大统一理论的唯象学和宇宙学研讨会

Workshop on Grand Unified Theories: Phenomenology and Cosmology (GUTPC)

Axion quality from the interplay of vertical and horizontal gauge symmetries

Giacomo Landini

19/04/25

Hangzhou, China

Based on arXiv [2503.16648] L. Di Luzio, **GL**, F. Mescia, V. Susic

CSIC

Universitat de València | IFIC Institut de Física Corpuscular

Generalitat Valenciana

The Strong CP problem

The QCD Lagrangian violates CP symmetry

$$\mathcal{L}_{\rm QCD} \supset \bar{\theta}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \longrightarrow$$

Gauge-invariant + renormalizable

Non-perturbative QCD (instantons)

$\longrightarrow$ Natural expectation

$$\bar{\theta} \sim \mathcal{O}(1)$$

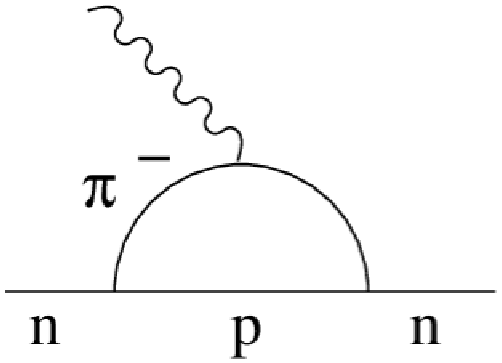
The Strong CP problem

The QCD Lagrangian violates CP symmetry

$$\mathcal{L}_{\mathrm{QCD}} \supset \bar{\theta}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$ → Gauge-invariant + renormalizable

Non-perturbative QCD (instantons) → Natural expectation $\bar{\theta} \sim \mathcal{O}(1)$

Prediction: electric dipole moment for the neutron



$$d_n \approx 10^{-16}|\bar{\theta}|e\ \mathrm{cm}$$

The Strong CP problem

The QCD Lagrangian violates CP symmetry

$$\mathcal{L}_{\rm QCD} \supset \bar{\theta}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \longrightarrow$$

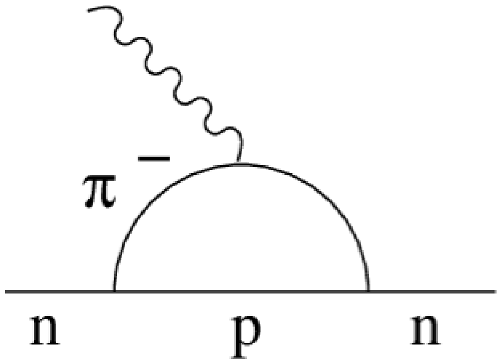
Gauge-invariant + renormalizable

Non-perturbative QCD (instantons) $\longrightarrow$

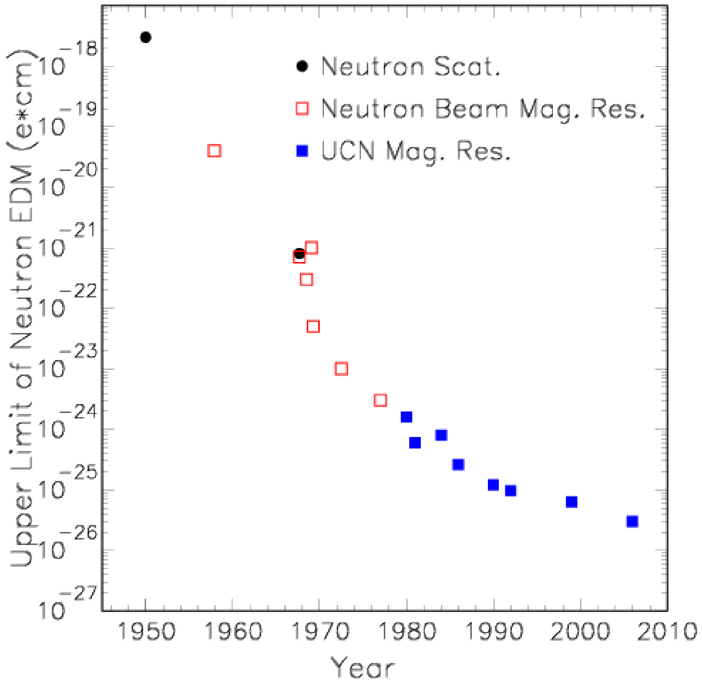
Natural expectation

$$\bar{\theta} \sim \mathcal{O}(1)$$

Prediction: electric dipole moment for the neutron



$$d_n \approx 10^{-16}|\bar{\theta}|e\ \mathrm{cm}$$



Upper bound

$$\bar{\theta} \leq 10^{-10}$$

Why so small??

Peccei-Quinn mechanism

[Peccei, Quinn 1977]
[Weinberg 1978]
[Wilczek 1978]

A new *global chiral* U(1) symmetry + scalar field Φ

Peccei-Quinn mechanism

[Peccei, Quinn 1977]
[Weinberg 1978]
[Wilczek 1978]

A new *global chiral* U(1) symmetry + scalar field Φ

1) Spontaneously broken → a new Goldstone boson: the ***axion***

$$\Phi(x) \simeq f_a e^{ia(x)/f_a}$$

$$a(x) \overset{PQ}{\to} a(x) + \gamma f_a$$

Peccei-Quinn mechanism

[Peccei, Quinn 1977]
[Weinberg 1978]
[Wilczek 1978]

A new *global chiral* U(1) symmetry + scalar field Φ

1) Spontaneously broken → a new Goldstone boson: the ***axion***

$$\Phi(x) \simeq f_a e^{ia(x)/f_a}$$

$$a(x) \overset{PQ}{\to} a(x) + \gamma f_a$$

2) QCD anomaly →

Colored fermions charged under U(1)

Explicit PQ breaking:
generates a potential for the axion
(shift-symmetry is broken!)

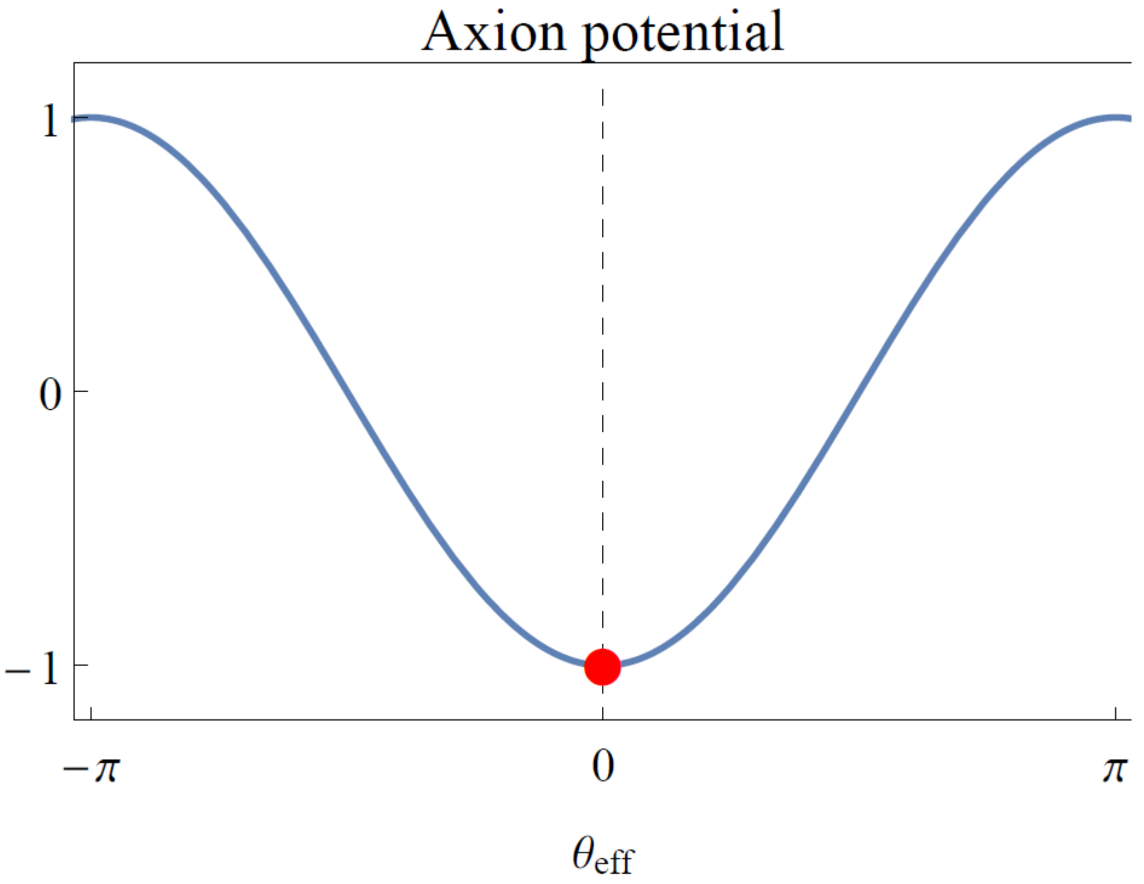
$$\mathcal{L}_{\mathrm{QCD}}^{\mathrm{PQ}} \supset \theta_{\mathrm{eff}}(x) \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$

$$\theta_{\mathrm{eff}}(x) \equiv \bar{\theta} + \frac{a(x)}{f_a}$$

Peccei-Quinn mechanism

The QCD potential relaxes to the CP-conserving minimum

$$V \approx -\underbrace{m_\pi^2 f_\pi^2}_{\Lambda_{\rm QCD}^4} \cos(\theta_{\rm eff})$$



$$m_a \sim \Lambda_{\rm QCD}^2 / f_a$$

[Weinberg 1978]

[Wilczek 1978]

$$\langle \bar\theta_{\rm eff} \rangle = 0$$

Vafa-Witten theorem [Vafa, Witten 1984]

$$E(\theta) \geq E(0)$$

Global symmetries

Global symmetries are not fundamental

Global symmetries

Global symmetries are not fundamental

Quantum Gravity conjectures

[Susskind 1995] — Black Hole physics

[Banks, Seiberg 2011] — No global symmetries in string theory

[Harlow, Ooguri 2018] — No global symmetries in ADS/CFT

Quantum Gravity must break all global symmetries

$$\Lambda_{\rm UV} = M_{\rm Pl} \simeq 1.2 \times 10^{19}\ {\rm GeV} \gg \Lambda_{\rm QCD}$$

Global symmetries

Global symmetries are not fundamental

Gauge Theory EFT perspective

Write down the most general gauge-invariant Lagrangian

$\Downarrow$

The renormalizable Lagrangian is invariant under **<u>accidental</u>** global symmetries…

Ex: baryon/lepton number in the SM

Global symmetries

Global symmetries are not fundamental

Gauge Theory EFT perspective

Write down the most general gauge-invariant Lagrangian

$\Downarrow$

The renormalizable Lagrangian is invariant under **accidental** global symmetries…

Ex: baryon/lepton number in the SM

…broken by higher-dimensional operators (parametrization of UV physics)

Ex: Quantum Gravity

$$\Delta\mathcal{L}_{\mathrm{UV}} \sim \frac{1}{\Lambda_{\mathrm{UV}}^{d-4}}\mathcal{O}^{[d]}$$

The quality problem

Good axion model: the PQ symmetry arises accidentally at low-energy *(How?)*

New gauge symmetries!

The quality problem

Good axion model: the PQ symmetry arises accidentally at low-energy *(How?)*

New gauge symmetries!

UV physics (gravity?) violates PQ symmetry

The quality problem

Good axion model: the PQ symmetry arises accidentally at low-energy *(How?)*

New gauge symmetries!

UV physics (gravity?) violates PQ symmetry

This generates Peccei-Quinn breaking operators at low-energy

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{1}{\Lambda_{\rm UV}^{d-4}}\mathcal{O}^{[d]}$$

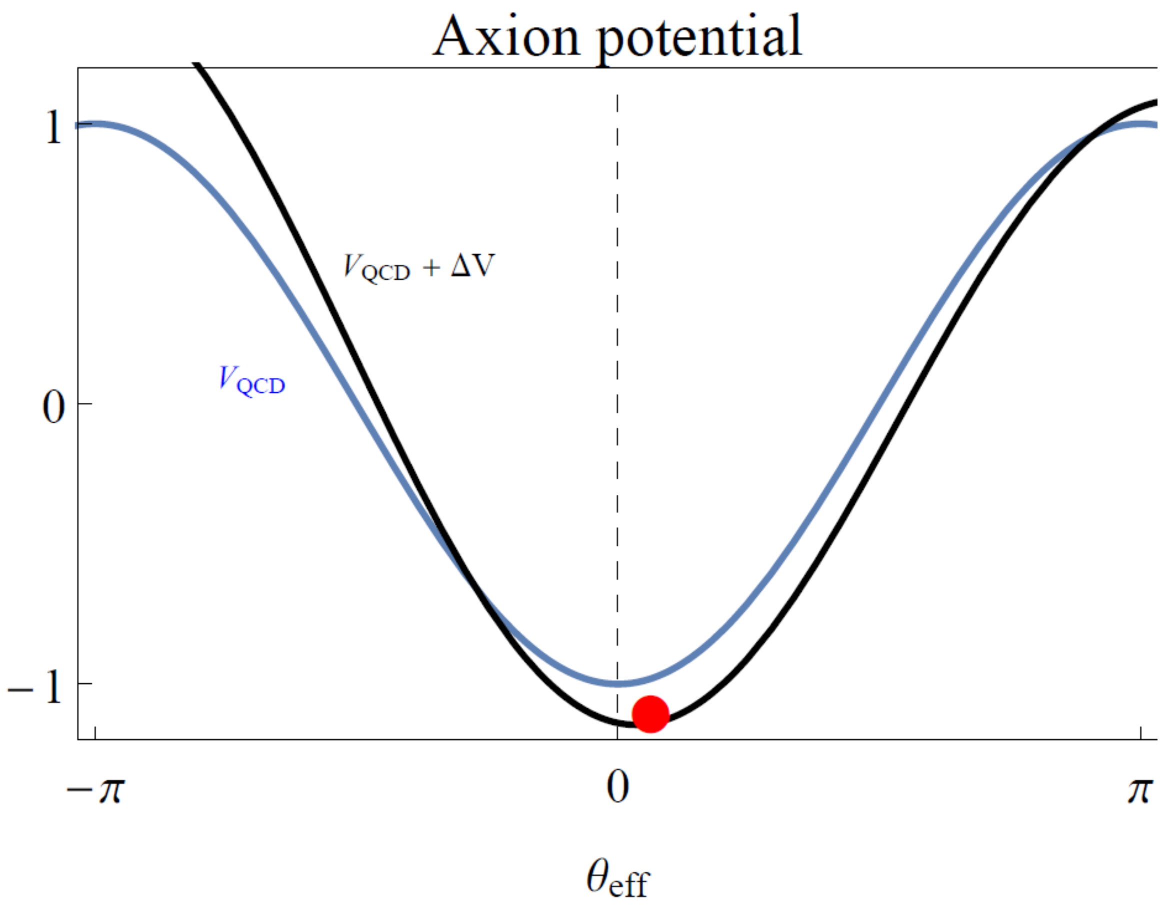
Explicit PQ breaking

New contribution to the axion potential (shift-symmetry is broken)

$$\Delta V_{\rm UV}(\theta_{\rm eff}) \sim \Lambda_{\rm UV}^4 \left(\frac{f_a}{\Lambda_{\rm UV}}\right)^d$$

The quality problem

The minimum of the potential is shifted



$\langle \theta_{\text{eff}} \rangle \neq 0$ → CP – violating minimum!

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\text{eff}} \rangle < 10^{-10}$$ (neutron EDM)

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10}$$ (neutron EDM)

Minimizing the full potental this translates to

$$\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \chi_{\rm QCD}^4$$

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potental this translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

UV PQ-breaking physics

The quality problem

We solve the Strong CP problem only if

$$\langle\theta_{\rm eff}\rangle < 10^{-10}$$

(neutron EDM)

Minimizing the full potental this translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4} \lesssim 10^{-10} \boxed{\chi_{\rm QCD}^4}$$

QCD anomalous breaking

$$\chi_{\rm QCD} = \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \simeq (76\ {\rm MeV})^4$$

UV PQ-breaking physics

The quality problem

We solve the Strong CP problem only if

$$\langle\theta_{\rm eff}\rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potental this translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4} \lesssim \boxed{10^{-10}}\boxed{\chi_{\rm QCD}^4}$$

UV PQ-breaking physics

Experimental bound

QCD anomalous breaking

$$\chi_{\rm QCD} = \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \simeq (76\ \text{MeV})^4$$

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\text{eff}} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potental this translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4}} f_a^4 \lesssim 10^{-10} \chi_{\text{QCD}}^4$$

$$f_a \ll \Lambda_{\text{UV}}$$

The lowest-dimensional PQ-breaking operators are the most dangerous!

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\text{eff}} \rangle < 10^{-10}$$

(neutron EDM)

Minimizing the full potental this translates to

$$\left(\frac{f_a}{\Lambda_{\text{UV}}} \right)^{d-4} f_a^4 \lesssim 10^{-10} \chi_{\text{QCD}}^4$$

For physically well-motivated scales

Dark Matter $f_a \sim 10^{11}\ \text{GeV}$

Gravity $\Lambda_{\text{UV}} \sim M_{\text{Pl}}$

PQ must be **preserved** up to operators of dimension **d ≥ 10 - 12**

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10}$$ (neutron EDM)

Minimizing the full potential this translates to

More generally $$\frac{f_a^n v^{d-n}}{\Lambda_{\rm UV}^{d-4}} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

For physically well-motivated scales

Dark Matter $f_a \sim 10^{11}\ \text{GeV}$

Gravity $\Lambda_{\rm UV} \sim M_{\rm Pl}$

PQ must be **preserved** up to operators of dimension **d ≥ 10 - 12**

We can reduce **d** if some $f_a \to v \ll f_a$

Solutions to the quality problem

New gauge symmetries

[Krauss and Wilczek ´89, Dias et al. ´03, Carpenter et al.´09, Harigaya et al. ´13, Barr and Seckel ´92, Di Luzio, Ubaldi and Nardi ´17, Ardu et al. ´20, Di Luzio ´20, Darmé, Nardi ´21, Darmé, Nardi, Smarra ´22,…]

Low-scale f_a

[Rubakov ‘97, Berezhiani, Gianfagna, Giannotti ‘01,
Gianfagna Giannotti Nesti ’04, Gaillard, Gavela et al. ‘18, …]

Gravitational suppression of coupling

[Lee ‘88, Giddings, Strominger ’88, Abbott, Wise ‘88,
Alvey, Escudero ’20,…]

Solutions to the quality problem

New gauge symmetries

[Krauss and Wilczek ´89, Dias et al. ´03, Carpenter et al.´09, Harigaya et al. ´13, Barr and Seckel ´92, Di Luzio, Ubaldi and Nardi ´17, Ardu et al. ´20, Di Luzio ´20, Darmé, Nardi ´21, Darmé, Nardi, Smarra ´22,…]

Gauge symmetries protect PQ breaking up to dimension N

Ex: Z_N , $SU(N) \otimes SU(N)$, $SU(N) \rightarrow SO(N)$, ...

Solutions to the quality problem

New gauge symmetries

[Krauss and Wilczek ´89, Dias et al. ´03, Carpenter et al.´09, Harigaya et al. ´13, Barr and Seckel ´92, Di Luzio, Ubaldi and Nardi ´17, Ardu et al. ´20, Di Luzio ´20, Darmé, Nardi ´21, Darmé, Nardi, Smarra ´22,…]

Gauge symmetries protect PQ breaking up to dimension N

Ex: Z_N, $SU(N) \otimes SU(N)$, $SU(N) \rightarrow SO(N)$, ...

However….

i) ad-hoc gauge symmetries, with no relation to the SM structure

ii) lack of testability of the UV mechanism addressing the quality problem

Interplay of vertical and horizontal symmetries

Idea

GUT extension of the SM = *vertical* structure

+

Gauged **flavor** symmetries = *horizontal* structure

interplay →

- No ad-hoc symmetries [addressing *i)*]
- PQ protection (quality problem)
- Axion – flavor connection (backup)
- Cosmological signatures [addressing *ii)*]

Interplay of vertical and horizontal symmetries

Examples: $SO(10) \otimes SU(3)_f$

$SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R}$ (Pati-Salam)

[Di Luzio 2020]

Field	Lorentz	SO(10)	$\mathbb{Z}_4$	$\mathrm{SU}(3)_f$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
ψ_{16}	$(1/2,0)$	16	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
ψ_1	$(1/2,0)$	1	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	16	0
ϕ_{10}	$(0,0)$	10	-1	$\overline{\mathbf{6}}$	$e^{i2\pi/3}$	1	-2
ϕ_{16}	$(0,0)$	16	i	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
$\phi_{\overline{126}}$	$(0,0)$	$\overline{126}$	-1	$\overline{\mathbf{6}}$	$e^{i2\pi/3}$	1	-2
ϕ_{45}	$(0,0)$	45	1	$\mathbf{1}$	1	1	0

[2503.16648]

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Accidental PQ symmetry (renormalizable)

Quality of the PQ symmetry (non-renormalizable)

Pati-Salam – flavor model

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Gauge group: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R}$

only "*right*" flavor is gauged

Avoid high-scale EW-breaking
or EW-scale flavor gauge bosons

Pati-Salam – flavor model

SM + RHNs

Flavor anomaly cancellation (see later)

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Pati-Salam – flavor model

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SM Higgs

SM flavor structure

Break PQ and B-L
at high scale

Avoid larger global symmetries (connecting Δ and χ)

Pati-Salam – flavor model

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Accidental global PQ symmetry !

$$-\mathcal{L}_Y = \sum_{\alpha=1}^{N_\Phi}\sum_{I=1}^{3} Y_I^{\Phi^\alpha}\,(\overline{Q}_L)^I Q_R\,\Phi^\alpha + \sum_{\alpha=1}^{N_\Sigma}\sum_{I=1}^{3} 2\sqrt{3}\,Y_I^{\Sigma^\alpha}\,(\overline{Q}_L)^I Q_R\,\Sigma^\alpha \quad + \frac{1}{2}\,Y_R\,Q_R Q_R\,\Delta^* + \mathrm{h.c.}$$

$$V = V(|\phi_i|^2) + \Phi\Sigma^*\xi + \Phi\Sigma^*\left(|\Sigma|^2 + |\Delta|^2 + |\chi|^2 + \xi^2\right) + \Sigma^{*2}\left(\Phi^2 + \Delta^2\right) + \Delta\chi^2\xi + \mathrm{h.c.}$$

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

Pati -Salam flavor Peccei-Quinn

Gauge **Accidental global**

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle \Delta, \chi, \xi \rangle} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $\underline{SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R} \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle \Delta,\chi,\xi \rangle} \underline{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$$

Pati-Salam breaking

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$\xrightarrow{\langle\Delta,\chi,\xi\rangle} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

flavor breaking

Possibly in two steps $SU(3)_{f_R} \xrightarrow{\langle\Delta,\chi\rangle_3} SU(2)_{f_R} \xrightarrow{\langle\Delta,\chi\rangle_{1,2}} \times$

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle \Delta, \chi, \xi \rangle} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

PQ breaking

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle \Delta,\chi,\xi \rangle} SU(3)_c \otimes \underline{SU(2)_L \otimes U(1)_Y} \xrightarrow{\langle \Phi,\Sigma \rangle} SU(3)_c \otimes \underline{U(1)_{\mathrm{EM}}}$$

EW breaking

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle\Delta,\chi,\xi\rangle} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\langle\Phi,\Sigma\rangle} SU(3)_c \otimes U(1)_{\mathrm{EM}}$$

Large vevs: $\langle\Delta,\chi,\xi\rangle \sim V \sim 10^{[9,14]}\ \mathrm{GeV}$

PS + flavor + PQ breaking

Small vevs: $\langle\Phi,\Sigma\rangle \sim v \sim 10^2\ \mathrm{GeV}$

EW-breaking

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

SSB chain: $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R} \otimes U(1)_{\mathrm{PQ}}$

$$\xrightarrow{\langle\Delta,\chi,\xi\rangle} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\langle\Phi,\Sigma\rangle} SU(3)_c \otimes U(1)_{\mathrm{EM}}$$

<u>Large vevs:</u> $\langle\Delta,\chi,\xi\rangle \sim V \sim 10^{[9,14]}\ \mathrm{GeV}$

PS + flavor + PQ breaking

<u>Small vevs:</u> $\langle\Phi,\Sigma\rangle \sim v \sim 10^2\ \mathrm{GeV}$

EW-breaking

Hierarchy

$$v \ll V \ll M_{\mathrm{Pl}}$$

Axion embedding

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Scalar field decomposition $\phi \supset V_\phi \exp\left(\frac{i a_\phi}{\sqrt{2}|V_\phi|}\right)$ $\Rightarrow$ $a \equiv a(a_\Delta, a_\chi)$ (physical) axion embedding

Axion embedding

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Scalar field decomposition $\quad \phi \supset V_\phi \exp\left(\frac{i a_\phi}{\sqrt{2}|V_\phi|}\right) \quad \Longrightarrow \quad a \equiv a(a_\Delta, a_\chi) \quad$ (physical) axion embedding

Peccei-Quinn scale $\quad f_a = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2 + 4V_\Delta^2}} \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{V}{3\sqrt{5}}$

Axion embedding

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Scalar field decomposition $\quad \phi \supset V_\phi \exp\left(\frac{i a_\phi}{\sqrt{2}|V_\phi|}\right) \quad \Longrightarrow \quad a \equiv a(a_\Delta, a_\chi) \quad$ (physical) axion embedding

Peccei-Quinn scale
$$f_a = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2 + 4V_\Delta^2}} \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{V}{3\sqrt{5}}$$

$\Longrightarrow$ related to PS and flavor breaking scales

Axion quality in PS – flavor model

Let's introduce the higher-dimensional operators

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{\mathcal{O}^{[d]}}{M_{\rm Pl}^{d-4}} \quad \Longrightarrow \quad \frac{\langle\mathcal{O}\rangle^{[d]}}{M_{\rm Pl}^{d-4}} \sim \frac{{\color{green}v}^n\,{\color{red}V}^{d-n}}{M_{\rm Pl}^{d-4}}$$

Axion quality in PS – flavor model

Let's introduce the higher-dimensional operators

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{\mathcal{O}^{[d]}}{M_{\rm Pl}^{d-4}} \longrightarrow \frac{\langle\mathcal{O}\rangle^{[d]}}{M_{\rm Pl}^{d-4}} \sim \frac{v^n V^{d-n}}{M_{\rm Pl}^{d-4}}$$

$\Big\downarrow$ which ones are dangerous for PQ quality?

Axion quality in PS – flavor model

Let's introduce the higher-dimensional operators

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{\mathcal{O}^{[d]}}{M_{\rm Pl}^{d-4}} \quad \longrightarrow \quad \frac{\langle\mathcal{O}\rangle^{[d]}}{M_{\rm Pl}^{d-4}} \sim \frac{v^n V^{d-n}}{M_{\rm Pl}^{d-4}}$$

$\Big\downarrow$ which ones are dangerous for PQ quality?

i) Gauge-invariant

ii) PQ- breaking

$\Big\}$ *Easy checks*

Axion quality in PS – flavor model

Let's introduce the higher-dimensional operators

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{\mathcal{O}^{[d]}}{M_{\rm Pl}^{d-4}} \longrightarrow \frac{\langle\mathcal{O}\rangle^{[d]}}{M_{\rm Pl}^{d-4}} \sim \frac{{\color{green}v}^n {\color{red}V}^{d-n}}{M_{\rm Pl}^{d-4}}$$

$\downarrow$ which ones are dangerous for PQ quality?

i) Gauge-invariant

ii) PQ- breaking

$\}$ *Easy checks*

iii) check non-vanishing explicit index contraction

iv) non-vanishing vacuum contribution $\langle\mathcal{O}\rangle \neq 0$

$\}$ *Not trivial*

Axion quality in PS – flavor model

An operator which satisfies *(i-to-iv)* provides a contribution to the axion potential

$$\frac{v^m {V_\Delta}^l {V_\chi}^{n-l}}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4 \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{v^m V^n}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4$$

Axion quality in PS – flavor model

An operator which satisfies *(i-to-iv)* provides a contribution to the axion potential

$$\frac{v^m {V_\Delta}^l {V_\chi}^{n-l}}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10} \chi_{\rm QCD}^4 \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{v^m V^n}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

$$(\Delta\Delta^*)\Delta^3\chi^{*6} \to V^{11}/M_{\rm Pl}^7$$

$$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2 V^6/M_{\rm Pl}^4$$

$$\Phi^{4-k}\Sigma^k\Delta\chi^{*2} \to v^4 V^3/M_{\rm Pl}^3$$

$$\Phi^{4-k}\Sigma^k\Sigma^2 \to v^6/M_{\rm Pl}^2$$

Set of rules based on group centers (see backup)

Axion quality in PS – flavor model

An operator which satisfies *(i-to-iv)* provides a contribution to the axion potential

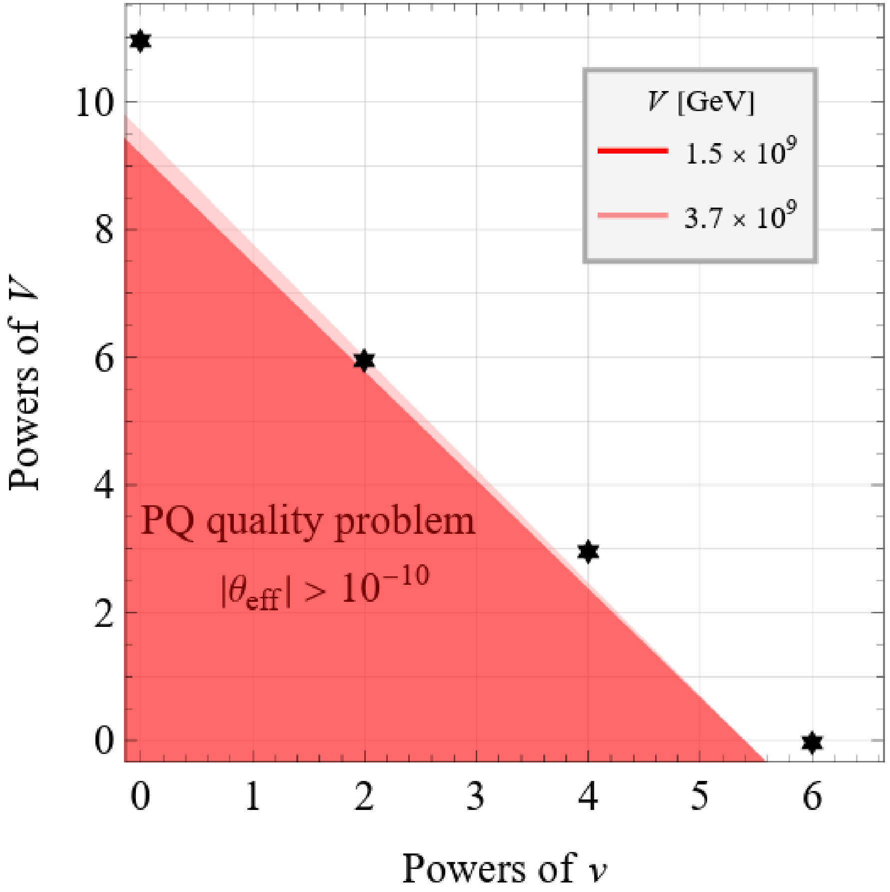
$$\frac{v^m {V_\Delta}^l {V_\chi}^{n-l}}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4 \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{v^m V^n}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4$$

$$(\Delta\Delta^*)\Delta^3\chi^{*6} \to V^{11}/M_{\rm Pl}^7$$

$$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2V^6/M_{\rm Pl}^4$$

$$\Phi^{4-k}\Sigma^k\Delta\chi^{*2} \to v^4V^3/M_{\rm Pl}^3$$

$$\Phi^{4-k}\Sigma^k\Sigma^2 \to v^6/M_{\rm Pl}^2$$



Axion quality in PS – flavor model

An operator which satisfies *(i-to-iv)* provides a contribution to the axion potential

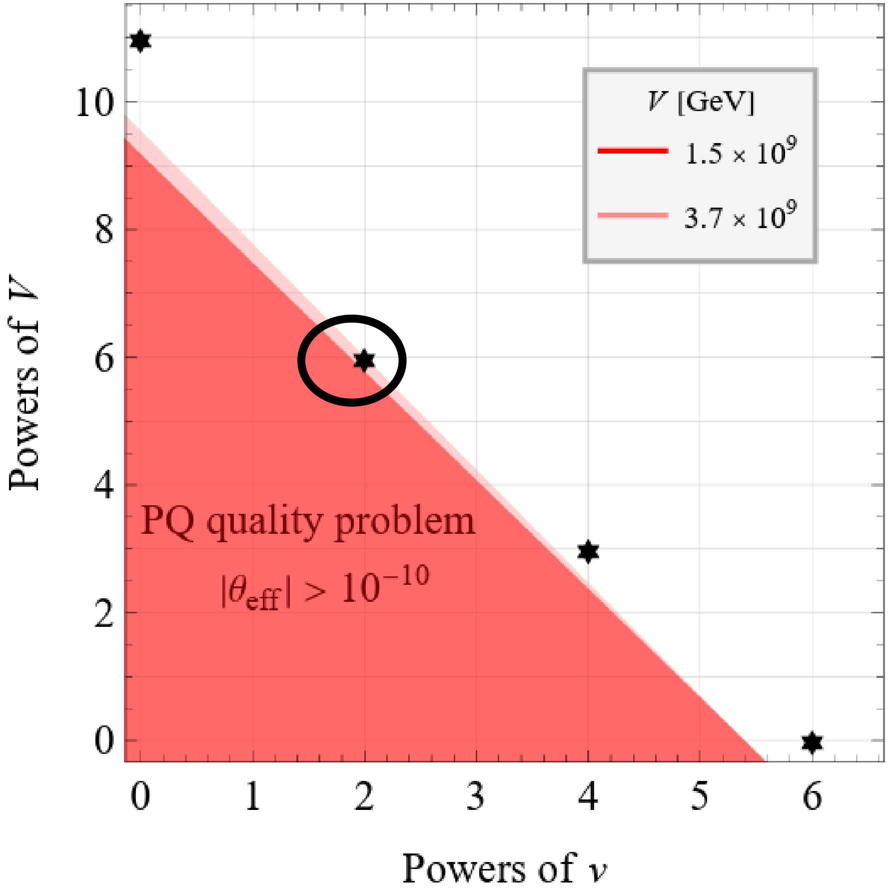
$$\frac{v^m {V_\Delta}^l {V_\chi}^{n-l}}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4 \xrightarrow{V_\chi \sim V_\Delta \sim V} \frac{v^m V^n}{M_{\rm Pl}^{n+m-4}} \lesssim 10^{-10}\chi_{\rm QCD}^4$$

$$(\Delta\Delta^*)\Delta^3\chi^{*6} \to V^{11}/M_{\rm Pl}^7$$

$$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2V^6/M_{\rm Pl}^4$$

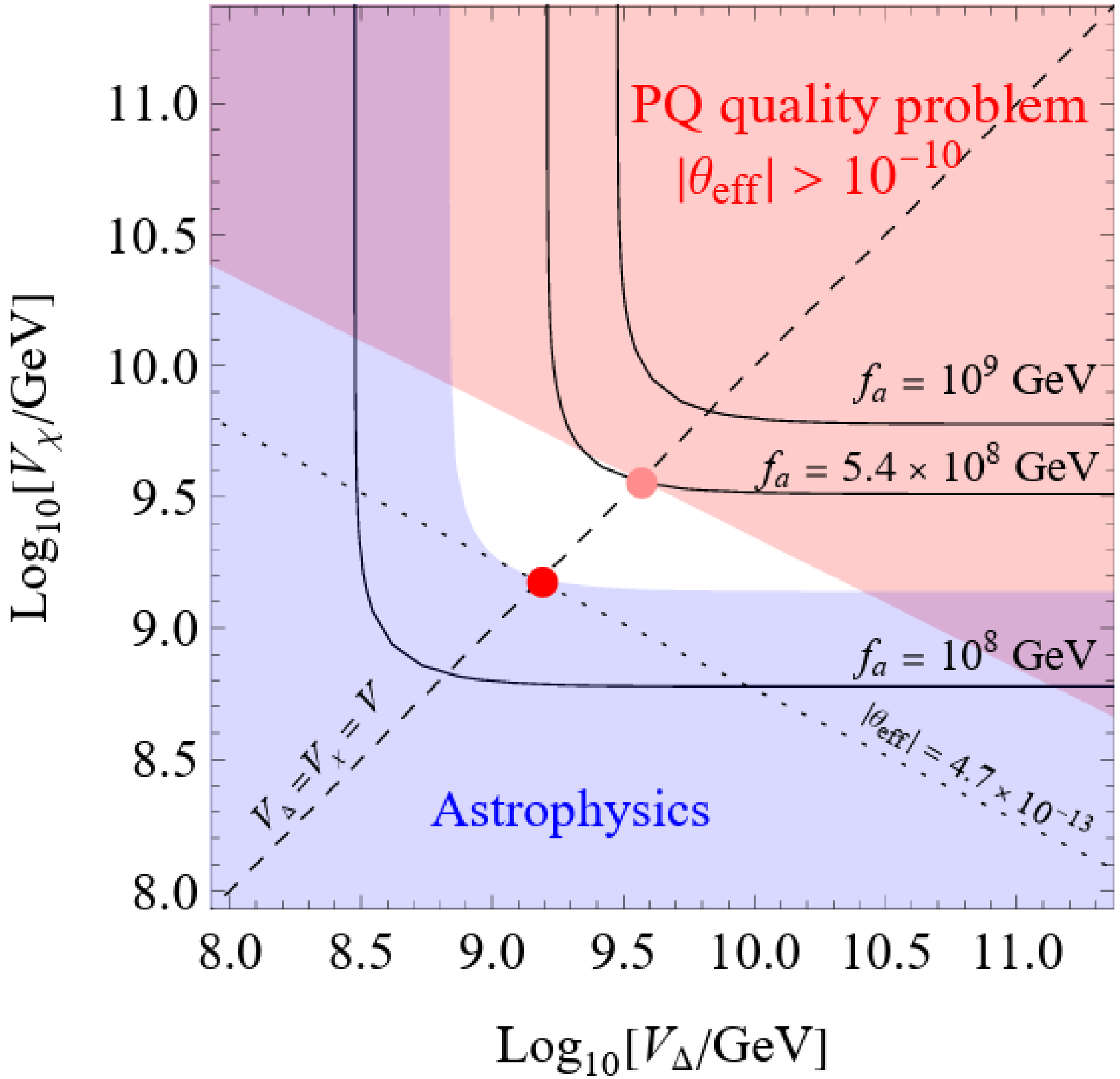
$$\Phi^{4-k}\Sigma^k\Delta\chi^{*2} \to v^4V^3/M_{\rm Pl}^3$$

$$\Phi^{4-k}\Sigma^k\Sigma^2 \to v^6/M_{\rm Pl}^2$$

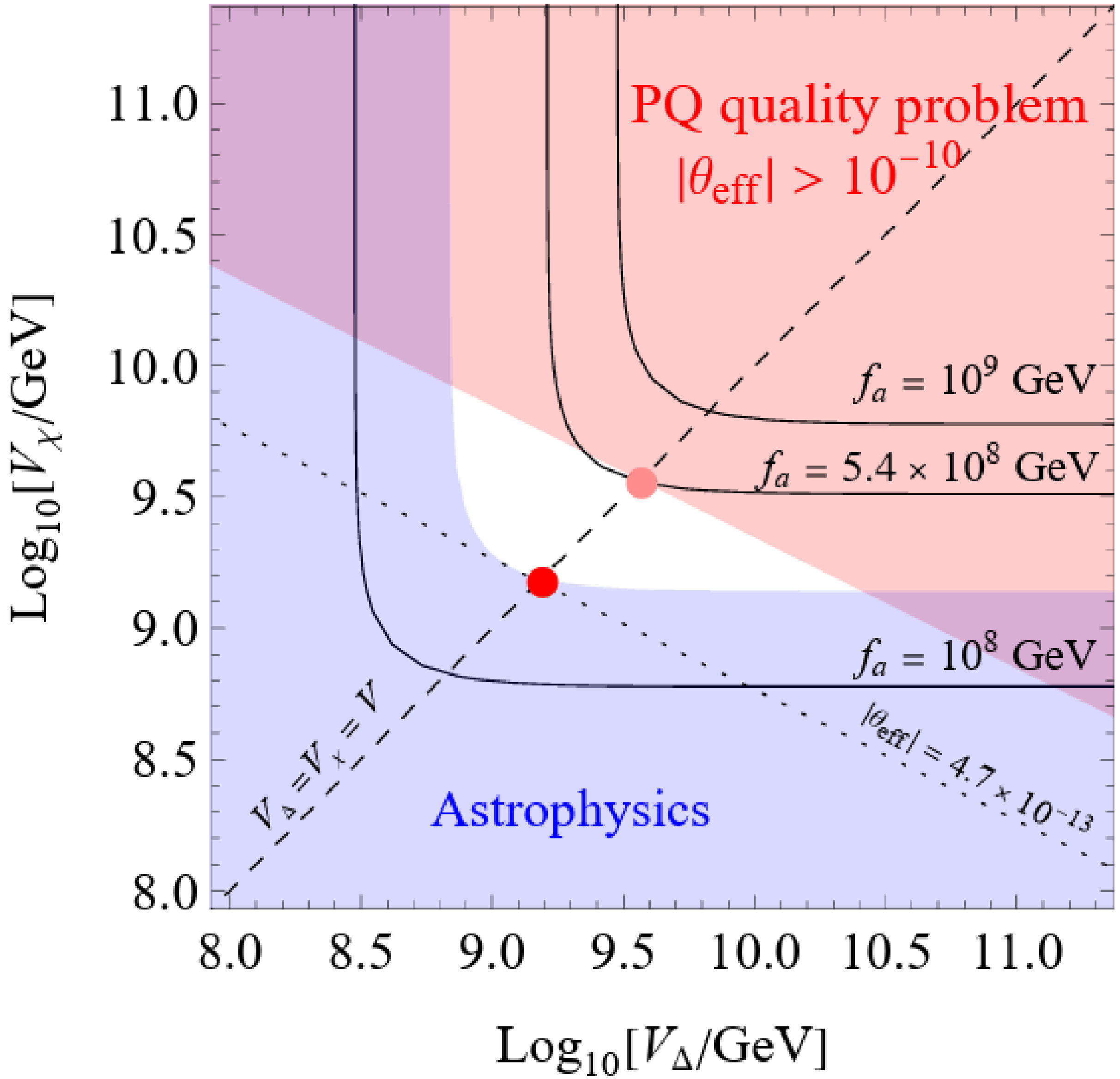


d = 8 operator is the most dangerous

Axion quality in PS – flavor model



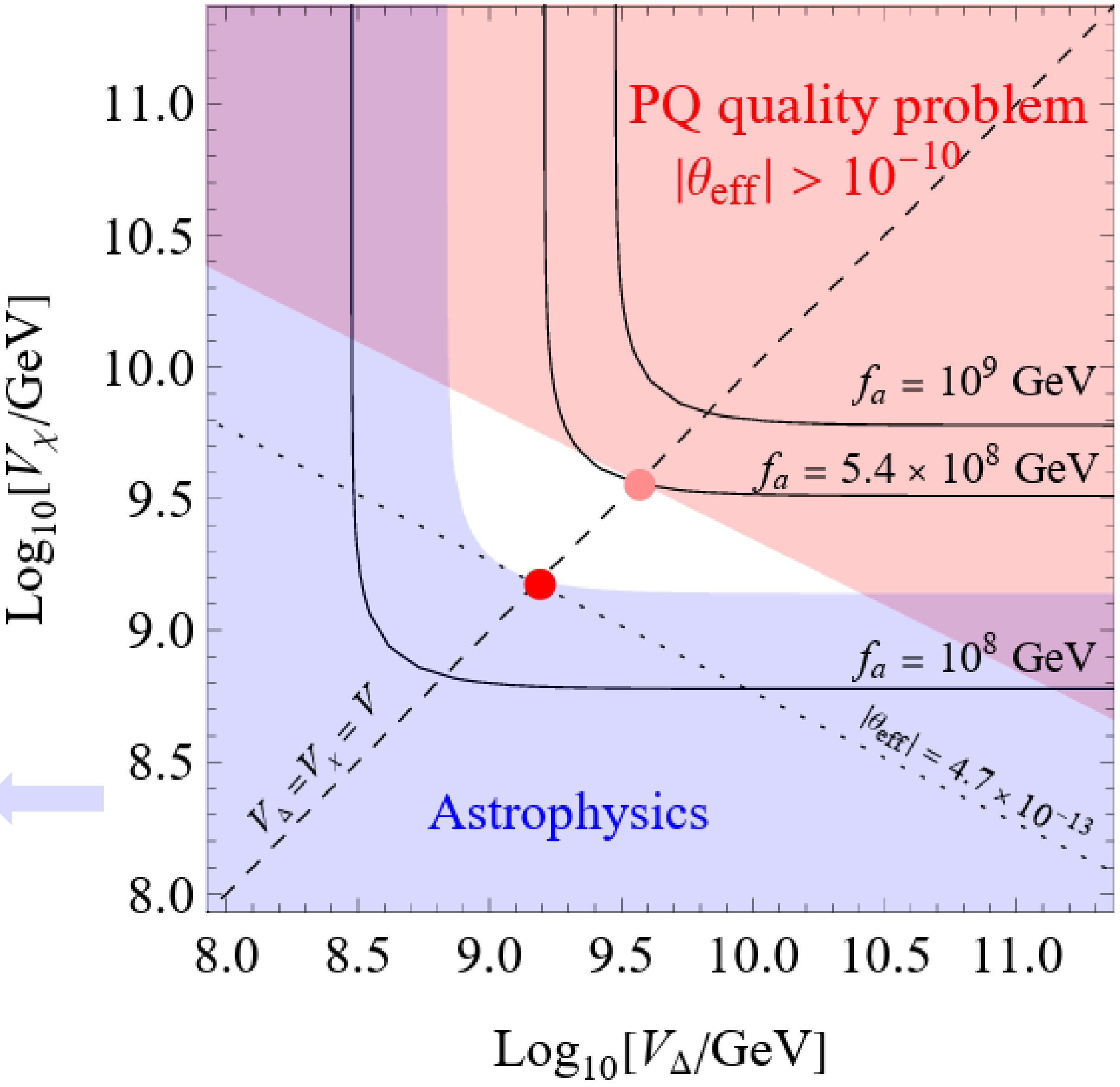
Axion quality in PS – flavor model



$$\frac{v^2 V_\Delta^2 V_\chi^4}{M_{\rm Pl}^4} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

$f_a \lesssim 5.6 \times 10^8$ GeV

Axion quality in PS – flavor model



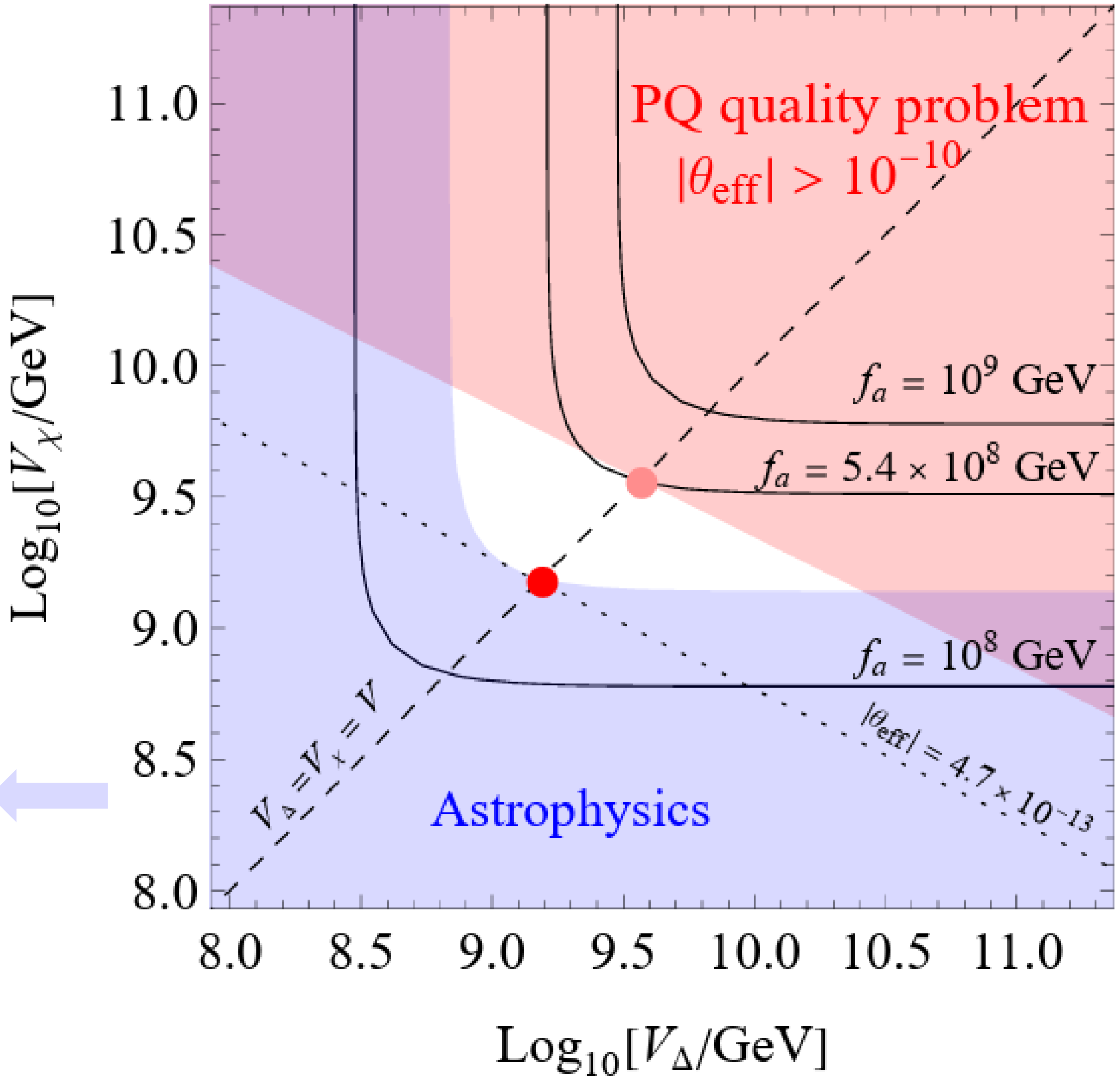
$$\frac{v^2 V_\Delta^2 V_\chi^4}{M_{\rm Pl}^4} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

$f_a \lesssim 5.6 \times 10^8$ GeV

axion-electron coupling
from
red giants+white dwarfs

$f_a \gtrsim 2.3 \times 10^8$ GeV

Axion quality in PS – flavor model



$$\frac{v^2 V_\Delta^2 V_\chi^4}{M_{\rm Pl}^4} \lesssim 10^{-10} \chi_{\rm QCD}^4$$

$f_a \lesssim 5.6 \times 10^8$ GeV

axion-electron coupling
from
red giants+white dwarfs

$f_a \gtrsim 2.3 \times 10^8$ GeV

Minimal value allowed
$|\theta_{\rm eff}^{\rm min}| \simeq 4.7 \times 10^{-13}$

Testable by future proton/neutron EDM experiments

Anomalons

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Flavor gauge anomaly $[SU(3)^3_{f_R}]$ cancellation requires extra fermions: *anomalons*

Anomalons

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Flavor gauge anomaly $[SU(3)^3_{f_R}]$ cancellation requires extra fermions: *anomalons*

GUT singlet – only charged under the flavor group

Anomalons

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Flavor gauge anomaly $[SU(3)^3_{f_R}]$ cancellation requires extra fermions: *anomalons*

GUT singlet – only charged under the flavor group

General prediction of these models

Field	Lorentz	SO(10)	$\mathbb{Z}_4$	$SU(3)_f$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
ψ_{16}	$(1/2, 0)$	16	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
ψ_1	$(1/2, 0)$	1	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	16	0

Anomalons

Anomalons are massless at the renormalizable level

$$\Psi_R\Psi_R \quad Q_R\Psi_R \quad \bar{Q}_L\Psi_R$$
$$\Psi_R\Psi_R\phi \quad Q_R\Psi_R\phi \quad \bar{Q}_L\Psi_R\phi$$

forbidden by gauge invariance

Higher-dimensional operators generate their mass

Anomalons

Anomalons are massless at the renormalizable level

$$\Psi_R\Psi_R \quad Q_R\Psi_R \quad \bar{Q}_L\Psi_R$$
$$\Psi_R\Psi_R\phi \quad Q_R\Psi_R\phi \quad \bar{Q}_L\Psi_R\phi$$

forbidden by gauge invariance

Higher-dimensional operators generate their mass

type	operator $\mathcal{O}$	d	$\langle M_{\mathcal{O}}\rangle$
$L\Psi$	$\overline{Q}_L\Psi_R\,\chi(\Phi+\Sigma+\Sigma')$	5	$vV/\Lambda_{\rm UV}$
$L\Psi$	$\overline{Q}_L\Psi_R\,\Delta\chi^*(\Phi^*+\Sigma^*+\Sigma'^*)$	6	$vV^2/\Lambda^2_{\rm UV}$
$R\Psi$	$Q_R\Psi_R\,\Delta^*\chi$	5	$V^2/\Lambda_{\rm UV}$
$R\Psi$	$Q_R\Psi_R\Delta\Delta^{*2}\chi$	7	$V^4/\Lambda^3_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Delta^*\chi^2$	6	$V^3/\Lambda^2_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Phi^{*2}$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,(\Sigma^{*2}+\Sigma^*\Sigma'^*+\Sigma'^{*2})$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\Delta\Delta^{*2}\chi^2$	8	$V^5/\Lambda^4_{\rm UV}$

Being SM-singlets, they mix with neutrinos !

Anomalons

Anomalons are massless at the renormalizable level

$$\Psi_R\Psi_R \quad Q_R\Psi_R \quad \bar{Q}_L\Psi_R$$
$$\Psi_R\Psi_R\phi \quad Q_R\Psi_R\phi \quad \bar{Q}_L\Psi_R\phi$$

forbidden by gauge invariance

Higher-dimensional operators generate their mass

type	operator $\mathcal{O}$	d	$\langle M_{\mathcal{O}}\rangle$
$L\Psi$	$\overline{Q}_L\Psi_R\,\chi(\Phi+\Sigma+\Sigma')$	5	$vV/\Lambda_{\rm UV}$
$L\Psi$	$\overline{Q}_L\Psi_R\,\Delta\chi^*(\Phi^*+\Sigma^*+\Sigma'^*)$	6	$vV^2/\Lambda^2_{\rm UV}$
$R\Psi$	$Q_R\Psi_R\,\Delta^*\chi$	5	$V^2/\Lambda_{\rm UV}$
$R\Psi$	$Q_R\Psi_R\Delta\Delta^{*2}\chi$	7	$V^4/\Lambda^3_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Delta^*\chi^2$	6	$V^3/\Lambda^2_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Phi^{*2}$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,(\Sigma^{*2}+\Sigma^*\Sigma'^*+\Sigma'^{*2})$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\Delta\Delta^{*2}\chi^2$	8	$V^5/\Lambda^4_{\rm UV}$

Being SM-singlets, they mix with neutrinos !

↓

New (light) fermions, **interact with the SM** via

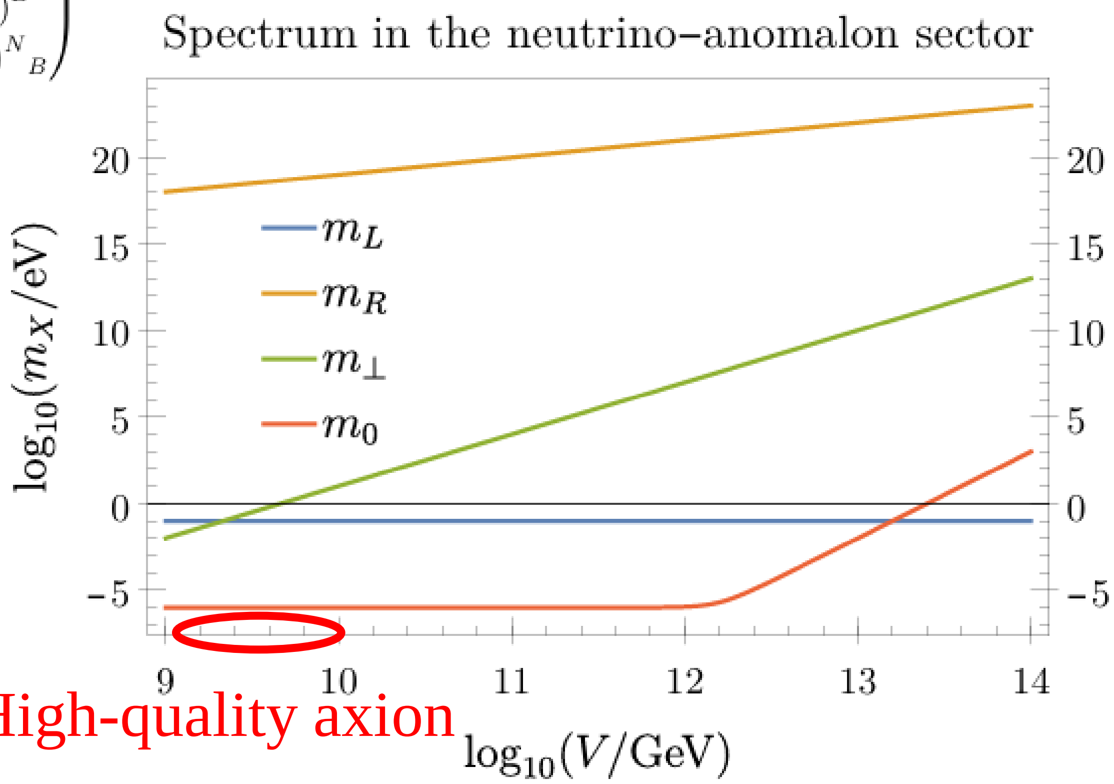
i) **flavor** gauge interactions

ii) **neutrino** mixing

Anomalon phenomenology = potential test of the quality mechanism !

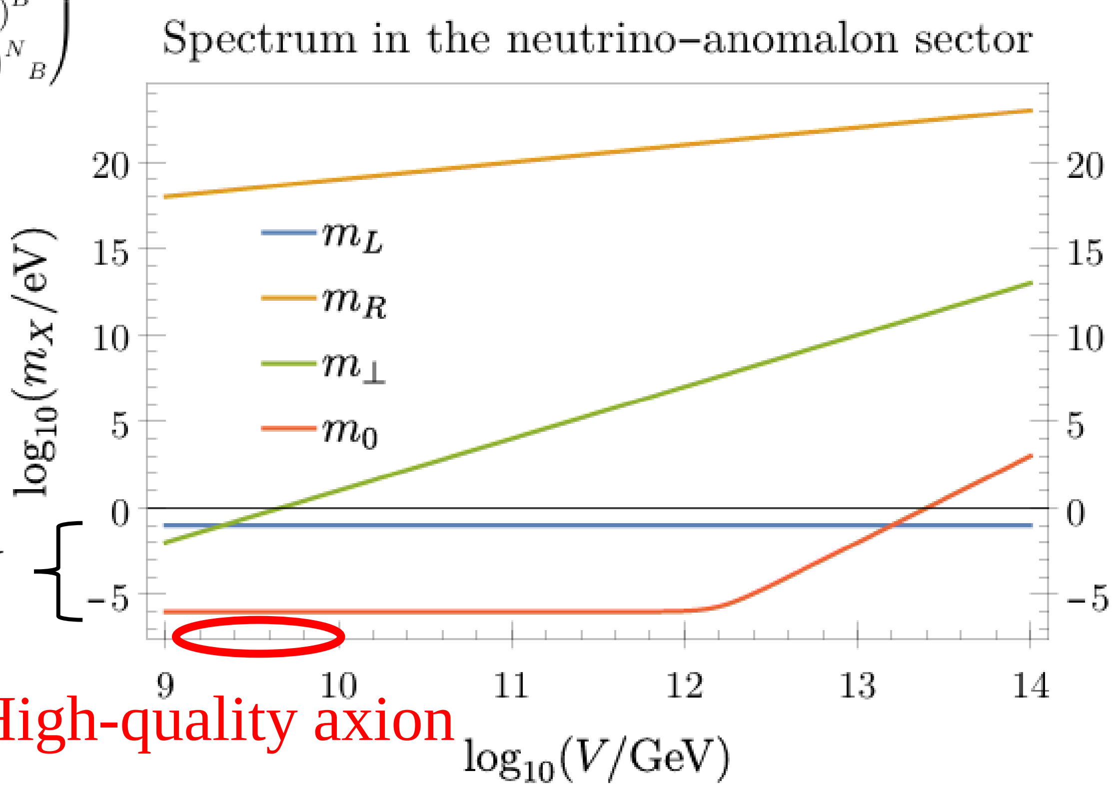
Anomalon spectrum

$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I\\(\nu_R)^A\\(\Psi_R)^K{}_A\end{pmatrix}^T \begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B\\ (M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B \\ (M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J\\(\nu_R)^B\\(\Psi_R)^N{}_B\end{pmatrix}$$



Anomalon spectrum

$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I\\(\nu_R)^A\\(\Psi_R)^K{}_A\end{pmatrix}^T\begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B\\(M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B\\(M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J\\(\nu_R)^B\\(\Psi_R)^N{}_B\end{pmatrix}$$



$m_\Psi \lesssim 0.1$ eV

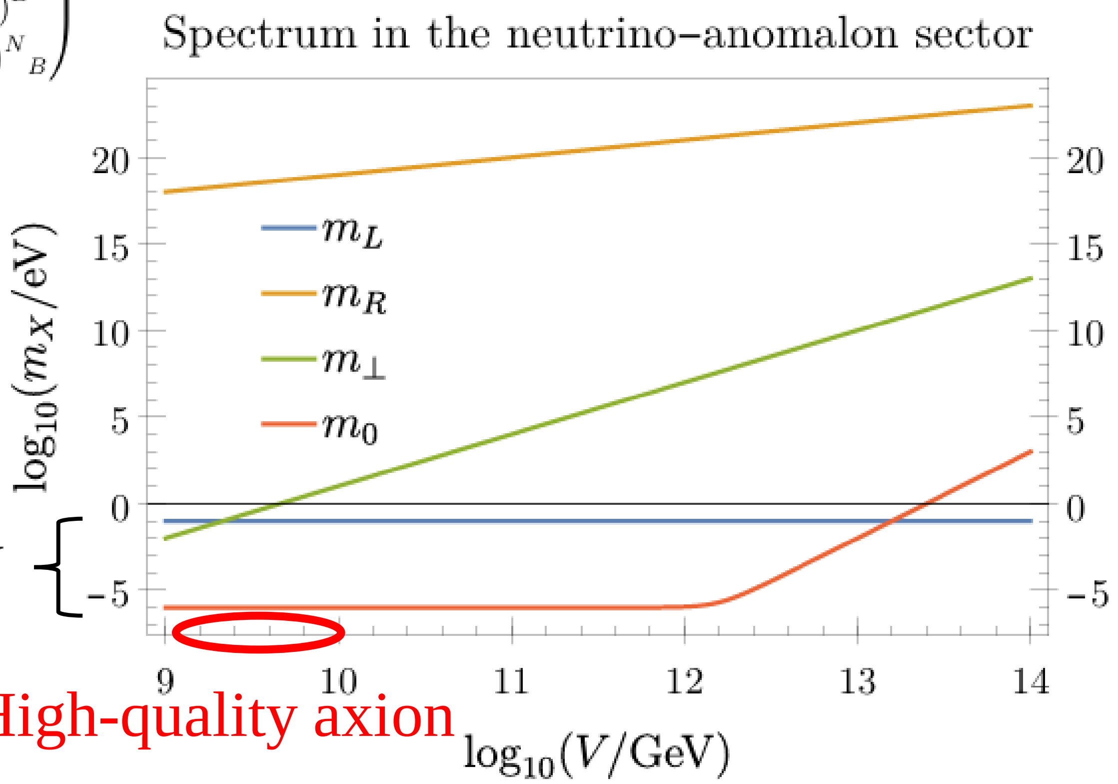
High-quality axion

<u>Dark Radiation</u>

$$\Delta N_{\mathrm{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\Psi}{\rho_\gamma}$$

Anomalon spectrum

$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I\\(\nu_R)^A\\(\Psi_R)^K{}_A\end{pmatrix}^T \begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B\\(M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B\\(M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J\\(\nu_R)^B\\(\Psi_R)^N{}_B\end{pmatrix}$$

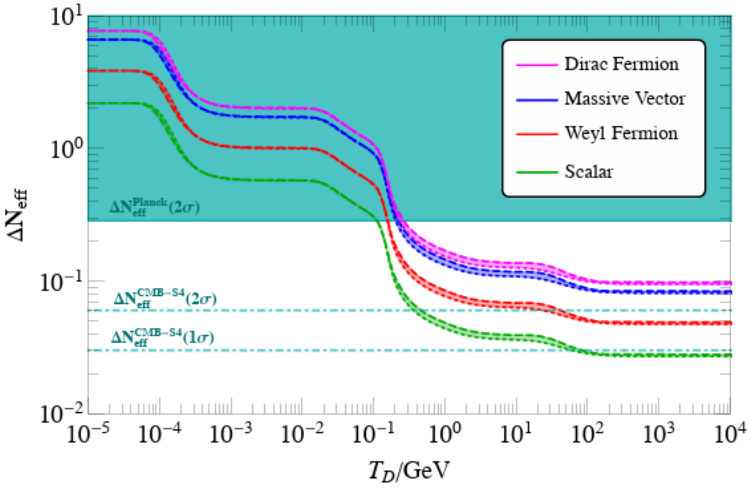


<u>Dark Radiation</u>

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\Psi}{\rho_\gamma}$$

$m_\Psi \lesssim 0.1$ eV

High-quality axion



Planck 18'

$\Delta N_{\rm eff} \leq 0.285$ (95%C.L.)

[D'Eramo, Hajkarim, Yun 2021]

	$\Delta N_{\rm eff}$ (1σ)	$\Delta N_{\rm eff}$ (2σ)
Planck 2018 [1]	$\Delta N_{\rm eff} < 0.115$	$\Delta N_{\rm eff} < 0.285$
SO [10, 11]	$\Delta N_{\rm eff} < 0.05$	$\Delta N_{\rm eff} < 0.1$
CMB-S4 [7, 8]	$\Delta N_{\rm eff} < 0.03$	$\Delta N_{\rm eff} < 0.06$
CMB-HD [14]	$\Delta N_{\rm eff} < 0.014$	$\Delta N_{\rm eff} < 0.028$

[Li, Xu 2023]

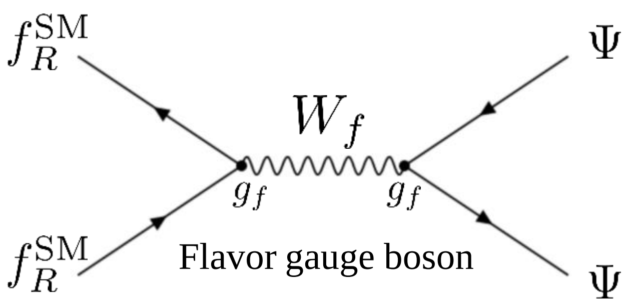
Anomalon production

How are they produced in the early Universe?

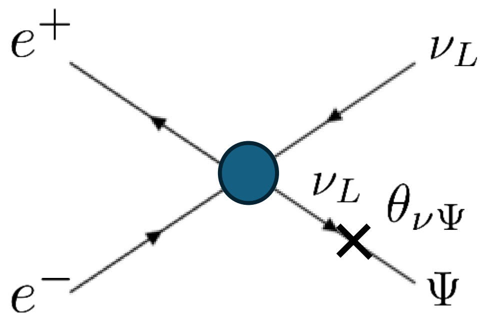
Anomalon production

How are they produced in the early Universe?

Flavor production



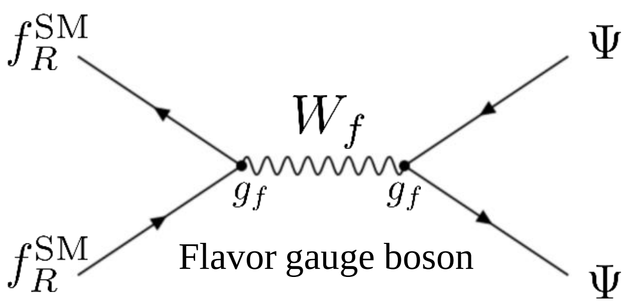
Neutrino mixing



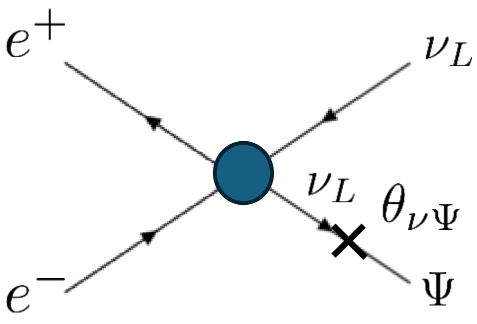
Anomalon production

How are they produced in the early Universe?

Flavor production



Neutrino mixing



Large couplings $g_f, \theta_{\nu\Psi} \sim \mathcal{O}(1)$

Thermalization with SM bath

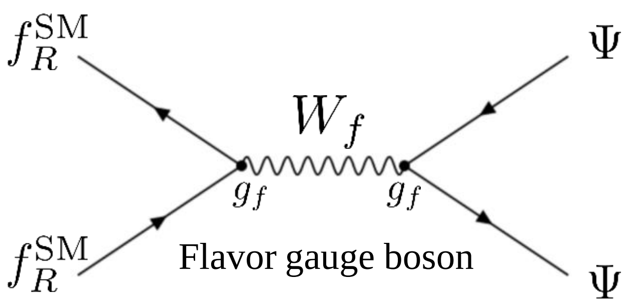
$$\Delta N_{\mathrm{eff}}^{\mathrm{TH}} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\mathrm{dec}})}\right)^{4/3} > 0.285$$

already excluded by PLANCK 18'!

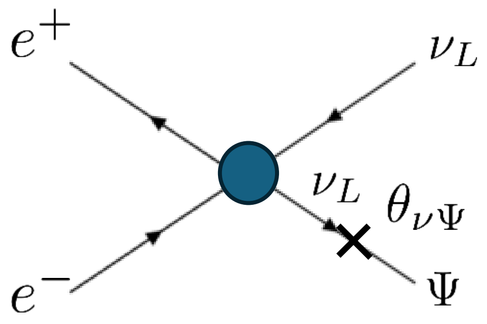
Anomalon production

How are they produced in the early Universe?

Flavor production



Neutrino mixing



Large couplings $g_f, \theta_{\nu\Psi} \sim \mathcal{O}(1)$

Thermalization with SM bath

$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\rm dec})} \right)^{4/3} > 0.285$$

already excluded by PLANCK 18'!

Very small couplings $g_f, \theta_{\nu\Psi} \ll \mathcal{O}(1)$

No thermalization / freeze-in production

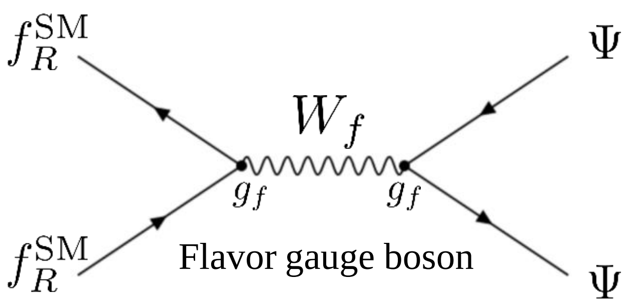
$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96\, Y_\Psi / g_s(T_{\rm FI})^{1/3} \ll 0.014$$

not observable!

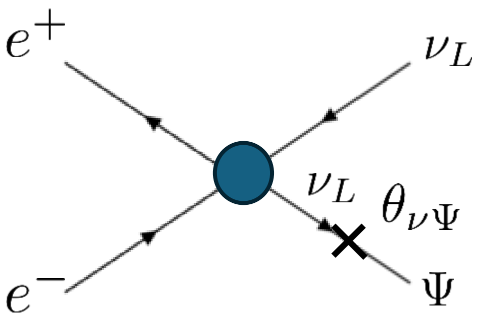
Anomalon production

How are they produced in the early Universe?

Flavor production



Neutrino mixing



Large couplings $g_f, \theta_{\nu\Psi} \sim \mathcal{O}(1)$

Thermalization with SM bath

$$\Delta N_{\mathrm{eff}}^{\mathrm{TH}} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\mathrm{dec}})} \right)^{4/3} > 0.285$$

already excluded by PLANCK 18'!

Intermediate regime

$$0.014 \lesssim \Delta N_{\mathrm{eff}}^{\mathrm{FI}} \lesssim 1.13$$

More precise computation is needed

Very small couplings $g_f, \theta_{\nu\Psi} \ll \mathcal{O}(1)$

No thermalization / freeze-in production

$$\Delta N_{\mathrm{eff}}^{\mathrm{FI}} \simeq 56.96\, Y_\Psi / g_s(T_{\mathrm{FI}})^{1/3} \ll 0.014$$

not observable!

Conclusions

The Peccei Quinn symmetry must be protected by UV sources (*quality problem*)

$\Downarrow$

GUT + flavor gauge symmetries provide the desired protection

Conclusions

The Peccei Quinn symmetry must be protected by UV sources (*quality problem*)

GUT + flavor gauge symmetries provide the desired protection

- No *ad-hoc* symmetries introduced

Connection between PQ and flavor structure

Conclusions

The Peccei Quinn symmetry must be protected by UV sources (*quality problem*)

GUT + flavor gauge symmetries provide the desired protection

- No *ad-hoc* symmetries introduced
  - Connection between PQ and flavor structure
- Phenomenological signatures
  - Cosmology of anomalons

Conclusions

The Peccei Quinn symmetry must be protected by UV sources (*quality problem*)

GUT + flavor gauge symmetries provide the desired protection

- No *ad-hoc* symmetries introduced
- Phenomenological signatures

Connection between PQ and flavor structure

Cosmology of anomalons

+more precise computations

+anomalons as extended neutrino sectors

+flavor observables?

Further investigation is needed

Conclusions

The Peccei Quinn symmetry must be protected by UV sources (*quality problem*)

GUT + flavor gauge symmetries provide the desired protection

- No *ad-hoc* symmetries introduced
- Phenomenological signatures

Connection between PQ and flavor structure

Cosmology of anomalons
+more precise computations
+anomalons as extended neutrino sectors
+flavor observables?

THANK YOU!

谢谢

Further investigation is needed

BACKUP SLIDES

Axion quality in PS – flavor model

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1

<u>Some criteria</u> **<u>Why ?</u>**

$$(\Delta\Delta^*)\Delta^3\chi^{*6}$$

$$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4}$$

$$\Phi^{4-k}\Sigma^k\Delta\chi^{*2}$$

$$\Phi^{4-k}\Sigma^k\Sigma^2$$

$\Phi^l\Sigma^m \Rightarrow (l+m) =$ even — $SU(2)_L$

$\chi^l(\chi^*)^m \Rightarrow (l+m) =$ even — $SU(2)_L +$ center$[SU(2)_R]$

$\chi, \Delta \Rightarrow \Delta\chi^{*2}$ — center$[SU(4)] = \mathbb{Z}_4$

$\phi^l(\phi^*)^m \Rightarrow |l-m|/3 \in \mathbb{Z}$ — center$[SU(3)_{f_R}] = \mathbb{Z}_3$

Explicit VEVs

$$\langle \Phi^{ii'}{}_A \rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes v_A^{u\Phi} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes v_A^{d\Phi} \,,$$

$$\langle \Sigma^a{}_b{}^{ii'}{}_A \rangle = \frac{1}{\sqrt{12}} \operatorname{diag}(1,1,1,-3) \otimes \left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes v_A^{u\Sigma} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes v_A^{d\Sigma} \right) ,$$

$$\langle \Sigma'^a{}_b{}^{ii'}{}_A \rangle = \frac{1}{\sqrt{12}} \operatorname{diag}(1,1,1,-3) \otimes \left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes v_A^{u\Sigma'} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes v_A^{d\Sigma'} \right)$$

$$\langle \Delta^{abi'j'AB} \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes Z^{AB} \,,$$

$$\langle \chi^{ai'}{}_A \rangle = (0,0,0,1) \otimes (1,0) \otimes V_A \,,$$

$$\langle \xi^a{}_b{}^{i'j'} \rangle = W \, \frac{1}{\sqrt{12}} \operatorname{diag}(1,1,1,-3) \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$

General SSB chain

$$
\begin{aligned}
&\mathrm{SU}(4)_{\mathrm{PS}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(3)_{f_R} \times \mathrm{U}(1)_{\mathrm{PQ}} \\
&\xrightarrow{\langle\Delta,\chi\rangle_3,\langle\xi\rangle} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(2)_{f_R} \\
&\xrightarrow{\langle\Delta,\chi\rangle_{1,2}} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\
&\xrightarrow{\langle\Phi,\Sigma,\Sigma'\rangle} \mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{EM}} \,,
\end{aligned}
$$

List of PQ-breaking operators with d < 10

$\mathcal{O}$ $(d=6)$	$\langle\mathcal{O}\rangle$	#
Φ^6	$v^6 \to 0$	1
$\Phi^{4-k}\Sigma^k\Sigma^2$	$v^6 \not\to 0$	5

$\mathcal{O}$ $(d=7)$	$\langle\mathcal{O}\rangle$	#
$\Phi^{4-k}\Sigma^k\Delta\chi^{*2}$	$v^4V^3 \not\to 0$	5
$\Phi^{5-k}\Sigma^k\Sigma\xi$	v^6V	6

$\mathcal{O}$ $(d=8)$	$\langle\mathcal{O}\rangle$	#
$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4}$	$v^2V^6 \not\to 0$	3
$\Phi^{4-k}\Sigma^k\Delta\chi^{*2}\xi$	v^4V^4	5
$\Phi^{6-k}\Sigma^k\Delta\Delta^*$	v^6V^2	7
$\Phi^{6-k}\Sigma^k\chi\chi^*$	v^6V^2	7
$\Phi^{6-k}\Sigma^k\xi^2$	v^6V^2	7
$\Phi^{4-k}\Sigma^k\Phi\Phi^*\Sigma^2$	v^8	5
$\Phi^{6-k}\Sigma^k\Sigma\Sigma^*$	v^8	7
$\Phi^7\Phi^*$	v^8	1
$\Phi\Sigma^{*7}$	v^8	1

$\mathcal{O}$ $(d=9)$	$\langle\mathcal{O}\rangle$	#
$\Delta^3\chi^{*6}$	$V^9 \to 0$	1
$\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4}\xi$	v^2V^7	3
$\Phi^{4-k}\Sigma^k\Delta^2\Delta^*\chi^{*2}$	v^4V^5	5
$\Phi^{4-k}\Sigma^k\Delta\chi^{*2}\xi^2$	v^4V^5	5
$\Phi^{4-k}\Sigma^k\Delta\chi\chi^{*3}$	v^4V^5	5
$\Phi^{6-k}\Sigma^k\xi^3$	v^6V^3	7
$\Phi^{6-k}\Sigma^k\Delta\Delta^*\xi$	v^6V^3	7
$\Phi^{6-k}\Sigma^k\chi\chi^*\xi$	v^6V^3	7
$\Phi^{4-k}\Sigma^k\Phi\Phi^*\Delta\chi^{*2}$	v^6V^3	5
$\Phi^{5-k}\Sigma^k\Sigma^*\Delta\chi^{*2}$	v^6V^3	6
$\Phi^*\Sigma^5\Delta\chi^{*2}$	v^6V^3	1
$\Phi^{5-k}\Sigma^k\Phi\Phi^*\Sigma\xi$	v^8V	6
$\Phi^{7-k}\Sigma^k\Sigma^*\xi$	v^8V	8
$\Phi\Sigma^{*7}\xi$	v^8V	1

Axion – photon coupling

$$\mathcal{L}_{a\gamma} = \frac{\alpha_{\rm em} C_{a\gamma}}{8\pi f_a} a\, F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$C_{a\gamma} = E/N - 1.92(4)\,,$$

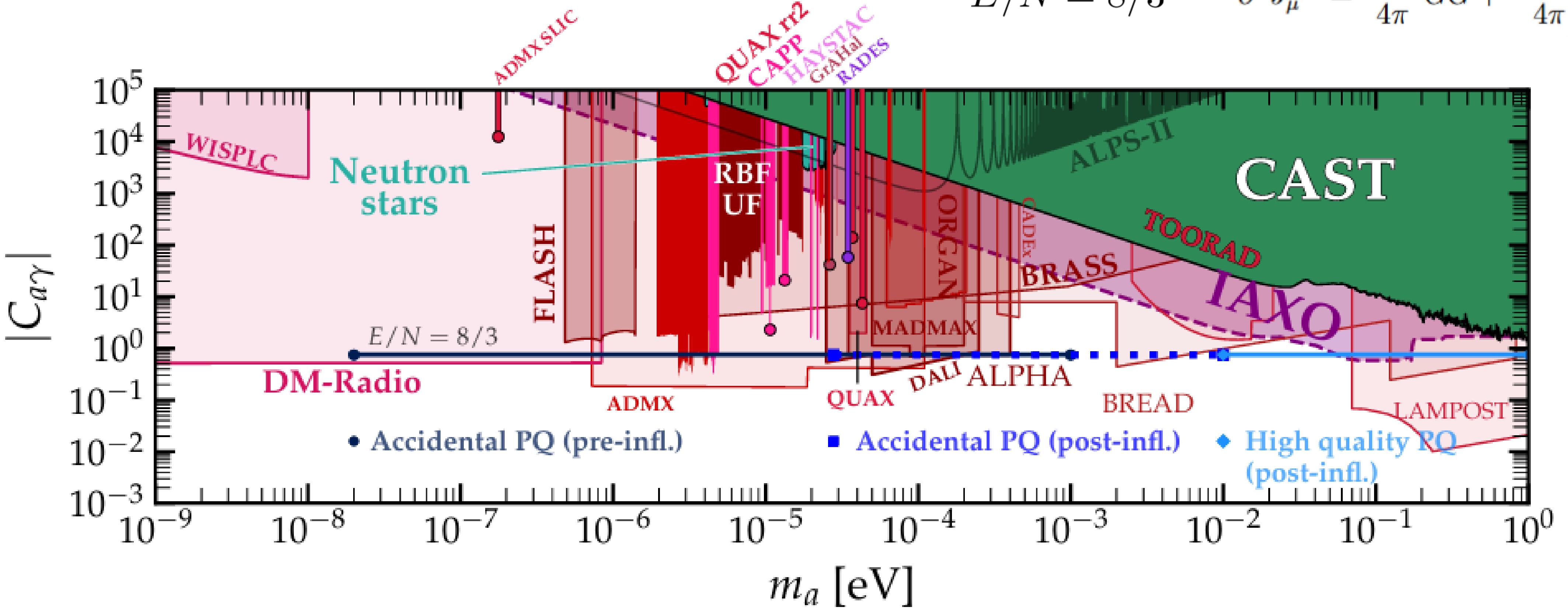
model independent

$a - \pi^0$ mixing

model dependent

$E/N = 8/3$

$$\partial^\mu J_\mu^{\rm PQ} = \frac{\alpha_s N}{4\pi} G\tilde{G} + \frac{\alpha_{\rm EM} E}{4\pi} F\tilde{F}$$



Axion astrophysical limits

Axion couplings with matter $g_{aX} = c_X m_X / f_a$

$$c_p = -0.45 \quad + 0.29 \cos^2\beta - 0.15 \sin^2\beta\,,$$

$$c_n = +0.013 - 0.14 \cos^2\beta + 0.27 \sin^2\beta\,.$$

$$c_e = 0.33 \sin^2\beta$$

SN 1987A $\Rightarrow$ $g_{an}^2 + 0.61 g_{ap}^2 + 0.53 g_{an} g_{ap} \lesssim 8.3 \times 10^{-19}$

Red giants+white dwarfs $\Rightarrow$ $g_{ae} \lesssim 2 \times 10^{-13}$

Saturating the perturbative bound $\tan\beta = 0.25$ $\Rightarrow$ $f_a \gtrsim 2 \times 10^8 \text{ GeV}$

dominated by SN 1987A

$$\tan\beta \equiv \frac{v_u}{v_d} = \frac{\sqrt{\sum_A |v_A^{u\Phi}|^2 + \sum_A |v_A^{u\Sigma}|^2 + \sum_A |v_A^{u\Sigma'}|^2}}{\sqrt{\sum_A |v_A^{d\Phi}|^2 + \sum_A |v_A^{d\Sigma}|^2 + \sum_A |v_A^{d\Sigma'}|^2}}\,.$$

Including radiative corrections to axion-electron coupling a stronger bound is obtained $\Rightarrow$

$$f_a \gtrsim 2.3 \times 10^8 \text{ GeV}$$

Axion embedding

$$J_\mu^{\rm PQ} = -\sum_\phi q_\phi \phi^\dagger i\overleftrightarrow{\partial}_\mu \phi\,,$$

$$J_\mu^{B-L} = -\sum_\phi (B-L)_\phi \phi^\dagger i\overleftrightarrow{\partial}_\mu \phi\,,$$

with $q = c_1 {\rm PQ} + c_2(B-L)$

$$J_\mu^{\rm PQ} = q_\chi \sum_A \sqrt{2}|V_A|\partial_\mu(a_\chi)_A + q_\Delta \sum_{AB}\sqrt{2}|Z^{AB}|\partial_\mu(a_\Delta)^{AB}\,,$$

$$J_\mu^{B-L} = (B-L)_\chi \sum_A \sqrt{2}|V_A|\partial_\mu(a_\chi)_A + (B-L)_\Delta \sum_{AB}\sqrt{2}|Z^{AB}|\partial_\mu(a_\Delta)^{AB}\,,$$

where $\phi \supset V_\phi \exp\left(\frac{i\,a_\phi}{\sqrt{2}\,|V_\phi|}\right)$, $V_\chi^2 = \sum_A |V_A|^2$, $V_\Delta^2 = \sum_{AB}|Z^{AB}|^2$.

Current orthogonality

$$q_\chi = c_1\frac{-8V_\Delta^2}{V_\chi^2+V_\Delta^2}\,,\qquad q_\Delta = c_1\frac{4V_\chi^2}{V_\chi^2+V_\Delta^2}$$

Canonical axion field

$$a = \frac{1}{V_a}\left(q_\chi \sum_A |V_A|(a_\chi)_A + q_\Delta \sum_{AB}|Z^{AB}|(a_\Delta)^{AB}\right)\,,$$

$$V_a^2 = q_\chi^2 V_\chi^2 + q_\Delta^2 V_\Delta^2 = c_1\frac{16V_\chi^2V_\Delta^2}{V_\chi^2+V_\Delta^2}\,,$$

Anomaly matching

$$\partial^\mu J_\mu^{\rm PQ} = \frac{\alpha_s N}{4\pi}G\tilde{G} + \frac{\alpha_{\rm EM}E}{4\pi}F\tilde{F}$$

$$N = 3\times T(3)\,(2\,{\rm PQ}(Q_L) - 2\,{\rm PQ}(Q_R)) = 6 \quad ({\rm UV})\,,$$

$$N = 3\times T(3)\,(2\,q(q_L) - 2\,q(q_R)) = 6c_1 \quad ({\rm IR})\,,$$

$$f_a \equiv \frac{\sqrt{2}V_a}{2N} = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2+4V_\Delta^2}}\,.$$

Axion embedding

Repeat with all fields to obtain **axion couplings with matter**

	χ	Δ	Φ_u	Φ_d	Σ_u	Σ_d	Σ'_u	Σ'_d
PQ	-1	2	2	2	2	2	2	2
$B-L$	-1	-2	0	0	0	0	0	0
Y	0	0	$-1/2$	$1/2$	$-1/2$	$1/2$	$-1/2$	$1/2$

$$q = c_1 \mathrm{PQ} + c_2(B-L) + c_3 Y$$

Current orthogonality

$$q_\chi = c_1 \frac{-8V_\Delta^2}{V_\chi^2 + V_\Delta^2}\,, \qquad q_\Delta = c_1 \frac{4V_\chi^2}{V_\chi^2 + V_\Delta^2}$$

Canonical axion field

$$a = \frac{1}{V_a}\left(q_\chi \sum_A |V_A| (a_\chi)_A + q_\Delta \sum_{AB} |Z^{AB}| (a_\Delta)^{AB} \right),$$

$$V_a^2 = q_\chi^2 V_\chi^2 + q_\Delta^2 V_\Delta^2 = c_1 \frac{16 V_\chi^2 V_\Delta^2}{V_\chi^2 + V_\Delta^2}\,,$$

Anomaly matching

$$\partial^\mu J_\mu^{\mathrm{PQ}} = \frac{\alpha_s N}{4\pi} G\tilde{G} + \frac{\alpha_{\mathrm{EM}} E}{4\pi} F\tilde{F}$$

$$N = 3 \times T(3)\,(2\,\mathrm{PQ}(Q_L) - 2\,\mathrm{PQ}(Q_R)) = 6 \quad (\mathrm{UV})\,,$$
$$N = 3 \times T(3)\,(2\,q(q_L) - 2\,q(q_R)) = 6c_1 \quad (\mathrm{IR})\,,$$

$$f_a \equiv \frac{\sqrt{2} V_a}{2N} = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2 + 4V_\Delta^2}}\,.$$

SM fermions embedding

$$(\overline{Q}_L)^I{}_{ai} = \begin{pmatrix} \overline{u}^I_{L1} & \overline{d}^I_{L1} \\ \overline{u}^I_{L2} & \overline{d}^I_{L2} \\ \overline{u}^I_{L3} & \overline{d}^I_{L3} \\ \overline{\nu}^I_L & \overline{e}^I_L \end{pmatrix}, \qquad (Q_R)^{ai'A} = \begin{pmatrix} u^A_{R1} & d^A_{R1} \\ u^A_{R2} & d^A_{R2} \\ u^A_{R3} & d^A_{R3} \\ \nu^A_R & e^A_R \end{pmatrix}$$

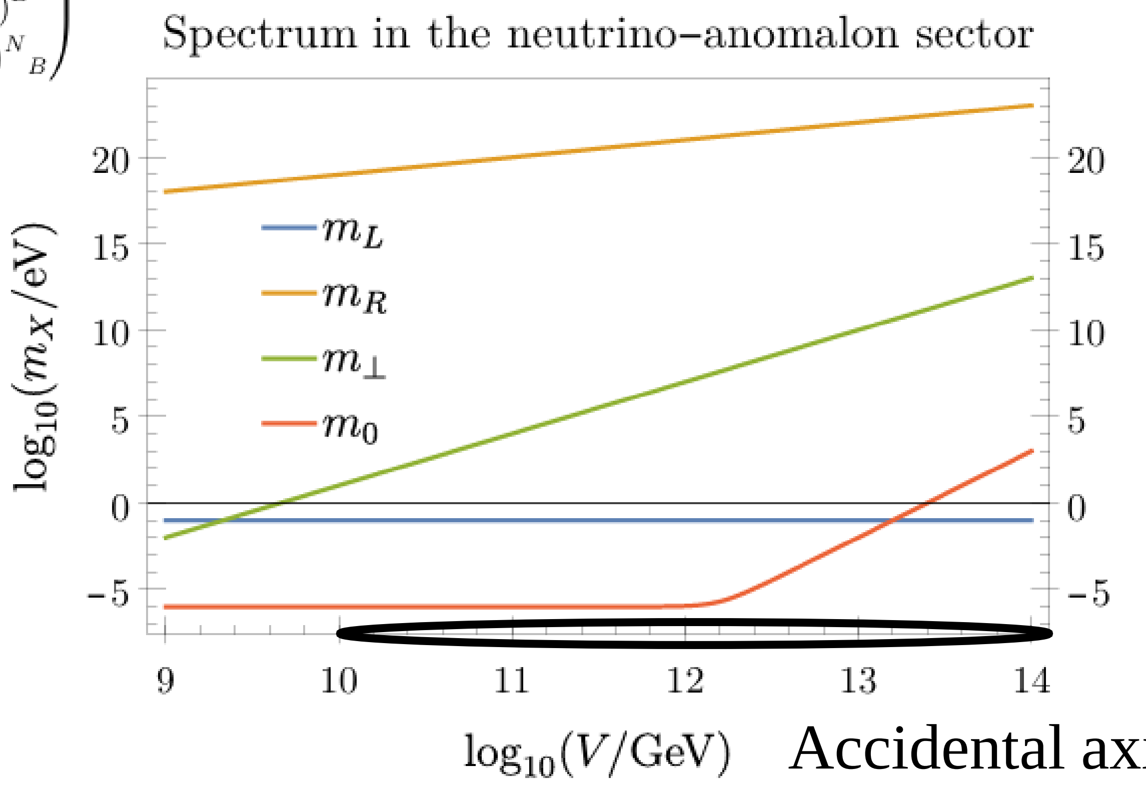
Anomalon – neutrino mixings

type	operator $\mathcal{O}$	d	$\langle M_{\mathcal{O}} \rangle$
LL	$\overline{Q}_L \overline{Q}_L\, \Delta(\Phi^2 + \Phi\Sigma + \Sigma^2)$	6	$v^2 V/\Lambda_{\rm UV}^2$
LR	$\overline{Q}_L Q_R\, (\Phi + \Sigma + \Sigma')$	4	v
RR	$Q_R Q_R\, \Delta^*$	4	V
$L\Psi$	$\overline{Q}_L \Psi_R\, \chi(\Phi + \Sigma + \Sigma')$	5	$vV/\Lambda_{\rm UV}$
$L\Psi$	$\overline{Q}_L \Psi_R\, \Delta\chi^*(\Phi^* + \Sigma^* + \Sigma'^*)$	6	$vV^2/\Lambda_{\rm UV}^2$
$R\Psi$	$Q_R \Psi_R\, \Delta^* \chi$	5	$V^2/\Lambda_{\rm UV}$
$R\Psi$	$Q_R \Psi_R \Delta\Delta^{*2}\chi$	7	$V^4/\Lambda_{\rm UV}^3$
$\Psi\Psi$	$\Psi_R \Psi_R\, \Delta^* \chi^2$	6	$V^3/\Lambda_{\rm UV}^2$
$\Psi\Psi$	$\Psi_R \Psi_R\, \Phi^{*2}$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R \Psi_R\, (\Sigma^{*2} + \Sigma^*\Sigma'^* + \Sigma'^{*2})$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R \Psi_R \Delta\Delta^{*2}\chi^2$	8	$V^5/\Lambda_{\rm UV}^4$

$$\begin{pmatrix} M_{LL} & M_{LR} & M_{L\Psi_\perp} & M_{L\Psi_0} \\ M_{RL} & M_{RR} & M_{R\Psi_\perp} & M_{R\Psi_0} \\ M_{\Psi_\perp L} & M_{\Psi_\perp R} & M_{\Psi_\perp \Psi_\perp} & M_{\Psi_\perp \Psi_0} \\ M_{\Psi_0 L} & M_{\Psi_0 R} & M_{\Psi_0 \Psi_\perp} & M_{\Psi_0 \Psi_0} \end{pmatrix} \sim \begin{pmatrix} \frac{v^2 V}{\Lambda_{\rm UV}^2} & yv & l\, \frac{vV}{\Lambda_{\rm UV}} & \tilde{l}\, \frac{vV^2}{\Lambda_{\rm UV}^2} \\ .. & V & r\, \frac{V^2}{\Lambda_{\rm UV}} & \tilde{r}\, \frac{V^4}{\Lambda_{\rm UV}^3} \\ .. & .. & \frac{V^3}{\Lambda_{\rm UV}^2} & \frac{v^2}{\Lambda_{\rm UV}} + \frac{V^5}{\Lambda_{\rm UV}^4} \\ .. & .. & .. & \frac{v^2}{\Lambda_{\rm UV}} + \frac{V^5}{\Lambda_{\rm UV}^4} \end{pmatrix}$$

Anomalon spectrum

$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I \\ (\nu_R)^A \\ (\Psi_R)^K{}_A\end{pmatrix}^T \begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B \\ (M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B \\ (M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J \\ (\nu_R)^B \\ (\Psi_R)^N{}_B\end{pmatrix}$$



Accidental axion

Anomalon spectrum

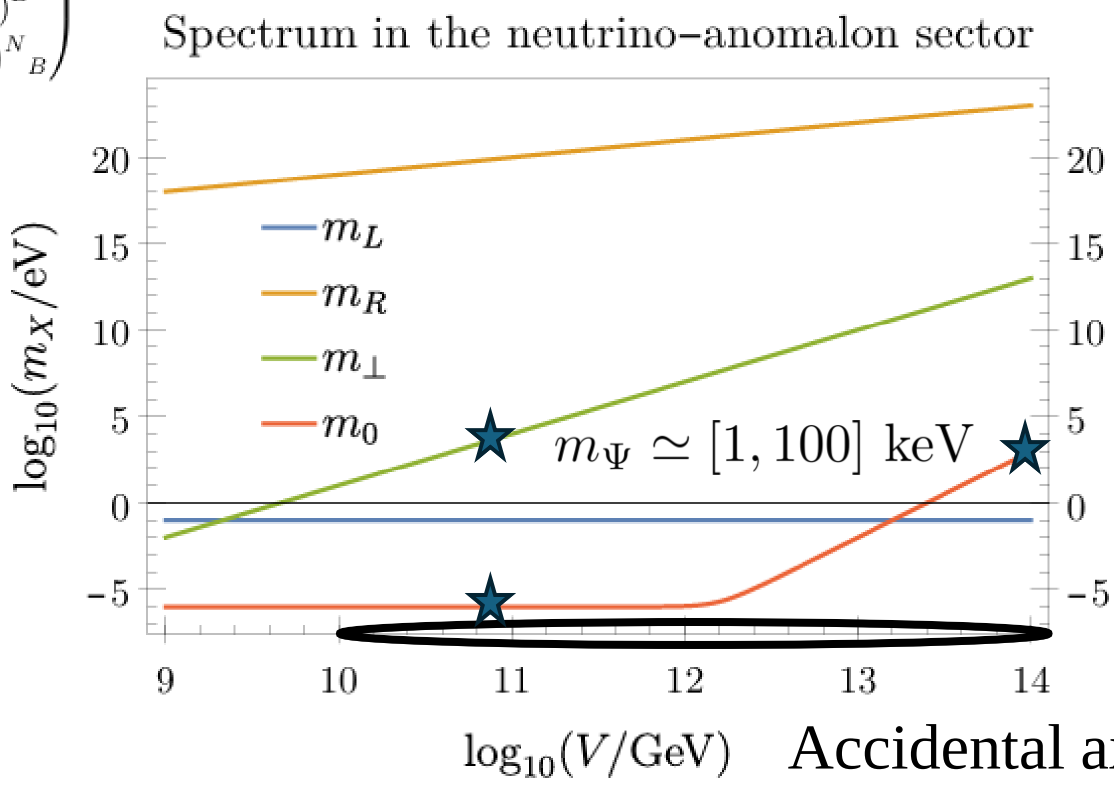
$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I\\(\nu_R)^A\\(\Psi_R)^K{}_A\end{pmatrix}^T\begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B\\(M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B\\(M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J\\(\nu_R)^B\\(\Psi_R)^N{}_B\end{pmatrix}$$

Dark radiation

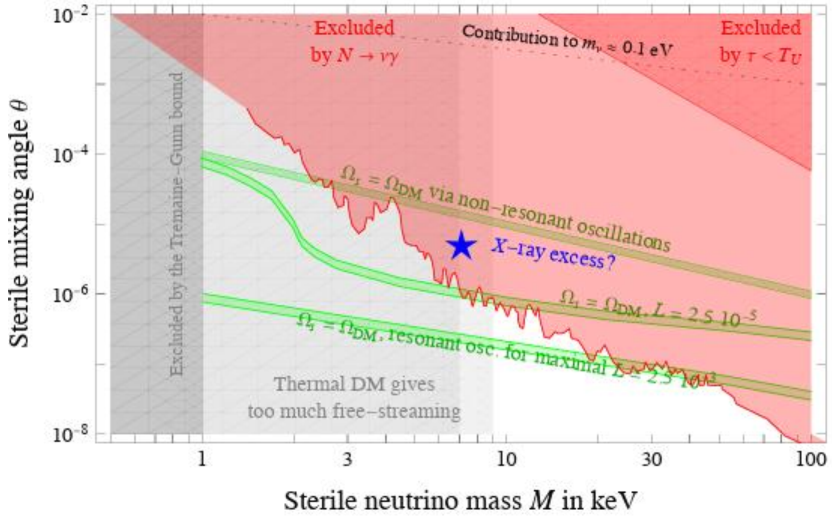
\+

Dark Matter

Analogous to sterile neutrino DM

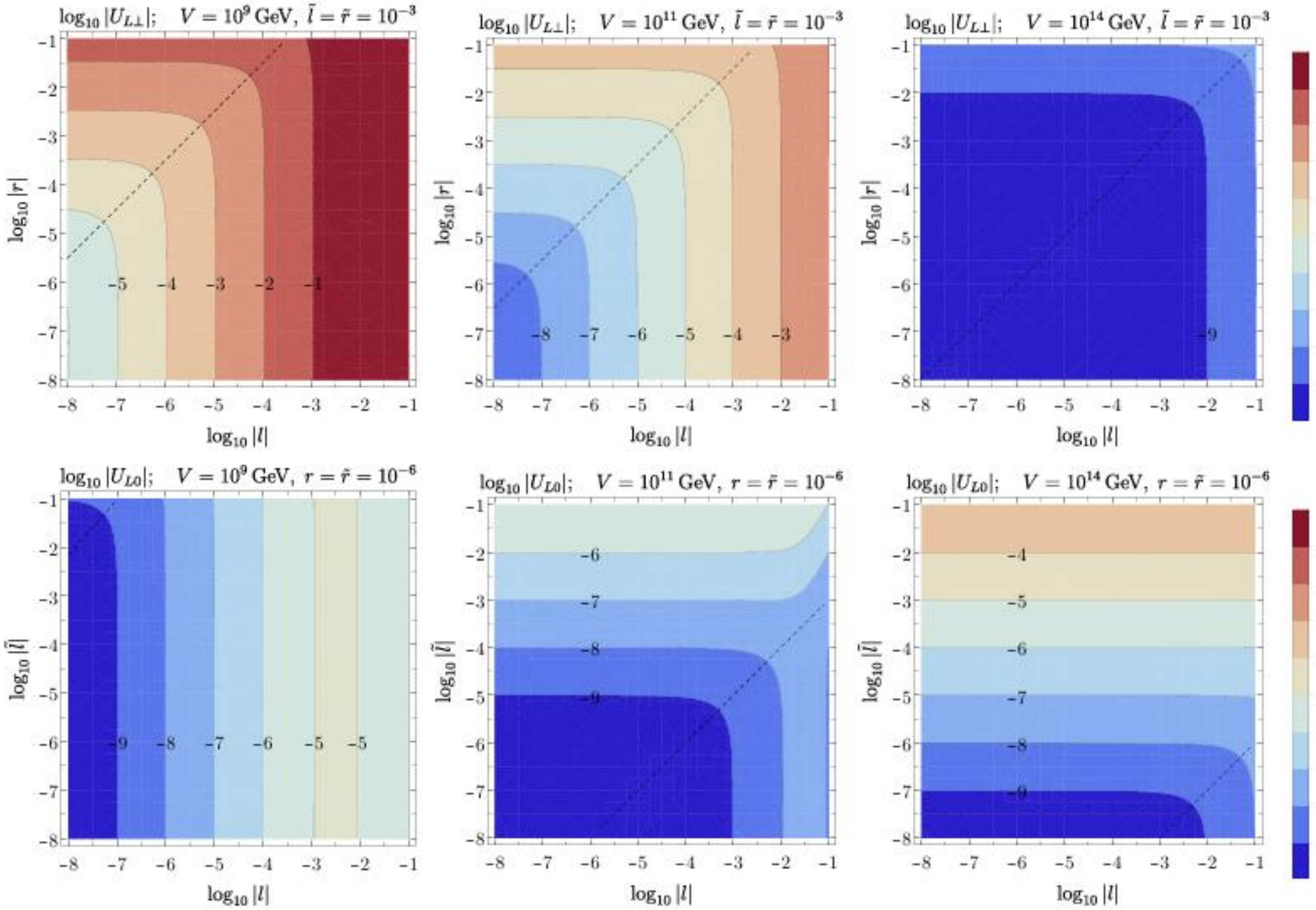


Accidental axion

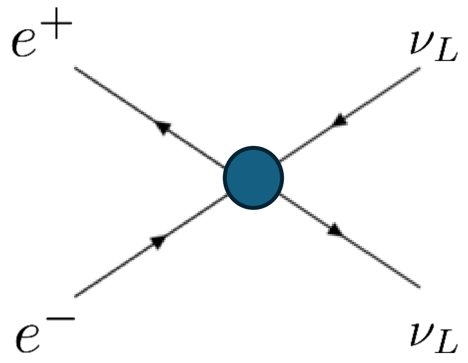


[Cirelli, Strumia, Zupan 2024]

Anomalon – neutrino mixings



Freeze – in via neutrino mixing



Neutrino interactions in the early Universe

$$\Gamma_\nu \sim G_F^2 T^5$$

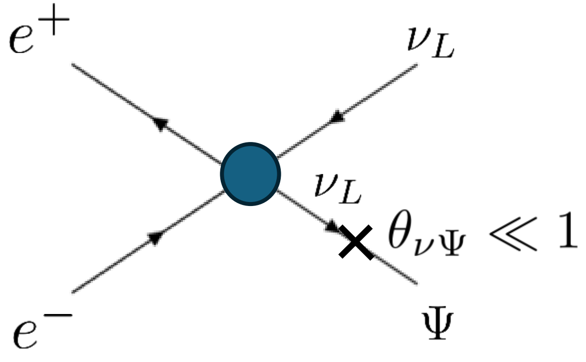
$$T \gtrsim \mathrm{MeV}$$

Neutrino-anomalon mixing

$$\Gamma_m \sim G_F^2 T^5 \overline{\theta}^2_{\nu\Psi_m}$$

$$\sum_{f=e,\mu,\tau} |\theta_{\nu_f \Psi_m}|^2 \qquad \overline{\theta}_{\nu\Psi_m} \simeq \frac{\theta_{\nu\Psi_m}}{\left(1+\mathcal{O}(1)\frac{T^6 G_F^2}{\alpha_{\mathrm{em}} m^2_{\Psi_m}}\right)}$$

finite temperature/density correction $m_\Psi > m_\nu$



$$T_{\mathrm{FI},m} \sim \max\{130\,\mathrm{MeV}(m_{\Psi_m}/\mathrm{keV})^{1/3}, \mathrm{MeV}\}$$

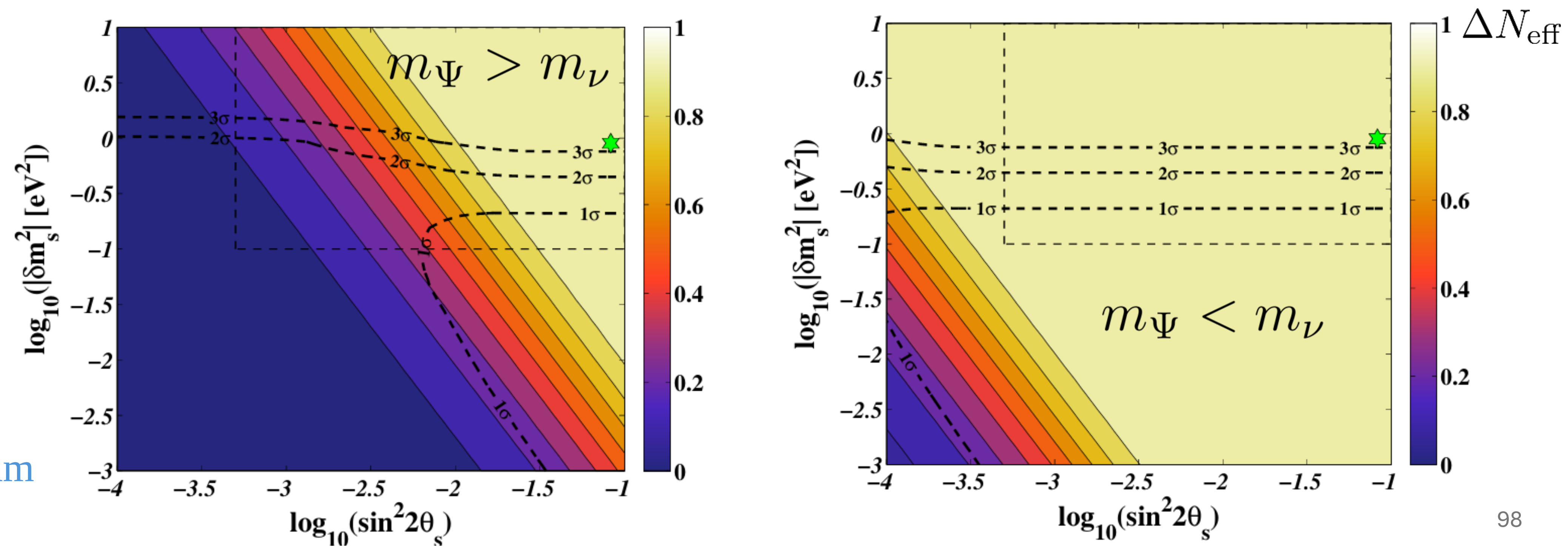
$$Y_\Psi^{\nu-\mathrm{mix}} \sim \sum_m (Y_{e,\mathrm{eq}}\Gamma_m/H)|_{T=T_{\mathrm{FI},m}}$$

Freeze – in via neutrino mixing

$$\theta_1^2 \to \theta_M^2(T) \simeq \frac{\theta_1^2}{\left(1 + \frac{2p}{M_1^2}\left(b(p,T) \pm c(T)\right)\right)^2 + \theta_1^2}$$

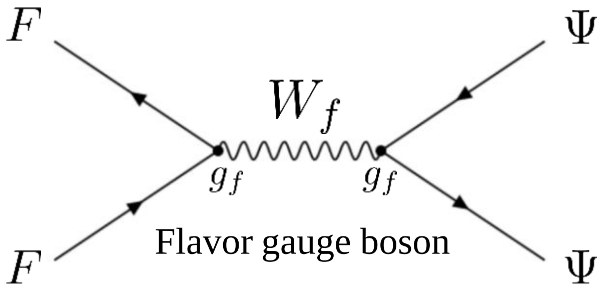
Boyarsky, Ruchayskiy, Shaposhnikov 2009

$m_\Psi < m_\nu$ ➡ resonant pole at $p^2 \sim \alpha_{\rm em}(m_\nu^2 - m_\Psi^2)/(G_F^2 T^4)$



[Hannestad, Tamborra, Tram 2012]

Anomalon thermalization



$$\frac{\Gamma_\Psi^{\rm flavor}}{H} \sim \frac{M_{\rm Pl}}{g_*^{1/2} T} \times \begin{cases} (g_{f_R}^2/4\pi)^2 & \text{if } T \gg m_{W_{f_R}} \\ (T/V)^4 & \text{if } T \ll m_{W_{f_R}} \end{cases}$$

High-reheating

→ resonant production

$$\left.\frac{\Gamma_\Psi^{\rm flavor}}{H}\right|_{\rm peak} \equiv \frac{\Gamma_\Psi^{\rm flavor}}{H}(T \simeq m_{W_{f_R}}) \sim \sqrt{\frac{g_{f_R}^2}{g_*(m_{W_{f_R}})}\frac{M_{\rm Pl}}{V}} \ll 1$$

$$\Rightarrow \quad g_{f_R} \ll 10^{-9} \left(\frac{V}{10^9\,{\rm GeV}}\right) \left(\frac{g_*(m_{W_{f_R}})}{106.75}\right)^{1/2} .$$

Low-reheating

$$\Rightarrow \quad \left.\frac{\Gamma_\Psi^{\rm flavor}}{H}\right|_{T_{\rm RH}} \sim \frac{M_{\rm Pl} T_{\rm RH}^3}{g_*^{1/2} V^4} \quad \Rightarrow \quad T_{\rm RH} \ll V \left(\frac{g_*^{1/2}(T_{\rm RH}) V}{M_{\rm Pl}}\right)^{1/3} \simeq 10^6\,{\rm GeV} \left(\frac{V}{10^9\,{\rm GeV}}\right)^{4/3} \left(\frac{g_*(T_{\rm RH})}{106.75}\right)^{1/6} .$$

Domain Wall problem

Bias term $\mathcal{V}_{\rm bias} = -2\Xi V^4 \cos\left(\frac{a}{V} + \delta\right)$ $V = N_{\rm DW} f_a$

$$\Rightarrow \quad t_{\rm decay} \approx 5 \times 10^{-5}\ {\rm s} \left(\frac{10^{-50}}{\Xi}\right)\left(\frac{12}{N_{\rm DW}}\right)^4 \left(\frac{m_a}{0.02\ {\rm eV}}\right)^3$$

Matching the bias term to $\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2 V^6/M_{\rm Pl}^4$

$$\Rightarrow \quad \Xi \sim \left(\frac{v}{M_{\rm Pl}}\right)^2 \left(\frac{V}{M_{\rm Pl}}\right)^2 \sim 10^{-52}(N_{\rm DW}/12)^2(0.02\ {\rm eV}/m_a)^2$$

$$\Rightarrow \quad t_{\rm decay} \sim 10^{-3}\ {\rm sec}\left(\frac{12}{N_{\rm DW}}\right)^6 \left(\frac{m_a}{0.02\ {\rm eV}}\right)^5$$

DW decay before BBN $\Rightarrow$ $f_a \gtrsim 4 \times 10^8\ {\rm GeV}$

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is <u>not</u> gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\psi_{16}\psi_{16}\phi_{10}^{*} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

3 copies

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is not gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\psi_{16}\psi_{16}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \to 0 \quad \Longrightarrow \quad SU(3)_f \otimes U(1)_{\rm PQ}$ global, accidental

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is <u>not</u> gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\psi_{16}\psi_{16}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \to 0 \quad \Longrightarrow \quad SU(3)_f \otimes U(1)_{\mathrm{PQ}}$ global, accidental

$\Downarrow$

SM flavor

$$\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}\}$$

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is <u>not</u> gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\psi_{16}\psi_{16}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \to 0$ ⟹ $SU(3)_f \otimes U(1)_{\rm PQ}$ global, accidental

SM flavor

$$\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}\}$$

PQ symmetry

$$\mathrm{PQ}(\psi_{16}) = -2\mathrm{PQ}(\phi_i) = +1$$

$$[SU(3)_c^2 U(1)_{\rm PQ}] \neq 0$$

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is <u>not</u> gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\psi_{16}\psi_{16}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \to 0$ ⟹ $SU(3)_f \otimes U(1)_{\rm PQ}$ global, accidental

↓ SM flavor

$$\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}\}$$

↓ PQ symmetry

$$\mathrm{PQ}(\psi_{16}) = -2\mathrm{PQ}(\phi_i) = +1$$

$$[SU(3)^2_c U(1)_{\rm PQ}] \neq 0$$

<u>**BUT**</u> $Y \neq 0$

Accidental origin of PQ symmetry

Example: SO(10) model

if flavor is <u>not</u> gauged

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \boxed{\tilde{y}_{10}\psi_{16}\psi_{16}\phi_{10}^*} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \to 0$ ⟹ $\cancel{SU(3)}_f \otimes \cancel{U(1)}_{\mathrm{PQ}}$ global, accidental

SM flavor

$\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{6}}, \bar{\mathbf{6}}\}$

PQ symmetry

$\mathrm{PQ}(\psi_{16}) = -2\mathrm{PQ}(\phi_i) = +1$

$[SU(3)_c^2 U(1)_{\mathrm{PQ}}] \neq 0$

<u>**BUT**</u> $Y \neq 0$ ⟹ **Explicitly** PQ and flavor breaking by $\tilde{y}_{10}$

Accidental origin of PQ symmetry

Example: SO(10) model

We gauge $SU(3)_f$ $\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{6}}, \bar{\mathbf{6}}\}$

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\cancel{\psi_{16}\psi_{16}}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

Accidental origin of PQ symmetry

Example: SO(10) model

We gauge $SU(3)_f$ $\{\psi_{16}, \phi_{10}, \phi_{1\bar{2}6}\} \sim \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}\}$

$$\mathcal{L} = \mathcal{L}_{Y=0} + y_{10}\psi_{16}\psi_{16}\phi_{10} + \tilde{y}_{10}\cancel{\psi_{16}\psi_{16}}\phi^*_{10} + y_{1\bar{2}6}\psi_{16}\psi_{16}\phi_{\overline{126}} + h.c.$$

$Y \neq 0 \quad \Longrightarrow \quad U(1)_{\mathrm{PQ}}$ global, accidental symmetry of the renormalizable Lagrangian

Similar for the Pati-Salam realization

GUT + flavor : (accidental) origin of the PQ symmetry

Flavor: quarks and charged leptons

$$-\mathcal{L}_Y = \sum_{\alpha=1}^{N_\Phi} \sum_{I=1}^{3} Y_I^{\Phi^\alpha} \, (\overline{Q}_L)^I Q_R \, \Phi^\alpha + \sum_{\alpha=1}^{N_\Sigma} \sum_{I=1}^{3} 2\sqrt{3}\, Y_I^{\Sigma^\alpha} \, (\overline{Q}_L)^I Q_R \, \Sigma^\alpha$$
$$+ \frac{1}{2} \, Y_R \, Q_R Q_R \, \Delta^* + \text{h.c.}\,,$$

$$(M_U)_{IA} = Y_I^\Phi v_A^{u\Phi} + Y_I^\Sigma v_A^{u\Sigma} + Y_I^{\Sigma'} v_A^{u\Sigma'}\,,$$
$$(M_D)_{IA} = Y_I^\Phi v_A^{d\Phi} + Y_I^\Sigma v_A^{d\Sigma} + Y_I^{\Sigma'} v_A^{d\Sigma'}\,,$$
$$(M_E)_{IA} = Y_I^\Phi v_A^{d\Phi} - 3\, Y_I^\Sigma v_A^{d\Sigma} - 3\, Y_I^{\Sigma'} v_A^{d\Sigma'}$$

$$M_U = \begin{pmatrix} Y_1^\Phi v_1^{u\Phi} & Y_1^\Phi v_2^{u\Phi} & Y_1^\Phi v_3^{u\Phi} \\ Y_2^\Phi v_1^{u\Phi} + Y_2^{\Sigma'} v_1^{u\Sigma'} & Y_2^\Phi v_2^{u\Phi} + Y_2^{\Sigma'} v_2^{u\Sigma'} & Y_2^\Phi v_3^{u\Phi} + Y_2^{\Sigma'} v_3^{u\Sigma'} \\ Y_3^\Phi v_1^{u\Phi} + Y_3^\Sigma v_1^{u\Sigma} + Y_3^{\Sigma'} v_1^{u\Sigma'} & Y_3^\Phi v_2^{u\Phi} + Y_3^\Sigma v_2^{u\Sigma} + Y_3^{\Sigma'} v_2^{u\Sigma'} & Y_3^\Phi v_3^{u\Phi} + Y_3^\Sigma v_3^{u\Sigma} + Y_3^{\Sigma'} v_3^{u\Sigma'} \end{pmatrix}$$

$$M_D = \begin{pmatrix} Y_1^\Phi v_1^{d\Phi} & Y_1^\Phi v_2^{d\Phi} & Y_1^\Phi v_3^{d\Phi} \\ Y_2^\Phi v_1^{d\Phi} & Y_2^\Phi v_2^{d\Phi} + Y_2^{\Sigma'} v_2^{d\Sigma'} & Y_2^\Phi v_3^{d\Phi} + Y_2^{\Sigma'} v_3^{d\Sigma'} \\ Y_3^\Phi v_1^{d\Phi} & Y_3^\Phi v_2^{d\Phi} + Y_3^{\Sigma'} v_2^{d\Sigma'} & Y_3^\Phi v_3^{d\Phi} + Y_3^\Sigma v_3^{d\Sigma} + Y_3^{\Sigma'} v_3^{d\Sigma'} \end{pmatrix},$$

Both Φ, Σ needed to avoid $M_E \propto M_D$

$$M_E = \begin{pmatrix} Y_1^\Phi v_1^{d\Phi} & Y_1^\Phi v_2^{d\Phi} & Y_1^\Phi v_3^{d\Phi} \\ Y_2^\Phi v_1^{d\Phi} & Y_2^\Phi v_2^{d\Phi} - 3Y_2^{\Sigma'} v_2^{d\Sigma'} & Y_2^\Phi v_3^{d\Phi} - 3Y_2^{\Sigma'} v_3^{d\Sigma'} \\ Y_3^\Phi v_1^{d\Phi} & Y_3^\Phi v_2^{d\Phi} - 3Y_3^{\Sigma'} v_2^{d\Sigma'} & Y_3^\Phi v_3^{d\Phi} - 3Y_3^\Sigma v_3^{d\Sigma} - 3Y_3^{\Sigma'} v_3^{d\Sigma'} \end{pmatrix}.$$

2 generations Σ, Σ' needed to distinguish down and charged lepton sectors in more than 1 family

Flavor: quarks and charged leptons

Independent parameters

$Y^{\Phi}_{1,2,3}$, Y^{Σ}_{3}, $Y^{\Sigma'}_{2,3}$, Y_R, real

$v^{u\Phi}_{1,2,3}$, $v^{u\Sigma}_{1,2,3}$, $v^{u\Sigma'}_{1,2,3}$, $v^{d\Phi}_{1,2,3}$, $v^{d\Sigma}_{3}$, $v^{d\Sigma'}_{2,3}$, $Z^*_{11,22,33,12,13,23}$

$\sum v_i^2 = (246\ \mathrm{GeV})^2/2$

$$(M_U)_{IA} = Y^{\Phi}_I v^{u\Phi}_A + Y^{\Sigma}_I v^{u\Sigma}_A + Y^{\Sigma'}_I v^{u\Sigma'}_A\,,$$
$$(M_D)_{IA} = Y^{\Phi}_I v^{d\Phi}_A + Y^{\Sigma}_I v^{d\Sigma}_A + Y^{\Sigma'}_I v^{d\Sigma'}_A\,,$$
$$(M_E)_{IA} = Y^{\Phi}_I v^{d\Phi}_A - 3\,Y^{\Sigma}_I v^{d\Sigma}_A - 3\,Y^{\Sigma'}_I v^{d\Sigma'}_A$$

$M_\nu = -M_{LR} M^{-1}_{RR} M^T_{LR}$. Completely determined by $Z^*_{11,22,33,12,13,23}$ (appearing nowhere else)

8 independent diagonal entries in (M_U, M_D, M_E) fix 8 masses

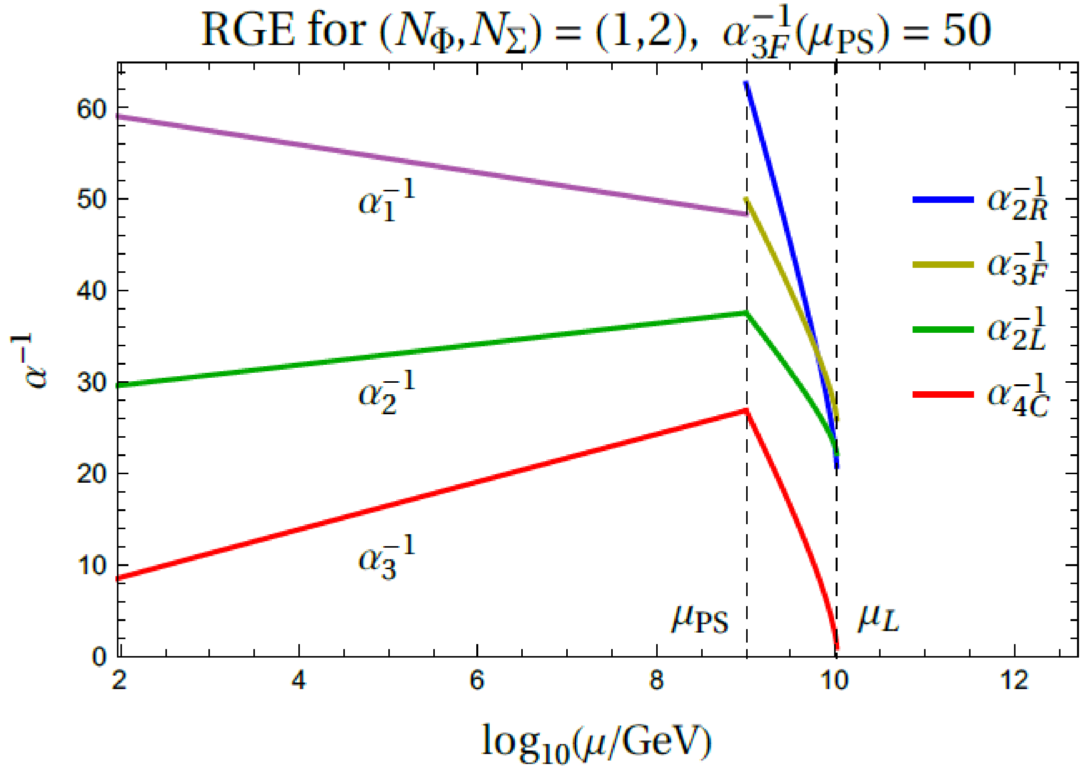
$+\,\mathrm{upp} M_U$ 3 extra entries to fix CKM angles

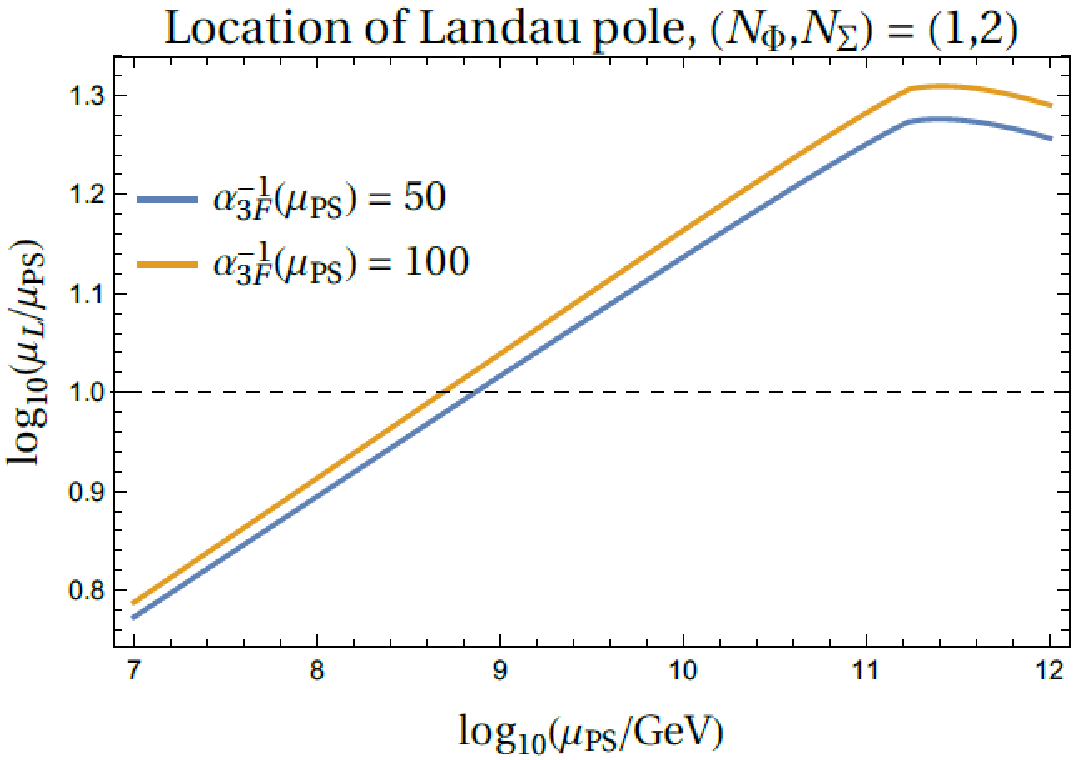
$+\,\mathrm{upp}(M_D, M_E)$ 3 extra entries, use 1 to fix last mass

Large number of phases fixes CKM and PMNS phases

set of entries	# independent
$\{\mathrm{diag}\, M_U, \mathrm{diag}\, M_D, \mathrm{diag}\, M_E\}$	8
$\{\mathrm{upp}\, M_U, \mathrm{diag}\, M_D, \mathrm{diag}\, M_E\}$	11
$\{\mathrm{upp}\, M_U, \mathrm{upp}\, M_D, \mathrm{upp}\, M_E\}$	14
$\{M_U, M_D, M_E\}$	18

Perturbativity





Landau pole of gauge couplings at scale O(10) x PS scale

New physics around the LP scale → We must require that this new physics is PQ-conserving in order not to worsten the quality problem!

Anomalon production

Thermal anomalons are already excluded → Freeze-in production of $Y_\Psi = n_\Psi / s$

$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\rm dec})} \right)^{4/3} > 0.285$$

$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96 \, Y_\Psi / g_s (T_{\rm FI})^{1/3}$$

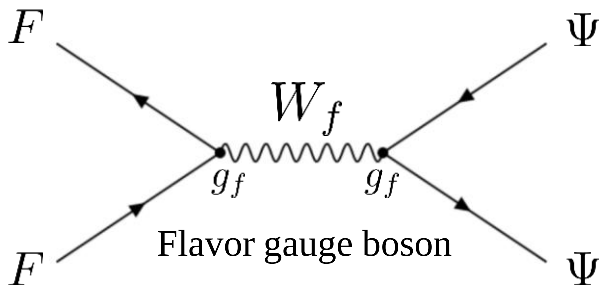
Anomalon production

Thermal anomalons are already excluded ➡ Freeze-in production of $Y_\Psi = n_\Psi / s$

$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24}\left(\frac{106.75}{g_s(T_{\rm dec})}\right)^{4/3} > 0.285$$

$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96\, Y_\Psi / g_s(T_{\rm FI})^{1/3}$$

<u>Flavor production</u>



$$Y_\Psi^{\rm flavor} \sim \begin{cases} 0.3\, g_f M_{\rm Pl} \,/\, g_s^{3/2} V & \text{if} \quad g_f \ll 10^{-9}\,(V \,/\, 10^9 \text{ GeV}) \\ 0.9\, M_{\rm Pl} T_{\rm RH}^3 \,/\, g_s^{3/2} V^4 & \text{if} \quad T_{\rm RH} \ll 10^6 \text{ GeV} \left(\frac{V}{10^9 \text{ GeV}}\right)^{4/3} \end{cases}$$

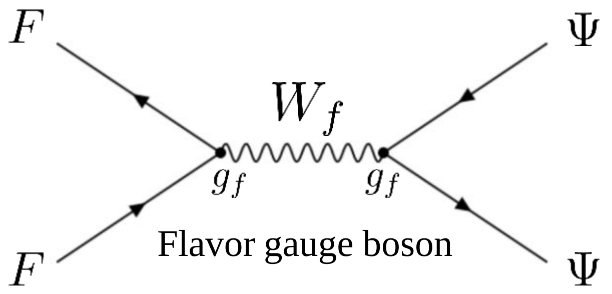
Anomalon production

Thermal anomalons are already excluded → Freeze-in production of $Y_\Psi = n_\Psi / s$

$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\rm dec})}\right)^{4/3} > 0.285$$

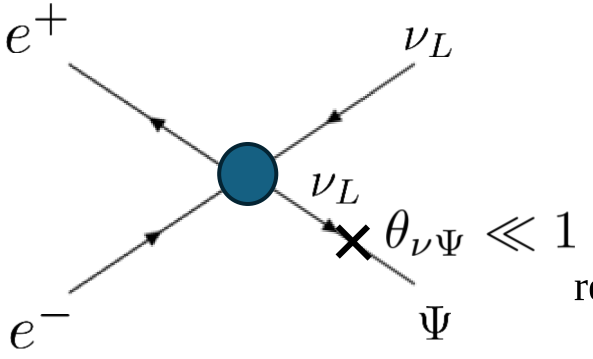
$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96\, Y_\Psi / g_s(T_{\rm FI})^{1/3}$$

Flavor production



$$Y_\Psi^{\rm flavor} \sim \begin{cases} 0.3\, g_f M_{\rm Pl} \,/\, g_s^{3/2} V & \text{if} \quad g_f \ll 10^{-9}\, (V \,/\, 10^9 \text{ GeV}) \\ 0.9\, M_{\rm Pl} T_{\rm RH}^3 \,/\, g_s^{3/2} V^4 & \text{if} \quad T_{\rm RH} \ll 10^6 \text{ GeV} \left(\frac{V}{10^9 \text{ GeV}}\right)^{4/3} \end{cases}$$

Neutrino mixing



$$Y_\Psi^{\nu-\rm mix} \sim 2.5 \times 10^4 \sum_{m=1}^{N_\Psi} \left(|\theta_{\nu\Psi_m}|^2 \frac{m_{\Psi_m}}{\rm keV}\right) \quad \text{if} \quad m_\Psi \gg m_\nu$$

require numerical solution of quantum kinetic equations if $m_\Psi \ll m_\nu$

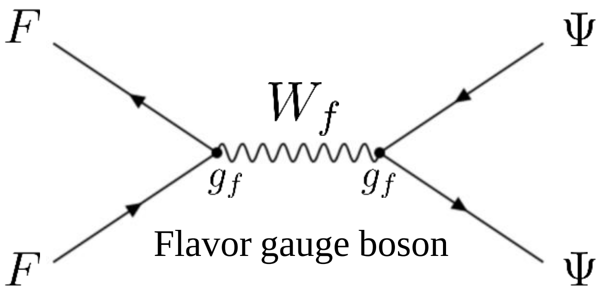
Anomalon production

Thermal anomalons are already excluded ➡ Freeze-in production of $Y_\Psi = n_\Psi / s$

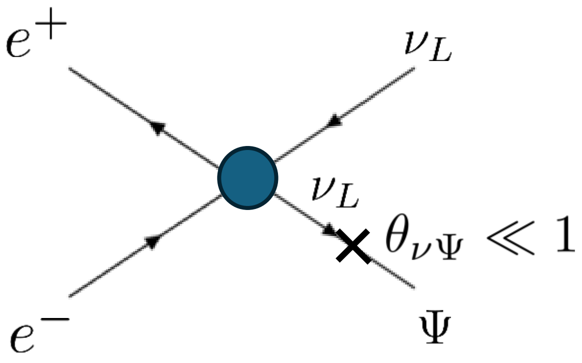
$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24}\left(\frac{106.75}{g_s(T_{\rm dec})}\right)^{4/3} > 0.285$$

$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96\, Y_\Psi / g_s(T_{\rm FI})^{1/3}$$

Flavor production



Neutrino mixing



<u>Our results</u>

$\Delta N_{\rm eff}^{\rm FI} \ll 0.014$ *deep* freeze-in regime

BUT

$0.014 \lesssim \Delta N_{\rm eff}^{\rm FI} \lesssim 1.13$ thermalization / freeze-in border

$g_f \lesssim 10^{-9}$ or $T_{\rm RH} \lesssim 10^6$ GeV

or $\theta_{\nu\Psi} \sim 10^{-[2,3]}$

⬇

More precise computation is needed

Freeze – in of dark radiation

Alternatively, we can drop the assumption $\rho_\Psi \propto n_\Psi^{4/3}$ and estimate $\Delta N_{\rm eff}$ as follows: the freeze-in production of the anomalons is maximally efficient at some temperature $T_{\rm FI}$ (which depends on the specific production mechanism). Since anomalons are produced by scatterings of SM bath particles, each one carrying energy $E \sim T$, their average energy immediately after production is also of order $\sim T$. Therefore, we can parametrize their energy density as $\rho_\Psi(T_{\rm FI}) \simeq \xi T_{\rm FI} n_\Psi(T_{\rm FI}) = \xi T_{\rm FI} s(T_{\rm FI}) Y_\Psi$, where ξ is an $\mathcal{O}(1)$ number.

The energy density of a relativistic particle redshifts as a function of the scale factor $a_T \equiv a(T)$ as $\rho_\Psi(T) = \rho_\Psi(T_{\rm FI})(a_{T_{\rm FI}}/a_T)^4 = \rho_\Psi(T_{\rm FI})(T/T_{\rm FI})^4(g_s(T)/g_s(T_{\rm FI}))^{4/3}$. At the time of the CMB, this corresponds to

$$\Delta N_{\rm eff}^{\rm FI2} \simeq \frac{16}{21}\left(\frac{11}{4}\right)^{4/3} \frac{g_s(T_{\rm CMB})^{4/3}}{g_s(T_{\rm FI})^{1/3}} \xi Y_\Psi \simeq 56.96\left(\frac{\xi}{3.151}\right)\frac{Y_\Psi}{g_s(T_{\rm FI})^{1/3}}\,, \tag{5.17}$$

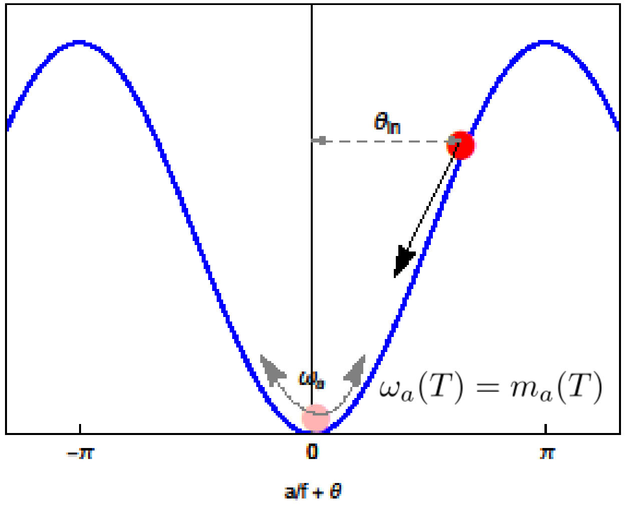
Axion production

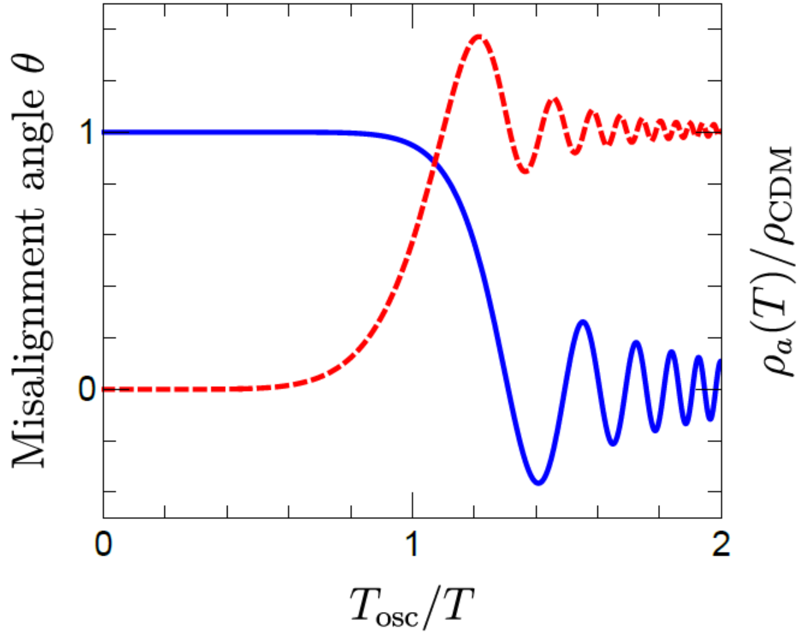
$$\ddot{\theta}_{\text{eff}} + 3H\dot{\theta}_{\text{eff}} + m_a^2(T)\theta_{\text{eff}} = 0$$

$H \propto T^2/M_{\text{Pl}}$

$m_a(T) \simeq m_a(T_c/T)^4$

$T_c \simeq 160$ MeV





$$\frac{\Omega_a h^2}{0.12} \approx \theta_{\text{in}}^2 \left(\frac{f_a}{2\times10^{11}\ \text{GeV}}\right)^{7/6}$$

Solutions to the quality problem

$$\lambda\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\rm QCD}^4$$

Particle Content	Gravitational Theory	Action Scaling	Quality Problem?	Refs.
Free Axion	Einstein Gravity	$M_{\rm pl}/f_a$	No	[32, 33]
Axion, Dynamical Radial Mode (incl. Arbitrary f Potential)	Einstein Gravity, *or* Kaluza-Klein/$f(\mathcal{R})$ Gravity	$\log M_{\rm pl}/f_a$	Yes	[30, 34]
Extended Global Symmetry	Einstein Gravity	$\sum_i \log M_{\rm pl}/f_{a,i}$	Yes	This Work
Axion, Dilaton	(Open*) String Theory	$(M_{\rm pl}/f_a)\cdot g_s^{-1}$	No	[35–40]
Axion, Dilaton, Dynamical Radial Mode		$\log(M_{\rm pl}/f_a)\cdot g_s^{-1}$	Yes	This Work

Studied in the context of Euclidian Wormholes

[Lee ‘88, Giddings, Strominger ‘88] $S \sim M_{\rm Pl}/f_a$ No quality problem

[Abbott, Wise ‘88, Alvey, Escudero ’20] $S \sim \log M_{\rm Pl}/f_a$ Quality problem

D = 9 operator vanishes on the vacuum

1. For $m = 0$ we have $n = 9$: the dominant candidate is V^9, which can come only from the operator $\Delta^3 \chi^{*6}$. This contribution in fact vanishes on the vacuum, essentially due to anti-symmetrization in flavor contraction while only having one copy of Δ and χ available. The argument applies to all 100 independent contractions of this type of invariant, where the counting was performed using condition 3 from App. B.2. To demonstrate this somewhat remarkable result, notice that there is only one SM-singlet in the Pati-Salam part of irreps Δ and χ, i.e. there is one flavor **6**-plet VEV in Δ and one flavor **3**-plet VEV in χ^*. Using `LiE` for only the $\mathrm{SU}(3)_{f_R}$ factor, the number of singlets **1** in the tensor product $\mathbf{6}^{\otimes_s 3} \otimes \mathbf{3}^{\otimes_s 6}$ is zero, where an exponent $\otimes_s n$ denotes the n-th symmetrized tensor power. This argument applies regardless of the choice of index contraction in $\Delta^3 \chi^{*6}$.

   It is instructive to demonstrate explicitly how this argument works. Given the VEV structure from Eqs. (B.7) and (B.8), the $\mathrm{SU}(4)_{\mathrm{PS}}$ indices must take the value 4 and $\mathrm{SU}(2)_R$ indices must take the value 1, and thus a contribution must necessarily have the VEV form

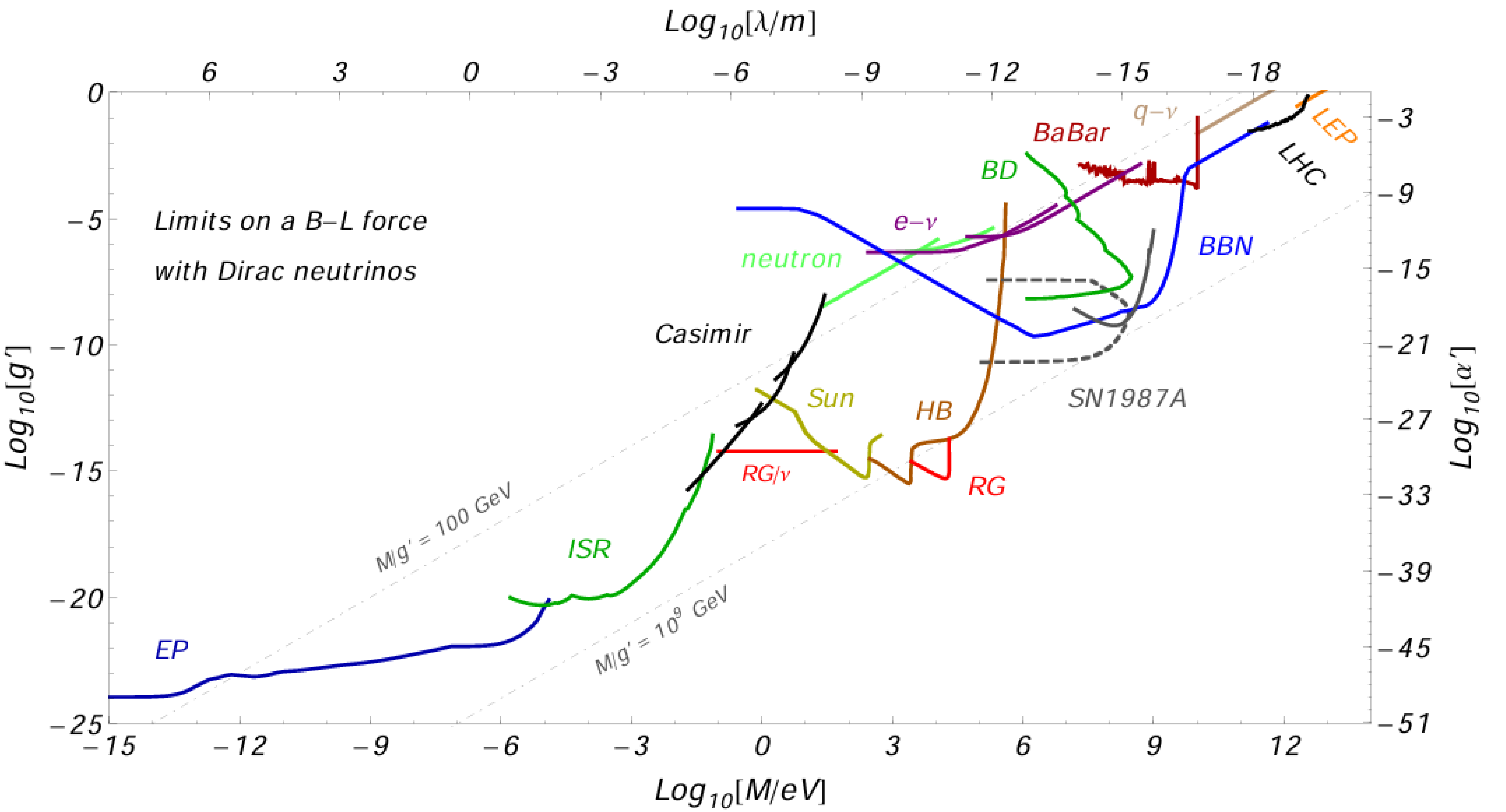
$$Z^{AB}\; Z^{CD}\; Z^{EF}\; V^{*G}\; V^{*H}\; V^{*I}\; V^{*J}\; V^{*K}\; V^{*L}, \tag{B.15}$$

   where the 12 flavor indices need to be contracted with an SU(3)-invariant tensor, for example four Levi-Civita tensors. Such a contraction will always contain sufficient anti-symmetry in index exchange contraction so that Eq. (B.15) vanishes.

Future directions

- A complete analysis of the PQ quality problem at the quantum level, including loop-induced effects, remains an essential next step (cf. App. B.3.2). Extending the current tree-level investigation to incorporate quantum corrections will provide a more robust understanding of the model's viability in addressing the PQ quality problem.

- The computation of anomalon abundance via neutrino mixing in the early universe, in the regime $m_\Psi \lesssim m_\nu$ (parametrically favored by the solution to the PQ-quality problem), requires a treatment based on quantum kinetic equations. This is a computationally intensive task, particularly when considering multiple sterile states and flavor mixing among active neutrino states. While analytical approximations have been employed for sterile neutrino dark matter, a full numerical solution is necessary to accurately determine the anomalon production rates in this regime. Addressing these challenges will refine the constraints on the parameter space of the model by providing a more robust determination of ΔN_{eff}.


Limits on a B–L force
with Dirac neutrinos
$Log_{10}[\lambda/m]$
$Log_{10}[g']$
$Log_{10}[\alpha']$
$Log_{10}[M/eV]$
EP
ISR
Casimir
neutron
Sun
RG/ν
HB
RG
e−ν
BD
BaBar
q−ν
LEP
LHC
BBN
SN1987A
M/g′ = 100 GeV
M/g′ = 10^9 GeV

keV – anomalon dark matter

For instance, if $V = 10^{14}\,\mathrm{GeV}$ ($f_a \sim 10^{13}\,\mathrm{GeV}$), the 8 light anomalons have mass $m_0 \sim 1 \div 10\,\mathrm{keV}$. As long as $T_{\mathrm{RH}} \lesssim V(V/M_{\mathrm{Pl}})^{1/3} \sim 10^{12}\,\mathrm{GeV}$ (pre-inflationary) or $g_{f_R} \ll 10^{-4}$ (post-inflationary), they are produced by freeze-in. The mixing with active neutrinos depends mostly on the parameters $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$, as shown in Fig. 4 (lower right). Mixings larger than roughly $10^{-(6\div5)}$ are already excluded by X-ray searches, while smaller angles are allowed. The population produced via mixing can still be the larger one even if mixings are small, $Y_{\mathrm{light}}^{\nu-\mathrm{mix}} \lesssim 10^{-8}(\theta_{\nu\Psi}/10^{-7})^2(m_0/10\,\mathrm{keV})$. This corresponds to a very small fraction of the total dark matter abundance, $\Omega_{\Psi,0}/\Omega_{\mathrm{DM}} \sim 10^{-3}$ (potentially up to 1% if we push T_{RH} up to $\sim 10^{12}\,\mathrm{GeV}$, where the computation is not fully reliable), which is not constrained by warm dark matter limits.

On the other hand, if $V = 10^{11}\,\mathrm{GeV}$, the picture reverses, as the 16 heavy anomalons are now the keV-ish ones, $m_\perp \sim 10\,\mathrm{keV}$. They possibly contribute to a small fraction of dark matter, analogous to the discussion above. The mixing angles are mostly determined by the coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$, see Fig. 4 (upper center), and tend to be slightly larger than in the previous case, so that X-ray searches would impose more severe constraints on the model parameters.

Anomalon decays

Anomalons with mass in the $1 \div 100\,\mathrm{keV}$ range dominantly decay via neutrino mixing to $3\nu_L$, with decay width

$$\Gamma_{3\nu} = \frac{G_F^2 m_\Psi^5 \theta_{\nu\Psi}^2}{96\pi^3} \,. \tag{5.12}$$

They are cosmologically stable if $\theta_{\nu\Psi} \lesssim 0.018(10\,\mathrm{keV}/m_\Psi)^{5/2}$. The decay channel $\Psi_R \to \nu_L\gamma$ is also open with rate

$$\Gamma_{\nu\gamma} = \frac{9\alpha_{\rm em} G_F^2 m_\Psi^5 \theta_{\nu\Psi}^2}{256\pi^4} \,. \tag{5.13}$$

Although subdominant, the latter process actually imposes the strongest constraints on the mixing; X-ray searches exclude $\theta_{\nu\Psi} \gtrsim 2\times 10^{-6}(10\,\mathrm{keV}/m_\Psi)^{5/2}$ (see [138]).

Ultra-light anomalons could decay via the same channels of keV-ish ones if $m_\Psi > m_\nu$, but they are cosmologically stable, as their decays are suppressed by their small mass $\Gamma \propto (m_\Psi/v)^4$. The same also applies to the decays of active neutrinos into anomalons when $m_\Psi < m_\nu$.

Finally, notice that the decays of heavy anomalons to lighter ones are suppressed, as they involve the exchange of a flavor gauge boson. These decays are completely irrelevant in all scenarios: heavy anomalons $m_\perp \gtrsim$ TeV decay much faster via the other decay channels discussed above; ultra-light anomalons are still cosmologically stable; for keV-ish anomalons we should compare these decays with Eq. (5.12). The exchange of a flavor gauge boson implies a decay rate, which is suppressed at least by $1/(G_F^2 V^4 \theta_{\nu\Psi}^2) \sim (v/V)^4 (1/\theta_{\nu\Psi})^2$ compared to the decay channel into $3\nu_L$. Thus, even in the most pessimistic case ($V \sim 10^9\,\mathrm{GeV}$), the decays to active neutrinos are dominant unless $\theta_{\nu\Psi} \lesssim 10^{-13}$.