Appeal and challenges of Natural SUSY GUT (with spontaneous SUSY breaking)

As a hint

for the model beyond the natural GUT

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Collaboration with Bando, Yamashita, …, Shigekami, ….
New part is collaborated with Taiju Tanii
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Contents of this talk

74’Georgi-Glashow

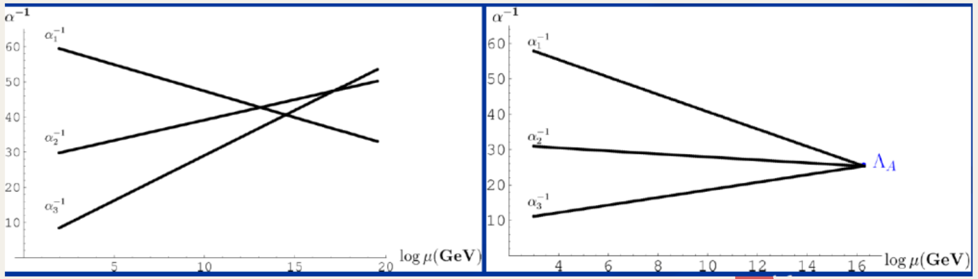
Introduction (Appeals of GUT)

Two unifications

1, Unification of forces, $SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6$

2, Unification of matters, $16 = 10(\mathrm{Q} + \mathrm{U}_\mathrm{R}^\mathrm{c} + \mathrm{E}_\mathrm{R}^\mathrm{c}) + \bar{5}(D_R^c + L) + 1(N_R^c)$

SUSY



$\Rightarrow$ A, Charge quantization ($C_P = -C_e$?)

B, Anomaly cancellation in the SM.

C, Gauge coupling unification ($g_3 > g_2 > g_1$)

D, Various hierarchies in Yukawa couplings (masses & mixings for matters)

3, Experimental supports for both unifications.(C&D)

10 fields induce stronger hierarchy in Yukawa couplings than $\bar{5}$ fields

Problems in SUSY GUT

1, The doublet-triplet(DT) splitting problem (Finetuning problem 10^{-14})

$$\text{Higgs } 5_H = \begin{pmatrix} 3_{H_T} \\ 2_{H_{SM}} \end{pmatrix} \quad \begin{matrix} m_{H_T} > 10^{16}\ \text{GeV} & \text{for long lifetime of proton} \\ m_{H_{SM}} \sim 100\ \text{GeV} & \text{to obtain the weak scale} \end{matrix}$$

This is possible if the GUT breaking is picked up, but finetuning is needed generically.

2, Unrealistic GUT relation for Yukawa matrices.

$$16 = 10(\mathrm{Q} + \mathrm{U_R^c} + \mathrm{E_R^c}) + \overline{5}(D_R^c + L) + 1(N_R^c)$$

$$Y_d = Y_e^T (= Y_u = Y_{\nu_D})$$

RGE effects ($y_b \sim y_\tau, 3y_s \sim y_\mu, y_d \sim 3y_e$ at GUT scale.)

SU(5) GUT relation for 3rd generation is good and the others are not so bad if $Y_d \sim Y_e^T$

01’N.M.
02’N.M.-T.Yamashita

Natural GUT solves these problems under “natural” assumption

All interactions allowed by the symmetry are introduced with O(1) coefficients.

(including infinite number of higher dimensional interactions.)

⇒Infinite number of interactions must be controlled.

SUSY zero mechanism plays an important role in controlling them.

The results are expected to be stable under loop corrections and/or gravity effects.

The assumption is opposite to finetuning.→natural

79'Froggatt-Nielsen
94' Ibanez-Ross

Froggatt-Nielsen mechanism for Yukawa hierarchies

U(1) gauge symmetry is broken by $\langle\Theta\rangle = \lambda\Lambda$, where the fravon Θ has U(1) charge $\theta = -1$. When Q_i, U^c_{Ri}, H have U(1) charges q_i, u^c_{Ri}, h, respectively, U(1) invariant interactions are written by

$$W_Y = \left(\frac{\Theta}{\Lambda}\right)^{q_i+u^c_{Rj}+h} Q_i U^c_{Ri} H \Rightarrow \lambda^{q_i+u_{Rj}{}^c+h} Q_i U^c_{Rj} H, \qquad \Lambda\text{: cutoff}$$

When $\lambda < 1$, hierarchical structure of Yukawa couplings can be obtained.

If we take $\lambda \sim sin\theta_C \sim 0.22$, $(q_1.q_2.q_3) = (u^c_{R1}.u^c_{R2}.u^c_{R3}) = (3{,}2{,}0)$ and $h = 0$,

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \qquad U_L \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Assumption: All interactions allowed by symmetry are introduced with O(1) coefficients.

SUSY zero mechanism: If $q_i + u^c_{Rj} + h < 0$, the Yukawa int. is forbidden.

Natural GUT in a nutshell

Principle: All interactions allowed by symmetry are introduced with O(1) coefficients.

This principle is applied to the Higgs sector in which GUT group is spontaneously broken.

Reckless?

Infinite number of terms can be controlled by the SUSY zero mechanism.

All int. are fixed by the symmetry except O(1) coefficients. $\langle Z\rangle \sim \begin{cases} 0 & (z>0) \\ \lambda^{-z} & (z\leq 0) \end{cases}$ with $\Lambda = 1$ unit

So, VEVs and mass spectrum of superheavy fields are fixed except O(1) coefficients.

Under this natural assumption, the various problems of SUSY GUT can be solved.

Realistic quark and lepton masses and mixings are obtained.

New explanation for the gauge coupling unification is given.

SO(10) natural GUT ($\boldsymbol{SO(10) \times U(1)_A \times Z_2}$)

$SO(10)$	negative charge	positive charge	matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-4,+)$	$C'(c'=4,-)$	$\Psi_i\left((\psi_1,\psi_2,\psi_3)=(\frac{9}{2},\frac{7}{2},\frac{3}{2}),+\right)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=-1,+)$	
10	$H(h=-3,+)$	$H'(h'=4,+)$	$T\left(t=\frac{5}{2},+\right)$
1	$\Theta(\theta=-1,+)$	$Z'(z'=5,+)$	
	$Z_i(z_1=z_2=-2,-)$		

$\langle A\rangle \sim \lambda$ $\langle C\rangle = \langle \bar{C}\rangle \sim \lambda^{2.5}$

$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to G_{SM}$

(Right-handed) neutrino masses

$$\mathbf{16}_{\Psi_i} = \mathbf{10}_{\Psi_i} + \overline{\mathbf{5}}_{\Psi_i} + \mathbf{1}_{\Psi_i}, \qquad \overline{\mathbf{16}}_{\bar{C}} = \overline{\mathbf{10}}_{\bar{C}} + \mathbf{5}_{\bar{C}} + \langle \mathbf{1}_{\bar{C}} \rangle$$

$$W = \lambda^{\psi_i+\psi_j+2\bar{c}} \mathbf{16}_{\Psi_i} \mathbf{16}_{\Psi_j} \langle \overline{\mathbf{16}}_{\bar{C}} \rangle \langle \overline{\mathbf{16}}_{\bar{C}} \rangle$$

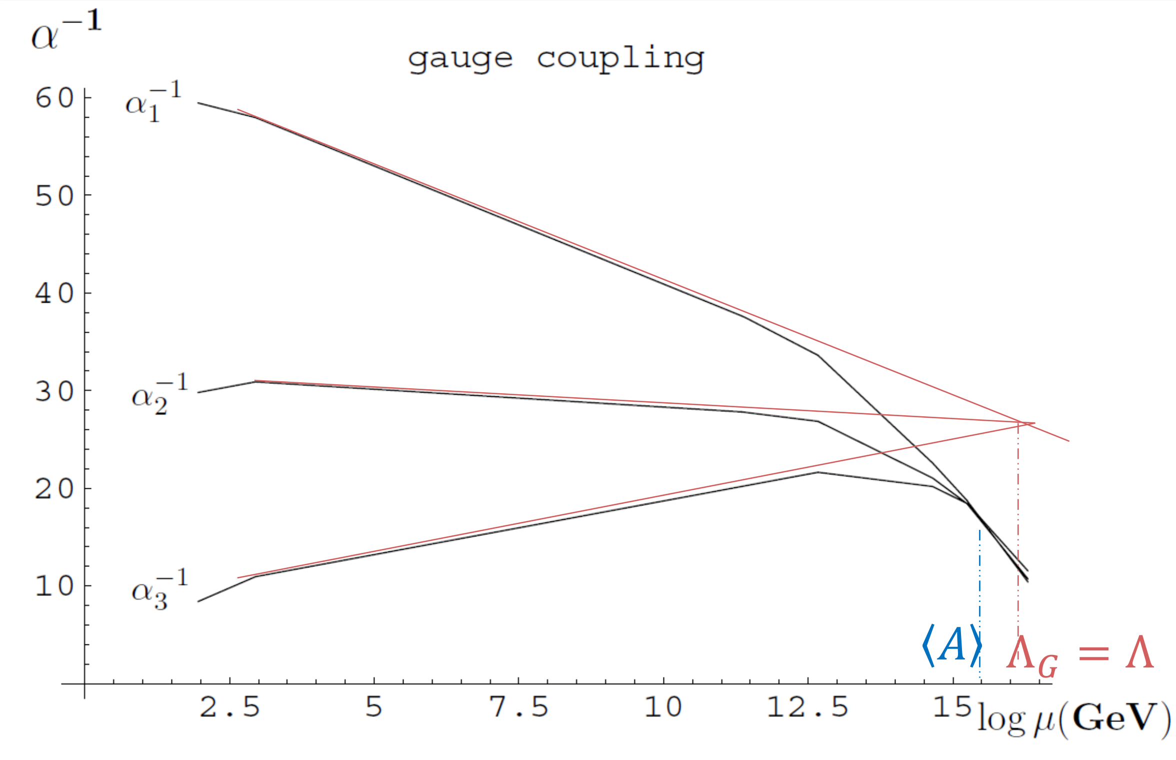
$$\Rightarrow \left(M_{\nu_R}\right)_{ij} \sim \lambda^{\psi_i+\psi_j+2\bar{c}} \langle C \rangle^2$$

$$\Rightarrow M_\nu = Y_{\nu_D} M_{\nu_R}^{-1} Y_{\nu_D}^T \langle H \rangle^2 \sim \lambda^{4+h+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H \rangle^2$$

To obtain sufficiently large neutrino masses, $h \leq -3$ is required for $m_{\nu 3} \sim 0.05$ eV.

01’N.M.
02’N.M.-T.Yamashita

Gauge coupling unification



All VEVs and mass spectrum of superheavy particles are fixed by their $U(1)_A$ charges.

⇒ The running gauge couplings can be calculated.

$$\alpha_1(\langle A\rangle) = \alpha_2(\langle A\rangle) = \alpha_3(\langle A\rangle)$$

$$\Rightarrow \quad \Lambda = \Lambda_G \sim 2\times 10^{16}\mathrm{GeV}, \quad h = 0$$

All $U(1)_A$ charges except h is cancelled in the conditions.

The real GUT scale $\langle A\rangle \sim \lambda^{-a}\Lambda$ is smaller than the usual GUT scale Λ_G.

⇒ Proton lifetime via superheavy gauge boson exchange becomes shorter than the usual GUT.

New explanation for the success of the gauge coupling unification in MSSM!

Predictions on proton decay in natural GUT

Proton decay via dim. 6 ($P \to e\pi$ mediated by gauge fields) is enhanced by smaller unification scale.

$$\Lambda_u \sim \lambda^{-a}\Lambda \sim \lambda\Lambda_G \sim 5\times 10^{15}\mathrm{GeV}. \quad (a=-1, \Lambda \sim \Lambda_G \sim 2\times 10^{16}\mathrm{GeV})$$

It can be found in near future experiment

Proton decay via dim. 5 ($P \to K\nu$ mediated by triplet Higgs) is suppressed.

$$m_{H_T}^{eff} \sim \lambda^{2h}\Lambda \sim 2\times 10^{20}\mathrm{GeV}. \ (h=-3.)$$

If $h = 0$, it is also important.

In original SUSY SU(5) GUT, the proton decay via dim. 5 is important.

Tension between neutrino masses ($h = -3$) & gauge coupling unification condition $h = 0$

Neutrino masses require $h \leq -3$.

The DT splitting needs $h < 0$ to forbid H^2 term by SUSY zero mechanism.

Gauge coupling unification can be recovered by choosing a lot of O(1) coefficients from 0.5 to 2 instead of 1 even if $h = -3$. (It is like 100 times 50% tuning.)

The model is not killed in that sense but unnatural. (The most unsatisfied issue.)

Unknown structure may be hiding behind this problem.

The unknown structure may lead to smaller coefficients.

Which terms have smaller coefficients to solve this tension?

Are the important predictions on proton decay changed? ($\Lambda_u \sim \lambda^{-a}\Lambda, m_{H_T}^{eff} \sim \lambda^{2h}\Lambda$)

N.M.-T. Tanii 2312.14214.

Several trials

Model 1, The easiest way to solve this problem is to enhance the neutrino masses by smaller right-handed neutrino masses as

$$W = \epsilon\lambda^{\psi_i+\psi_j+2\bar{c}} 16_{\Psi_i} 16_{\Psi_j} \langle\overline{16}_{\bar{C}}\rangle\langle\overline{16}_{\bar{C}}\rangle \qquad \epsilon \ll 1$$

$$\Rightarrow \left(M_{\nu_R}\right)_{ij} \sim \epsilon\lambda^{\psi_i+\psi_j+2\bar{c}}\langle C\rangle^2$$

The natural GUT with $h = 0$ is not good because the mass term H^2 is not forbidden by the SUSY zero mechanism.

The model with $h = -1$ is more reasonable, that requires $\epsilon \sim 0.001$.

Unfortunately, we have not found any approximate symmetry to realize this suppression.

Several trials(many possibilities)

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2, Changing RGEs. We assume that the suppression factors depends only on the positively charged fields so that the VEVs of fields $\langle Z\rangle \sim \lambda^{-z}$ do not change.

$$W = \epsilon_{A'}\left(\lambda^{a'+a}A'A + \lambda^{a'+3a}A'A^3\right) + \epsilon_{2A'}\lambda^{2a'}A'^2 + \epsilon_{C'}\lambda^{c'+\bar{c}}C'(\lambda^a A + \lambda^{z_1}Z_1)\bar{C}$$
$$+\epsilon_{\bar{C}'}\lambda^{\bar{c}'+c}\bar{C}'(\lambda^a A + \lambda^{z_2}Z_2)C + \epsilon_{\bar{C}'C'}\bar{C}'C' + \epsilon_{H'}\lambda^{h'+a+h}H'AH + \epsilon_{2H'}H'^2$$

+Correct neutrino masses and gauge coupling unification ($\Lambda_u \sim \lambda\Lambda(a=-1)$)

⇒ Model 2: $\epsilon_{H'} \sim 10^{-2}$ and the other $\epsilon \sim 1 \Rightarrow m_{H_T}^{\text{eff}} \sim \Lambda \sim 2\times 10^{16}$GeV.

Model 3: $\epsilon_{A'} \sim 10^{-3}, \epsilon_{H'} \sim 2\times 10^{-4} \Rightarrow \Lambda \sim 2\times 10^{18}$GeV. $m_{H_T}^{\text{eff}} \sim 2\times 10^{16}$GeV.

Model 4: $\epsilon_{H'} \sim 10^{-4},\ \epsilon_{2H'} < \epsilon_{H'} \Rightarrow \Lambda \sim 2\times 10^{16}$GeV. $m_{H_T}^{\text{eff}} \sim \epsilon_{H'}/\epsilon_{2H'} > 2\times 10^{16}$GeV.

Predictions on proton decay can change by these suppression factors. Generically $m_{H_T}^{\text{eff}} \sim \Lambda_G$.

Approximate Z_2 symmetry(odd fields A' and/or H') makes solutions 2 and 3 natural.

Spontaneously broken Z_2 does not work in natural GUT. (Breaking is maximal as $\lambda^z\langle Z\rangle \sim 1$)

⇒It may suggest hiding structure (for example, extra dimension.)

19'N.M.-Y.Omura-Y.Shigekami-M.Yoshida

Natural SO(10) GUT with spontaneous SUSY breaking

$SO(10)$	negative charge	positive charge	matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-4,+)$	$C'(c'=3,-)$	$\Psi_i\left((\psi_1,\psi_2,\psi_3)=(\frac{9}{2},\frac{7}{2},\frac{3}{2}),+\right)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=7,+)$	
10	$H(h=-3,+)$	$H'(h'=4,+)$	$T\left(t=\frac{5}{2},+\right)$
1	$\Theta(\theta=-1,+)$	$Z'(z'=5,+)$	
	$Z_i(z_1=\cancel{z_2}=-2,-)$		

$\frac{\partial W}{\partial z_j^+}=0$ determine $\langle Z_j^-\rangle$. $(N_+\sim N_-)$

If a singlet field Z_2 is removed, then SUSY is spontaneously broken in this model.

When $\bar{c}'$ is larger as $\bar{c}'=18$, $F_{\bar{c}'}\sim\lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})}$ breaks SUSY.

19'N.M.-Y.Omura-Y.Shigekami-M.Yoshida

Natural SO(10) GUT with spontaneous SUSY breaking

Prediction Split(or High scale) SUSY&D term dominance

Approximate $U(1)_R$ symmetry → very small gaugino masses $m_{1/2} \propto F_{\bar{C}'}^2$.

$m_{1/2} \sim m_{3/2} = \frac{F_{\bar{C}'}}{M_{Pl}} \sim 1$ TeV ⇐ SUGRA effect 17'N.M.-Omura-Shigekami-Yoshida

$\mathrm{m}_0^2 \sim D_A \sim (10 m_{SUSY})^2 \sim O((1000\ \mathrm{TeV})^2)$ Sfermion&Higgs mass square

$m_{SUSY} \equiv \frac{F_{\bar{C}'}}{\Lambda} \sim 100$ TeV $(\Lambda \sim \Lambda_G \sim 2 \times 10^{16}\mathrm{GeV})$

Long life heavy charged lepton with O(1) TeV mass is predicted.

Split SUSY, which leads to destabilize m_W, mainly because $\Lambda \ll M_{Pl}$

Natural SO(10) GUT with spontaneous SUSY breaking

Split SUSY due to $\Lambda \ll M_{Pl}$

In Model 3, $\Lambda \sim M_{Pl}$.

$$\rightarrow m_{1/2} \sim m_{\frac{3}{2}} \sim m_{SUSY} = \frac{F_{\bar{C}'}}{M_{Pl}} \sim 1 \text{ TeV}$$

$$\mathrm{m}_0^2 \sim D_A \sim (10 m_{SUSY})^2 \sim O((10\ \mathrm{TeV})^2)$$

The finetuning becomes milder but SUSY flavor and CP problems reappears.

D term dominance is critical. Smaller D term is possible?

If possible, $E_6 \times SU(2)_F \times U(1)_A \times CP$ GUT can solve SUSY flavor and CP problem.

02'N.M., 09'Ishiduki-Kim-N.M.-Sakurai, 14'N.M.-Muramatsu-Shigekami

D term dominance is a generic feature 25'N.M.-Tanii

Big Challenge in Natural GUT with sp. SUSY breaking

How to obtain the stability of the electroweak scale in the natural GUT with spontaneous SUSY breaking?

A collaboration with T.Tanii with an idea is running.

Summary

Natural GUT solves various problems in SUSY GUT under natural assumption that all interactions allowed by symmetry are introduced with O(1) coefficients.

Proton decay via dim. 5 op. (colored Higgs) is suppressed, while proton decay via dim. 6 op. (massive gauge boson) is enhanced.

But there is a tension between neutrino masses and gauge coupling unification.

Additional suppression factors may avoid the tension, although the prediction on proton decay may change.

When a singlet is removed from a natural GUT, SUSY is spontaneously broken.

Split SUSY, D term dominance, and long-lived charged lepton are predicted.

Although SUSY flavor and CP problems can be avoided by split SUSY, the weak scale is destabilized.

It is a big challenge to realize the stability in natural GUT with sp. SUSY breaking.

Considering problems in natural GUT, we may obtain hints for the model beyond the natural GUT.

Other problems of natural GUT

Why do not the terms which spoils the SUSY zero mechanism appear?

Even if $x+y+z<0$, the term $\frac{\mathrm{XYZ}}{\Theta^{-(x+y+z)}}$ is allowed by the symmetry.

$U(1)_A$ has gauge anomaly which is cancelled by shift $U(1)_A$ transformation of a dilaton (or moduli) as

$$D \to D + \frac{i}{2}\delta_{GS}\Lambda_A$$

Then, $\mathrm{e}^{\frac{\mathrm{z}}{\delta_{\mathrm{GS}}}\mathrm{D}}$ transforms like positively charged field with non-vanishing VEV.

Such (non-perturbative?) terms in the superpotential must be forbidden.

This may become a constraint to higher theory like superstring.

Cosmology

If the natural GUT is real, there must be consistent history of the universe.

But we do not understand

1, Inflation The natural assumption spoils the flatness conditions generically.

2, the dark matter (WIMP, axion, axino, etc.) Many candidates, but which is the DM?

Gravitino problem

Thermal leptogenesis works well in E_6 natural GUT which has 6 right-handed neutrinos.

15'T.Ishihara-N.M.-M.Takegawa-M.Yamanaka

Thermal leptogenesis in natural SO(10) is possible.

25' N.M.-Shibata-Yamanaka to appear soon.