Minimal dark matter in SU(5) grand unification

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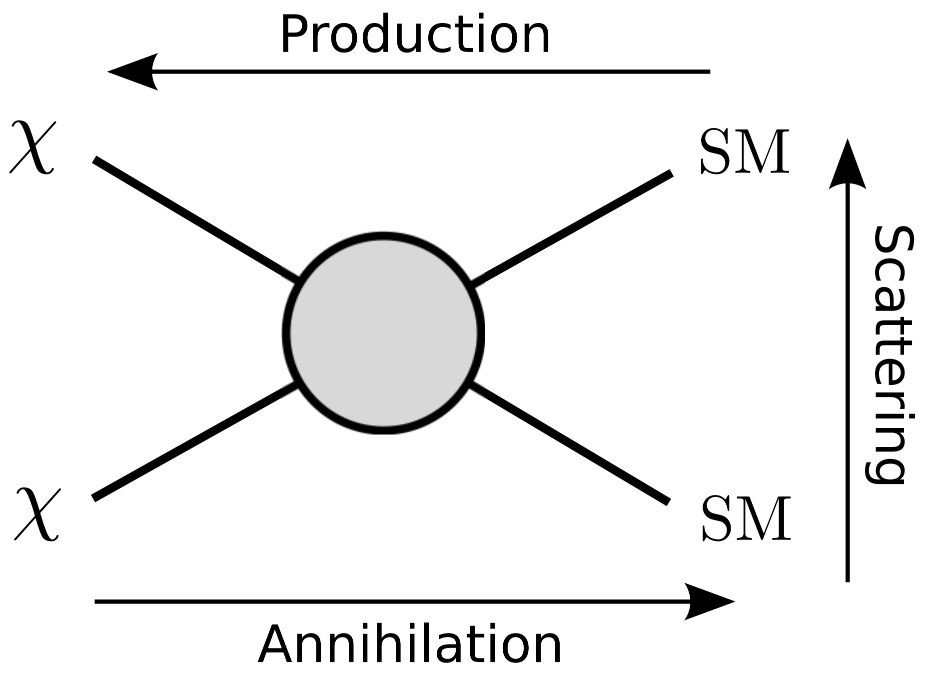
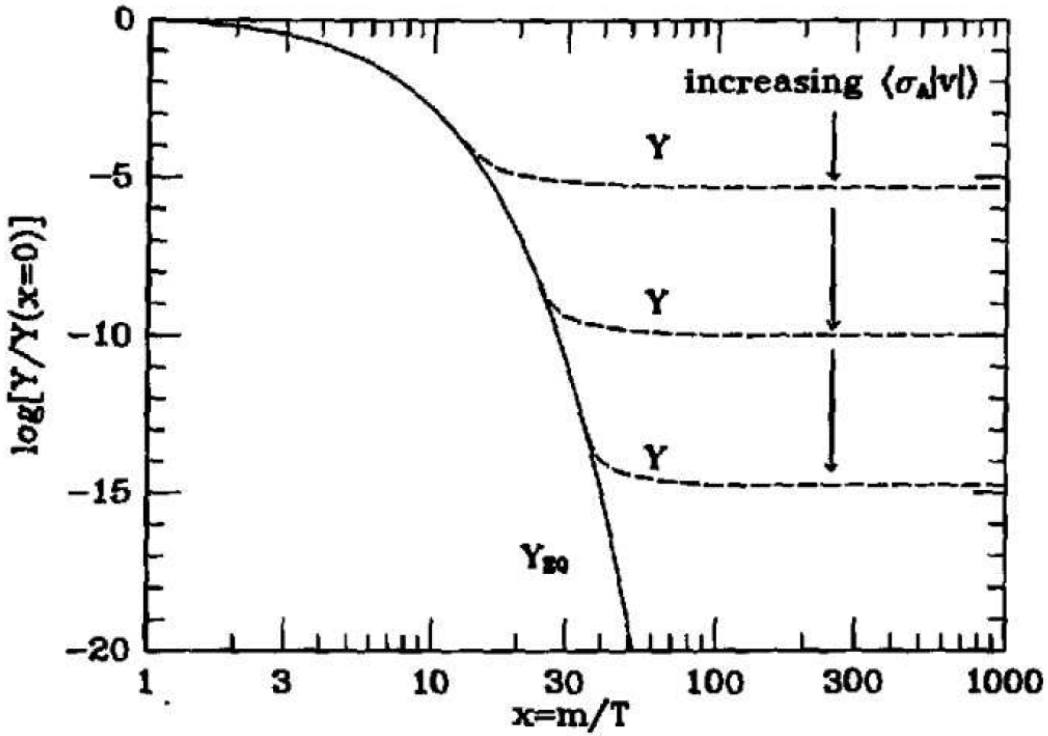
Outline

1. Thermal DM and experimental status
   Minimal dark matter (MDM)
2. Embedding MDM in SU(5) GUT and gauge coupling unification
3. Additional comments
4. Summary

Thermal dark matter

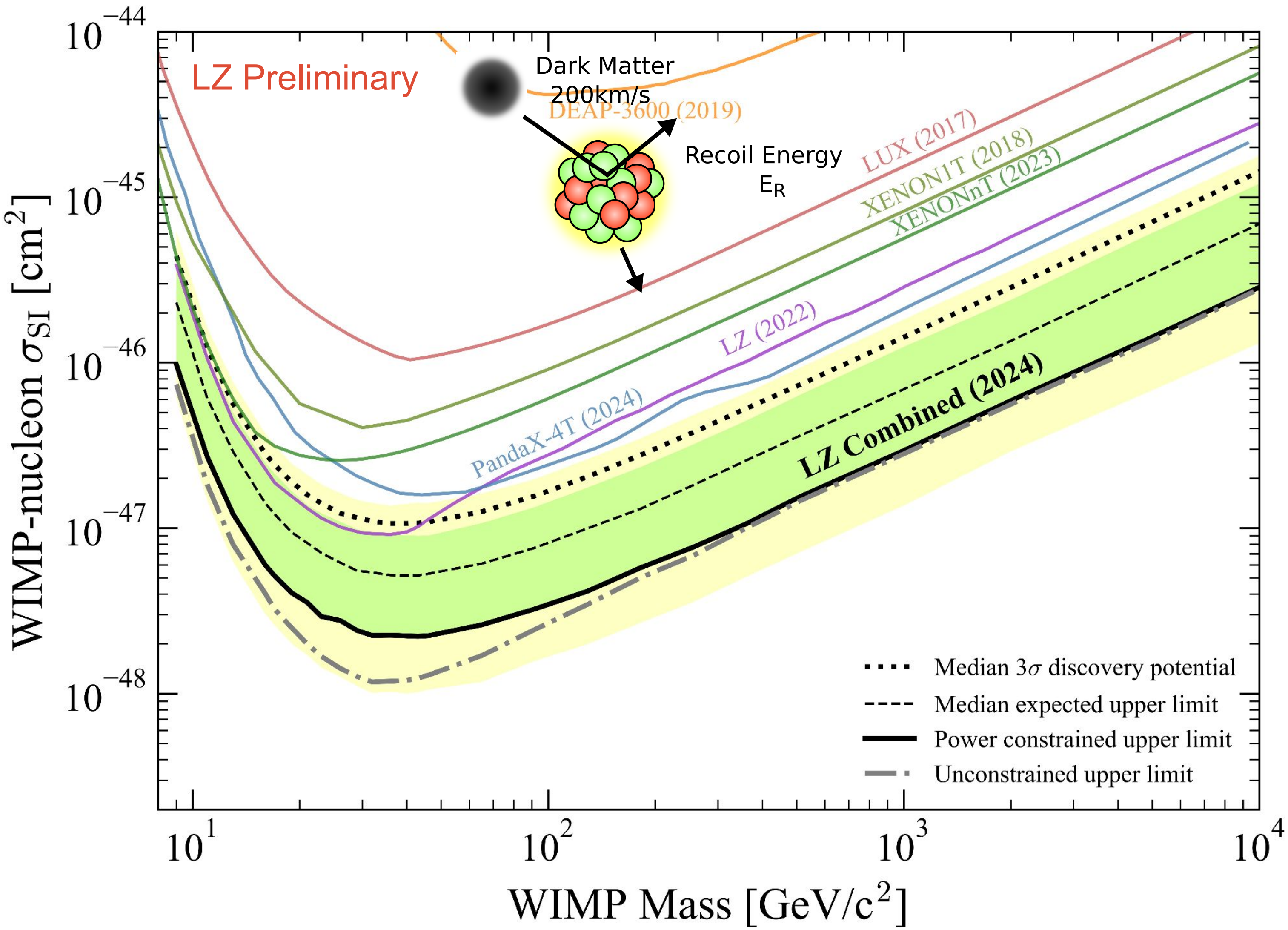


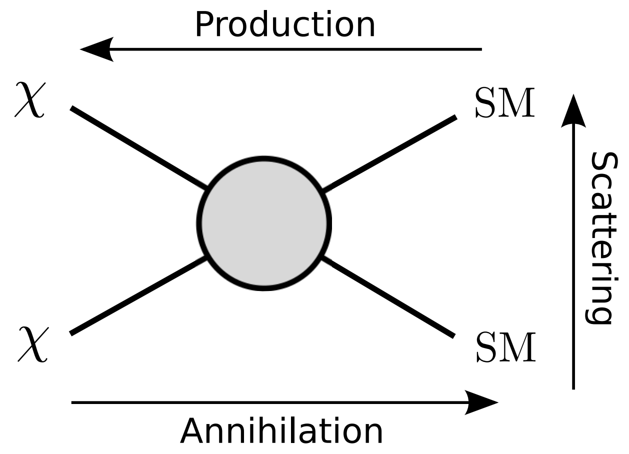


$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_\chi^{\mathrm{eq}2}\right)$$

- Thermalized with SM particles in early universe.
- To get $\Omega_\chi h^2 = 0.12$, roughly $\sigma \sim$ 1pb $\sim 10^{-26}\mathrm{cm}^3/\mathrm{s} \sim 10^{-36}\mathrm{cm}^2$ (only log dependent on DM mass)
- Mass range: 10 MeV – 100 TeV

Status of direct detection experiments





- $\Omega h^2 = 0.12$
  $\Leftrightarrow \sigma_{\rm el} \sim 10^{-36}$ cm^2
  $(\sigma_{\rm el} \lesssim 10^{-45}$ cm$^2)$
  (loop)

LZ talk @ TeVPA2024

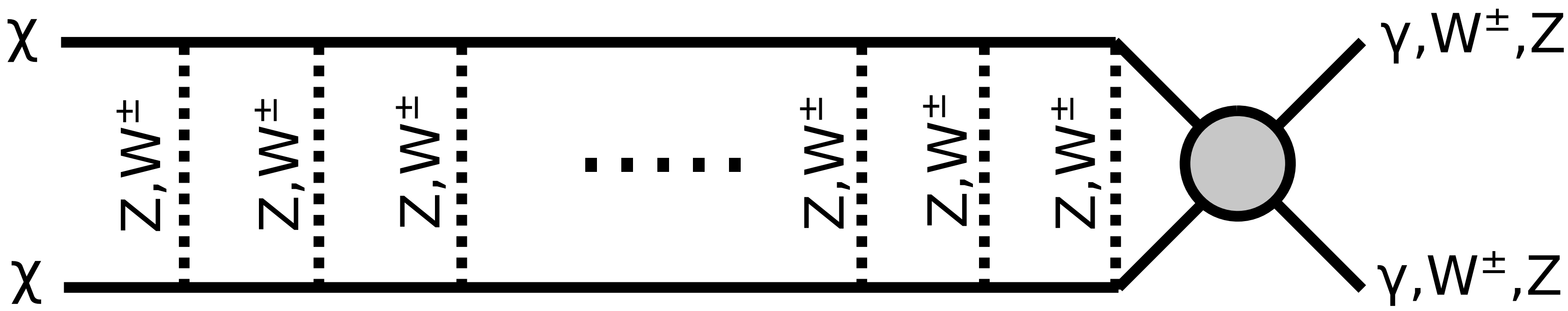
- LZ gives the strongest bound 2.2×10^{-48} cm^2 at 43 GeV.

Minimal dark matter (MDM)

M. Cirelli et al., Nucl.Phys.B (2005) [hep-ph/0512090]
Farina et al., JHEP (2013) [arxiv:1303.7244]

Quantum numbers $SU(2)_L$	$U(1)_Y$	Spin	DM could decay into	DM mass in TeV	$m_{DM^\pm} - m_{DM}$ in MeV	σ_{SI} in 10^{-46} cm^2
5	0	1/2	stable	$4.4 \to 14$	166	1.0 ± 0.2
7	0	0	stable	$8 \to 25$	166	4 ± 1

- Lagrangian: $\mathcal{L} = \frac{1}{2}\overline{\chi}\left(i\not{D} - M_5\right)\chi$ or $\mathcal{L} = \frac{1}{2}\left(D_\mu\chi\right)^*\left(D^\mu\chi\right) - \frac{1}{2}M_7\chi^2$
- No other interactions $\Rightarrow$ χ^0 is stabilized
  (Thermal relic via by $\chi\chi \to$ gauge bosons, mass $M_5 \sim$ 14 TeV)
- Sommerfeld enhancement and bound state formation



MDM in SU(5) GUT

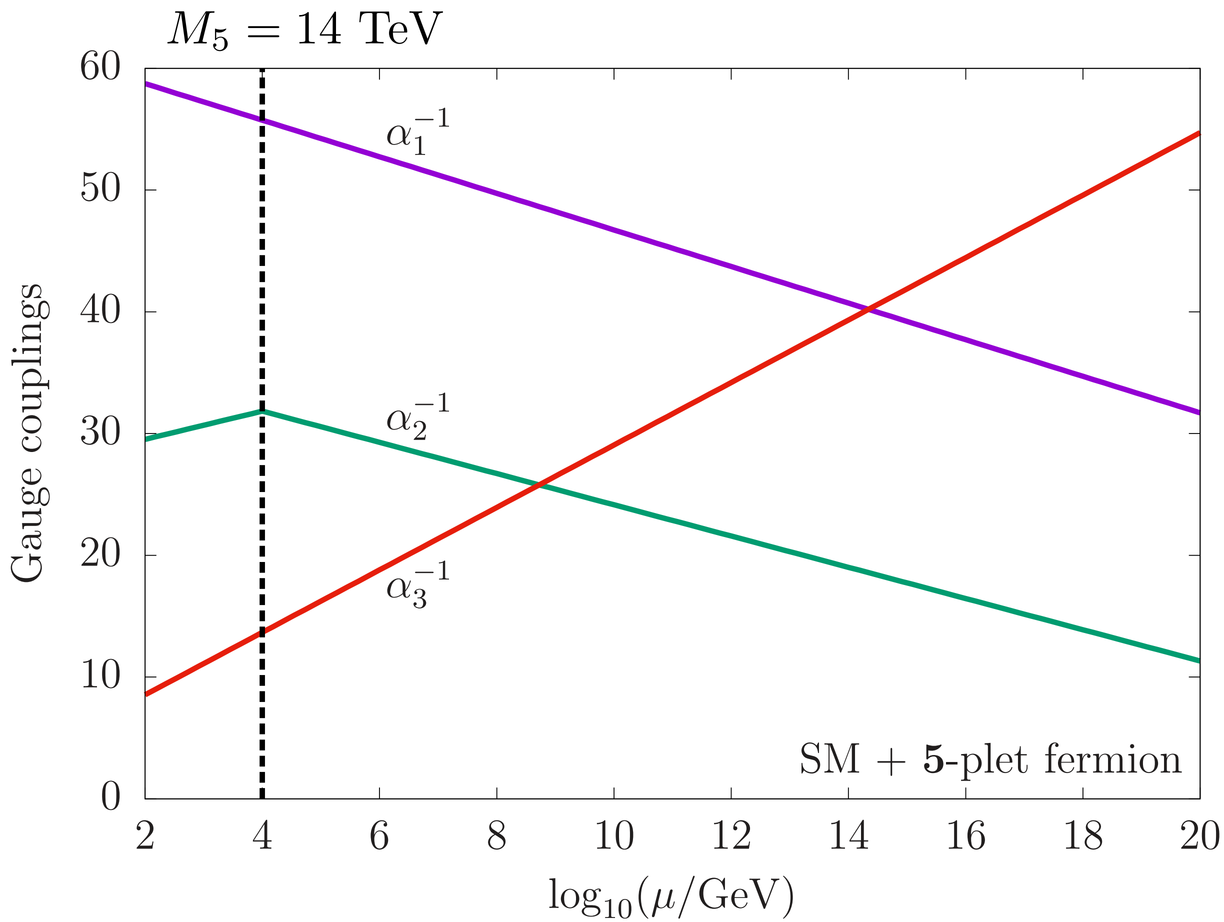
Motivation

- **Can fermionic MDM achieve gauge coupling unification?**
- Big change of the running at $\mu = M_5 \sim$ 14 TeV
  $\Rightarrow$ need to add colored fields with non-zero Y, but singlet for $SU(2)_L$.

$$(\boldsymbol{n}, \mathbf{1})_Y$$

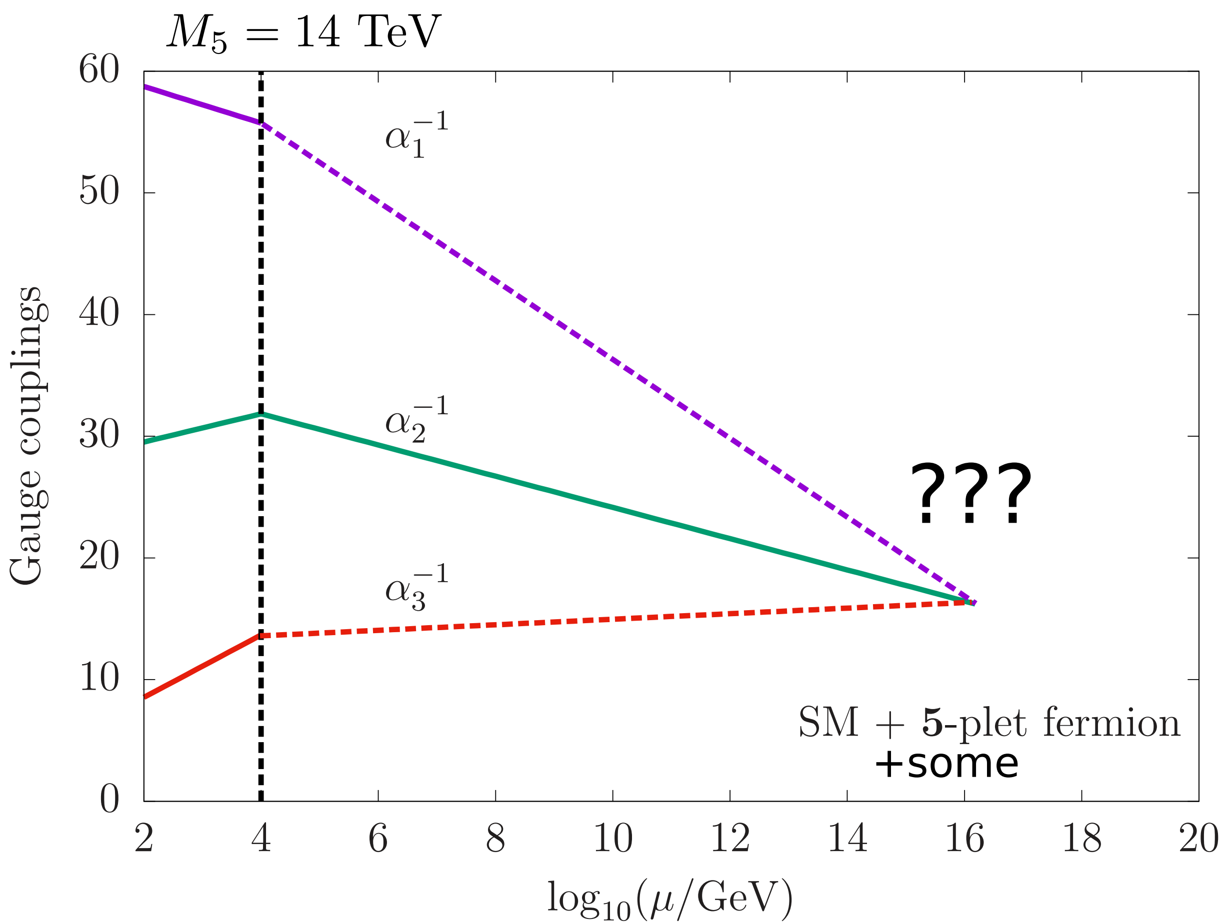
Required conditions

- No additional scale other than $\mathcal{O}(10)$ TeV
- No Landau pole up to Planck scale
- Extra fields have to be embedded in SU(5).
  $\Rightarrow$ possible reps. are limited. $\mathbf{5}$-plet $\subset \mathbf{200}$



MDM in SU(5)

- $\mathbf{200} = (\mathbf{6}, \mathbf{3})_{-5/3} + (\mathbf{15}, \mathbf{2})_{-5/6} + (\mathbf{3}, \mathbf{4})_{-5/6} + (\mathbf{3}, \mathbf{2})_{-5/6}$
  $+(\mathbf{27}, \mathbf{1})_0 + (\mathbf{8}, \mathbf{3})_0 + (\mathbf{8}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{5})_0 + (\mathbf{1}, \mathbf{3})_0 + (\mathbf{1}, \mathbf{1})_0$
  $+(\overline{\mathbf{6}}, \mathbf{3})_{5/3} + (\overline{\mathbf{15}}, \mathbf{2})_{5/6} + (\overline{\mathbf{3}}, \mathbf{4})_{5/6} + (\overline{\mathbf{3}}, \mathbf{2})_{5/6}$
- Only $(\mathbf{1}, \mathbf{5})_0$ is light.
- Other states cannot be light.
  Otherwise Landau pole for g_2 appears below Planck scale.



Rough estimate at one-loop level

N. Yamatsu, arXiv:1511.08771 ($>$ 10000 pages)

- β functions:

  $$\frac{dg_i}{dt} = \frac{b_i g_i^3}{(4\pi)^2} \text{ where } b_1 = \frac{41}{10},\ b_2 = -\frac{19}{6},\ b_3 = -7 \text{ in SM}$$

  and $\Delta b_2 = \dfrac{20}{3}$ from $\mathbf{5}$-plet MDM

- Requirement for gauge coupling unification

  $\Rightarrow -2.9 \lesssim \Delta b_1 - \Delta b_3 \lesssim -1.6$ (additional contribution)

- Look for possible states $(\boldsymbol{n}, \mathbf{1})_Y$

  $(\mathbf{3},\mathbf{1})_{-1/3},\ (\overline{\mathbf{3}},\mathbf{1})_{1/3} \subset \mathbf{5}, \overline{\mathbf{5}}$, $(\overline{\mathbf{3}},\mathbf{1})_{-2/3},\ (\mathbf{3},\mathbf{1})_{2/3} \subset \mathbf{10}, \overline{\mathbf{10}}$,

  $(\mathbf{6},\mathbf{1})_{-2/3},\ (\overline{\mathbf{6}},\mathbf{1})_{2/3} \subset \mathbf{15}, \overline{\mathbf{15}}$, $\cdots$

  If two pairs of $(\mathbf{6},\mathbf{1})_{-2/3},\ (\overline{\mathbf{6}},\mathbf{1})_{2/3}$ are added

  $\Rightarrow \Delta b_1 - \Delta b_3 = -\frac{12}{5} = -2.4 \Leftarrow$ likely to work

- Two-loop calculation

β functions at two loop level

- Gauge couplings in SM

$$\frac{dg_i}{dt} = \frac{b_i g_i^3}{(4\pi)^2} + \sum_{j=1}^{3} \frac{b_{ij} g_i^3 g_j^2}{(4\pi)^4} + \frac{c_i g_i^3 y_t^2}{(4\pi)^4}$$

$$b_i = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}, \quad c_i = \begin{pmatrix} -17/10 \\ -3/2 \\ -2 \end{pmatrix}$$

- Top Yukawa y_t and Higgs self-coupling λ in SM

$$\frac{dy_t}{dt} = \frac{\beta_{y_t}^{(1)}}{(4\pi)^2} + \frac{\beta_{y_t}^{(2)}}{(4\pi)^4}, \quad \frac{d\lambda}{dt} = \frac{\beta_{\lambda}^{(1)}}{(4\pi)^2} + \frac{\beta_{\lambda}^{(2)}}{(4\pi)^4}$$

- Additional contribution due to a $\mathbf{5}$-plet fermion

$$\Delta b_2 = \frac{20}{3}, \quad \Delta b_{22} = \frac{560}{3}, \quad \Delta\beta_{y_t}^{(2)} = 5g_2^4, \quad \Delta\beta_{\lambda}^{(2)} = -2g_2^4\left(4g_1^2 + 20g_2^2 - 25\lambda\right)$$

β functions at two loop level 2

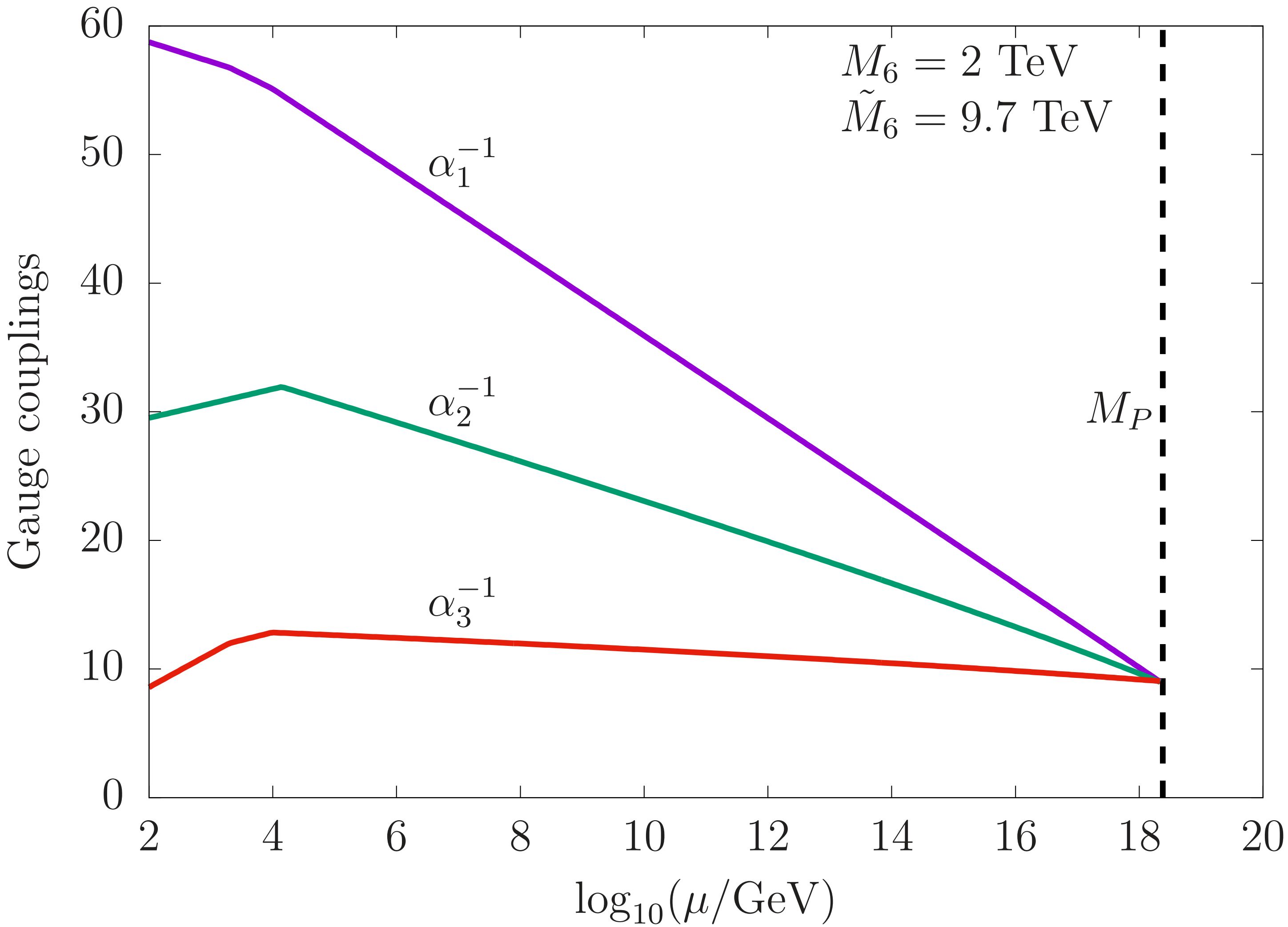
- Additional contribution due to a pair of sextet fermions $\Psi = (\mathbf{6}, \mathbf{1})_{-2/3}$, $\overline{\Psi} = (\overline{\mathbf{6}}, \mathbf{1})_{2/3}$

$$\Delta b_1 = \frac{32}{15}, \quad \Delta b_{11} = \frac{128}{75}, \Delta b_{13} = \frac{64}{3},$$

$$\Delta b_3 = \frac{10}{3}, \quad \Delta b_{31} = \frac{8}{3}, \quad \Delta b_{33} = \frac{250}{3},$$

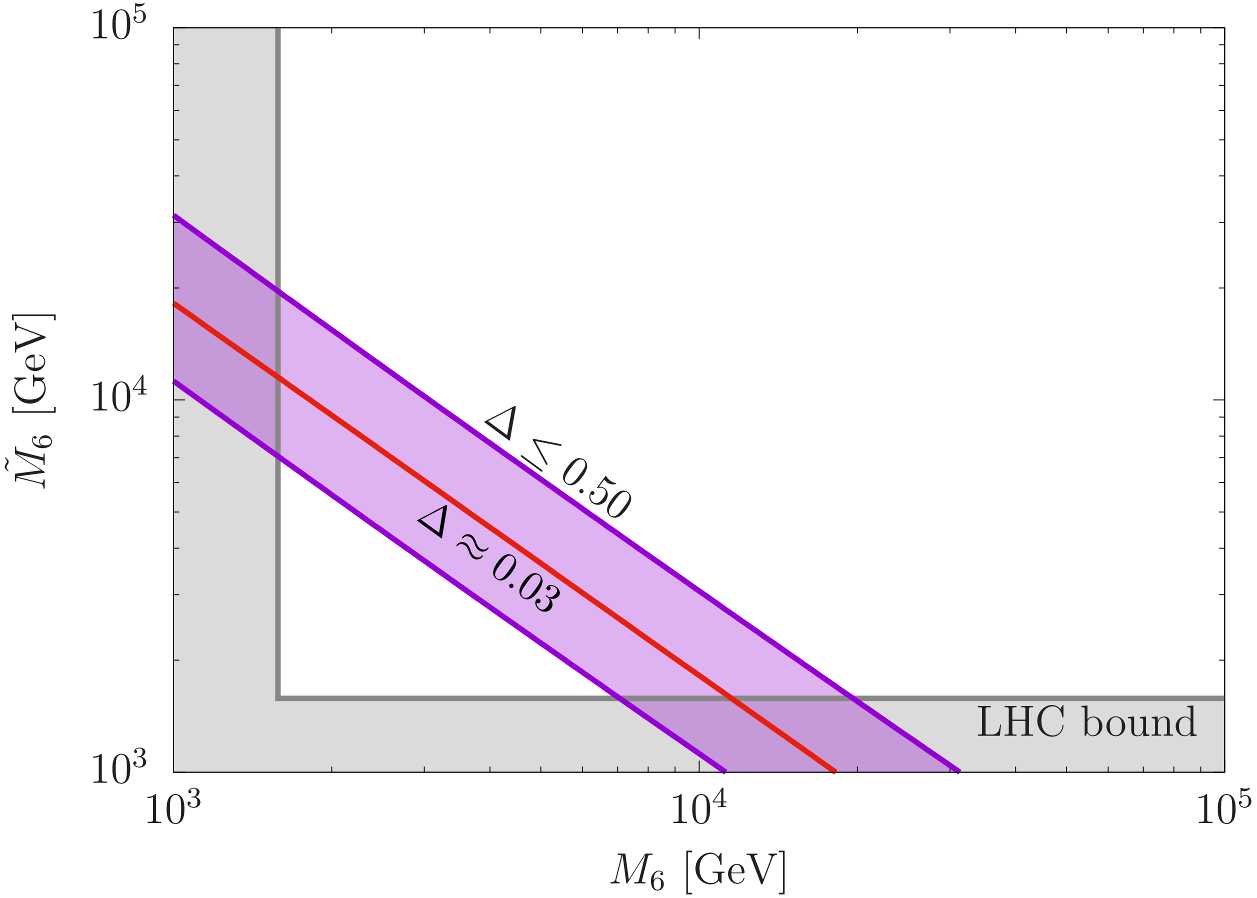
$$\Delta \beta_{y_t}^{(2)} = \frac{232}{225} g_1^4 + \frac{200}{9} g_3^4, \quad \Delta \beta_\lambda^{(2)} = -\frac{16}{125} g_1^4 \left(12 g_1^2 + 20 g_2^2 - 25\lambda\right)$$

Gauge coupling unification



- Unification scale M_U is close to the reduced Planck scale M_P
- No proton decay, $\tau_p \gtrsim 10^{39}$yrs $\gg \tau_{\rm exp} = 10^{34}$yrs

Gauge coupling unification 2



- Define criterion of unification (complete unification if $\Delta = 0$)

$$\Delta(\mu) \equiv \sqrt{\Delta\alpha_{12}^{-2}(\mu) + \Delta\alpha_{23}^{-2}(\mu)}, \quad \Delta\alpha_{ij}^{-1} = \alpha_i^{-1} - \alpha_j^{-1}$$

- 1.6 TeV $\lesssim M_6, \tilde{M}_6 \lesssim$ 20 TeV

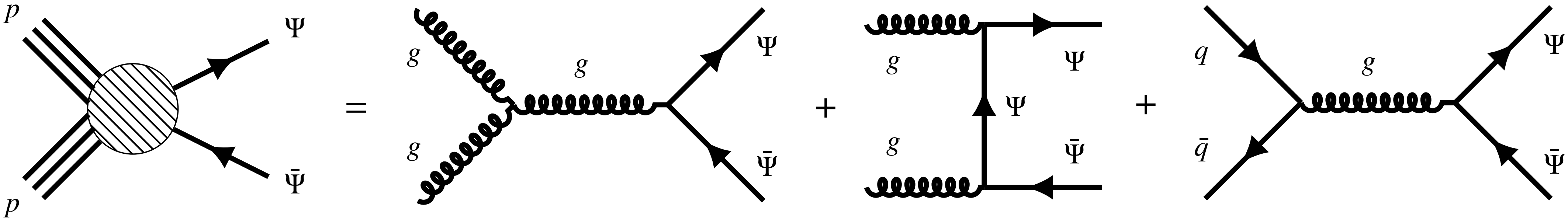
Mass splitting

- $\mathcal{L} = -\frac{M_{200}}{2}\mathbf{200}_F\mathbf{200}_F + \frac{Y_{200}}{2}\mathbf{24}_H\mathbf{200}_F\mathbf{200}_F$
- $\mathbf{5}$-plet:
  $M_{200} - Y_{200}\langle\mathbf{24}_H\rangle =$ 14 TeV
  Other states:
  $M_{200} - C_i Y_{200}\langle\mathbf{24}_H\rangle \sim M_U$
- Same for $(\mathbf{15}, \overline{\mathbf{15}})$ pairs.
- Dimopoulos-Wilczek mech.
  $\rightarrow$ no fine-tuning?
  $(\mathbf{1},\mathbf{5})_0 \subset \mathbf{2640}$ in SO(10)

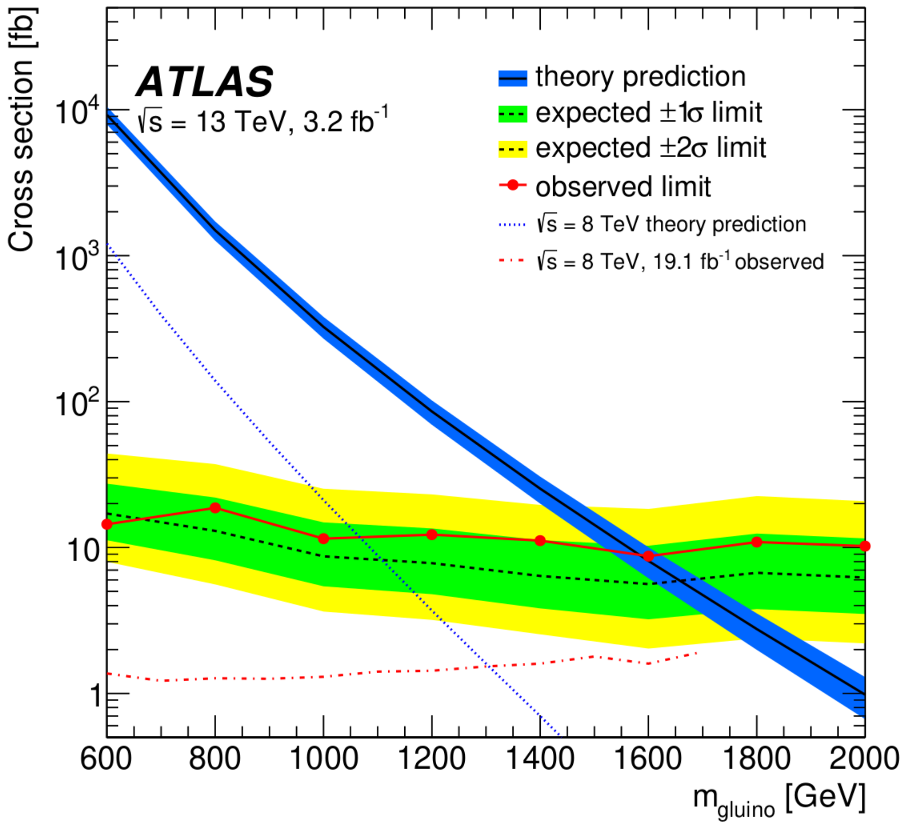
Representation			C_i
$\mathbf{24}_H$	$\mathbf{200}_F$	$\mathbf{200}_F$	
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{6},\mathbf{3})_{-5/3}$	$(\overline{\mathbf{6}},\mathbf{3})_{5/3}$	$1/6$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{15},\mathbf{2})_{-5/6}$	$(\overline{\mathbf{15}},\mathbf{2})_{5/6}$	$1/4$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{3},\mathbf{4})_{-5/6}$	$(\overline{\mathbf{3}},\mathbf{4})_{5/6}$	$-7/12$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{3},\mathbf{2})_{-5/6}$	$(\overline{\mathbf{3}},\mathbf{2})_{5/6}$	$-19/84$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{27},\mathbf{1})_0$	$(\mathbf{27},\mathbf{1})_0$	$2/3$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{8},\mathbf{3})_0$	$1/6$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{1})_0$	$1/14$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{5})_0$	$(\mathbf{1},\mathbf{5})_0$	1
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{3})_0$	$11/21$
$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_0$	$-2/7$
$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{2})_{5/6}$	$(\mathbf{3},\mathbf{2})_{-5/6}$	$-19/84$
$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{4})_{5/6}$	$(\mathbf{3},\mathbf{4})_{-5/6}$	$-7/12$
$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{15}},\mathbf{2})_{5/6}$	$(\mathbf{15},\mathbf{2})_{-5/6}$	$1/4$
$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{6}},\mathbf{3})_{5/3}$	$(\mathbf{6},\mathbf{3})_{-5/3}$	$1/6$

Exotic colored fermion search

Carpenter et al., arXiv:2110.11359, ATLAS Coll., arXiv: 1606.05129



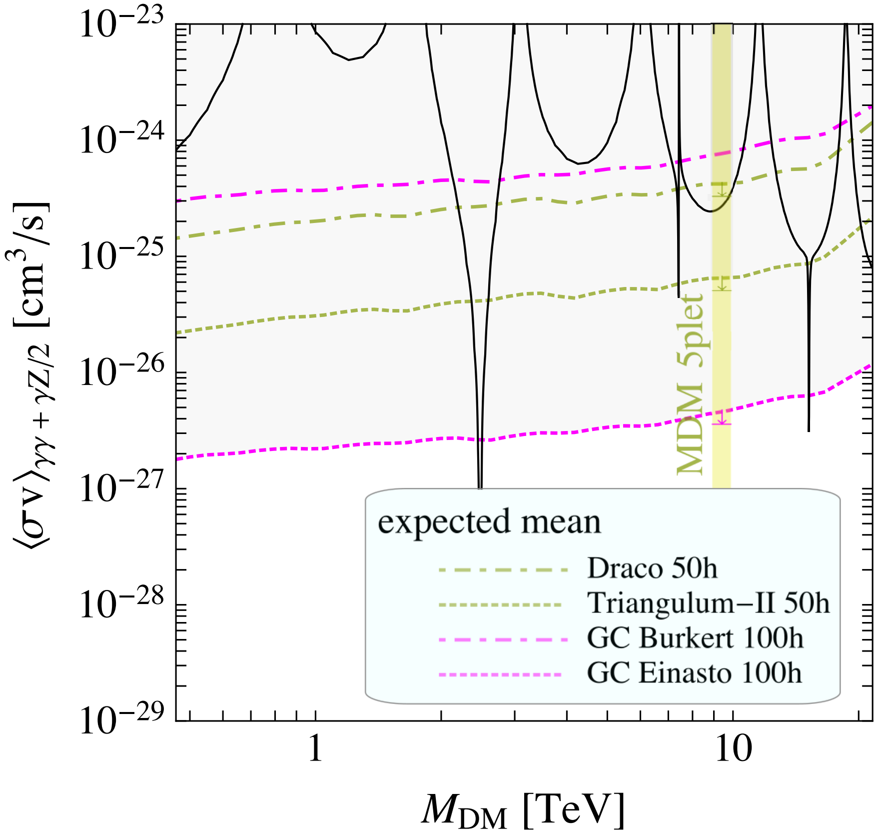
- $\Psi = (\mathbf{6}, \mathbf{1})_{-2/3}$, $\overline{\Psi} = (\overline{\mathbf{6}}, \mathbf{1})_{2/3}$
- Metastable sextet production at LHC
- Mass bound is 1.6 TeV for metastable gluino.
- R-hadrons are produced (bound state of Ψ, $\overline{\Psi}$ and q/g)
- Detectable via large ionization losses dE/dx and slow propagation velocities

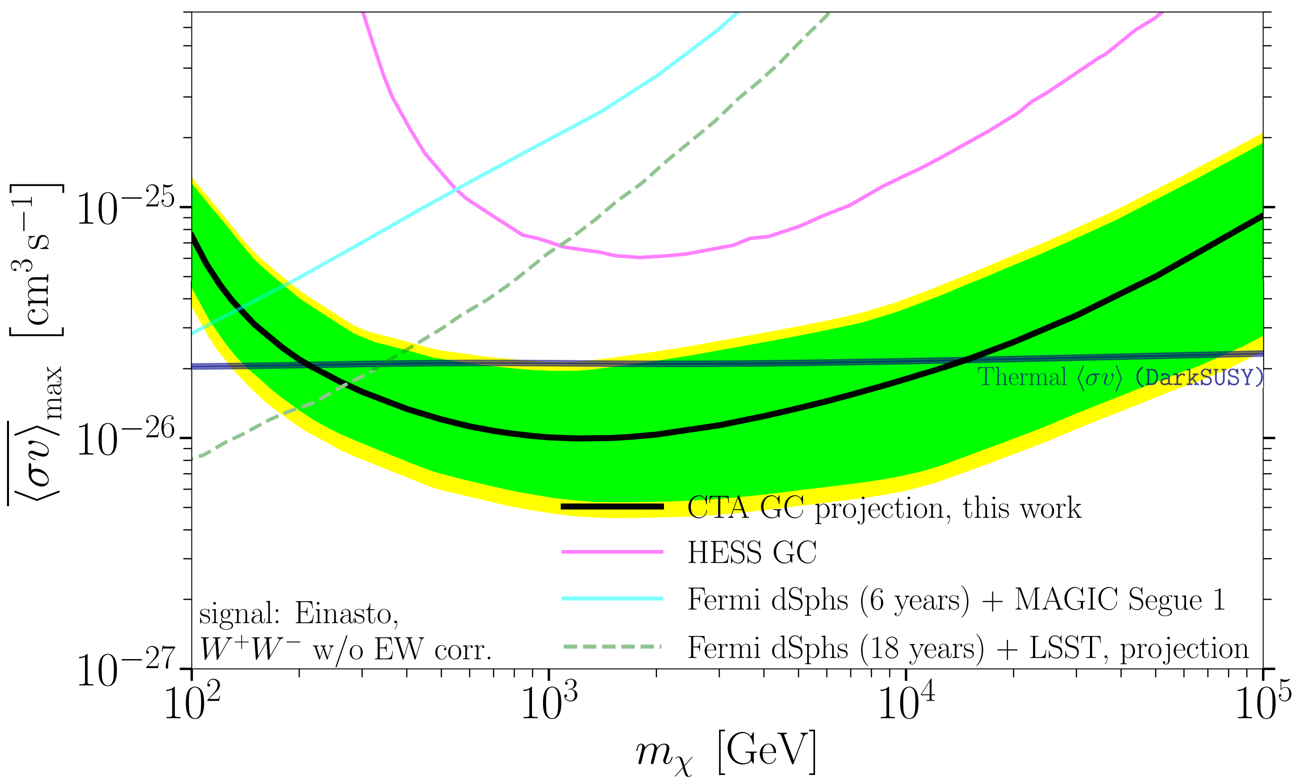


Gamma-ray search

Lefranc et al., arXiv:1608.00786,   CTA Collaboration, arXiv:2007.16129

- CTA prospect （expected to start in 2026）
  Energy range: 20 GeV – 300 TeV
- Line (left) and continuum (right)





Additional comments

Neutrino masses

A. Ibarra, P. Strobl, T.T., Phys.Rev.Lett. 122 (2019)

- Neutrino masses can be generated from Planck scale.
- Democratic mass matrix for N: $M_P \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow M_P \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

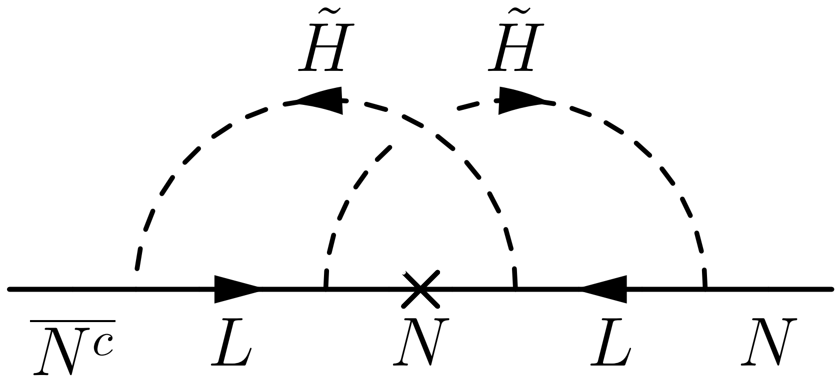
  Rank-1 mass matrix

- M_1 and M_2 are generated at loop level.

  $M_2 \sim \frac{y_\nu^4 M_P}{(4\pi)^4} \sim 10^{14}$ GeV (seesaw scale)

  $M_1 \sim 10^9$ GeV with $y_\nu = \mathcal{O}(1)$

  $\Rightarrow$ seesaw $m_\nu = \frac{m_D^2}{M_2} = \mathcal{O}(0.1)$ eV

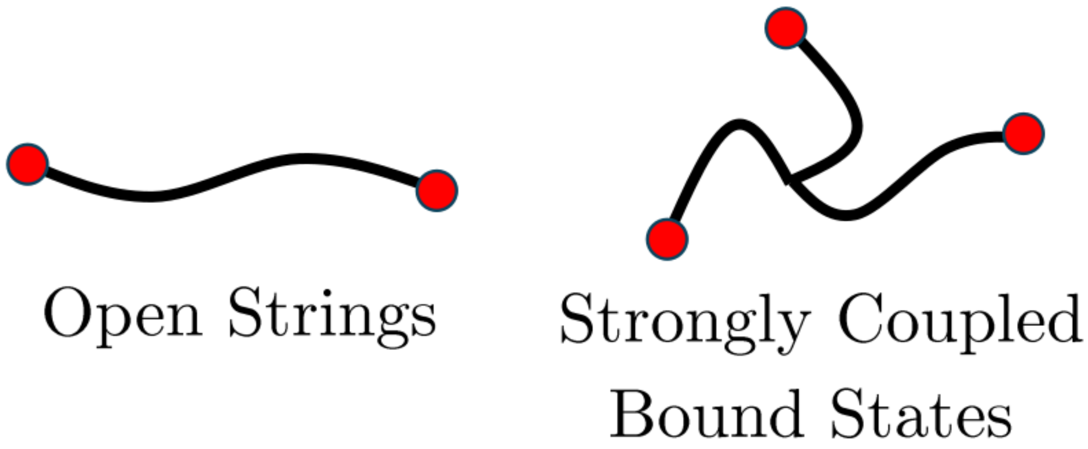


- Another small neutrino mass has to be generated somehow.

Implication to string theory

M. Baumgart et al., arXiv: 2412.13192

- String endpoints correspond to fundamental indices of gauge groups.
  Ex. symmetric $ij \Rightarrow$ adjoint
  symmetric $ijk\ell \Rightarrow$ $\mathbf{5}$-plet
- Maximally three indices even if strongly coupled strings are considered.



- Any examples with four indices are not known so far, and such construction seems to be difficult.
- Isolation of high dimensional reps is also difficult.
  $\Rightarrow$ Once $\mathbf{5}$-plet MDM is experimentally confirmed, most of known string theories have to be falsified.

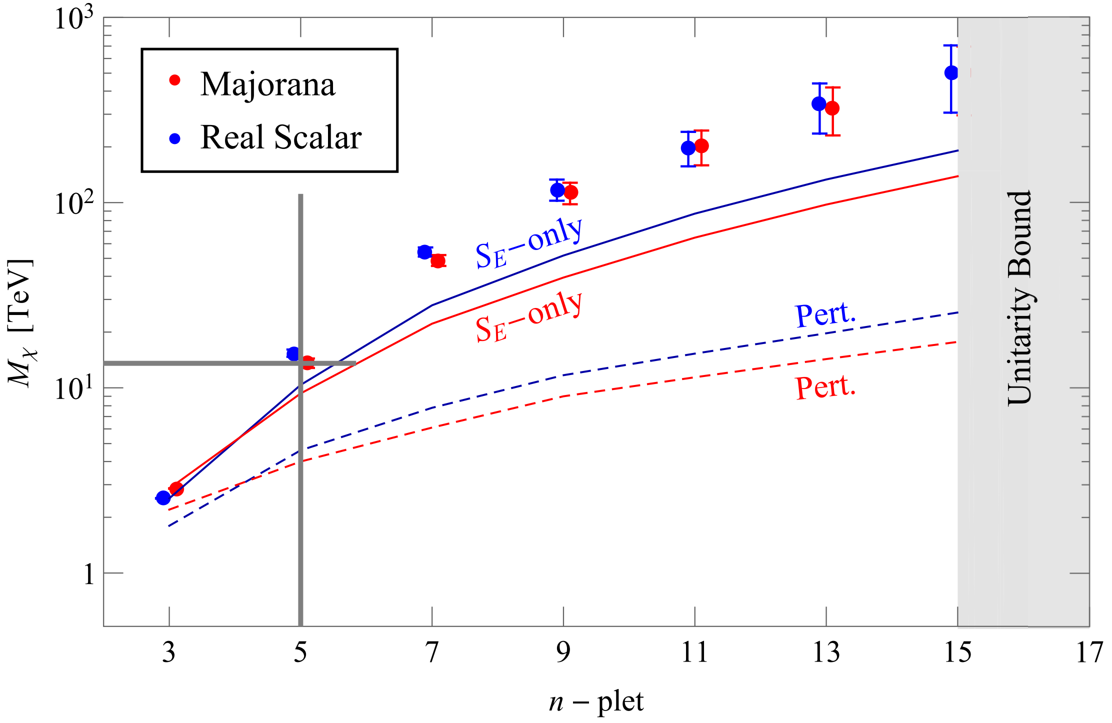
Summary

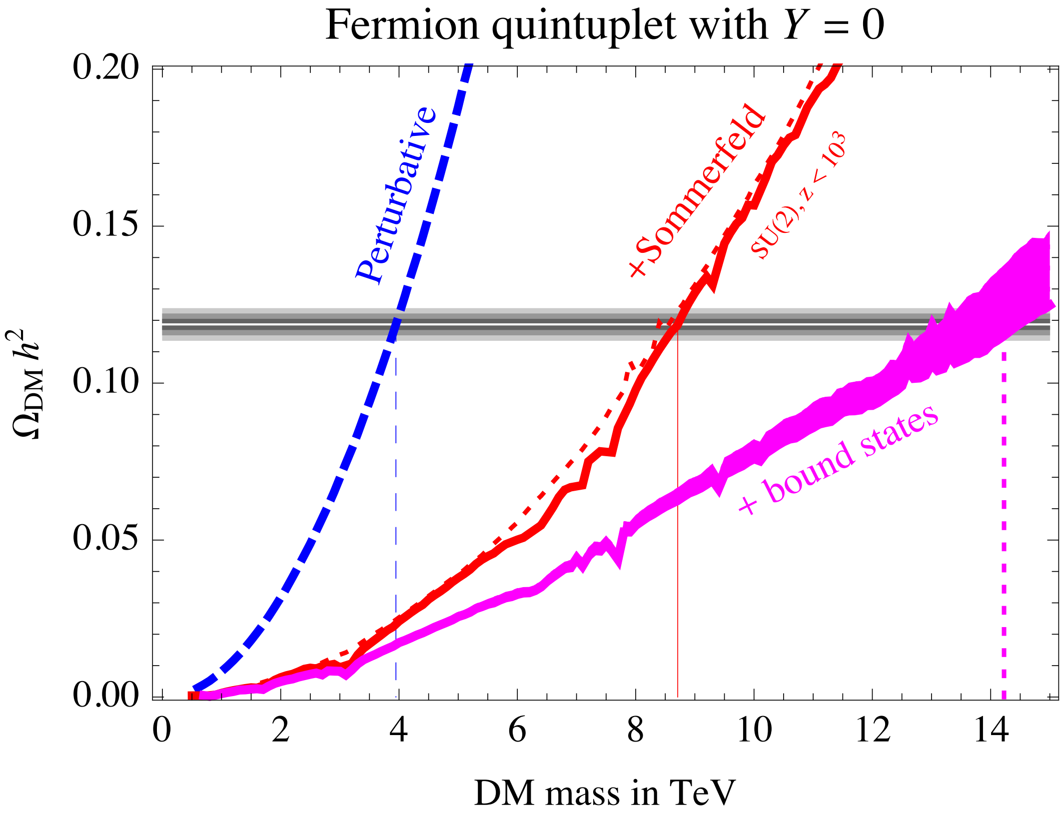
1. $\mathbf{5}$-plet MDM is phenomenologically promising DM candidate from bottom up approach. The mass is 14 TeV.
2. The model can accommodate gauge coupling unification by adding 2 pairs of exotic colored fermions.
3. The model can be tested via searching for the exotic colored fermions, and MDM through gamma-ray observation + direct detection.
4. If an isolated MDM is found by experiments, most of known string theories are excluded, and need to be modified.

Backup

MDM thermal mass

S. Bottaro et al., Eur.Phys.J.C (2022)
A. Mitridate et al., JCAP (2017) [arxiv:1702.01141]





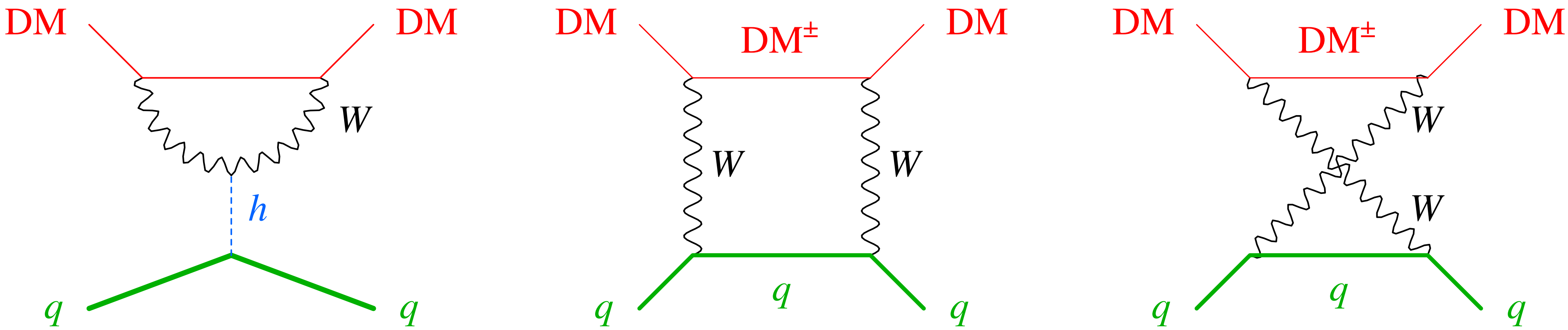
- Perturbative cross section $(n = 5)$

$$\langle \sigma_{\text{eff}} v \rangle = \frac{\pi \alpha_2^2}{80 n M_\chi^2} \left(2n^4 + 17n^2 - 19\right)$$

- Plus Sommerfeld effects and bound state formation

Direct detection of MDM

M. Cirelli et al., Nucl.Phys.B (2005) [hep-ph/0512090]
Farina et al., JHEP (2013) [arxiv:1303.7244]



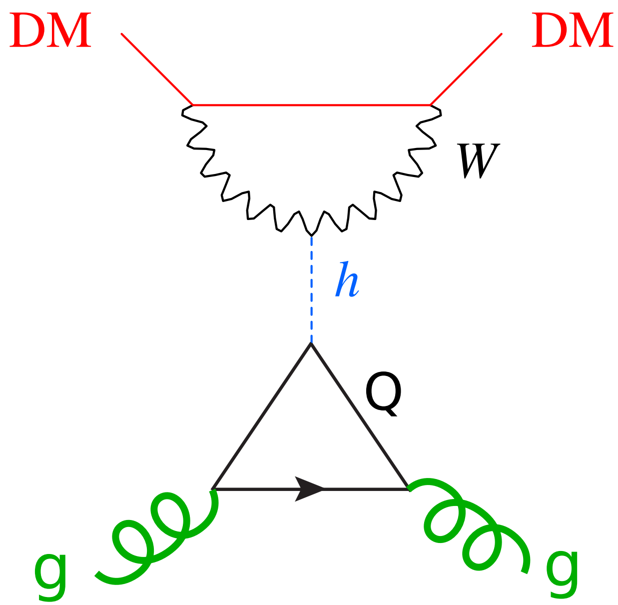
- Cross section at one-loop level ($n = 5$)

$$\sigma_{\rm SI} = (n^2 - 1)^2 \frac{\pi \alpha_2^4 m_N^4 f^2}{64 m_W^2} \left(\frac{1}{m_W^2} + \frac{1}{m_h^2} \right)^2$$

- Plus gluon contribution at two-loop level

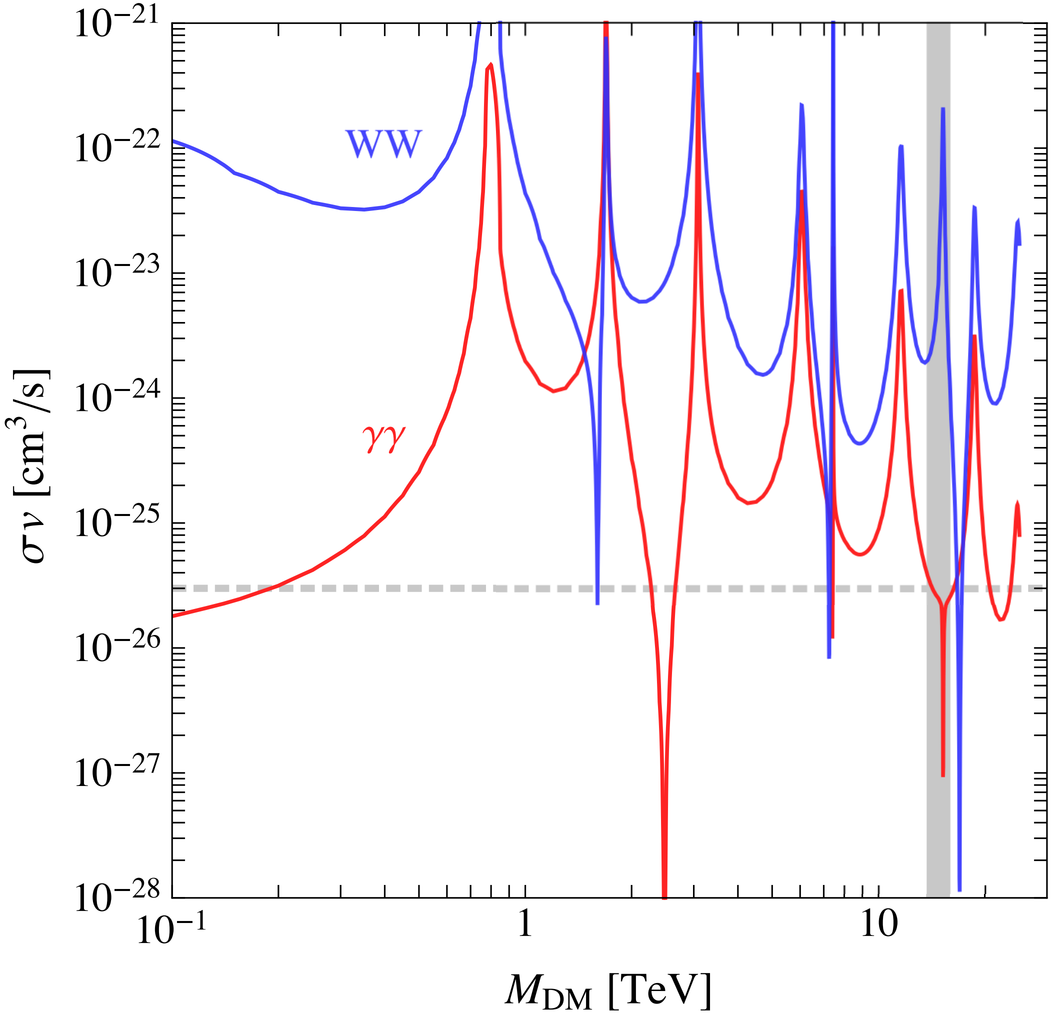
J. Hisano et al., Phys.Rev.D82 (2010) [hep-ph/1007.2601]

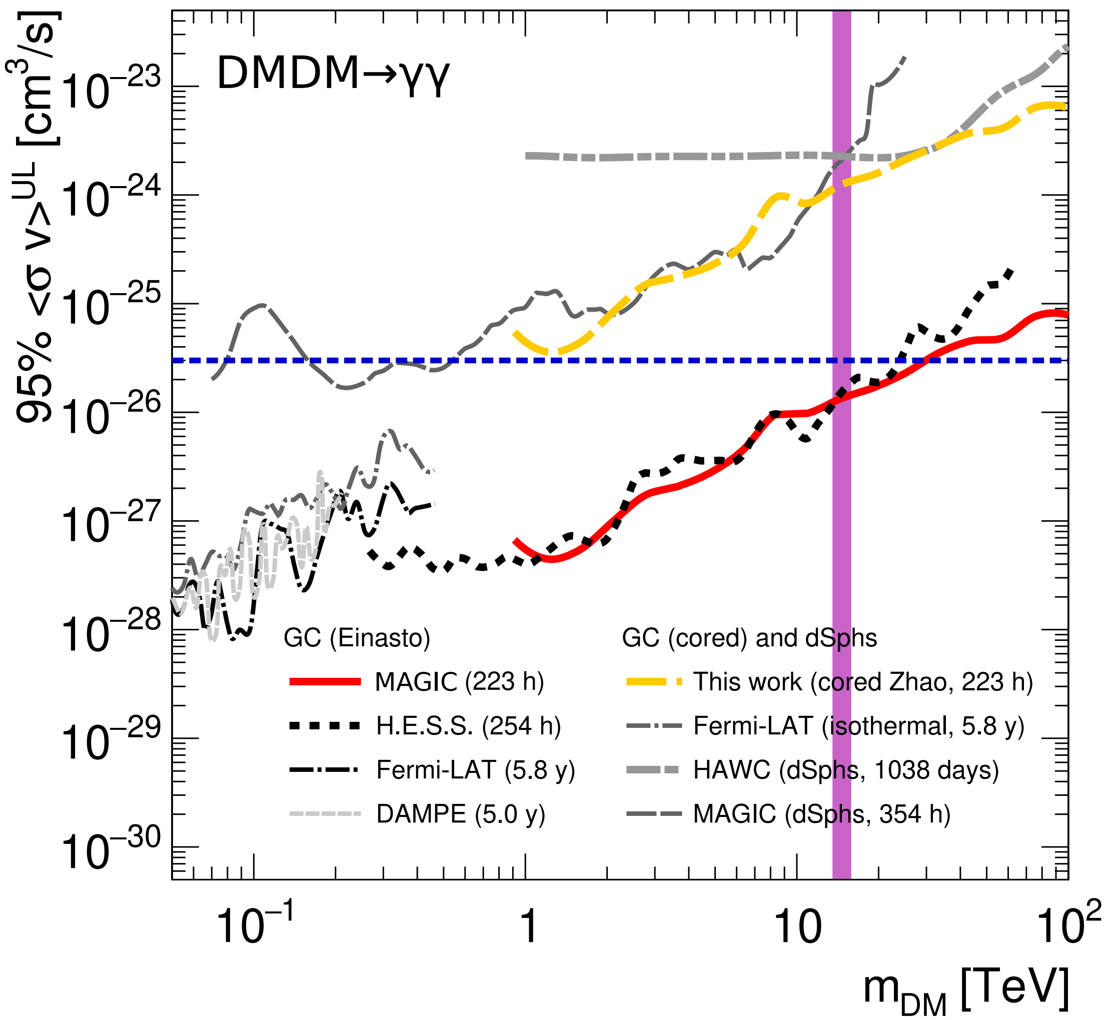
$\Rightarrow$ Partial cancellation of $\sigma_{\rm SI}$
$\Rightarrow$ $\sigma_{\rm SI} \sim 10^{-46}$ [cm^2]



Indirect detection of MDM

M. Cirelli et al., JCAP (2015) [1507.05519 [hep-ph]]
Abe et al., Phys.Rev.Lett. (2023) [arxiv:2212.10527]

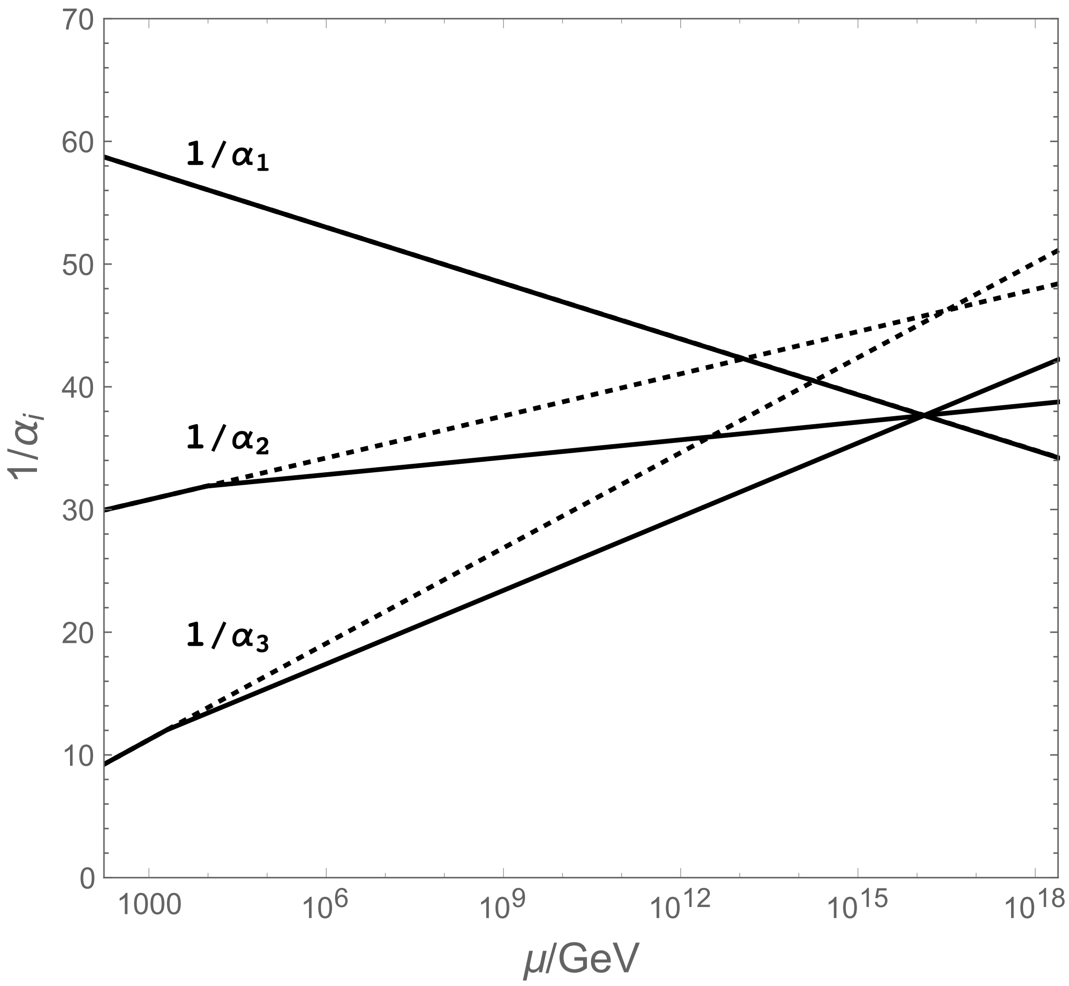




- Large fluctuation due to Sommerfeld effects ($v_\chi \sim 10^{-3}$)
- 5-plet MDM is already constrained for cuspy profiles.

Previous attempt

G. C. Cho, K. Hayami, N. Okada, Phys.Rev.D 105 (2022) 1, 015027



Their conditions

- α_1^{-1} should unchange.
- SM + one $(\mathbf{1},\mathbf{5})_0$ real scalar + three $(\mathbf{8},\mathbf{1})_0$ real scalars
- $(\mathbf{1},\mathbf{5})_0$ is DM candidate, but not automatically stabilized. DM $\to 4H$
- Thermal DM mass 9.4 TeV
- Calculate β functions at two-loop level