河南師範大學
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Proton Lifetime in Minimal SUSY SU(5) with Gauge Mediation

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Based on 2409.06239

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Introduction

- Grand unified theory (GUT) is one of promising extension of the Standard Model (SM)
- All SM particles are embedded into larger multiplets

$$Q_L \quad u_R^c \quad d_R^c \quad H \quad B^0$$

$$L_L \quad e_R^c \quad W^\pm \quad W^3$$

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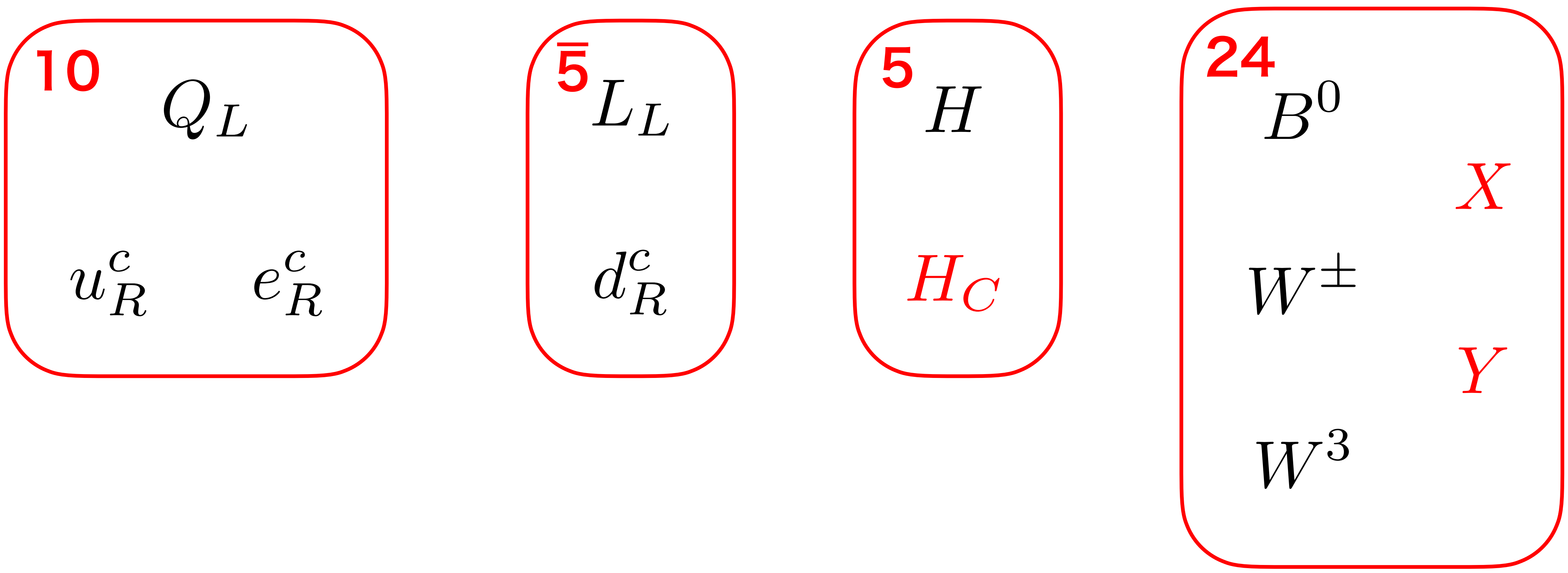

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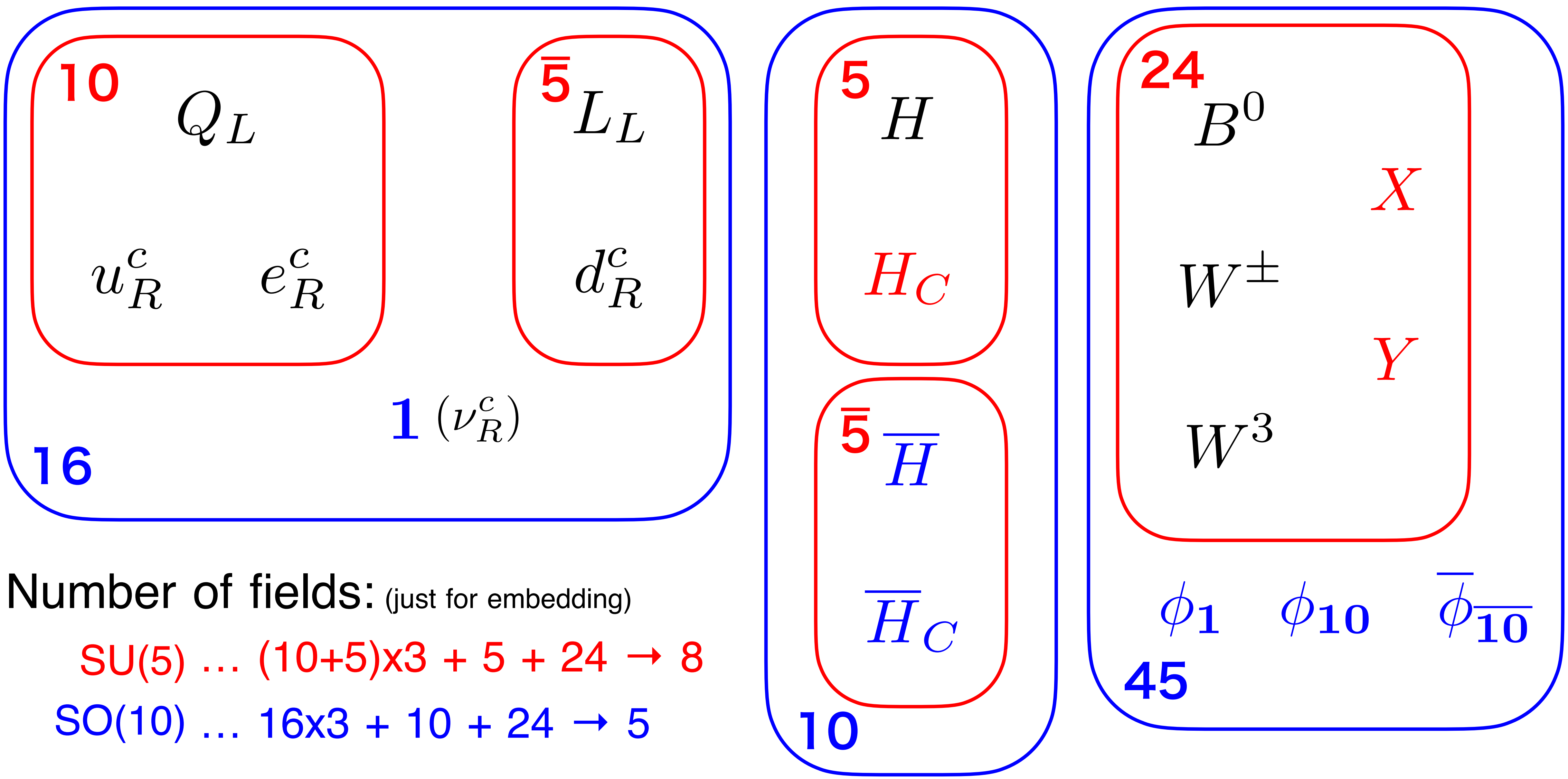


Number of fields: (just for embedding)

SU(5) … (10+5)x3 + 5 + 24 → 8

SO(10) … 16x3 + 10 + 24 → 5

Introduction

- Some of SM parameters are unified

  ✓ Yukawa couplings $(i, j:\ \text{generation indices})$

  SU(5): $(y_u)_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H + (y_{d,e})_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$

  SO(10): $(y_f)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H$

  Note: for SM fermion mass hierarchy, these will not be “minimal” Yukawa’s

  ✓ gauge couplings

  $g_Y\ ,\ g_2\ , g_s \to g_5$ for SU(5) GUT case

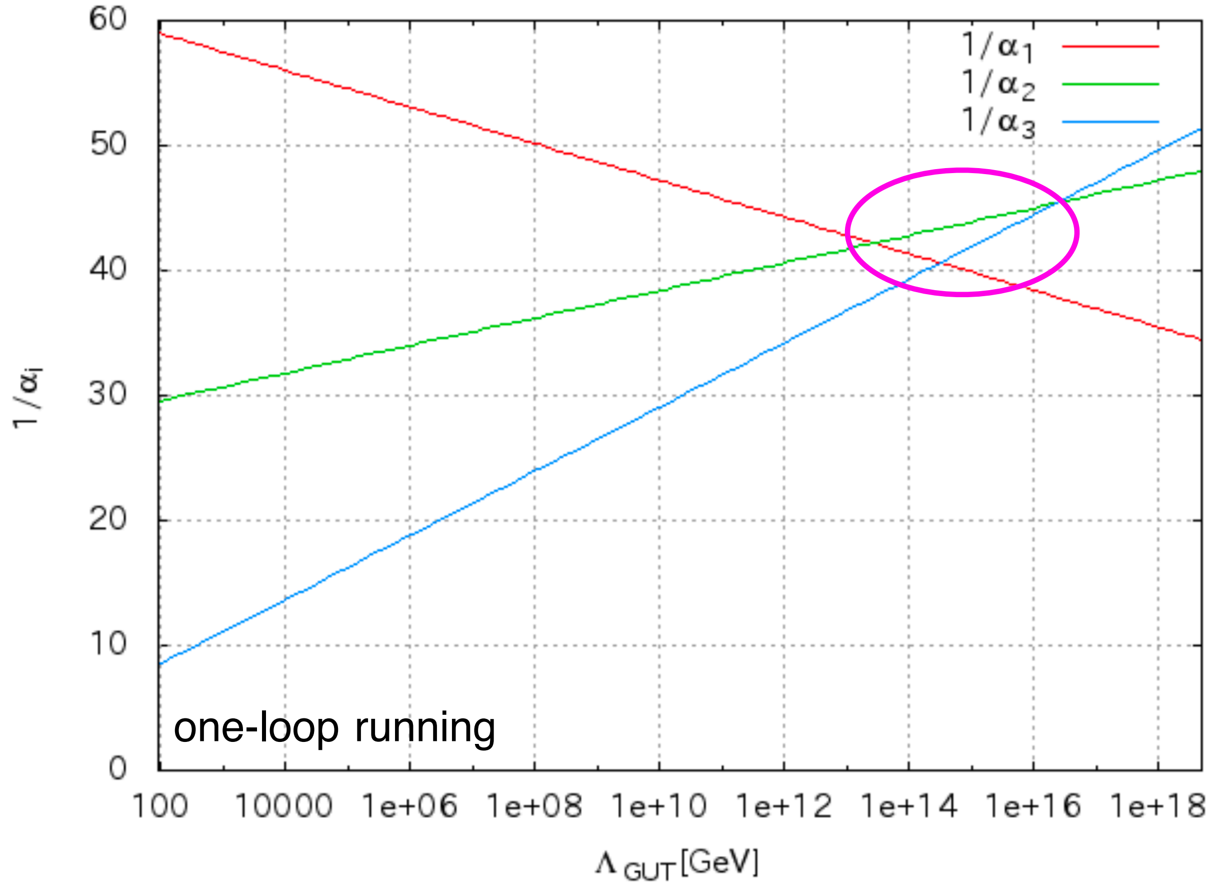
  SM can be obtained from fewer parameters

- This feature is interesting, but GUT has challenging issues

  SM fermion mass hierarchy, doublet-triplet splitting, hierarchy problem, gauge coupling unification, nucleon decays, …

Introduction

- Gauge coupling unification



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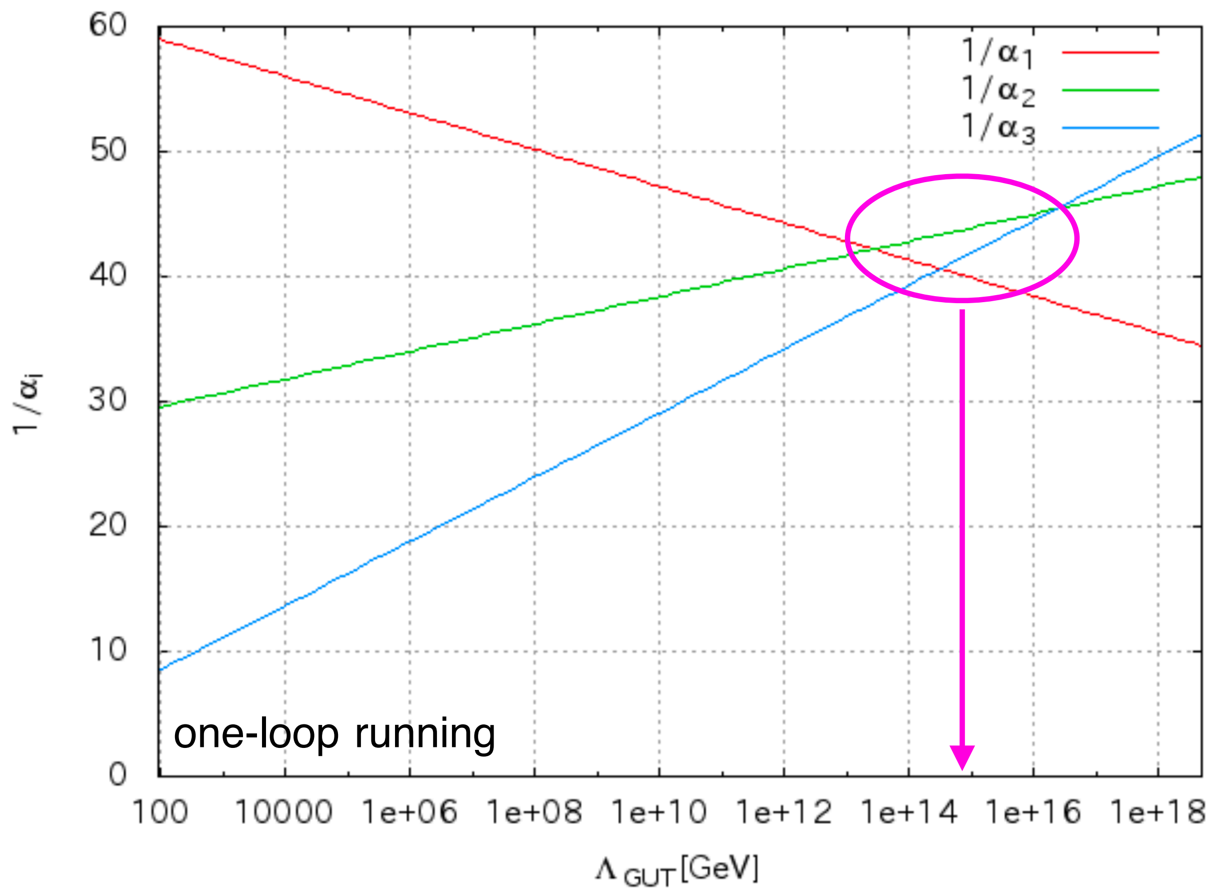

⇨ tends to “unify”, but not accurately

- GUT scale ~ 10^{15} GeV → scale of heavy particle masses

  quite heavy! but, slightly “light” for nucleon decays (depends on gauge coupling)

  another issue: huge hierarchy, M_Z <<<< M_{GUT}!!!

Introduction

- Supersymmetry (SUSY): symmetry of fermion ←→ boson

example:

spin	$1/2, 1$	$1/2, 0$		$1/2, 0$	
fermion	$\widetilde{B}$	Q_L	e_R	$\widetilde{H}_u$	
boson	B^0	$\widetilde{Q}_L$	$\widetilde{e}_R$	H_u	SUSY particles

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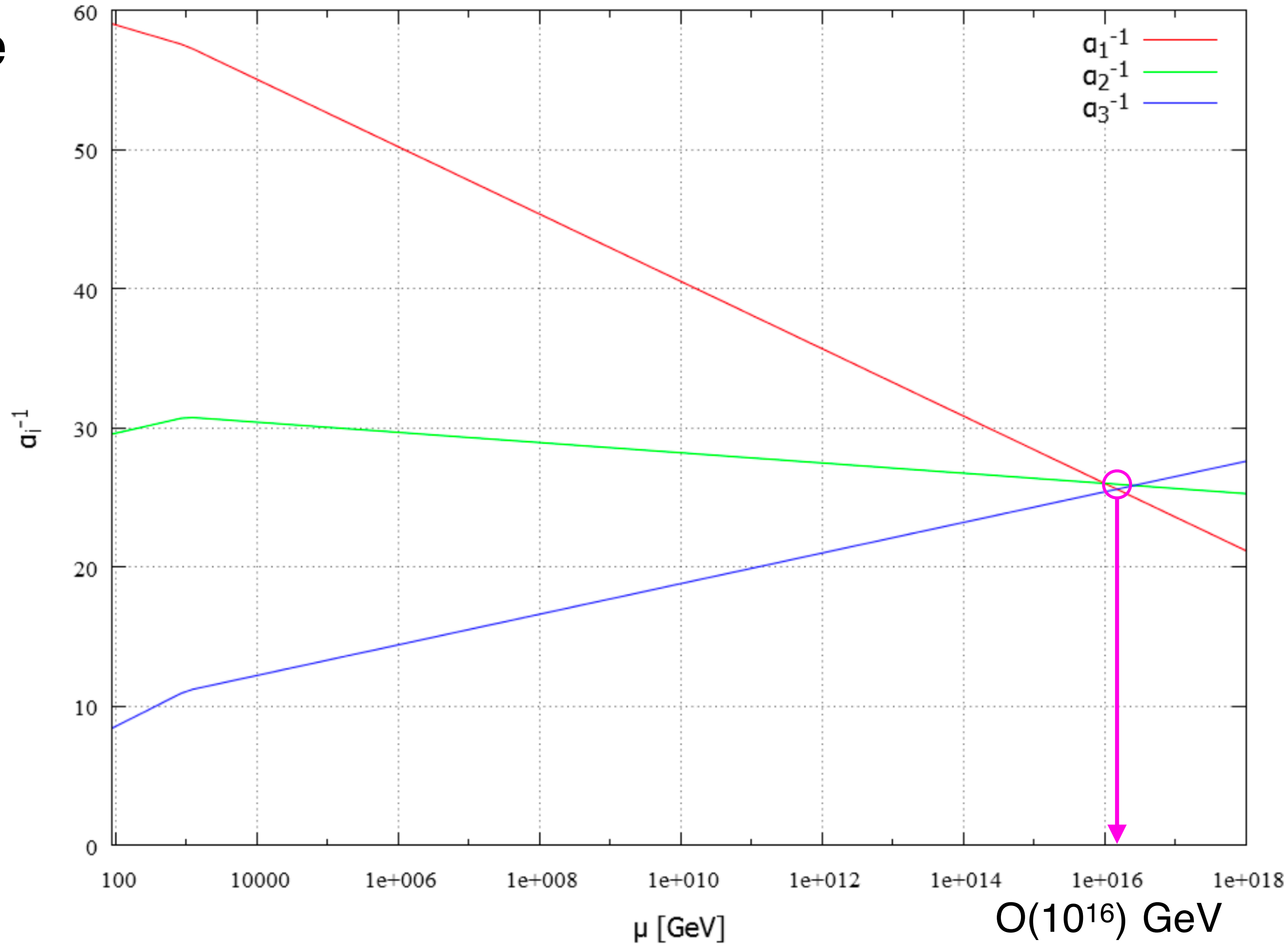

O(10^{16}) GeV

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- SUSY requires the same mass for fermion and boson

$$m_{Q_L} = m_{\widetilde{Q}_L} \,, \; m_{e_R} = m_{\widetilde{e}_R} \,, \cdots$$

No light gauginos, sfermions, Higgsinos

- SUSY should be broken at low energy scale

✓ How to break?

✓ What's the origin?

Introduction

- Soft-breaking terms in minimal SUSY SM (MSSM)

$$\mathcal{L}_{\rm soft} = -\frac{1}{2}M_a\lambda^a\lambda^a - \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k - \frac{1}{2}b^{ij}\phi_i\phi_j + {\rm c.c.} - (m^2)^i_j\phi^{j\,*}\phi_i$$

λ^a : gaugino , ϕ_i : any scalar

- Lots of parameters, and they are arbitrary can be real and/or complex

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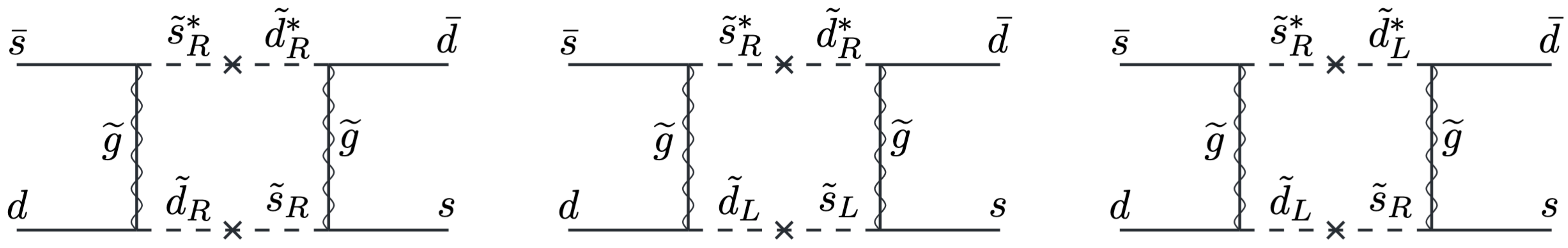
  - ✓ these lead to SUSY flavor/CP-violating processes

e.g.) K-Kbar mixing (figure from S.P. Martin's SUSY primer)



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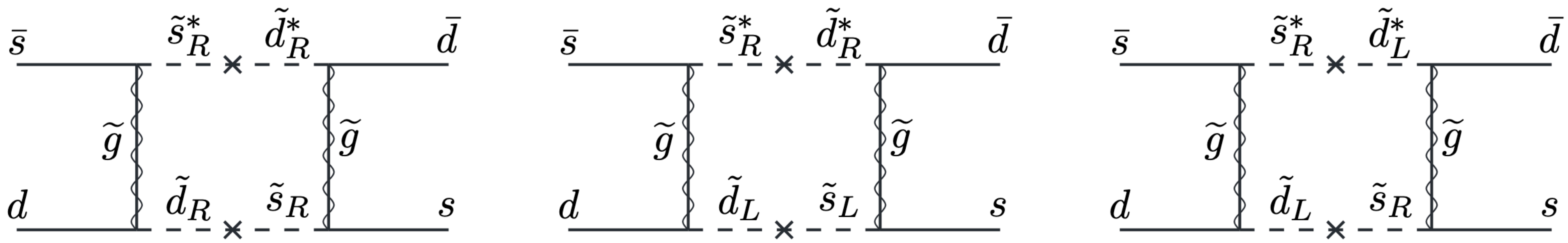

- Soft terms are constrained by flavor/CP violating physics when sfermion masses are close to the weak scale

(collider experiments increase lower bound of the SUSY scale > O(1) TeV)

Introduction

- SUSY breaking mechanisms (with mediation)
  - ◉ Gravity mediation
    - it is straightforward from supergravity
    - there is still SUSY flavor problems, due to long running of parameters
  - ◉ Anomaly mediation
    - high predictability, low SUSY breaking scale
    - dangerous negative sfermion mass square, color-charge breaking minima
  - ◉ Gauge mediation
    - safe from SUSY flavor problems, low SUSY breaking scale
    - vacuum structure will be complicated, (heavy) messenger sector is required

✓ Also, it's possible to break without additional sector (see, e.g., Maekawa-san's talk)

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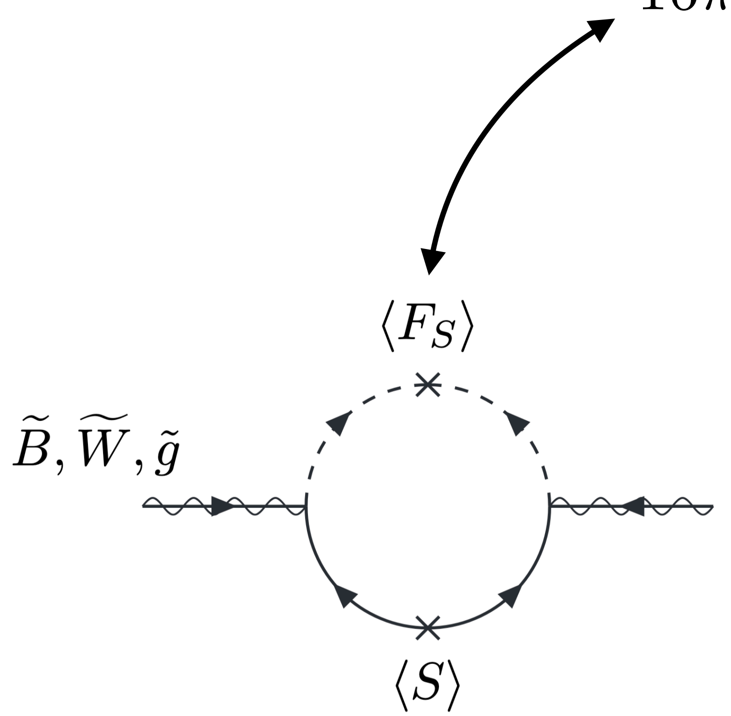
Introduction

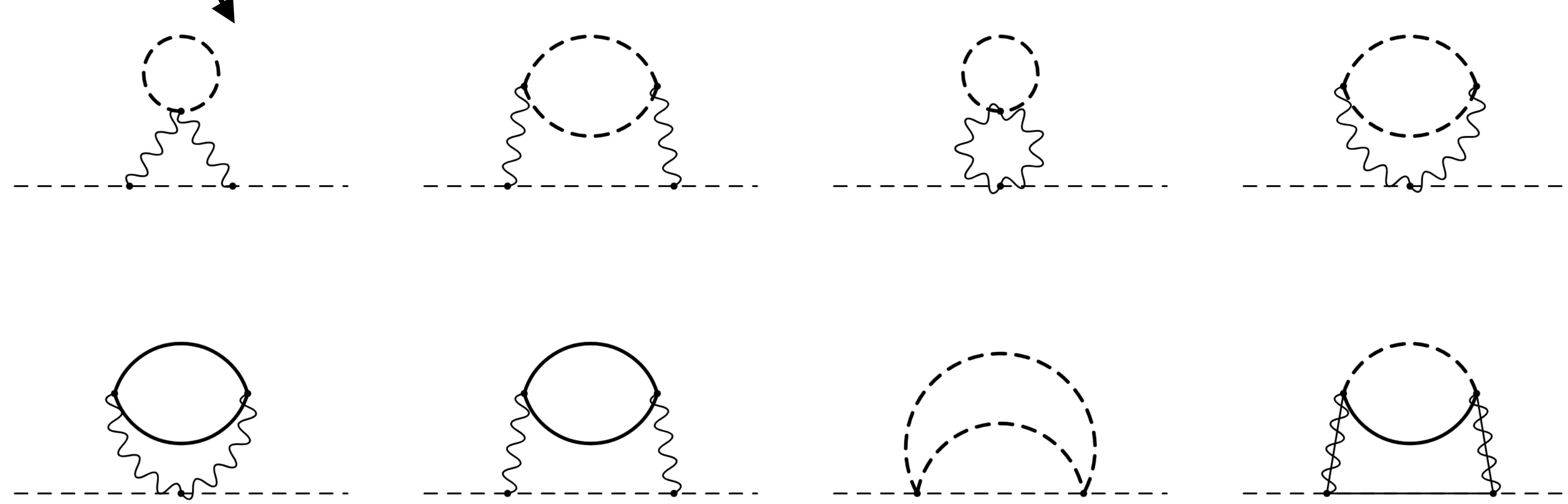
- We focus on gauge mediated SUSY breaking (GMSB)

The key feature: gaugino and sfermion masses generated at M_{mess} due to SM charges of messengers

e.g.) $$M_2 \simeq \frac{g_2^2}{16\pi^2}\Lambda_L\ , \ m^2_{\widetilde{Q}_L} \simeq \frac{2}{(16\pi^2)^2}\left[\frac{4}{3}g_3^4\Lambda_D^2 + \frac{3}{4}g_2^4\Lambda_L^2 + \frac{3}{5}\frac{1}{6^2}g_1^4\Lambda_Y^2\right]$$

$\Lambda_{L,\ D}$: related to the messenger scale M_{mess}





(figures from S.P. Martin's SUSY primer)

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$\Lambda_{L,\,D}$: related to the messenger scale M_{mess}

- Introduce 5+$\overline{5}$ as messengers

minimal model: $\mathbf{10}_i\,,\ \bar{\mathbf{5}}_i\,,\ H\,,\ \overline{H}\,,\ \Sigma\,;\ \mathbf{5} = (\Psi_D, \Psi_L)\,,\ \bar{\mathbf{5}} = (\Psi_{\bar{D}}, \Psi_{\bar{L}})\,,\ Z$

scalar and auxiliary components get VEV, leads to SUSY breaking

- In the viewpoint of GUT, these messengers contribute to RGE of gauge coupling

but expected to be unified, because of vector-like messenger

Introduction

- We are interested in the prediction of proton decay

  our main focus is $p \to K^+ \bar{\nu}$ (dim. 5 operator, mediated by H_C)

  but also check $p \to \pi e$ (dim. 6 operator, mediated by X)

- Check the current Super-Kamiokande (SK) bound

  most of interesting parameter space is safe, but some of it not

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  most of interesting parameter space is safe, but some of it not

- Also check the future prospects (@90% C.L., 10(20) years running)

  Bhattiprolu, Martin, Wells [PRD107(2023)055016]

  ✓ Hyper-Kamiokande (HK): $3.1(4.8) \times 10^{34}$ years

  ✓ Deep Underground Neutrino Experiment (DUNE): $2.2(4.2) \times 10^{34}$ years

  ✓ Jiangmen Underground Neutrino Observatory (JUNO): $0.96(1.8) \times 10^{34}$ years

Most of parameter space can be tested!

Model

- Visible sector … minimal SUSY SU(5)

Evans, Yanagida [PLB833(2022)137359]
Evans, YS [2409.06239]

$$W_{\rm vis} = \mu_\Sigma {\rm Tr}\Sigma^2 + \frac{\lambda'}{6}{\rm Tr}\Sigma^3 + \mu_H \overline{H} H + \lambda \overline{H}\Sigma H + (h_{\bf 10})_{ij} {\bf 10}_i {\bf 10}_j H + (h_{\bar{\bf 5}})_{ij} {\bf 10}_i \bar{\bf 5}_j \overline{H}$$

H : fundamental Higgs superfield
$\overline{H}$: anti-fundamental Higgs superfield
Σ : adjoint superfiled
${\bf 10}_i$: matter superfiled $(Q_L,\, u_R^c,\, e_R^c)$
$\bar{\bf 5}_i$: matter superfiled $(L_L,\, d_R^c)$

$h_{\bf 10}, h_{\bar{\bf 5}}$ are Yukawa couplings for SM fermions

- Scalar component of the adjoint acquires non-zero VEV

SU(5) → $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\langle \Sigma \rangle = V \cdot {\rm diag}(2,2,2,-3,-3) \quad \text{with } V = 4\mu_\Sigma/\lambda'$$

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- This VEV scales GUT scale fields as

$$\text{GUT gauge boson}: \ M_X = 5g_5 V\,; \quad \text{color triplet Higgs}: \ M_{H_C} = 5\lambda V\,;$$

$$\text{color octet of } \Sigma: \ M_\Sigma = \frac{5}{2}\lambda' V$$

g_5 is unified gauge coupling

Model

- Matching conditions for gauge coupling unification

$$\frac{1}{g_1^2(Q)} = \frac{1}{g_5^2(Q)} + \frac{1}{8\pi^2}\left[\frac{2}{5}\ln\frac{Q}{M_{H_C}} - 10\ln\frac{Q}{M_X}\right] - \epsilon\,,$$
$$\frac{1}{g_2^2(Q)} = \frac{1}{g_5^2(Q)} + \frac{1}{8\pi^2}\left[2\ln\frac{Q}{M_\Sigma} - 6\ln\frac{Q}{M_X}\right] - 3\epsilon\,,$$
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Hisano, Murayama, Goto [PRD49(1994)1446]
Tobe, Wells [PLB588(2004)99]
Evans, Nagata, Olive [PRD91(2015)055027]

$g_{1,2,3}$: SM gauge couplings with $g_1^2 = (5/3)g_Y^2$

- $\epsilon \equiv \frac{8dV}{M_P}$ is a contribution from Planck suppressed operator:

$$W_{\rm eff}^{\Delta g} = \frac{d}{M_P}{\rm Tr}\left[\Sigma\mathcal{W}\mathcal{W}\right]$$

with $\mathcal{W} = \mathcal{W}^A T^A$ ($SU(5)$ field strength superfield)

This contribution should be included because of the fact that $V/M_P \simeq 10^{-2}$
→ it is comparable to one-loop threshold corrections!

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- With this ϵ, we can find viable model … 2 d.o.f. (3 fixed)

Without ϵ, the proton lifetime tends to be short (Evans, Yanagida [PLB833(2022)137359])

Colored Higgs mass is determined by $\frac{1}{g_1^2(Q)} - \frac{3}{g_2^2(Q)} + \frac{2}{g_3^2(Q)} = \frac{3}{10\pi^2}\ln\frac{Q}{M_{H_C}} + 12\epsilon$

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- Other constraints:

$$\frac{5}{g_1^2(Q)} + \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} = \frac{6}{g_5^2(Q)} - \frac{15}{2\pi^2}\ln\frac{Q}{M_X} - 18\epsilon$$

$$\frac{5}{g_1^2(Q)} - \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} = -\frac{3}{2\pi^2}\ln\frac{Q^3}{M_X^2 M_\Sigma}$$

Model

- Messenger sector … introduce $\mathbf{5}+\bar{\mathbf{5}}$ messengers

$$W_{\rm mess} = (M_L + k_L Z)\Psi_L \Psi_{\bar{L}} + (M_D + k_D Z)\Psi_D \Psi_{\bar{D}} - \xi_Z Z$$

$\Psi_L : SU(2)_L$ doublet ; $\Psi_D : SU(3)_C$ triplet ; $Z :$ singlet superfield

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- After integrating out messengers, we obtain all gauginos and scalar masses as

$$M_1 \simeq \frac{g_1^2}{16\pi^2}\left(\frac{2}{5}\Lambda_D + \frac{3}{5}\Lambda_L\right) , \ M_2 \simeq \frac{g_2^2}{16\pi^2}\Lambda_L , \ M_3 \simeq \frac{g_3^2}{16\pi^2}\Lambda_D ,$$

$$m_{\varphi_i}^2 \simeq \frac{2}{(16\pi^2)^2}\left[C_3(i) g_3^4 \Lambda_D^2 + C_2(i) g_2^4 \Lambda_L^2 + \frac{3}{5} Y_i^2 g_1^4 \Lambda_Y^2\right]$$

$$\Lambda_{L,D} \equiv \frac{k_{L,D}}{M_{L,D}}\langle F_Z \rangle = \frac{k_{L,D}}{M_{L,D}}\xi_Z$$

$$\Lambda_Y^2 = \frac{2}{5}\Lambda_D^2 + \frac{3}{5}\Lambda_L^2$$

Quadratic Casimir invariant for SU(N): $C_N(i) = \frac{N^2-1}{2N}$ (fundamental), N (adjoind)

- These are generated at messenger scale, $M_{\rm mess} \sim M_{L,D}$

Proton lifetime

- $p \to K^+ \bar{\nu}$ is relevant

More details: [Ellis, Evans, Nagata, Olive, Velasco-Sevilla [EPJC80(2020)332]]

$$W_{p-\text{decay}} = \frac{1}{2} C_{5L}^{ijkl} \epsilon_{abc} \left(Q_i^a \cdot Q_j^b\right) \left(Q_k^c \cdot L_l\right) + C_{5R}^{ijkl} \epsilon_{abc} \left(\bar{u}_{ia} \bar{e}_j \bar{u}_{kb} \bar{u}_{lc}\right)$$

Each coefficients: $C_{5L}^{ijkl} = \frac{\sqrt{8}}{M_{H_C}} h_{\mathbf{10},i} e^{i\phi_i} \delta^{ij} V_{kl}^* h_{\bar{\mathbf{5}},l}$ and $C_{5R}^{ijkl} = \frac{\sqrt{8}}{M_{H_C}} h_{\mathbf{10},i} V_{ij} V_{kl}^* h_{\bar{\mathbf{5}},l} e^{-i\phi_k}$

- Our choice for Yukawa basis:

$$(h_{\mathbf{10}})_{ij} = e^{i\phi_i} \delta^{ij} h_{\mathbf{10},i} \,, \quad (h_{\bar{\mathbf{5}}})_{ij} = V_{ij}^* h_{\bar{\mathbf{5}},j} \,.$$

ϕ_i : GUT phases with $\phi_1 + \phi_2 + \phi_3 = 0$

V_{ij} : CKM matrix elements

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⇩ Running down to QCD scale by RGE

- Decay rate: $\Gamma(p \to K^+ \bar{\nu}_i) = \frac{m_p}{32\pi} \left(1 - \frac{m_K^2}{m_p^2}\right)^2 \left|\mathcal{A}(p \to K^+ \bar{\nu}_i)\right|^2$

Summing up all neutrino flavors, taking inverse gives lifetime

$$\tau(p \to K^+ \bar{\nu}_i) = \left(\sum_{i=1}^{3} \Gamma(p \to K^+ \bar{\nu}_i)\right)^{-1} > 5.9 \times 10^{33} \,\mathrm{years} \quad @90\% \ \mathrm{C.L.}$$ [SK (2014)]

Proton lifetime

- $p \to K^+ \bar{\nu}$ amplitude

More details: Ellis, Evans, Nagata, Olive, Velasco-Sevilla [EPJC80(2020)332]

$$\mathcal{A}(p \to K^+ \bar{\nu}_i) = C_{LL_i} \left(\langle K^+ | (us)_L d_L | p \rangle + \langle K^+ | (ud)_L s_L | p \rangle \right) + C_{RL_1} \langle K^+ | (us)_R d_L | p \rangle + C_{RL_2} \langle K^+ | (ud)_R s_L | p \rangle$$

Approx. form of coefficients:

$$C_{LL_i} \simeq \frac{2\alpha_2^2}{\sin 2\beta} \frac{m_t m_{d_i} M_2}{m_W^2 M_{H_C} M_{\rm SUSY}^2} V_{ui}^* V_{td} V_{ts} e^{i\phi_3} \left(1 + e^{i(\phi_2 - \phi_3)} \frac{m_c V_{cd} V_{cs}}{m_t V_{td} V_{ts}} \right)$$

$$C_{RL_1} \simeq -\frac{\alpha_2^2}{\sin^2 2\beta} \frac{m_t^2 m_s m_\tau \mu}{m_W^4 M_{H_C} M_{\rm SUSY}^2} V_{tb}^* V_{us} V_{td} e^{i\phi_1}$$

$$C_{RL_2} \simeq -\frac{\alpha_2^2}{\sin^2 2\beta} \frac{m_t^2 m_d m_\tau \mu}{m_W^4 M_{H_C} M_{\rm SUSY}^2} V_{tb}^* V_{ud} V_{ts} e^{i\phi_1}$$

Hadronic matrix elements:

$$\langle K^+ | (us)_L d_L | p \rangle = 0.041(2)(5)\,\text{GeV}^2\,,$$
$$\langle K^+ | (ud)_L s_L | p \rangle = 0.139(4)(15)\,\text{GeV}^2\,,$$
$$\langle K^+ | (us)_R d_L | p \rangle = -0.049(2)(5)\,\text{GeV}^2\,,$$
$$\langle K^+ | (ud)_R s_L | p \rangle = -0.134(4)(14)\,\text{GeV}^2\,.$$

Proton lifetime

- $p \to K^+ \bar{\nu}$ amplitude

More details: [Ellis, Evans, Nagata, Olive, Velasco-Sevilla [EPJC80(2020)332]]

$$\begin{aligned}\mathcal{A}(p \to K^+ \bar{\nu}_i) =& C_{LL_i} \left(\langle K^+ | (us)_L d_L | p \rangle + \langle K^+ | (ud)_L s_L | p \rangle \right) \\ &+ C_{RL_1} \langle K^+ | (us)_R d_L | p \rangle + C_{RL_2} \langle K^+ | (ud)_R s_L | p \rangle\end{aligned}$$

Approx. form of coefficients:

$$C_{LL_i} \simeq \frac{2\alpha_2^2}{\sin 2\beta} \frac{m_t m_{d_i} M_2}{m_W^2 M_{H_C} M_{\mathrm{SUSY}}^2} V_{ui}^* V_{td} V_{ts} e^{i\phi_3} \left(1 + e^{i(\phi_2 - \phi_3)} \frac{m_c V_{cd} V_{cs}}{m_t V_{td} V_{ts}} \right)$$

$$C_{RL_1} \simeq -\frac{\alpha_2^2}{\sin^2 2\beta} \frac{m_t^2 m_s m_\tau \mu}{m_W^4 M_{H_C} M_{\mathrm{SUSY}}^2} V_{tb}^* V_{us} V_{td} e^{i\phi_1}$$

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- Longer lifetime needs small coefficients … means

✓ Large M_{HC} … we cannot choose small λ

✓ Large M_{SUSY} … determined by stop masses, not so small (O(10) TeV)

✓ Small M_2 … determined by Λ_L, smaller tends to longer lifetime

✓ Small μ … numerically determined, difficult to control

Numerical analysis

- We have eight free parameters of the model:

$$\tan\beta\,, \quad \Lambda_D\,, \quad \Lambda_L\,, \quad M_{\text{mess}}\,, \quad \lambda\,, \quad \lambda'\,, \quad \phi_{2,3}$$

- Parameter setting for analysis

$\tan\beta$: ratio of two VEVs, choose $\tan\beta = 5$ from previous work

$\Lambda_{D,L}$: inputs of analysis

M_{mess} : messenger scale, choose $M_{\text{mess}} = 1500$ TeV

λ, λ' : determine M_{H_C}, M_Σ, choose $\lambda = 0.6$ and $\lambda' = 10^{-3}$ in analysis

$\phi_{2,3}$: in range of $0 \leq \phi_{2,3} \leq 2\pi$, and find longest $\tau(p \to K^+\bar{\nu})$

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- Check list:

Proton lifetime, SM Higgs mass, EWSB, collider bound, …

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- Check list:

Proton lifetime, SM Higgs mass, EWSB, collider bound, …

Requires $\Lambda_D = O(10^3)$ TeV

Ibe, Matsumoto, Yanagida, Yokozaki [JHEP03(2013)078]
Asano, Yokozaki [PRD93(2016)095002]
Bhattacharyya, Yanagida, Yokozaki [PLB784(2018)118]

Note: Using FeynHiggs to calculate the Higgs mass

DM pheno., muon g-2, … → future work

Numerical analysis

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$$\tan\beta = 5\,, \quad M_{\rm mess} = 1500\,{\rm TeV}\,, \quad \lambda = 0.6\,, \quad \lambda' = 10^{-3}\,,$$

$$\phi_{2,3} \cdots \text{ optimized for longer } \tau(p \to K^+\bar{\nu})\,, \quad \Lambda_{D,L} \cdots \text{ inputs}$$

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- Other setting for calculation
  - Gaugino and scalar masses are given at messenger scale $M_{\rm mess}$
  - A-terms and B parameter are set to be zero at messenger scale
  - Non-zero A-terms are generated via RGE from messenger to SUSY scale
  - B parameter at low scale is determined by EWSB conditions
  - Iterate until μ is converged (bottom-up <—> top-down)

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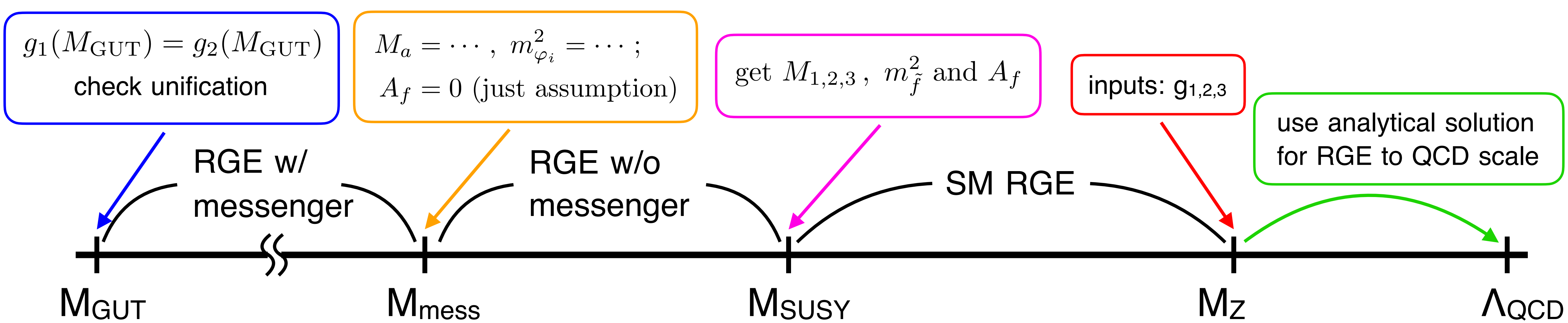
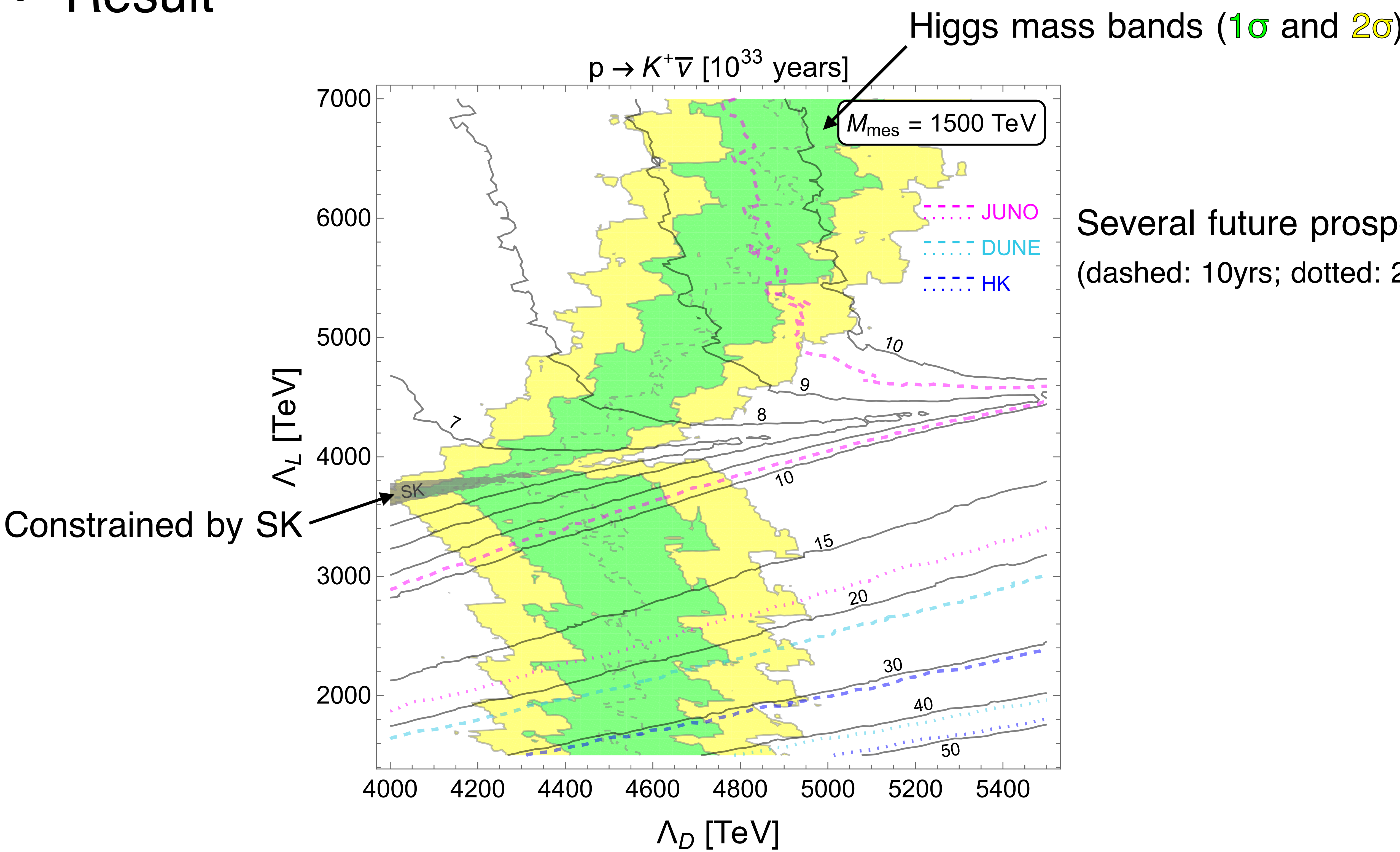


Numerical analysis

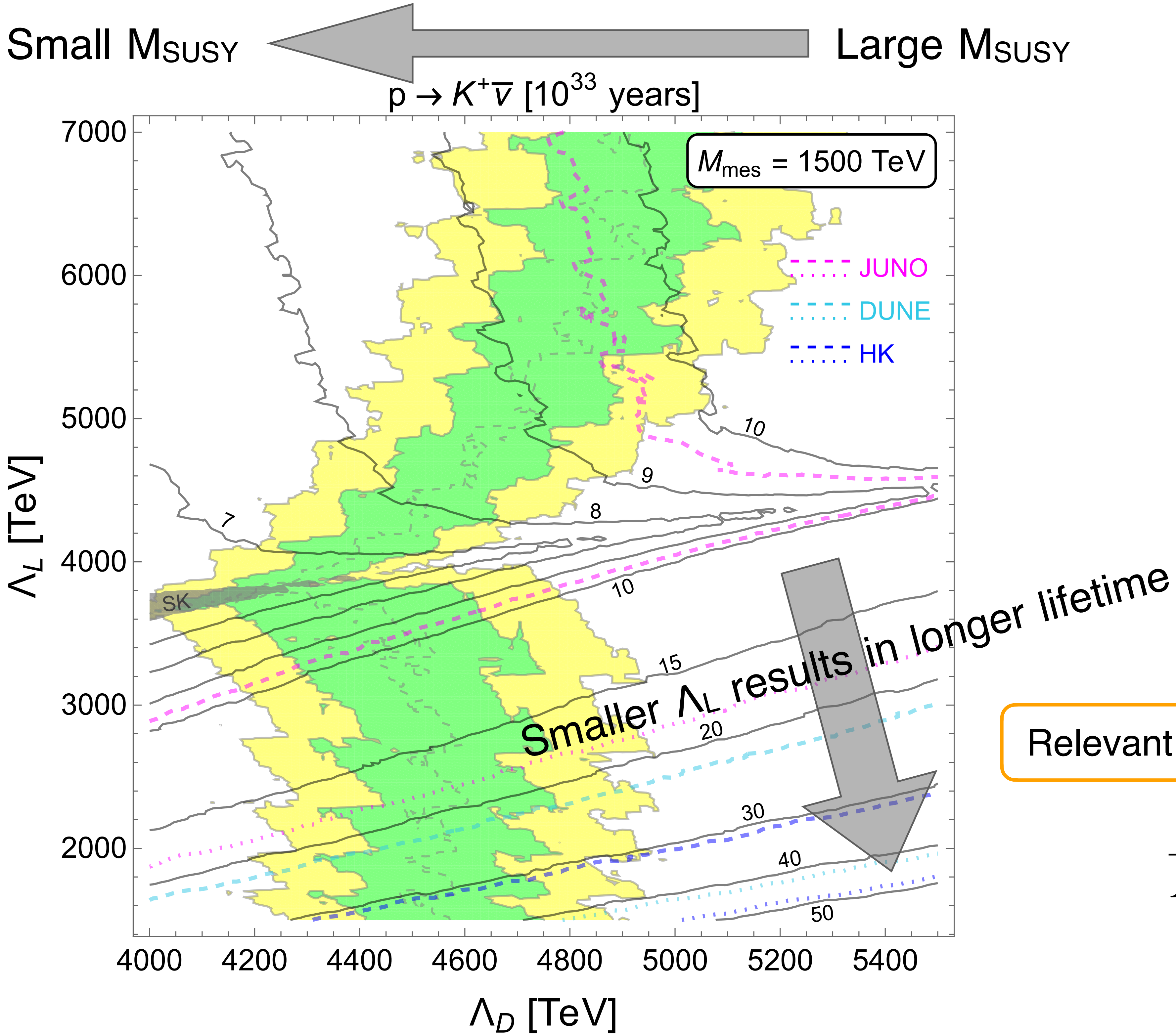
- Result



Several future prospects
(dashed: 10yrs; dotted: 20yrs)

Numerical analysis

- Result



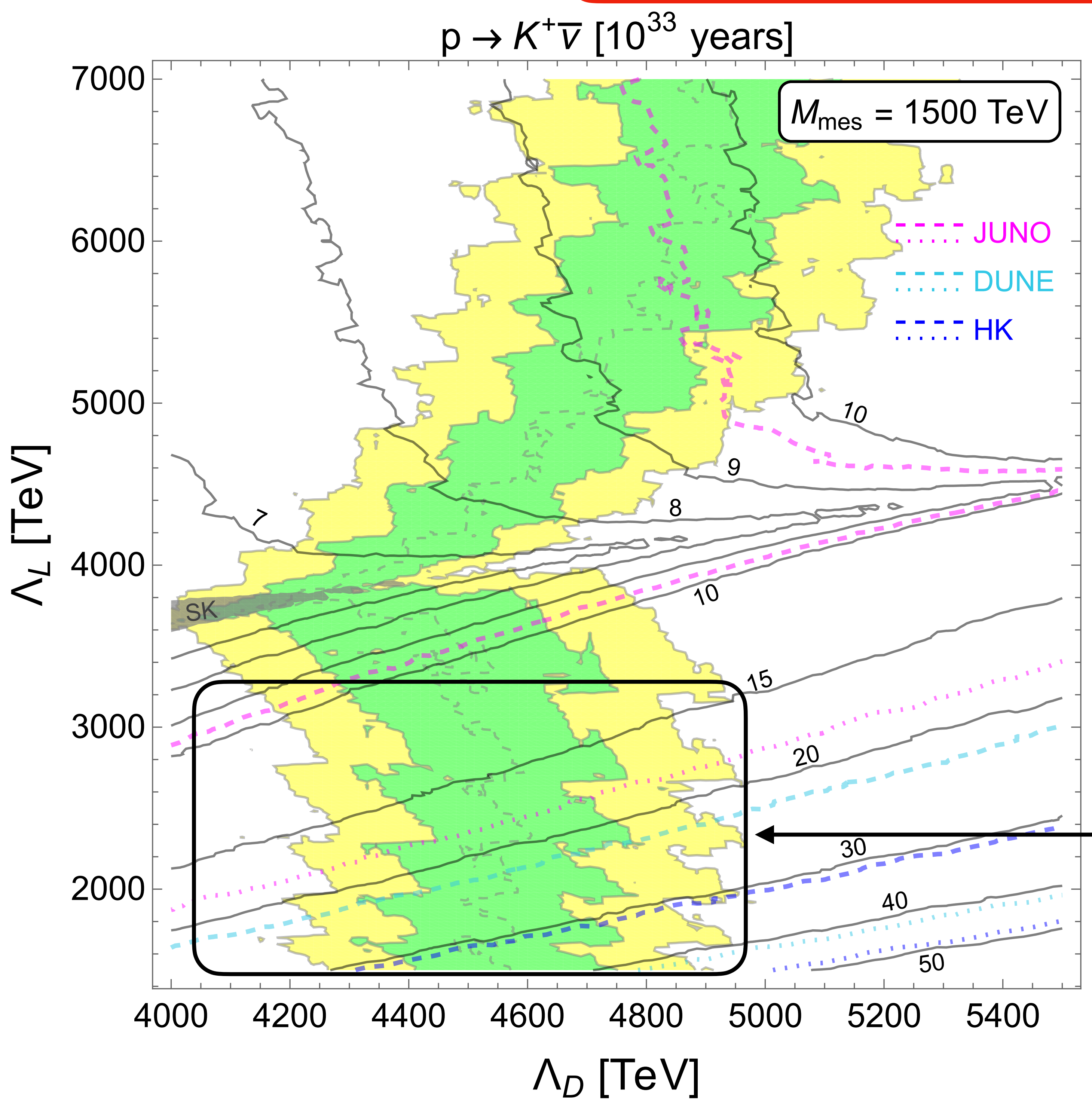
Relevant combination:

$$\frac{M_2 \text{ or } \mu}{M_{H_C} M_{\text{SUSY}}^2}$$

Numerical analysis

- Result

Higgs mass … determined by stops

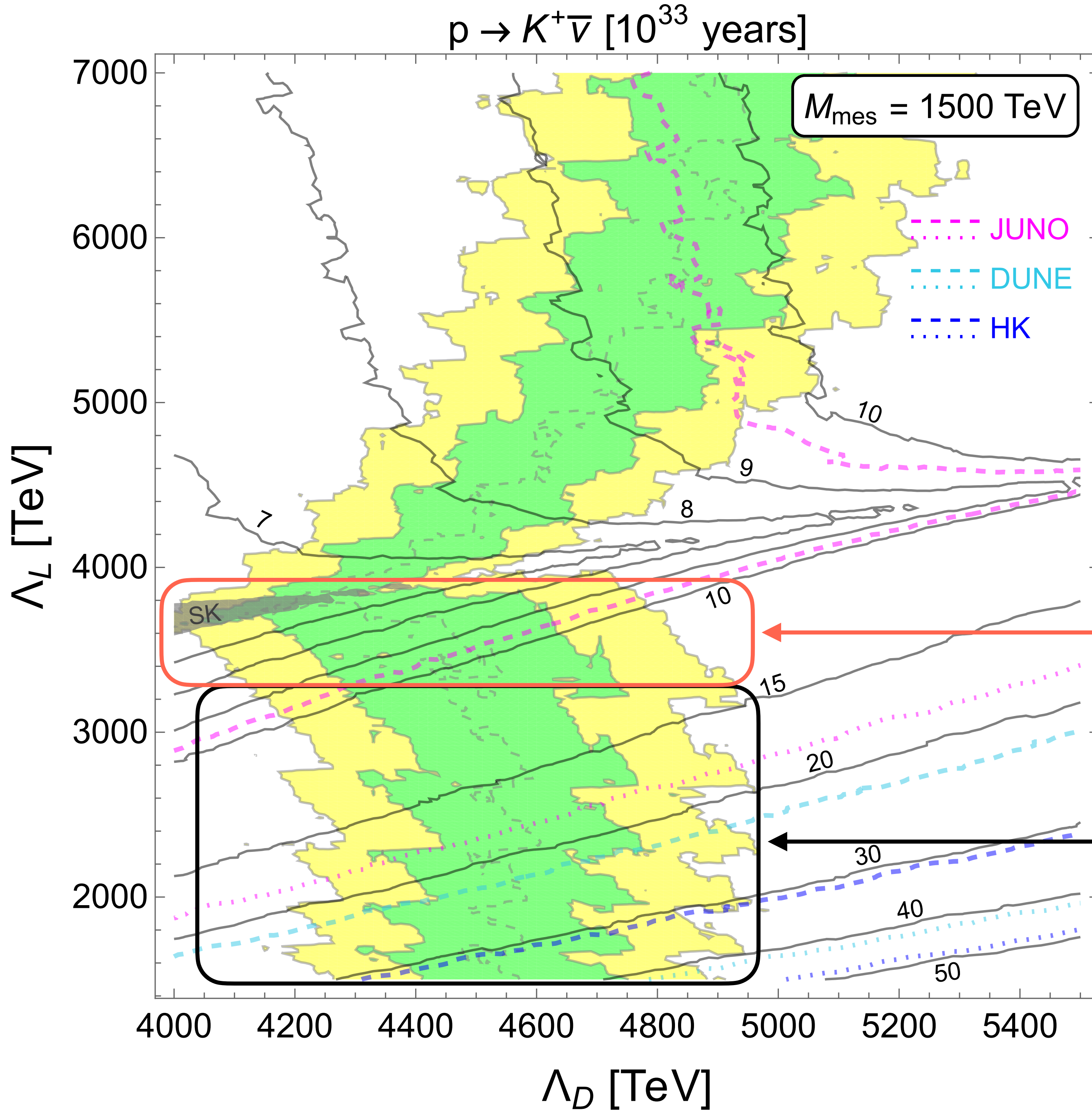


For small Λ_L, stop mass is almost determined by Λ_D

Numerical analysis

- Result

Higgs mass … determined by stops



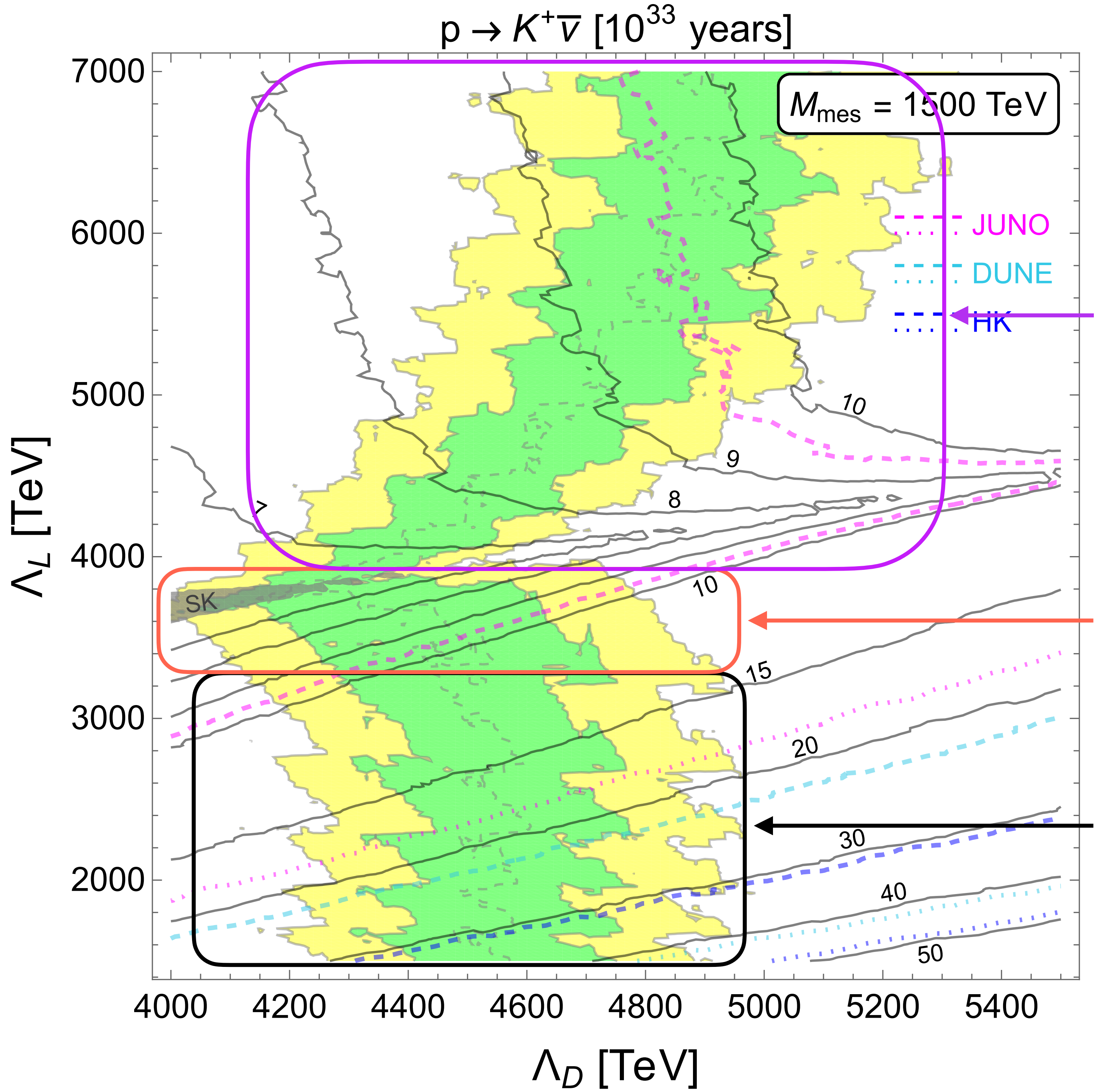
For this region, both Λ_L and Λ_D contribute to stop mass

For small Λ_L, stop mass is almost determined by Λ_D

Numerical analysis

- Result

Higgs mass … determined by stops



For large Λ_L, radiative corrections from stop mixing parameter affect m_H

$$X_t = A_t - \mu/\tan\beta$$

For this region, both Λ_L and Λ_D contribute to stop mass

For small Λ_L, stop mass is almost determined by Λ_D

Comments

- We do not check,

  Dark matter (gravitino LSP)

  - minimal model of GMSB suffers from the gravitino problem

  anomalous magnetic moment of muon

  - since most of relevant particles are heavy (O(10) TeV), $\Delta a_\mu = O(10^{-12})$

  μ-B_μ problem in GMSB

  - it is better than gravity mediated model and non-SUSY GMSB, but still need to be tuned for μ

- Go beyond the minimal, what happens for proton decay?

  additional fields will change RGE

  light sleptons and/or light bino or wino will change lifetime

  …

Summary

- We explore the proton lifetime in SU(5) SUSY GUT with gauge mediation, $5+\bar{5}$ messenger case

- M_{mess} = O(1000) TeV leads to M_{SUSY} = O(10) TeV
  - ✓ This results in reproduction of the SM Higgs mass ~ 125 GeV!

- Focus: $p \to K^+\bar{\nu}$, bounded by SK (exclude small region)
  - ✓ Most of parameter space can be tested by HK, DUNE, JUNO

- Further extension will be required for several issues
  - DM pheno., gravitino problem, muon g-2, μ-B_μ problem, …

Thank you!

Back up

Future prospects of proton decay

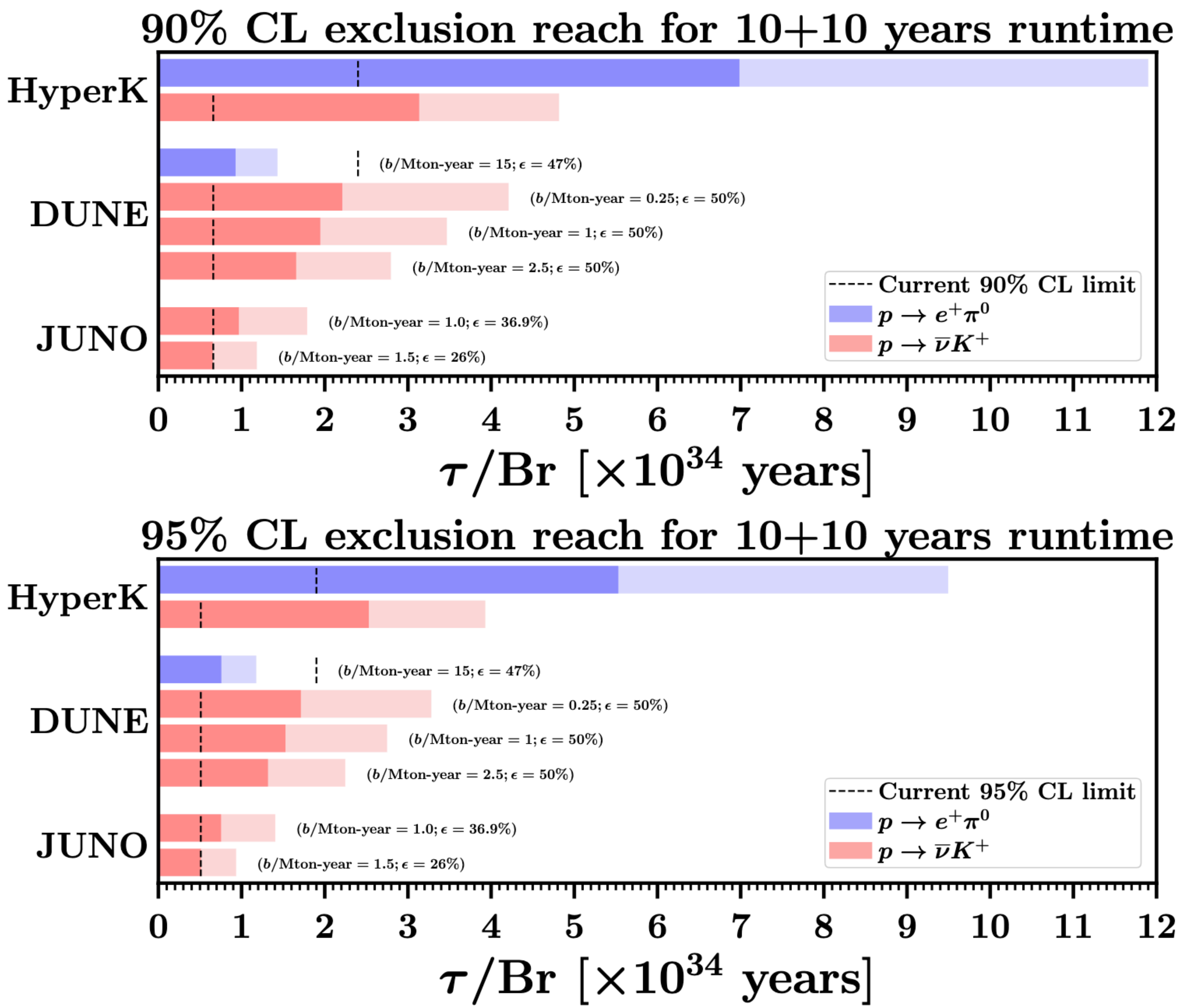


Fig. 4.1 of Bhattiprolu, Martin, Wells [PRD107(2023)055016]

Breaking of explicit GUT relation

- Messenger sector: should be coming from GUT interaction

$$W_{\rm mess} = (M_L + k_L Z)\Psi_L \Psi_{\bar{L}} + (M_D + k_D Z)\Psi_D \Psi_{\bar{D}}$$
$$\leftarrow M\Psi\bar{\Psi} + kZ\Psi\bar{\Psi}$$

- Basically, there is no large hierarchy btw. Λ_L and Λ_D

$\Lambda_D \sim 4500$ TeV, while smaller Λ_L is favored for longer proton lifetime

- Adjoint field helps to split Λ_L and Λ_D with term of

$$W \supset \lambda_\Psi \Psi \Sigma \bar{\Psi} \to \lambda_\Psi \Psi \langle \Sigma \rangle \bar{\Psi}$$

$$\Rightarrow \quad M_L = M - 3\lambda_\Psi V \,; \;\; M_D = M + 2\lambda_\Psi V$$

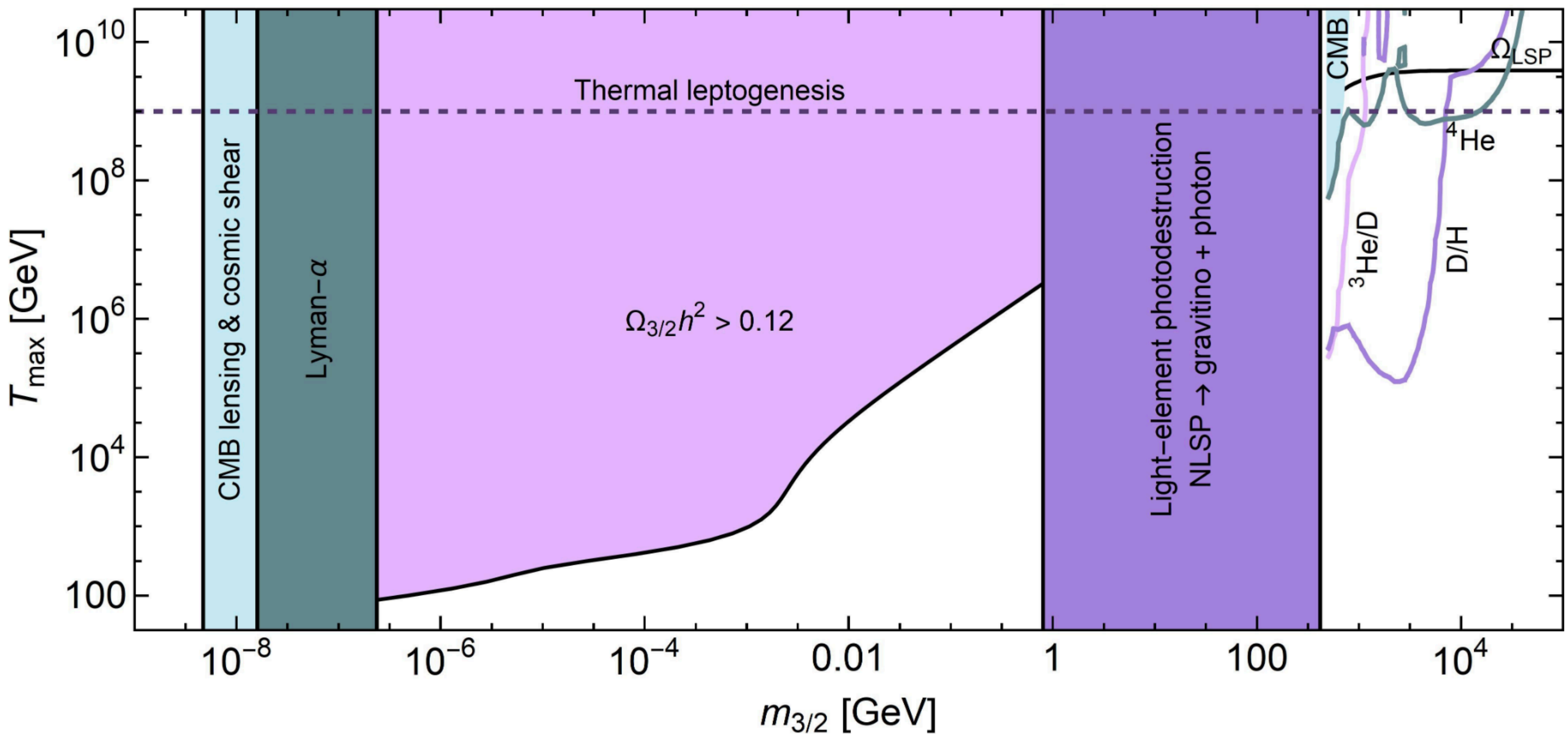
- Appropriate choice of M, λ_Ψ, k reproduces desired (Λ_L, Λ_D)

Gravitino mass

- Rough estimate

$$m_{3/2} \simeq \frac{F_Z}{\sqrt{3}M_{\rm pl}} \simeq 16\ \mathrm{keV} \left(\frac{\Lambda_D}{4500\ \mathrm{TeV}}\right)\left(\frac{M_D}{1500\ \mathrm{TeV}}\right)\left(\frac{0.1}{k_D}\right)$$

- Cosmologically, severely constrained: Hook, McGehee, Murayama [PRD98(2018)115036]



Muon g-2

- In minimal GMSB model, muon g-2 tends to be small

  125 GeV Higgs requires M_{mess} ~ O(1000) TeV
  → naively, all sfermions and gauginos are heavy
  → muon g-2 is suppressed

- Additional Higgs soft squared mass will help

  introduce two singlets X, Y

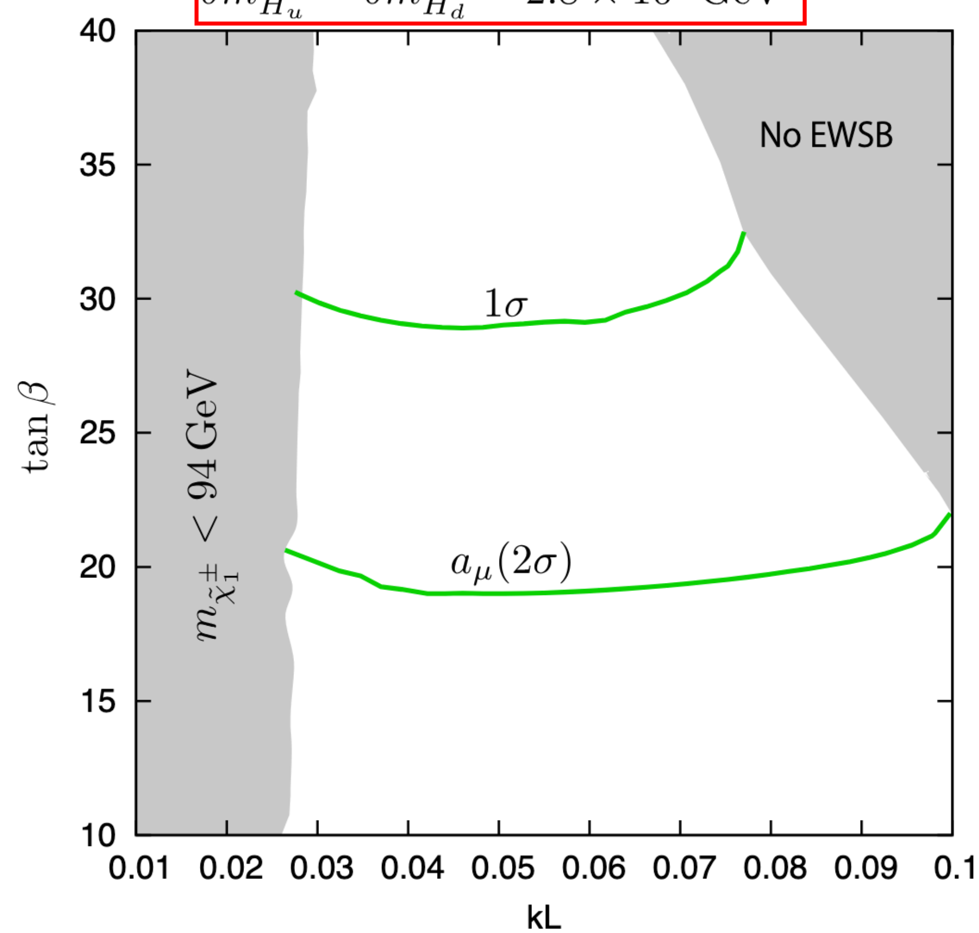
$$W \supset \frac{\kappa}{2} Z X^2 + M_{XY} XY + \lambda X H_u H_d$$

Ibe, Matsumoto, Yanagida, Yokozaki [JHEP03(2013)078]

- For large tanβ, there is a chance to explain the discrepancy!

$M_{\rm mess} = 2000{\rm TeV}, \quad \Lambda_D = 1250\,{\rm TeV} \quad N_5 = 1$

$\delta m^2_{H_u} = \delta m^2_{H_d} = 2.8 \times 10^7\,{\rm GeV}^2$



μ-B_μ problem

- Tree-level equations for EWSB:

$$\frac{m_Z^2}{2} = -|\mu|^2 - \frac{m_{H_u}^2 \tan^2\beta - m_{H_d}^2}{\tan^2\beta - 1}, \quad \sin 2\beta = \frac{2B_\mu}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}$$

Dvali, Giudice, Pomarol [NPB478(1996)31]

- In GMSB models, one obtains $B_\mu = \mu\Lambda$ ($m_{soft} \sim g^2/(16\pi^2)\times\Lambda$)

  for large Λ, it is straightforward $B_\mu \gg \mu^2$

cannot satisfy both

- Lots of attempts to solve the problem

  e.g.) Higgs superfields couple to messenger (O_u, O_d include messenger field(s))

$$W \supset \lambda_u H_u \mathcal{O}_u + \lambda_d H_d \mathcal{O}_d$$

See, e.g.,
Csaki, Falkowski, Nomura, Volansky [PRL102(2009)111801]
De Simone, Franceschini, Giudice, Pappadopulo, Rattazzi [JHEP05(2011)112]
Asano, Yokozaki [PRD93(2016)095002]

- Contributes to μ, B_μ as well as $m_{H_u}^2$, $m_{H_d}^2$

  taking $\lambda_u \ll \lambda_d$ helps to satisfy both equations for EWSB!