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A simple and safe way to break PQ at the GUT scale

GUTPC, HIAS HANGZHOU, APRIL 2025

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Based on *JCAP* 03 (2024) 061, *JCAP* 03 (2025) 014 and arXiv:2504.xxxx
With J. Osinski, L. Roszkowski, U. Min, A. Ghoshal and D. Paul

QCD axion, a simple solution?

- Peccei-Quinn can explain the smallness of CP violating terms in QCD and dark matter in a simple low energy theory.
- Spontaneously broken $U(1)_{\mathrm{PQ}}$ global symmetry

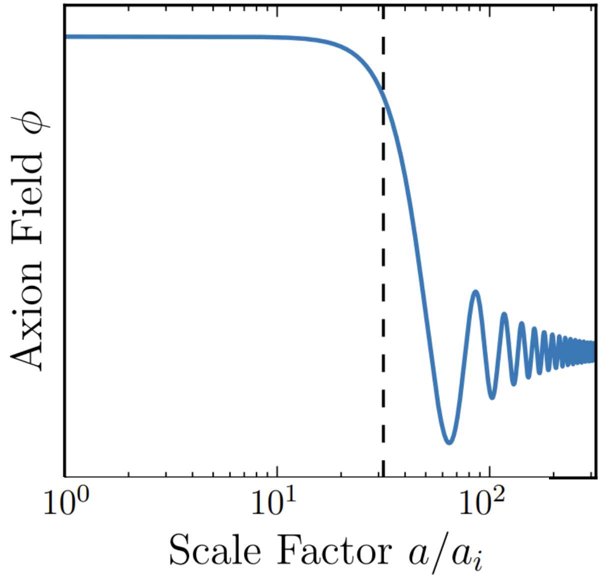
$$\mathcal{L}_{\mathrm{eff}}^{a} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a} = \theta\,(t,x)\,\frac{\alpha_s}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a}$$

- This anomaly term leads to a small axion mass

$$V_{\mathrm{QCD}}(\theta) = -m_\pi^2 f_\pi^2\sqrt{1-\frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\frac{N\theta}{2}} \longrightarrow m_a \approx 6\,\mu\mathrm{eV}\,\frac{10^{12}\,\mathrm{GeV}}{f/N}.$$

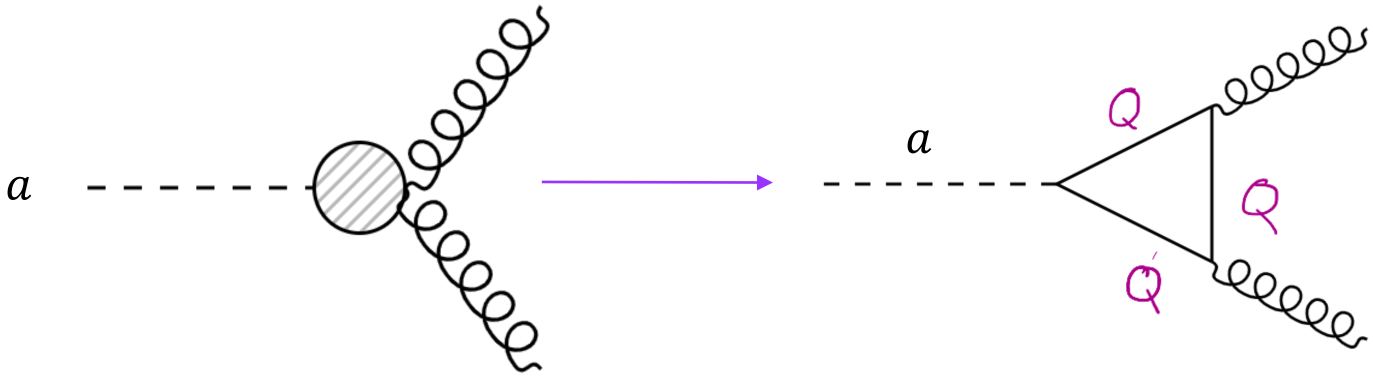
- This axion will be produced via the misalignment mechanism

D.J.E. Marsh
[arXiv:1510.07633]



Complicated by completions

$$\mathcal{L}^a_{\text{eff}} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



UV completions must involve strongly coupled particles.

KSVZ: SM fields are $U(1)_{\mathrm{PQ}}$ neutral $\longrightarrow$ heavy quarks which need to decay

DFSZ: SM fields are charged under $U(1)_{\mathrm{PQ}}$ $\longrightarrow$ suffer from a domain wall problem

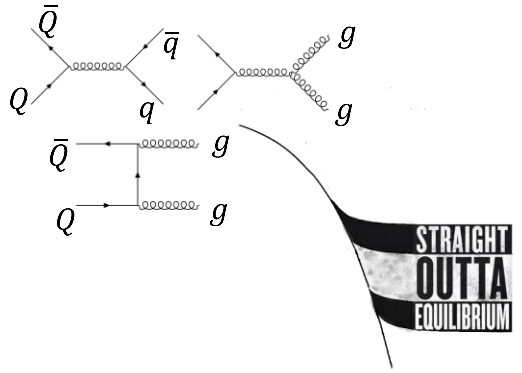
These problems can be avoided with pre-inflationary PQ breaking (need low H_{inf})

Heavy Quark Abundance

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q \; - (y_Q \overline{Q}_L Q_R \Phi + \mathrm{H.c})$$

$\Phi \to (v_\varphi/\sqrt{2}) e^{ia/f_a}$ so $m_Q = y_Q f_a/\sqrt{2}$ where f_a is constrained to be $\geq 10^8$ GeV

These strongly coupled Q particles will **freeze-out like dark matter**



$$Y_Q^\infty \approx \frac{x_f}{\lambda} \approx \frac{10\, H(m_Q)}{m_Q^3 \langle \sigma v \rangle}$$

$$H(m_Q) \sim m_Q^2/M_{\rm pl}$$

$$\langle \sigma v \rangle \sim 1/m_Q^2$$

$$Y_Q^\infty \sim 10 \frac{m_Q}{M_{\rm pl}}$$

$\rho_Q = m_Q n_Q \Rightarrow \rho_Q \propto m_Q^2 T^3$ and $\rho_{\rm SM} \propto T^4$ leads to $\dfrac{\rho_Q}{\rho_{\rm R}^{\rm SM}} \sim 10^{10} \left(\dfrac{m_Q}{10^{12}\,\mathrm{GeV}}\right)^2 \left(\dfrac{1\,\mathrm{MeV}}{T}\right)$

Heavy Quarks must decay

- Such heavy quarks will be overabundantly produced via freeze-out
- Can even dominate before BBN
- They must decay.
- Must introduce Q-breaking term

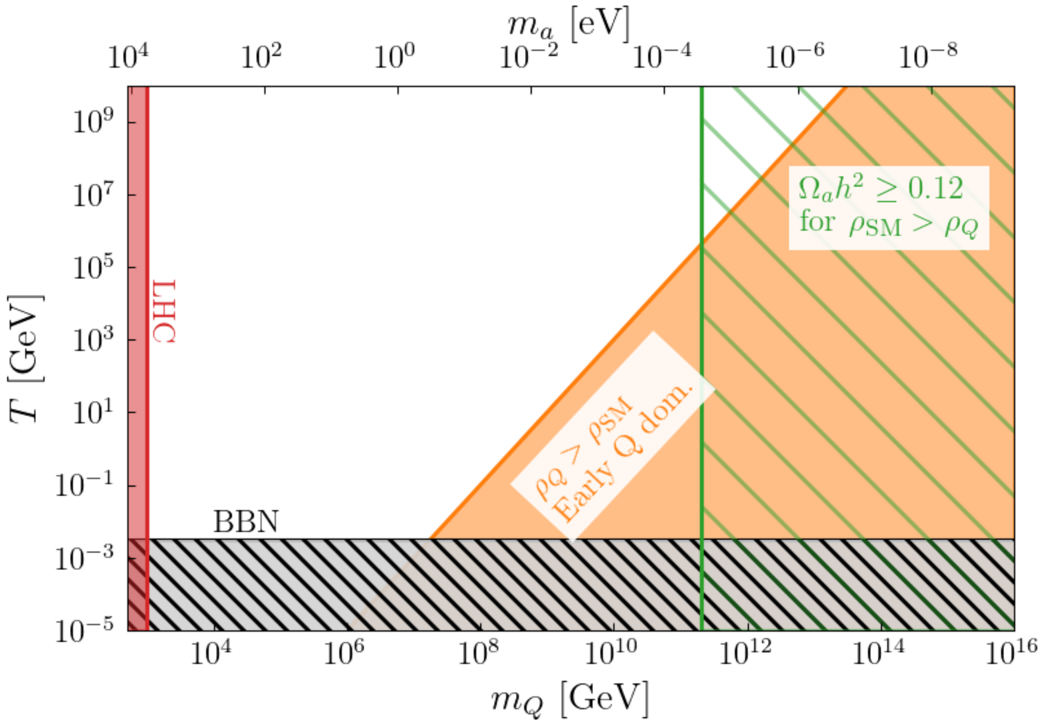
$$\mathcal{L}_{\rm PQ} = \underbrace{|\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q}_{U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\Phi} - (\underbrace{y_Q \overline{Q}_L Q_R \Phi}_{U(1)_{PQ} \times U(1)_Q} + \mathrm{H.c})$$

$$U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\Phi \longrightarrow U(1)_{PQ} \times U(1)_Q$$

- This is only possible for some charge assignments

$$R_Q : (\underset{SU(3)}{3}, \underset{SU(2)}{1}, \underset{U(1)_Y}{-\tfrac{1}{3}}) \quad \text{and} \quad \underbrace{(\chi_L, \chi_R)}_{\text{PQ charge}} = (\tfrac{1}{2}, -\tfrac{1}{2})$$

$\Rightarrow$ decay impossible



Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially $N_{\text{DW}} = 1$

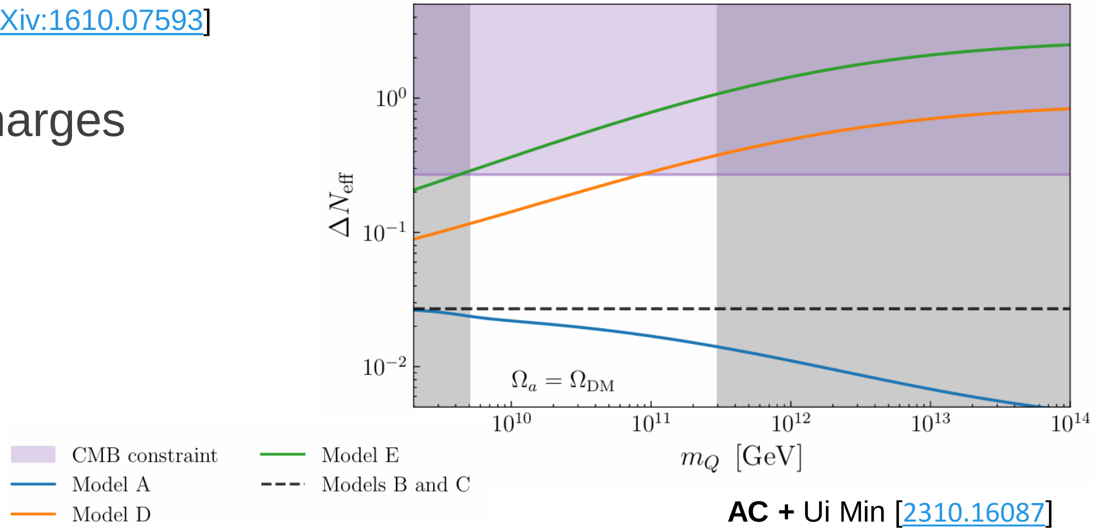
R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_1: $(3,1,-\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$	1
R_2: $(3,1,+\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$	1
R_3: $(3,2,+\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$	2
R_4: $(3,2,-\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_5: $(3,2,+\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$	2
R_6: $(3,3,-\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$	3
R_7: $(3,3,+\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3

L. Di Luzio et. al.
[arXiv:1610.07593]

From here, can determine distinct models from PQ charges

$$
\begin{aligned}
\mathcal{O}_4^M &= M_d \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (0,-1), && \text{Model A,}\\
\mathcal{O}_4^H &= y_{1,q} H \overline{q}_L Q_R, && \text{for } (\chi_L, \chi_R) = (1,0), && \text{Model B,}\\
\mathcal{O}_4^\Phi &= y_{2,d} \Phi \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (1,0), && \text{Model B,}\\
\mathcal{O}_4^{\Phi^\dagger} &= y_{3,d} \Phi^\dagger \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (-1,-2), && \text{Model C.}
\end{aligned}
$$



AC + Ui Min [2310.16087]

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L. Di Luzio et. al. (PRL)
[arXiv:1610.07593]

Can also go to the non-renormalizable level to determine the limit?

$$\mathcal{L}_{Qq} = \mathcal{L}_{Qq}^{d\leq 4} + \mathcal{L}_{Qq}^{d>4} = \mathcal{L}_{Qq}^{d\leq 4} + \frac{1}{\Lambda^{(d-4)}}\mathcal{O}^{d>4} + \text{h.c.}$$

This leads to decays which are suppressed by powers of $\Lambda \neq f_a$

$$\Gamma_{d,n_f} = \frac{m_Q}{4\,(4\pi)^{2n_f-3}\,(n_f-1)!\,(n_f-2)!}\left(\frac{m_Q^2}{\Lambda^2}\right)^{d-4}$$

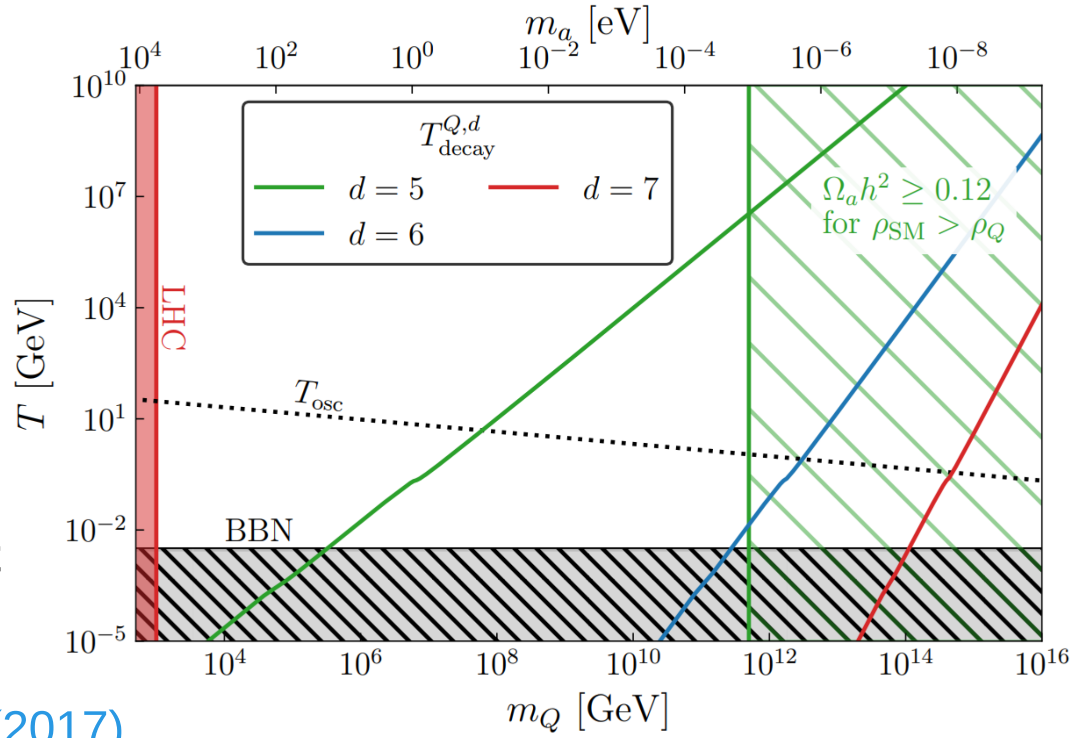
Use Standard Cosmology

Decay terms are limited by

1) Ensuring misalignment production doesn't overproduce axions
2) Ensuring Q decay occurs before BBN

approximately

$$\tau \lesssim 0.01\,\mathrm{s}$$



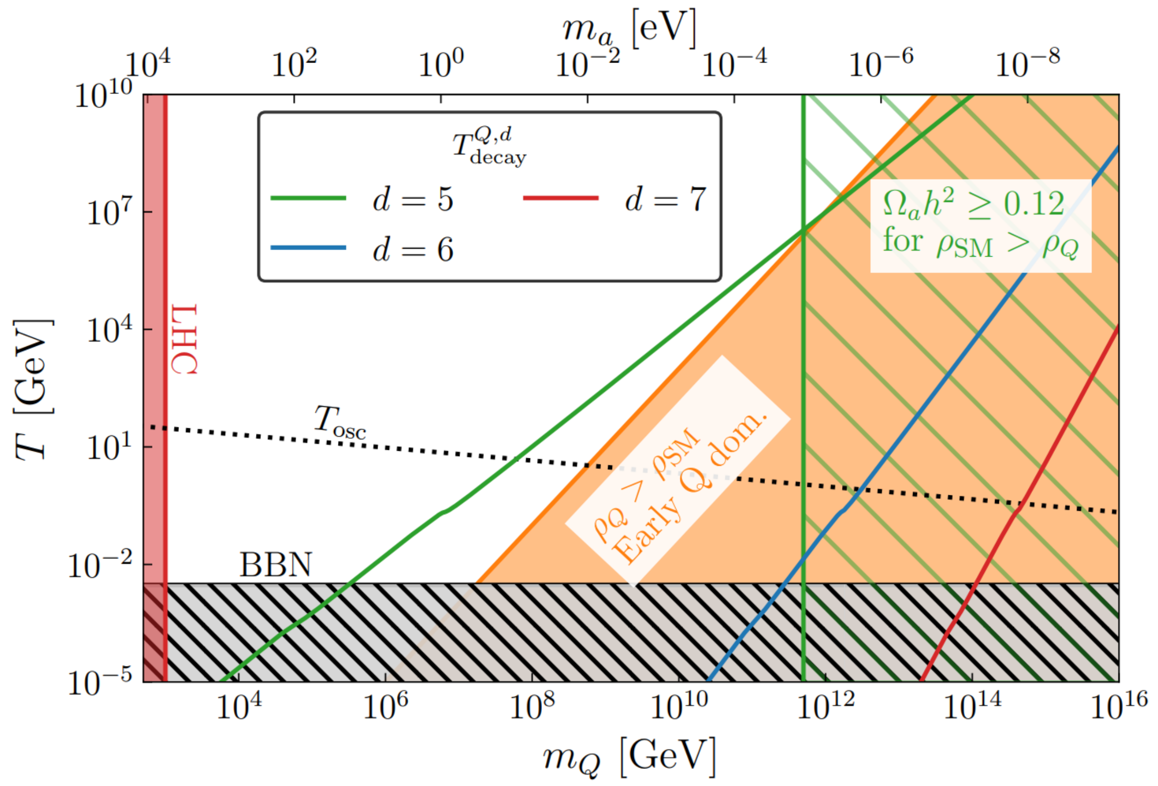
***Preferred axion models* decay via dimension 5 at most!**

Put forward by Luzio, Mescia and Nardi in [*PRL* 118 (2017) 3, 031801] and [*PRD* 96 (2017) 7, 075003].

But we know HQs can dominate

- For these higher dimensional decaying models, the heavy quarks will dominate.
- Through $Q \to SM$ decay reheating may slightly alter the BBN bound.
- More importantly, the misalignment mechanism will be affected.
- We show T_{osc}, temperature when axion field oscillations begin

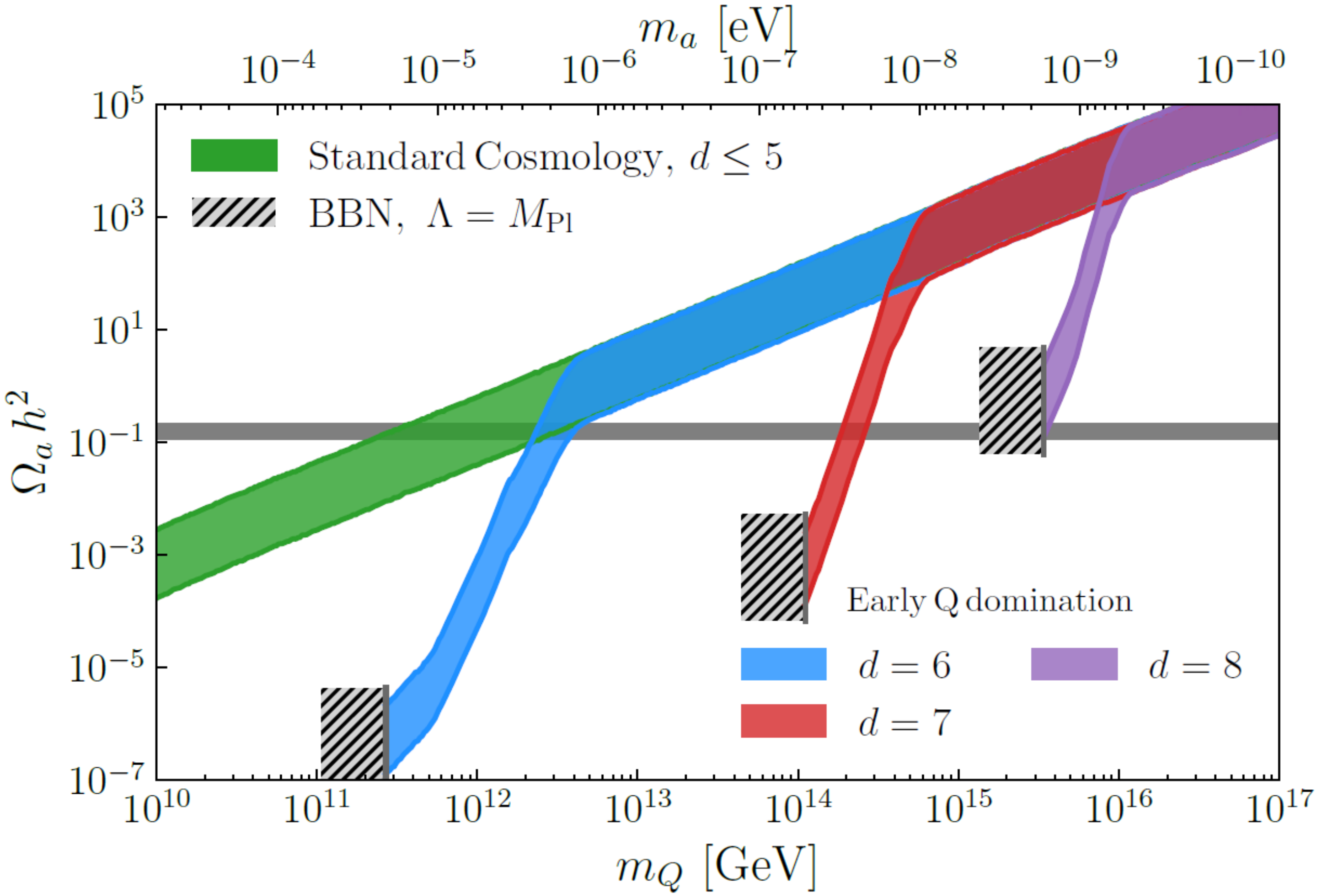
$$3H\left(T_{\rm osc}\right) = \tilde{m}_a\left(T_{\rm osc}\right)$$



Heavy Quark domination dilutes Ω_a

- Early matter domination can alter misalignment

- We were the first to point out the axion models themselves could provide this phase.
- Plotting band of different initial angles $\theta_i \in \left[\frac{1}{2}, \frac{\pi}{\sqrt{3}}\right]$
- Dimension 6-7-8 now are viable.
- More axion models available and parameter space.



AC, J. Osinski, L. Roszkowski [2310.16087]

More models without domain walls

Recently, Di Luzio et. al. confirmed my findings and catalogued higher dimensional models

L. Di Luzio et. al. [arXiv:2412.17896]

Rep. ($\mathcal{C}$,	$\mathcal{I}$,	$6\mathcal{Y}$)	E/N	N_{DW}	Min. d	Example operator	LP [GeV]
3	1	−2	2/3	1	3	$\bar{\mathcal{Q}}_L\, d_R$	2.0×10^{39}
3	1	4	8/3	1	3	$\bar{\mathcal{Q}}_L\, u_R$	6.8×10^{35}
3	1	−14	98/3	1	6	$\bar{\mathcal{Q}}_L\, d_R\, (\bar{e}^c_R\, e_R)$	2.2×10^{22}
$\bar{3}$	1	8	32/3	1	6	$\bar{u}_R\, \gamma_\mu\, e_R\, \bar{d}_R\, \gamma^\mu\, \mathcal{Q}_R$	3.0×10^{28}
$\bar{3}$	1	−10	50/3	1	6	$(\bar{d}_R\, d^c_R)\, \bar{e}_R\, \mathcal{Q}_L$	6.4×10^{25}
3	1	16	128/3	1	6	$\bar{\mathcal{Q}}_L\, u_R\, (\bar{e}_R\, e^c_R)$	1.8×10^{21}
$\bar{3}$	1	20	200/3	1	9	$(\bar{d}^c_R\, d_R)\, (\bar{e}^c_R\, e_R)\, \bar{u}_R\, \mathcal{Q}_L$	6.2×10^{19}
3	1	22	242/3	1	9	$\bar{\mathcal{Q}}_L\, u_R\, (\bar{\ell}_L\, \ell^c_L)\, (\bar{e}_R\, e^c_R)$	2.0×10^{19}

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi} \frac{1}{f_a} \left(\frac{E}{N} - 1.92(4) \right)$$

GUT-Scale PQ breaking

Previously taken $m_Q = f_a$ and $\Lambda = M_{\text{pl}}$

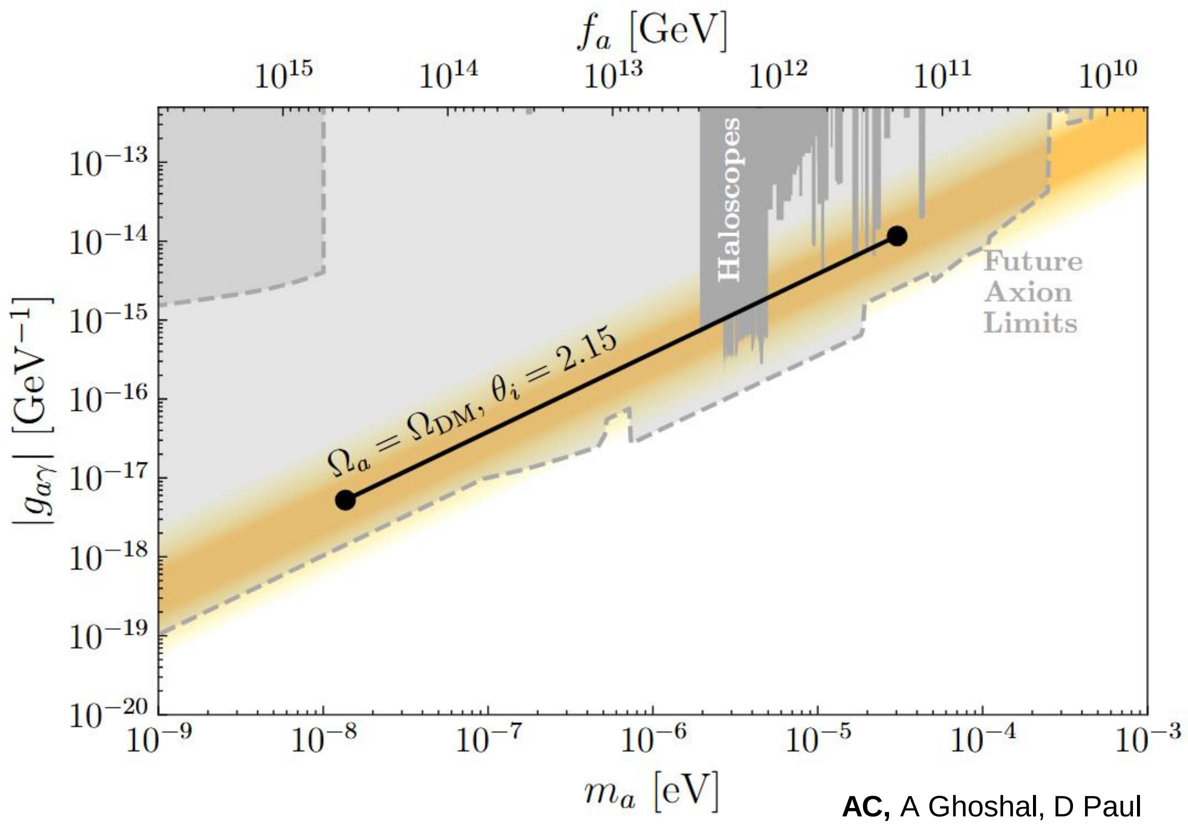
Can relax this and obtain order of magnitude lower mass.

The plot shows dimension 6 models

The point with smallest m_a corresponds to:

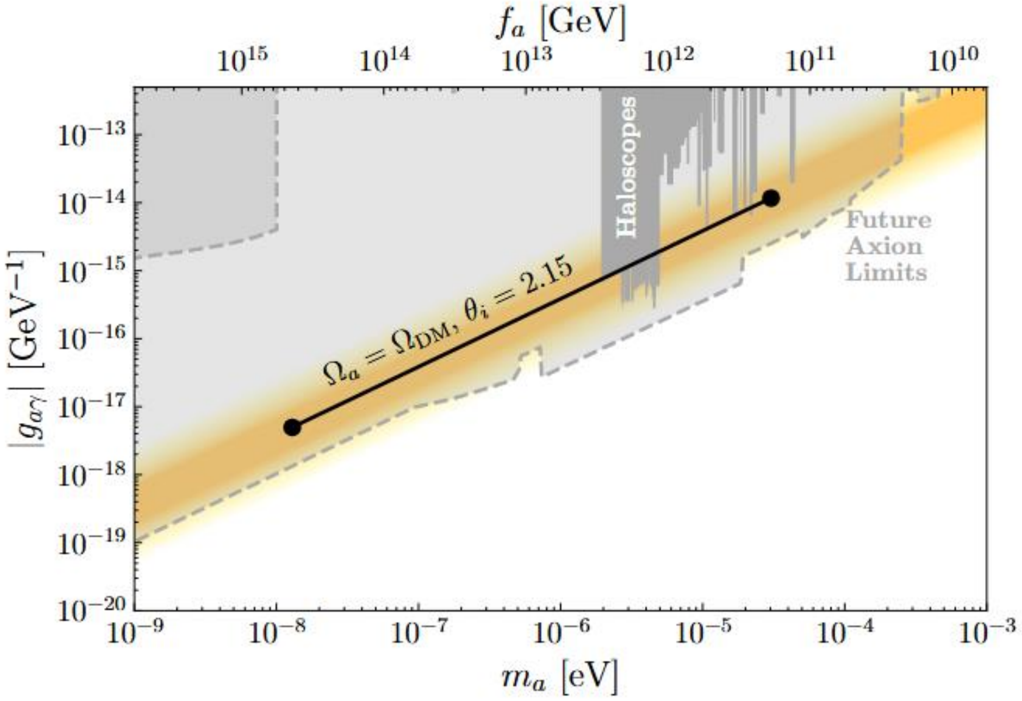
$$f_a = 4 \times 10^{14}\ \text{GeV}$$
$$m_Q = 4 \times 10^{11}\ \text{GeV}$$
$$\Lambda = 4 \times 10^{18}\ \text{GeV}$$

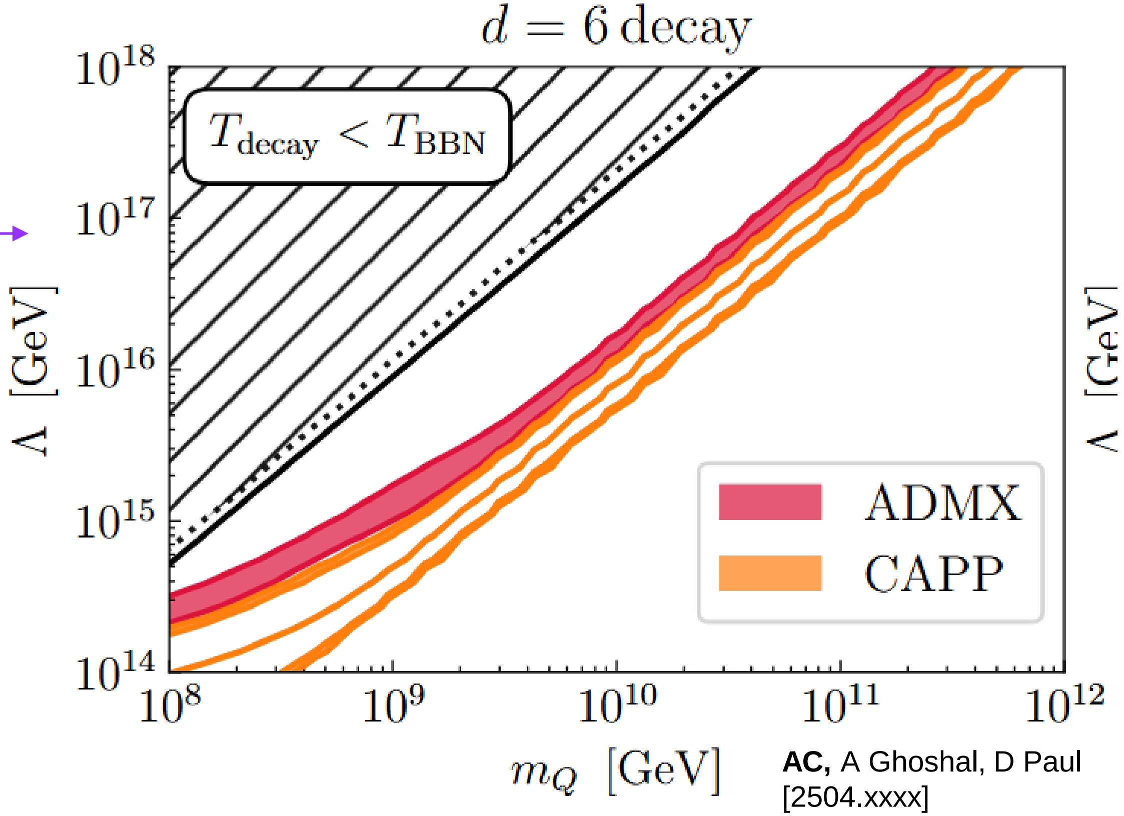
PQ breaking can be close to the GUT-scale!



AC, A Ghoshal, D Paul [2504.xxxx]

GUT-Scale PQ breaking





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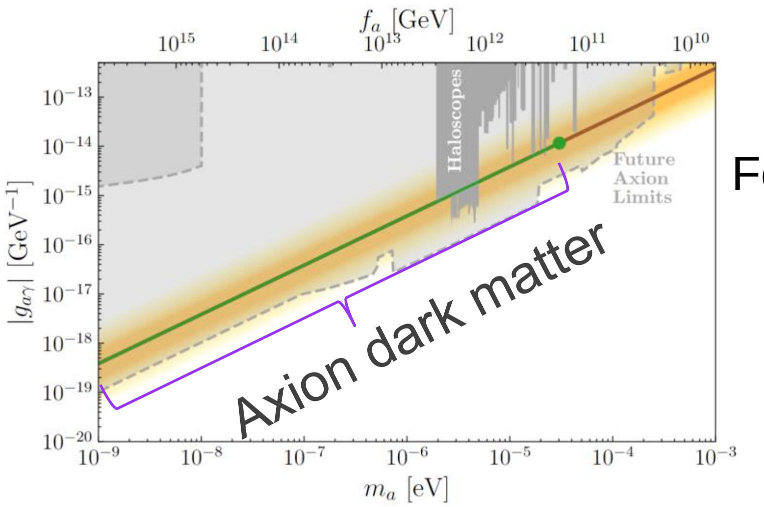
When does PQ break?

BEFORE INFLATION

Can have $m_a \leq 10\ \mu\text{eV}$

No detectable dark radiation component from axion.



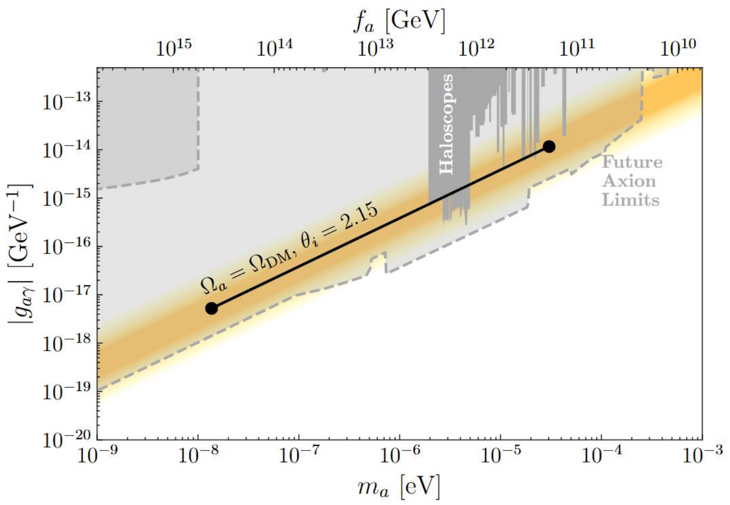
For $m_a = 10^{-8}$ eV

$\theta_i = 10^{-2}$

AFTER INFLATION

Now can have $m_a \leq 10\ \mu\text{eV}$ with HQD

The only models that survive with have no detectable dark radiation component from axion.



Both scenarios have the same phenomenological output.

Primordial GWs as a tracer of HQD

- We propose using inflationary gravitational waves (IGWs) to probe whether HQD happened

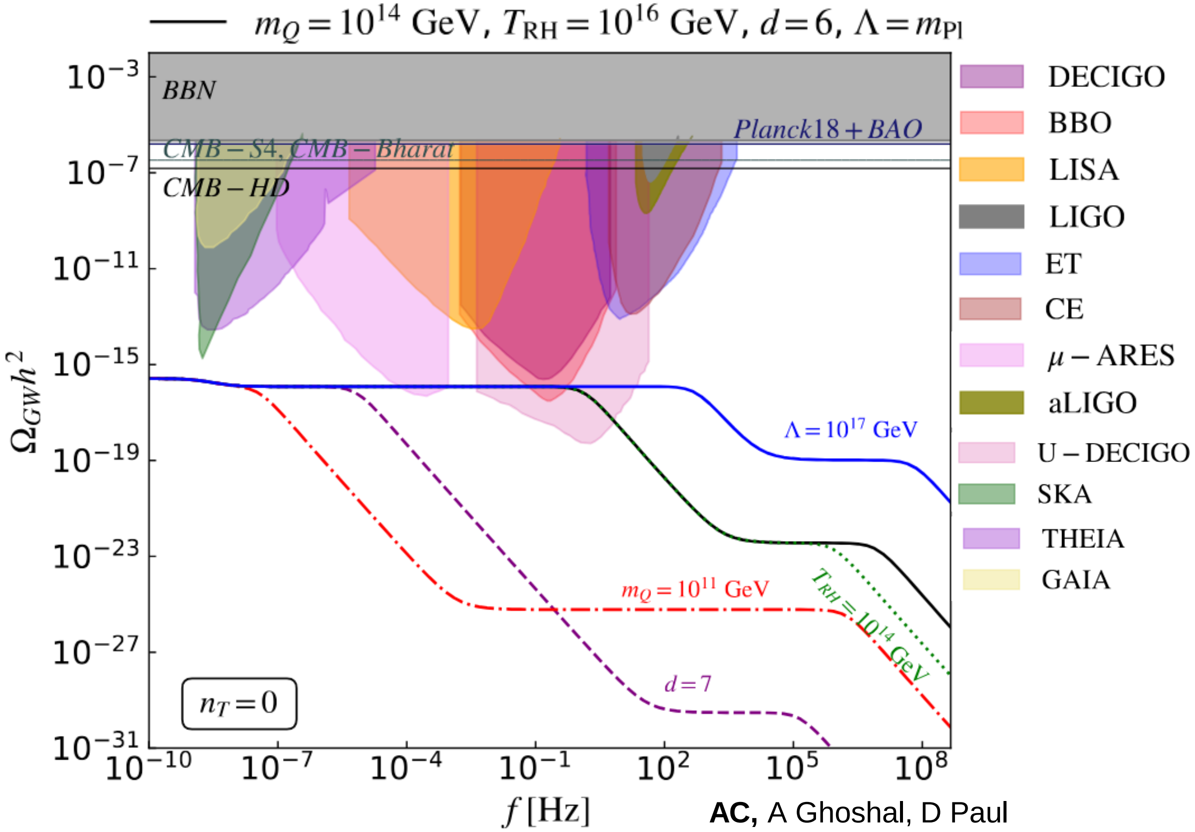
- These are tensor perturbations from inflation, its power spectrum is parameterized by

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

- Current constraints of measurements of B-modes in the CMB constrain tensor to scalar ratio

$$r = \frac{A_T}{A_S} < 0.036$$

- The tilt n_T is determined in vanilla slow-roll inflation to be $n_T = -\frac{r}{8}$, but there are numerous alternatives (string inflation, ekpyrotic particle production).



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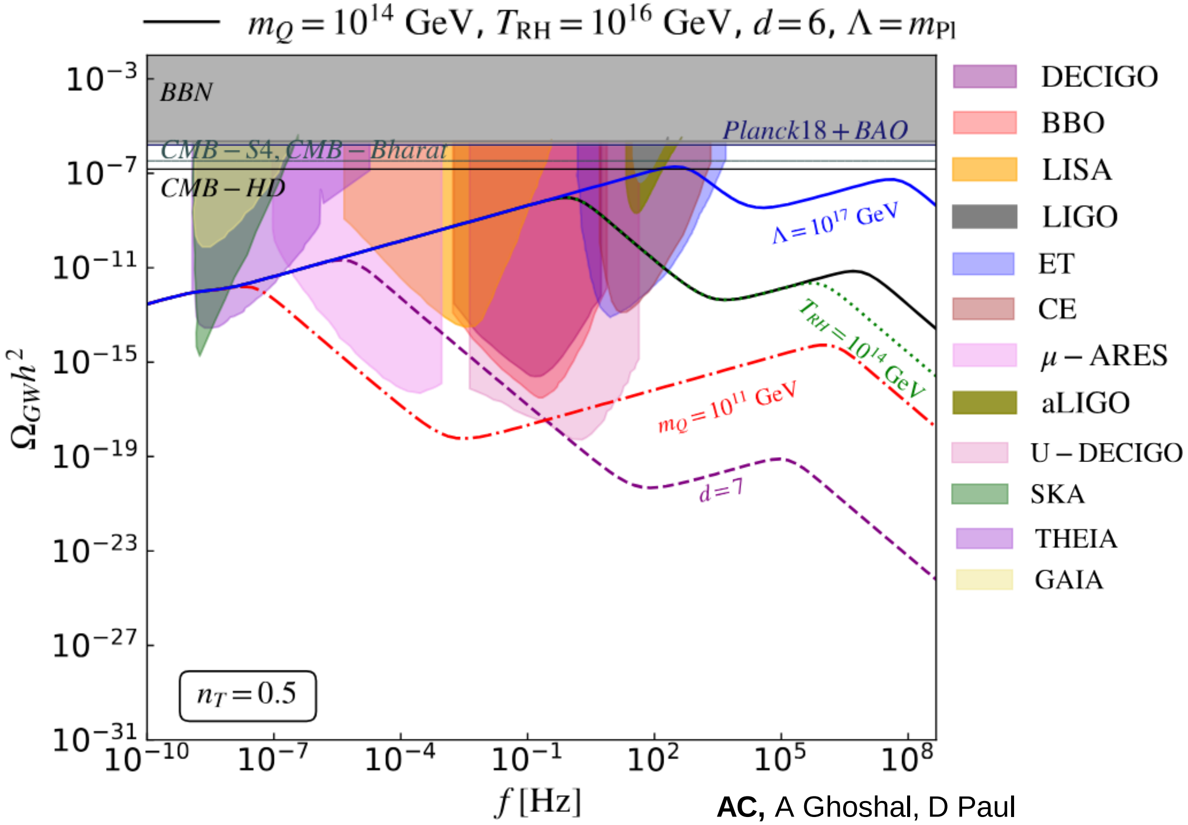
[2504.xxxx]

Maximum (blue-)tilt

- We take an optimistically blue-tilted power spectrum to assess the maximum sensitivity of GW experiments.
- The sensitivity plots shown on the right are just illustrative (from GWplotter for example)
- We perform our forecasts using the signal-to-noise ratio for each detector

$$\mathrm{SNR} \equiv \sqrt{\tau_{\mathrm{obs}} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left(\frac{\Omega_{\mathrm{GW}}(f, \{\theta\})h^2}{\Omega_{\mathrm{GW}}^{\mathrm{noise}}(f)h^2} \right)^2}$$

Detectors	Frequency range	τ_{obs}
SKA	$[10^{-9} - 4 \times 10^{-7}]$ Hz	15 years
μ-ARES	$[10^{-7} - 1]$ Hz	4 years
LISA	$[10^{-4} - 1]$ Hz	4 years
BBO	$[10^{-3} - 7]$ Hz	4 years
ET	$[1 - 10^{3}]$ Hz	5 years



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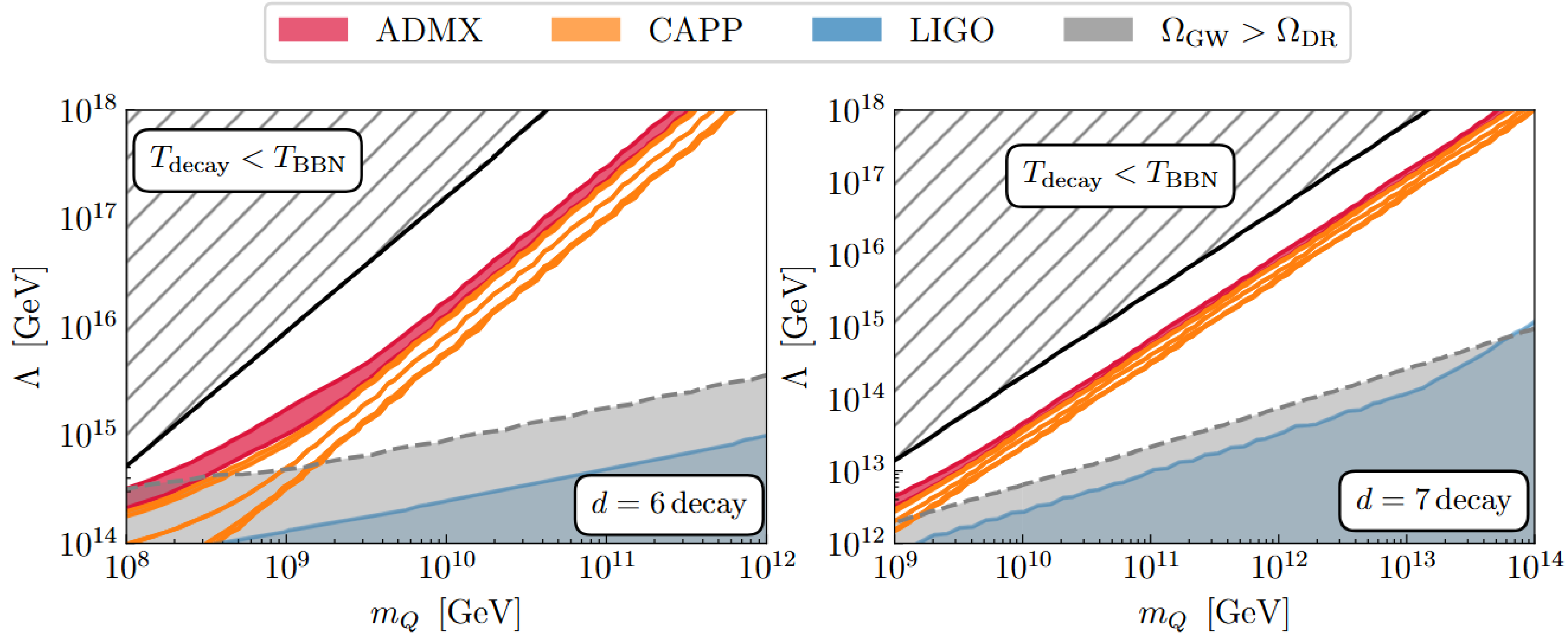
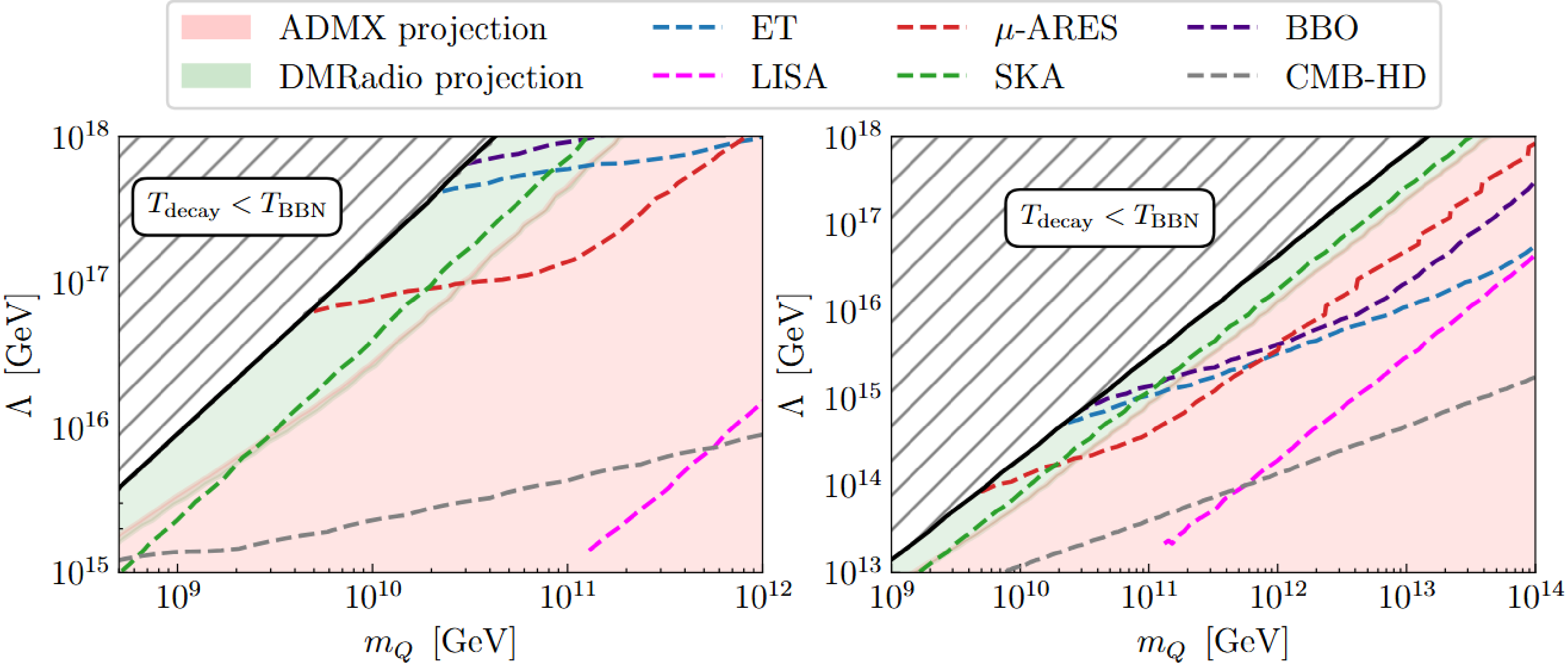
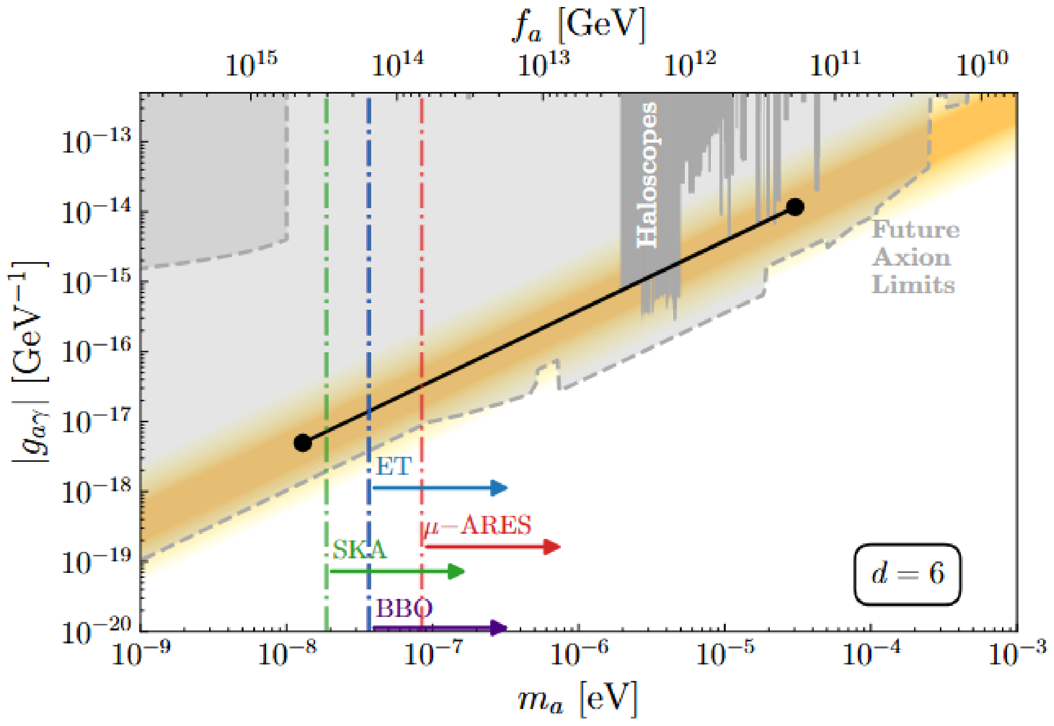
Current GW-axion landscape

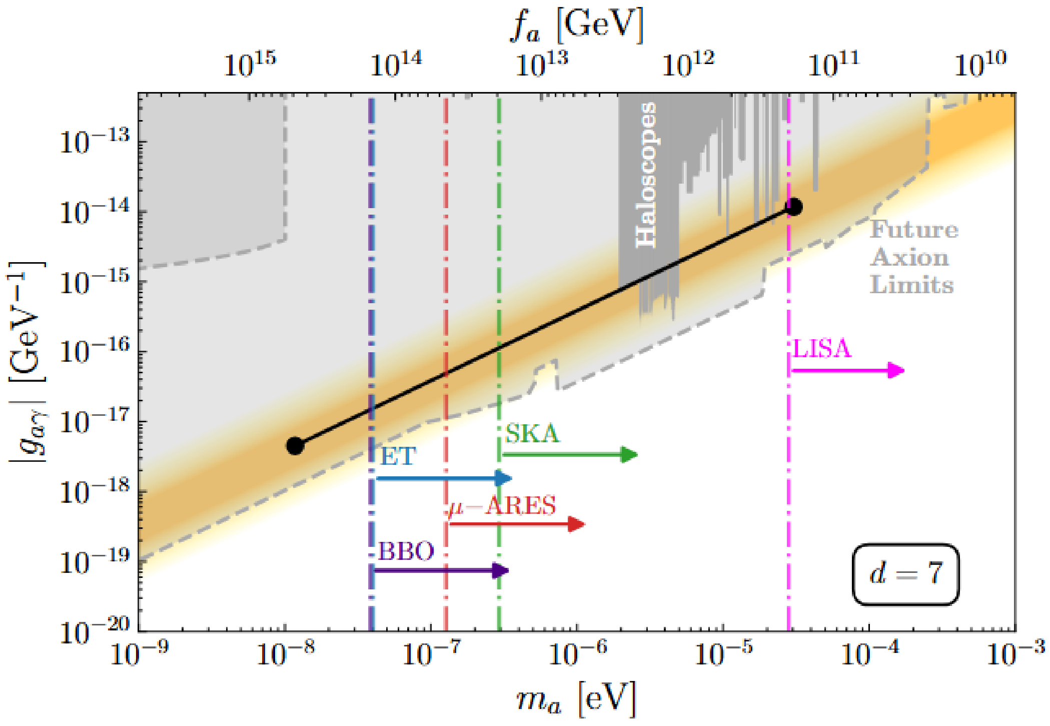


Future prospects



Where GWs will be able to probe





Conclusions

- Although the axion solution to strong CP problem and dark matter seems simple, the high energy completions can have phenomenological consequence.
- We have shown that heavy quark domination allows for a greater number of models to be viable. Including those without a domain wall problem.
- Axion dark matter as light as $m_a = 10^{-8}\ \mathrm{eV}$ can be achieved through a natural set of parameters.
- This corresponds to GUT-scale PQ breaking!
- The phenomenological signals of pre- and post-inflationary PQ breaking is now more similar.
- We propose using inflationary GWs to learn about heavy quark domination.

Thanks for listening!