D 76 (2007) 094028.


$G_X(E)$
$Z = 1$
$Z = 0$
$Z \neq 1; Z \neq 0$
$D_{BW}(E)$
$(-\sqrt{2\mu B}/\mu)\, D_{LE}(E)$
$D_{FL}(E)$

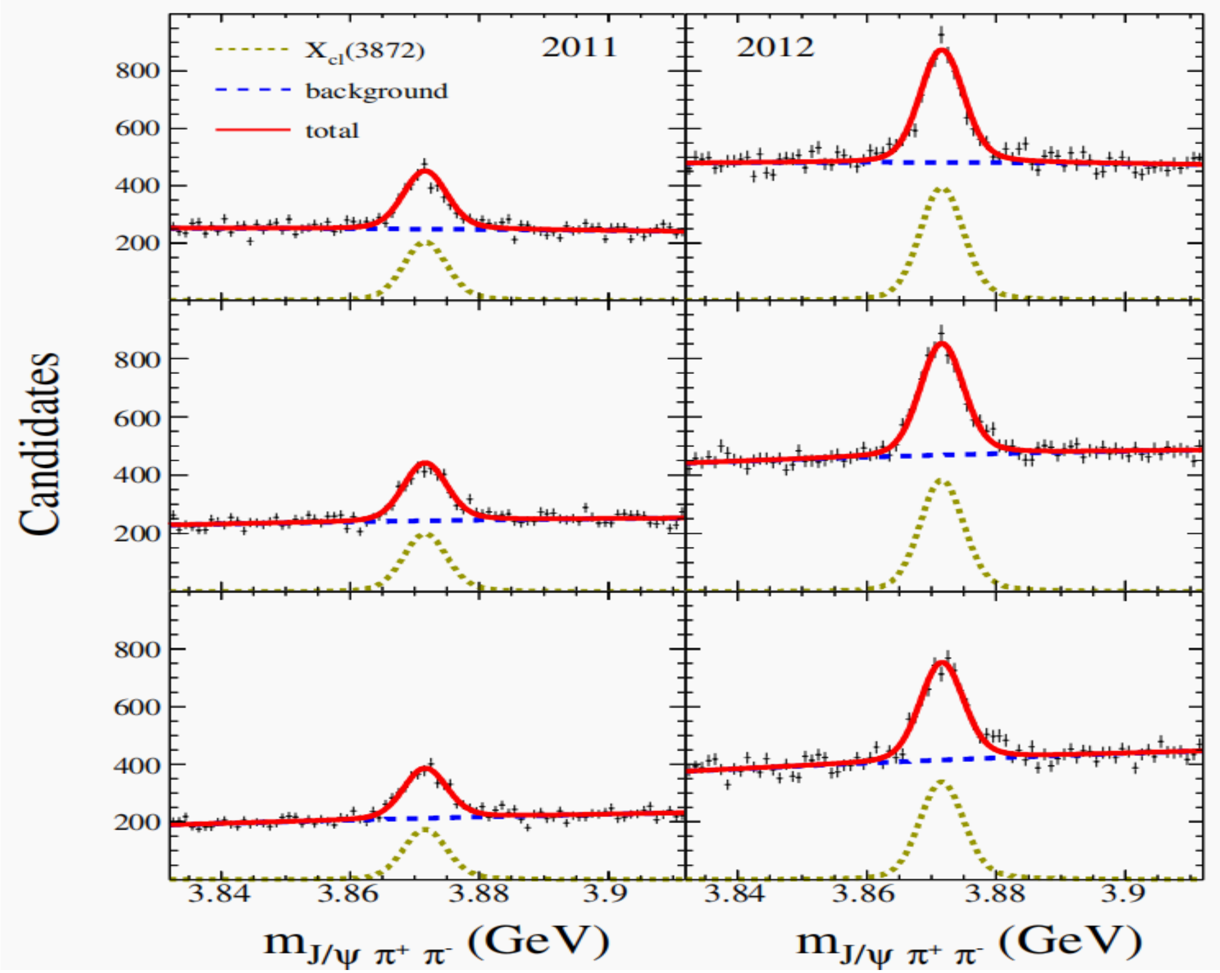


$P_{\pi^+\pi^-} \leq 12GeV$

$20GeV \leq P_{\pi^+\pi^-} \leq 12GeV$

$12GeV \leq P_{\pi^+\pi^-} \leq 50GeV$

The mass distributions of $X(3872)$ with the parameters determined by fitting data of LHCb, considering only neutral channel in the propagator. The red solid line shows the total fit result, blue dashed line shows the contribution of background.



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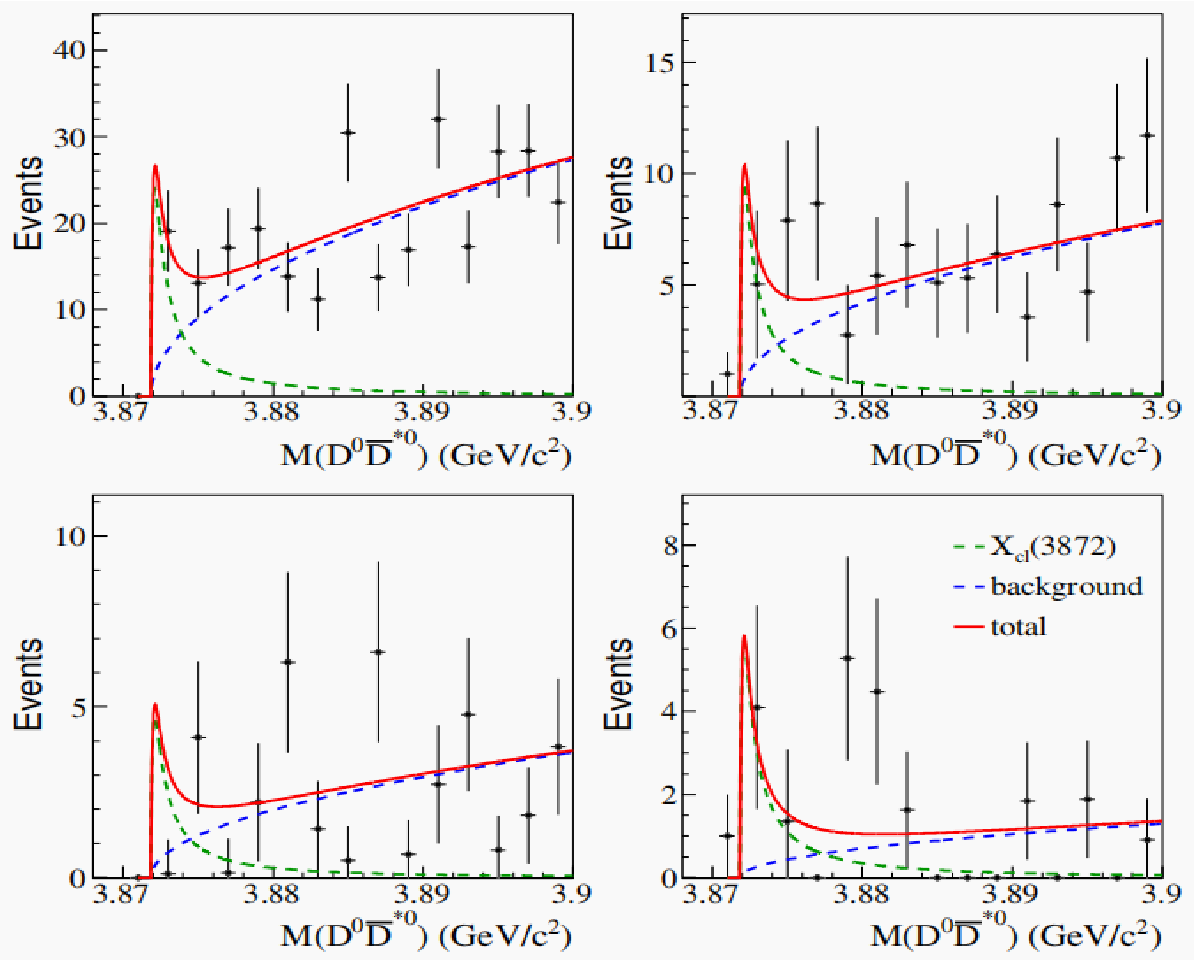
The mass distributions of $X(3872)$ with the parameters determined by fitting LHCb experiment data, considering charge channel in the propagator.

The parameters from fitting lineshape of $X(3872)$ based on LHCb data

Fitting scheme	only neutral channel	With charged channel
Z	0.42 ± 0.16	0.49±0.27
Γ(MeV)	0.57 ± 0.23	0.78±0.40
B(MeV)	0.19 ± 0.05	0.18±0.06
χ^2/ndf	$76.7/77$	$79.3/77$

H. Xu, N. Yu, and Z. Zhang, arxiv:2401.00411.
R. Aaij et al. (LHCb), Physical Review D 102, 092005 (2020).
H. Hirata et al. (Belle)， Phys. Rev. D 107, 112011
A. Esposito, L. Maiani, A. Pilloni, et al., Phys. Rev. D 105, L031503 (2022).
M. Ablikim, et al. (BESIII Collaboration), Phys.Rev.Lett. 132 (2024) 15, 151903



The $M(D^0\overline{D}^{0*})$distributions based on Belle data, with the parameters determined by LHCb's fitting. $\overline{D}^{0*} \to \overline{D}^0\gamma$(left) and $\overline{D}^{0*} \to \overline{D}^0\pi^0\gamma$ (right).

Summary

- The propagator for near-threshold states in EFT incorporating Weinberg's compositeness theorem is general.
- The fitting result of Z for $X(3872)$ is non-vanishing based on LHCb data.
- We are analyzing other exotic states using the propagator.

Thanks for your attention!