Yukawa textures from selection rules without group actions

Hajime Otsuka (Kyushu University)

References :

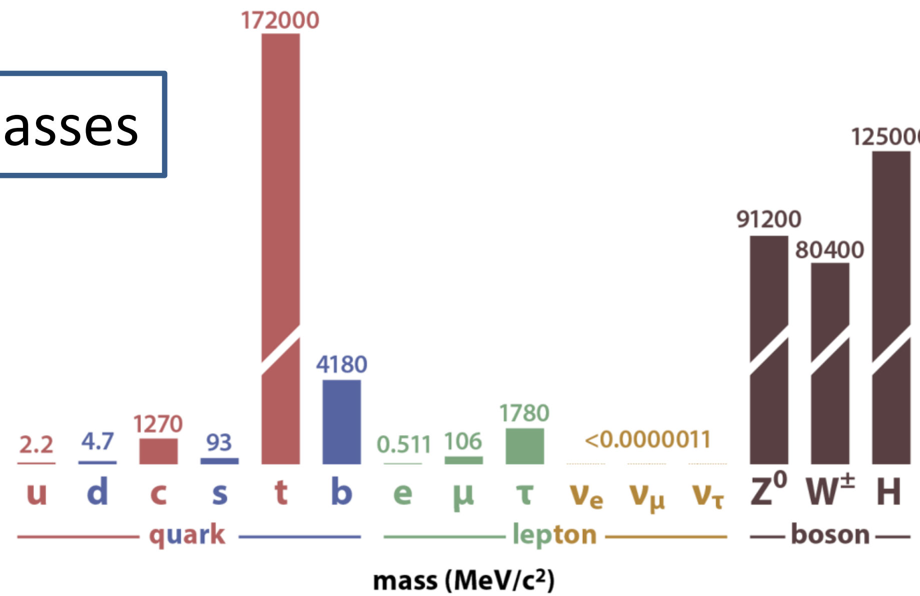
Top-down approach (string compactifications):
T. Kobayashi, H.O., 2408.13984 [hep-th], S. Funakoshi, T. Kobayashi, H.O., 2412.12524 [hep-th]
J. Dong, T. Kobayashi, R. Nishida, S. Nishimura and H.O., 2504.09773 [hep-th]

Bottom-up approach :
T. Kobayashi, H.O., M. Tanimoto, 2409.05270 [hep-ph],
T. Kobayashi, Y. Nishioka, H.O., M. Tanimoto, 2503.09966 [hep-ph]

"The 2nd Workshop of GUTPC 2025" @ HIAS, UCAS, 2025/4/19

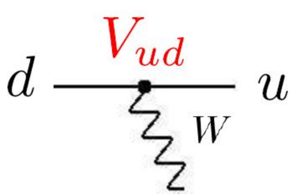
Flavor puzzle

- Hierarchy of quark and lepton masses



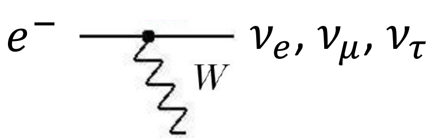
- Different mixings for quarks/leptons

PDG ('24)



$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}$$

NuFIT 6.0 (2024)



$$|U|^{\text{IC19 w/o SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.842 & 0.519 \to 0.580 & 0.142 \to 0.155 \\ 0.248 \to 0.505 & 0.473 \to 0.682 & 0.649 \to 0.764 \\ 0.270 \to 0.521 & 0.483 \to 0.690 & 0.628 \to 0.746 \end{pmatrix}$$

Texture zeros approach

Two generations of quarks

- In the basis in which the up-type quark mass matrix is diagonal,

$$m_{\text{down}} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$\xrightarrow{M \gg m} \begin{pmatrix} m^2/M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}$$

The Cabibbo angle is successfully predicted to be

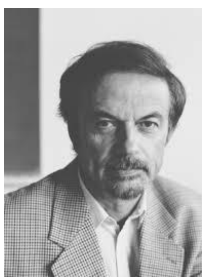
$$\sin\theta_c \sim \tan\theta_c = m/M = \sqrt{m_d/m_s}$$

S. Weinberg (1977)

Texture zeros approach

Three generations of quarks

- Fritzsch extended this approach to the three family case (1978)



$$m_{u,d}^{(\mathrm{Fritzsch})} = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix}$$

leading to various relations between masses and mixing angles

See for systematic approach with four or five zeros for symmetric or hermitian quark mass matrices by Ramond-Roberts-Ross (1993))

These textures are not viable today under the precise experimental data

Texture zeros approach

Three generations of quarks

The Fritzsch texture belongs to the more generic
Nearest-neighbor-interaction (NNI) form of quark mass matrices :

G.C.Branco, L.Lavoura, F.Mota (89),...

$$m_{u,d}^{(\mathrm{NNI})} = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & e \end{pmatrix}$$

consistent with observed CKM matrices and quark masses

(also for leptons)

(obtained by choosing a weak-basis transformation
from general 3×3 matrices of Yukawa matrices)

Q : Origin of these textures ?

A : They cannot be derived by the conventional symmetry

“Traditional” flavor symmetry

- Fusion rule of symmetry operator :

$$U_{g_1}U_{g_2} = U_{g_1 g_2} \qquad g_1, g_2 \in G$$

Ex., $U(1) : e^{i\theta_1}e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

- Transformations of fields:

$$\phi_i \overset{g}{\rightarrow} \rho_{ij}(g)\phi_j \qquad g \in G_{\mathrm{flavor}}$$

- G_{flavor} : continuous or discrete

"Traditional" flavor symmetry

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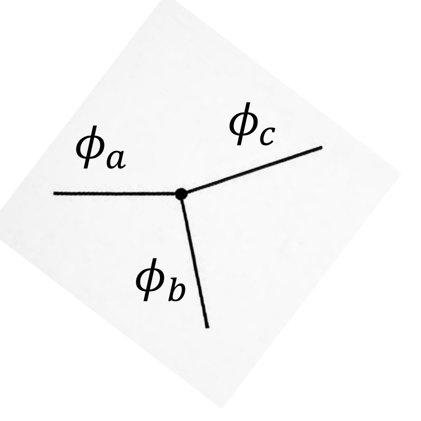
- Suppose that fields $\{\phi_a, \phi_b, \phi_c\}$ correspond to rep. of G

if $\rho(a)\rho(b)\rho(c) \supset \mathbb{I} \ \rightarrow \ \phi_a\phi_b\phi_c$ is allowed
if $\rho(a)\rho(b)\rho(c) \not\supset \mathbb{I} \ \rightarrow \ \phi_a\phi_b\phi_c$ is forbidden



"Coupling selection rules"

(When G is Abelian, it is the charge conservation law)

Coupling selection rules without group actions

Selection rules without group actions

- Fusion rule of symmetry operator :

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Ex., $U(1) : e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}$

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⇓

$$U_i U_j = \sum_k c_{ijk} U_k$$

Ex., $U_{\theta_1} U_{\theta_2} = U_{\theta_1 + \theta_2} + U_{\theta_1 - \theta_2}$

Many examples in 2D CFT
E.P.Verlinde ('88),
G.W.Moore and N. Seiberg ('89),

$$U_\theta \equiv \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

- The right hand side in multiplication laws is not unique

Selection rules without group actions

- Suppose that

1. Each field ϕ_i is labeled by U_i obeying the fusion rules

$$U_i U_j = \sum_k c_{ijk} U_k$$

2. Interactions in the classical Lagrangian are allowed when the identity $\mathbb{I}$ is contained in the fusion product of fields

Selection rules without group actions

- Ex., the Fibonacci fusion rules (describing anyons) :

$$\mathbb{I} \otimes \mathbb{I} = \mathbb{I}$$
$$\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau$$
$$\tau \otimes \tau = \mathbb{I} + \tau$$

- τ does not have the inverse

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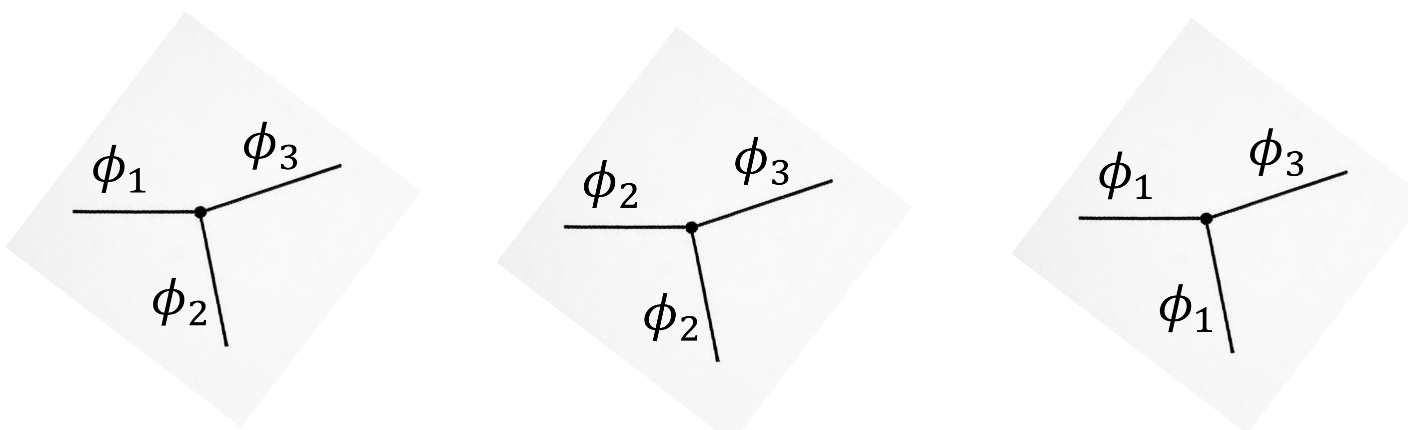
- Suppose that fields are labed by $\{\phi_1, \phi_2, \phi_3\} = \{\mathbb{I}, \tau, \tau\}$



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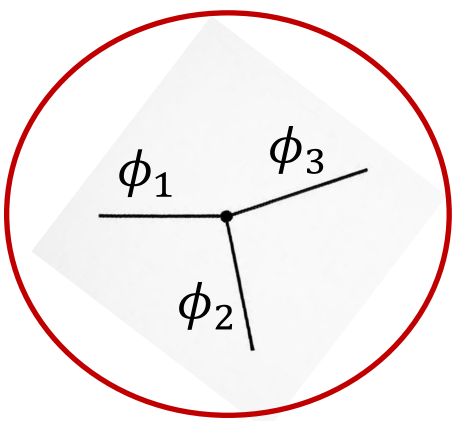

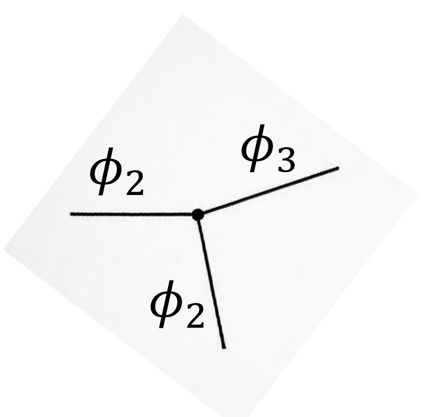

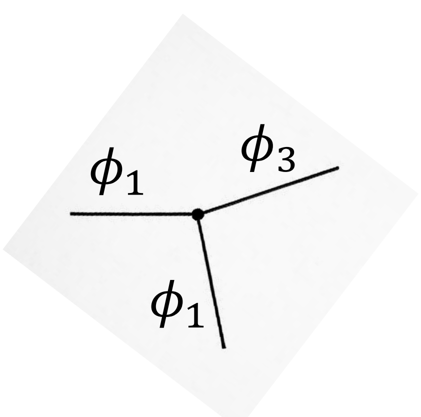

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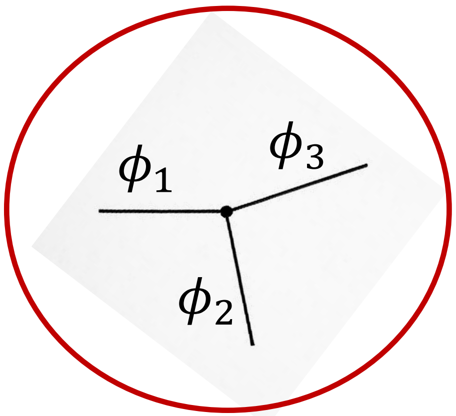

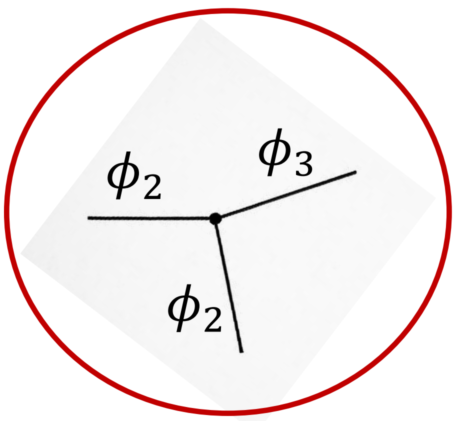

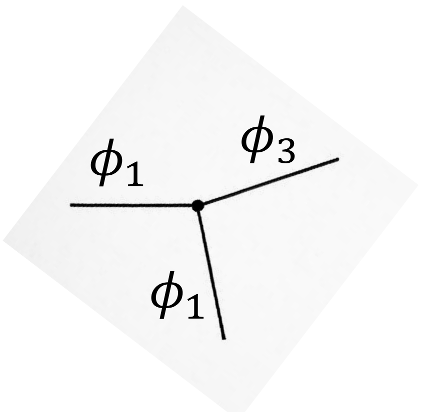

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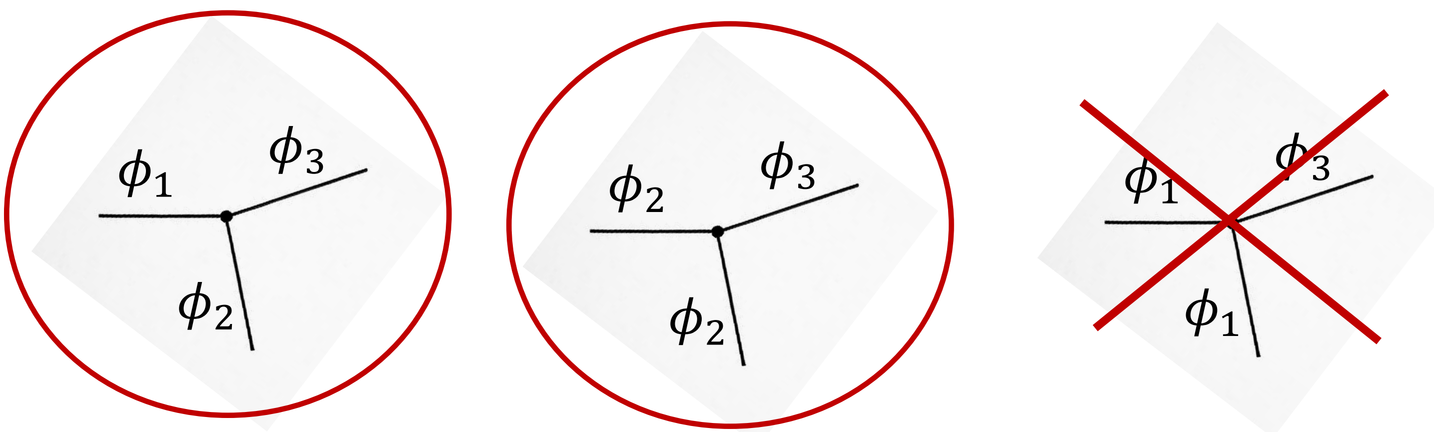

Application of the non-trivial fusion rules to flavor physics

The Fibonacci fusion rules

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- τ does not have the inverse

- *As a simplest case, let us consider two generations of fermions:*

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

Left : $(\mathbb{I}, \tau)$
Right : $(\mathbb{I}, \tau)$
Higgs : τ

$$Y = \begin{pmatrix} 0 & a \\ b & c \end{pmatrix}$$

One cannot derive this pattern by U(1) or $\mathbb{Z}_M$ for any M due to the charge conservation law.

How do we realize the non-trivial fusion rules ?

Selection rules without group actions（1/3)

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

- Let us start with $\mathbb{Z}_M$ *symmetry (generators :* $g = e^{2\pi i/M}$*)*

Transformation of fields ϕ_j *in 4D QFT :*

$$\phi_j \rightarrow g^{k_j} \phi_j$$

- Charge k_j under $\mathbb{Z}_M$

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- Charge k_j under $\mathbb{Z}_M$

Selection rules :

$$g^{k_1}g^{k_2} = g^{k_1+k_2}, \qquad g^{k_1}g^{-k_2} = g^{k_1-k_2}$$

The condition to allow n-point couplings :

$$g^{k_1}g^{k_2} \ldots g^{k_n} = e$$

Selection rules without group actions（2/3)

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

- Let us consider the following automorphism of $\mathbb{Z}_M$:

$$ege^{-1} = g$$

$$rgr^{-1} = g^{-1}$$

- We define the following "class" :

$$[g^k] = \{hg^kh^{-1} \mid h = e, r\}$$

- $[g^k]$ includes g^k and $g^{-k} = g^{M-k}$

  ($\mathbb{Z}_2$ symmetry associated with r is gauged)

Intuitively, $[g^k]$ has both k and $M-k$ charges

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($\mathbb{Z}_2$ symmetry associated with r is gauged)

corresponding to $\mathbb{Z}_2$ *invariant modes labeled by* $[g^{k_j}]$ *:*

$$\Phi_j = \phi_j + \phi_{M-j}$$

Selection rules without group actions (3/3)

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

- Product of the classes :

$$[g^{k_1}][g^{k_2}] = [g^{k_1+k_2}] + [g^{M-k_1+k_2}]$$

if $[g^0]$ appears in the right-hand side, 2-point couplings are allowed

i.e., $\pm k_1 \pm k_2 = 0 \ (\mathrm{mod}\ M)$

→ $[g^{k_1}] = [g^{k_2}]$ (no kinetic mixings)

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- The condition to allow three-point (Yukawa) couplings :

$$[g^{k_1}][g^{k_2}][g^{k_3}] = [g^{k_1+k_2+k_3}] + [g^{M-k_1+k_2+k_3}] + [g^{M+k_1-k_2+k_3}] + [g^{M+k_1+k_2-k_3}]$$

$$\pm k_1 \pm k_2 \pm k_3 = 0 \ (\mathrm{mod}\ M)$$

Overview

- $\mathbb{Z}_M$ *symmetry :*

$$\phi_j \to g^{kj}\phi_j$$

- Gauging $\mathbb{Z}_2$ *(*the automorphism of $\mathbb{Z}_M$*), i.e.,* $D_M \cong \mathbb{Z}_M \rtimes \mathbb{Z}_2$

$$[g^k] = \{hg^k h^{-1} \mid h = e, r\}$$

  - $[g^k]$ includes g^k and $g^{-k} = g^{M-k}$

- $\mathbb{Z}_2$ *invariant modes labeled by* $[g^k]$

$$\Phi_j = \phi_j + \phi_{M-j}$$

$M = 3$ ($\mathbb{Z}_3$ symmetry)

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

- Two different classes :

$$[g^0], \qquad [g^1] (= [g^2])$$

$$\supset g^0 \qquad \supset g^1,\, g^2$$

$$[g^k] \supset g^k, g^{M-k}$$

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- *Fusion rules :*

Fibonacci fusion rules

$$[g^0][g^0] = [g^0]$$

$$[g^0][g^1] = [g^1]$$

$$[g^1][g^1] = [g^0] + [g^1]$$

$$\mathbb{I} \otimes \mathbb{I} = \mathbb{I}$$

$$\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau$$

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- *Previous example for two generations of fermions:*

Left : ($[g^0], [g^1]$)
Right : ($[g^0], [g^1]$)
Higgs : $[g^1]$

$$Y = \begin{pmatrix} 0 & a \\ b & c \end{pmatrix}$$

$M = 5$ ($\mathbb{Z}_5$ symmetry)

T. Kobayashi, H.O.,, M. Tanimoto, 2409.05270 [hep-ph]

- Three different classes :

$$[g^0], \quad [g^1], \quad [g^2]$$

$$[g^k] \supset g^k, g^{M-k}$$

$$\supset g^0 \quad \supset g^1, g^4 \quad \supset g^2, g^3$$

$M = 5$ ($\mathbb{Z}_5$ symmetry)

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- Three different classes :

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$$\supset g^0 \quad \supset g^1, g^4 \quad \supset g^2, g^3$$

- *Yukawa couplings for 3 generations of fermions :*

Left : ($[g^0], [g^1], [g^2]$)
Right : ($[g^0], [g^1], [g^2]$)
Higgs : $[g^1]$

$$Y_{[g^1]} = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & e \end{pmatrix}$$

Fusion rules :
$[g^0][g^0] = [g^0]$
$[g^1][g^1] = [g^0] + [g^2]$
$[g^2][g^2] = [g^0] + [g^1]$
$[g^1][g^2] = [g^1] + [g^2]$

Texture with "nearest neighbor intereraction"
consistent with observed CKM and quark masses (also for leptons)

Extensiton to two flavor symmetries

T. Kobayashi, Y. Nishioka, H.O., M. Tanimoto, 2503.09966 [hep-ph]

- $\mathbb{Z}_2$ *gaugings of* $\mathbb{Z}_M \times \mathbb{Z}_{M'}$ symmetries with generators g_1, g_2

classes : $\left[g_1^k\right][g_2^m]$, $\quad (k, m = 0,1, \ldots, M-1)$ for $M = M'$

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- *Ex., when* $M = 3$,

  Left : $([g_1^0][g_2^0], [g_1^1][g_2^0], [g_1^1][g_2^1])$
  Right : $([g_1^0][g_2^0], [g_1^0][g_2^1], [g_1^1][g_2^1])$
  Higgs : $[g_1^1][g_2^1]$

$$Y_u,\, Y_d = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

A non-trivial weak CP which does not appear in its determinant
Available to address the strong CP problem without axion

UV completions ?

Overview

T. Kobayashi, H.O., 2408.13984 [hep-th]

- $\mathbb{Z}_M$ *symmetry :*

$$\phi^j \to g^k{}_j \phi^j$$

- Gauging $\mathbb{Z}_2$ *(*the automorphism of $\mathbb{Z}_M$*),*
 i.e., $D_M \cong \mathbb{Z}_M \rtimes \mathbb{Z}_2$

$$[g^k] = \{h g^k h^{-1} \mid h = e, r\}$$

$[g^k]$ includes g^k and $g^{-k} = g^{M-k}$

- $\mathbb{Z}_2$ *invariant modes labeled by* $[g^k]$

$$\Phi_j = \phi_j + \phi_{M-j}$$

Top-down approach (1/2)

T. Kobayashi, H.O., 2408.13984 [hep-th]

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- *$\mathbb{Z}_2$ invariant modes labeled by $[g^k]$*

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Flavor symmetry on Super Yang-Mills on T^2 with magnetic fluxes

(EFT of type IIB Magnetized D-brane models)

$T^2/\mathbb{Z}_2$ orbifold with gauged $\mathbb{Z}_2$

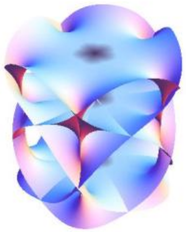
Fusion rules of momentum ops.:

$$U_{\lambda_1}U_{\lambda_2} = U_{\lambda_1+\lambda_2} + U_{\lambda_1-\lambda_2}$$

$\mathbb{Z}_2$ invariant modes on $T^2/\mathbb{Z}_2$

Top-down approach (2/2)

- E_6 GUT in heterotic string theory on Calabi-Yau threefolds:



Candelas-Horowitz-Strominger-Witten ('85)

- Matters (E_6 : 27 or $\overline{27}$) ≈ Moduli

$$27^i \approx \text{Kaehler Moduli } t^i$$

(2-cycle volume)

$(i = 1,2, \ldots , h^{1,1})$

- Yukawa couplings ($27^{\mathbf{3}}$)

$$W = F_{ijk} 27^i 27^j 27^k$$

determined by topological data of CY

- Non-trivial fusion rules :

$$V_{27^i} V_{27^j} = \sum_k F_{ijk} V_{27^k}$$

Top-down approach (2/2)

- Classification of selection rules on complete intersection CYs:

We find that all of the selection rules on CYs (with # of moduli ≤ 5) are understood by combinations of only five types of fusion rules.

J. Dong, T. Kobayashi, R. Nishida, S. Nishimura and H.O., 2504.09773 [hep-th]

Conclusions

Phenomenological applications of selection rules without group actions
(including the the Fibonacci fusion rule and its extension)

- leading to the non-trivial structure of mass matrices,
  NNI form of fermion mass matrix and its extension:

$$m_{u,d}^{(\mathrm{NNI})} = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & e \end{pmatrix}$$

- available to address the strong CP problem without axion

$$m_{u,d} = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Conclusions

- UV completions (1/2):

1. Supersymmetric Yang-Mills theory on $T^2/\mathbb{Z}_N$ with fluxes
   (Magnetized D-branes in type IIB string theory)

   - Fusion rules of $\mathbb{Z}_N$-invariant operators :

   For $T^2/\mathbb{Z}_2$, $U_{\lambda_1} U_{\lambda_2} = U_{\lambda_1+\lambda_2} + U_{\lambda_1-\lambda_2}$

   - Chiral zero modes will correspond to three generations of quarks/leptons
     Non-invertible symmetries → flavor symmetries

Conclusions

- UV completions (2/2):

2. E_6 GUT in heterotic string theory on Calabi-Yau threefolds

   - Non-trivial fusion rules of moduli fields

     → Coupling selection rules for 27 or $\overline{27}$ of E_6

Discussions

- Selection rule of n-point couplings（higher-dimensional operators）

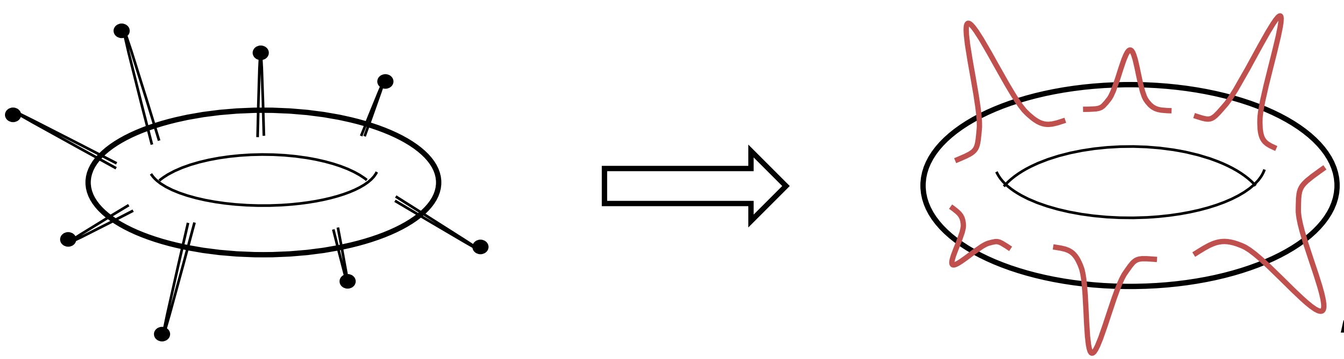
- Other applications of Non-Invertible Symmetries (NIS)
  - Ex., Tiny neutrino masses from UV instantons breaking NIS

C.Cordova, K.Ohmori 2205.05086, C.Cordova, S.Hong, S.Koren, K. Ohmori 2211.06243

- Breaking of non-invertible symmetries in toroidal orbifolds

  - would be realized by resolutions of $T^{2n}/\mathbb{Z}_N$ orbifolds

  - blowup modes → the breaking of non-invertible symmetries



$Thank\ you!$