Exotic proton decays and the GUT Models

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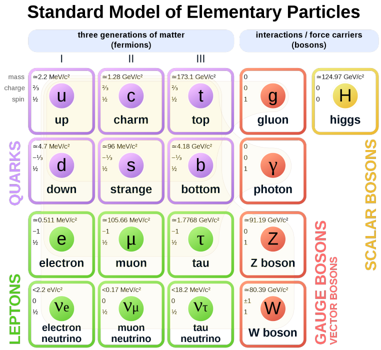
In collaboration with Lisa L. Everett
Work in progress

Overview

Standard Model



Gauge symmetry group $\mathbb{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}^Y = \bar{u} Y_u Q H + \bar{Q} Y_d d H + \text{h.c.}$$

$$\langle h^0 \rangle = v \to \quad m_f = Y_f v$$

Introduction/Motivation

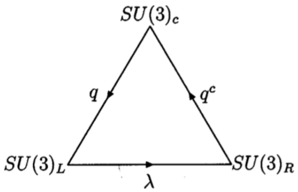
- Exotic proton decays
- Stability of proton
- Unification of gauge forces
- Accommodate dark matter
- Different options for neutrino mass sector

$SU(5)$ and $SO(10)$ examples

- $\bar{5}_F, 10_F + 5_S, 24_S$ in $SU(5) \to \mathscr{G}_{\mathrm{SM}}$ naturally predicts 2 body proton decay via $(\mathbf{3}, 2, -\frac{5}{3})$ vector leptoquark. All decays in $SU(5)$ case accidentally conserve $B - L$ symmetry.
- 16 spinnor representation of $SO(10)$ with $SO(10) \to SU(5) \times U(1)_{B-L}$. Proton decays similar to $SU(5)$ case.
- Trinification and quartification (partial unification) with extension to E_6 and E_8, respectively. No gauge mediated proton decay

$[SU(3)]^3$ trinification, $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$

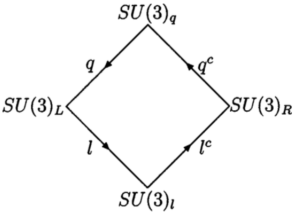
$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \quad \lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix}$$



$[SU(3)]^4$ quartification, $SU(3)_C \otimes SU(3)_L \otimes SU(3)_l \otimes SU(3)_R$

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \qquad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}$$

$$l = \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & N \end{pmatrix} \qquad l^c = \begin{pmatrix} x_1^c & y_1^c & z_1^c \\ x_2^c & y_2^c & z_2^c \\ \nu^c & e^c & N^c \end{pmatrix}$$



*K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. D69, 051301(R) (2004).

Scotogenic model in $SU(6)$ GUT*

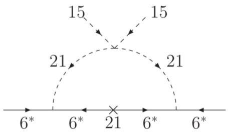
Simple case: Extend $SU(5)$ to $SU(6)$ to incude BSM particles needed

$\underline{5}^*_F \times \underline{10}_F \times \underline{5}^*_S$, $\underline{10}_F \times \underline{10}_F \times \underline{5}_S$ $SU(5)$ Yukawa terms are extended to

$\underline{6}^*_F \times \underline{15}_F \times \underline{6}^*_S$, $\underline{15}_F \times \underline{15}_F \times \underline{15}_S$ $SU(6)$ Yukawas

Anomaly free combinations: $\underline{5}^*_F + \underline{10}_F$ for $SU(5)$, $\underline{6}^*_F + \underline{6}^*_F + \underline{15}_F$ for $SU(6)$. New $SU(6)$ $\underline{21}_S$ scalar is added to obtain 2'nd Higgs doublet ($\mathbb{Z}_2 \sim -$) with new interactions

$\underline{6}^*_F \times \underline{6}^*_F \times \underline{21}_S$, $\underline{15}^*_S \times \underline{15}^*_S \times \underline{21}_S \times \underline{21}_S$



*10.1088/1742-6596/539/1/012001

Scotogenic model in $SU(7)$ GUT†

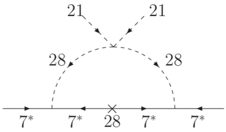
Less simple case: Extend $SU(5)$ to $SU(7)$ to incude BSM particles needed

$\underline{5}^*_F \times \underline{10}_F \times \underline{5}^*_S$, $\underline{10}_F \times \underline{10}_F \times \underline{5}_S$ $SU(5)$ Yukawa terms are extended to

$\underline{7}^*_F \times \underline{21}_F \times \underline{7}^*_S$, $\underline{21}_F \times \underline{21}_F \times \underline{35}_S$ $SU(7)$ Yukawas

Anomaly free combination for $SU(7)$: $\underline{7}^*_F + \underline{7}^*_F + \underline{7}^*_F + \underline{21}_F$.

New $\underline{28}_S$ scalar needed to accomodate $SU(2)_N$ doublet, with new interactions:$\underline{7}^*_F \times \underline{7}^*_F \times \underline{28}_S$, $\underline{21}^*_S \times \underline{21}^*_S \times \underline{28}_S \times \underline{28}_S$



†10.1088/1742-6596/539/1/012001

Symmetry and Fields

- $\bar{7}_F$, 21_F, $\bar{35}_F$
- 7_S, 48_S, 84_S, $(210_S, 21_S)$
-

$$
\begin{aligned}
SU(7) &\underset{\langle 48_S \rangle}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_{F,1} \times U(1)_{1'} \times U(1)_Y \\
&\underset{\langle 7_{S,1'} \rangle}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_{F,1} \times U(1)_Y \\
&\underset{\langle 84_S \vee 210_S \rangle}{\longrightarrow} SU(3)_c \times SU(2)_L \times \times U(1)_Y \times \mathbb{Z}_n \\
&\underset{\langle 7_{S,2} \rangle}{\longrightarrow} SU(3)_c \times U(1)_{em} \times \mathbb{Z}_n
\end{aligned}
$$

Anomalies

- $-\frac{1}{2}\left(\bar{7}_F\right)+\frac{3}{2}\left(21_F\right)-1\left(\bar{35}_F\right)=0$
- After $SU(7)$ breaking there are two SM−like sectors and two SM*−like sectors, each of which is anomaly free under $\mathbb{G}_{SM}$

$SU(7)$ Lagrangian

- $$-\mathscr{L}^{\text{Yuk}} = Y_d \bar{7}_F 21_F 7_S^\dagger + Y_u 21_F \bar{35}_F 7_S + Y_e \bar{35}_F \bar{35}_F 7_S^\dagger + \text{h.c.}$$

- $$-\mathscr{L}_{21}^{\text{Yuk}} = Y_{21} \bar{7}_F \bar{7}_F 21_S + \text{h.c.}$$

$SU(7)$ Potential

- $$V_{\text{main}} = \left(7_S^\dagger 7_S\right) + \left(7_S^\dagger 7_S\right)^2 + \text{Tr}\left[48_S 48_S\right] + \text{Tr}\left[48_S 48_S 48_S\right] + \left(\text{Tr}\left[48_S 48_S\right]\right)^2 + \text{Tr}\left[48_S 48_S 48_S 48_S\right] + 7_S^\dagger 48_S 7_S + 7_S^\dagger 7_S \text{Tr}\left[48_S 48_S\right] + 7_S^\dagger 48_S 48_S 7_S$$

- $$V_{84} = \left(84_S^\dagger 84_S\right) + \left(84_S^\dagger 84_S\right)^2 + \left(7_S^\dagger 7_S\right)\left(84_S^\dagger 84_S\right) + \left(84_S^\dagger 7_S 7_S 7_S + \text{h.c.}\right)$$

- $$V_{210} = \left(210_S^\dagger 210_S\right) + \left(210_S^\dagger 210_S\right)^2 + \left(7_S^\dagger 7_S\right)\left(210_S^\dagger 210_S\right)$$

- $$V_{21} = 21_S^\dagger 21_S + \left(21_S^\dagger 21_S\right)^2 + 7_S^\dagger 7_S 21_S^\dagger 21_S + 7_S^\dagger 21_S 21_S^\dagger 7_S + \left(7_S 21_S^\dagger 7_S + 21_S 21_S 21_S 7_S \varepsilon + \text{h.c.}\right)$$

- $$V_{\text{rest}} = V_{\times\,\text{trivial}} + \left(210_S^\dagger 84_S 7_S + \text{h.c.}\right)$$

Vacuum expectation values

- $\langle 48_S \rangle \sim \mathrm{Diag}\left[v_c, v_c, v_c, v_1, v_{1'}, v_2, v_2\right]$
- $\langle 7_S \rangle \sim (0, 0, 0, 0, v_{1'}, 0, v_2)$
- $\left\langle 84_{S,111'} \right\rangle \sim v_{84}$ (2 body)
- $\left\langle 210_{S,1111} \right\rangle \sim v_{210}$ (3 body)

Currents

$$\begin{pmatrix} T^1_{SU3} \\ T^2_{SU3} \\ Q_{F=A} \\ Q_B \\ Y \\ I_3^{SU2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\sqrt{\frac{5}{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\sqrt{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{5}{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{\sqrt{10}} & -\frac{1}{5\sqrt{6}} & -\frac{7}{30} & -\frac{\sqrt{\frac{7}{5}}}{6} \\ 0 & 0 & 0 & \sqrt{\frac{14}{15}} & -\frac{\sqrt{\frac{7}{5}}}{6} & -\frac{1}{6} \\ 0 & 0 & \frac{1}{\sqrt{10}} & \frac{\sqrt{\frac{3}{2}}}{5} & \frac{7}{10} & \frac{\sqrt{\frac{7}{5}}}{2} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{\frac{5}{3}}}{2} & \frac{\sqrt{\frac{7}{3}}}{2} \end{pmatrix} \begin{pmatrix} T^1_{SU7} \\ T^2_{SU7} \\ T^3_{SU7} \\ T^4_{SU7} \\ T^5_{SU7} \\ T^6_{SU7} \end{pmatrix}$$

$SU(7)$ Decomposition

Fermions

SU(7)	$SU(3)_C$ x $SU(2)_L$ x $U(1)_A$ x $U(1)_B$ x $U(1)_C$ content
$\overline{\mathbf{7}}$	$\{\bar{\mathbf{3}}\otimes\mathbf{1}\otimes-\frac{1}{3}\otimes-1\otimes\frac{1}{3},\ \mathbf{1}\otimes\mathbf{1}\otimes\frac{5}{3}\otimes-1\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes 6\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes-\frac{1}{3}\otimes-1\otimes-\frac{1}{2}\}$
$\mathbf{21}$	$\{\bar{\mathbf{3}}\otimes\mathbf{1}\otimes\frac{2}{3}\otimes 2\otimes-\frac{2}{3},\ \mathbf{3}\otimes\mathbf{1}\otimes-\frac{4}{3}\otimes 2\otimes-\frac{1}{3},\ \mathbf{3}\otimes\mathbf{1}\otimes\frac{1}{3}\otimes-5\otimes-\frac{1}{3},\ \mathbf{3}\otimes\mathbf{2}\otimes\frac{2}{3}\otimes 2\otimes\frac{1}{6},$ $\mathbf{1}\otimes\mathbf{1}\otimes-\frac{5}{3}\otimes-5\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes-\frac{4}{3}\otimes 2\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{2}\otimes\frac{1}{3}\otimes-5\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{1}\otimes\frac{2}{3}\otimes 2\otimes 1\}$
$\overline{\mathbf{35}}$	$\{\mathbf{1}\otimes\mathbf{1}\otimes-1\otimes-3\otimes 1,\ \mathbf{3}\otimes\mathbf{1}\otimes 1\otimes-3\otimes\frac{2}{3},\ \mathbf{3}\otimes\mathbf{1}\otimes-\frac{2}{3}\otimes 4\otimes\frac{2}{3},$ $\mathbf{3}\otimes\mathbf{2}\otimes-1\otimes-3\otimes\frac{1}{6},\ \bar{\mathbf{3}}\otimes\mathbf{1}\otimes\frac{4}{3}\otimes 4\otimes\frac{1}{3},\ \bar{\mathbf{3}}\otimes\mathbf{2}\otimes 1\otimes-3\otimes-\frac{1}{6},\ \bar{\mathbf{3}}\otimes\mathbf{2}\otimes-\frac{2}{3}\otimes 4\otimes-\frac{1}{6},$ $\bar{\mathbf{3}}\otimes\mathbf{1}\otimes-1\otimes-3\otimes-\frac{2}{3},\ \mathbf{1}\otimes\mathbf{2}\otimes\frac{4}{3}\otimes 4\otimes-\frac{1}{2},\ \mathbf{1}\otimes\mathbf{1}\otimes 1\otimes-3\otimes-1,\ \mathbf{1}\otimes\mathbf{1}\otimes-\frac{2}{3}\otimes 4\otimes-1\}$

Scalars

SU(7)	$SU(3)_C$ x $SU(2)_L$ x $U(1)_A$ x $U(1)_B$ x $U(1)_C$ content
$\mathbf{7}$	$\{\mathbf{3}\otimes\mathbf{1}\otimes\frac{1}{3}\otimes 1\otimes-\frac{1}{3},\ \mathbf{1}\otimes\mathbf{1}\otimes-\frac{5}{3}\otimes 1\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes-6\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes\frac{1}{3}\otimes 1\otimes\frac{1}{2}\}$
$\mathbf{48}$	$\{\mathbf{3}\otimes\mathbf{2}\otimes 0\otimes 0\otimes-\frac{5}{6},\ \mathbf{3}\otimes\mathbf{1}\otimes\frac{1}{3}\otimes 7\otimes-\frac{1}{3},\ \mathbf{3}\otimes\mathbf{1}\otimes 2\otimes 0\otimes-\frac{1}{3},\ \mathbf{8}\otimes\mathbf{1}\otimes 0\otimes 0\otimes 0,$ $\mathbf{1}\otimes\mathbf{3}\otimes 0\otimes 0\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes 0\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes 0\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes 0\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes-2\otimes 0\otimes-\frac{1}{2},$ $\mathbf{1}\otimes\mathbf{1}\otimes-\frac{5}{3}\otimes 7\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes-\frac{1}{3}\otimes-7\otimes-\frac{1}{2},\ \mathbf{1}\otimes\mathbf{2}\otimes\frac{1}{3}\otimes 7\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{1}\otimes\frac{5}{3}\otimes-7\otimes 0,$ $\mathbf{1}\otimes\mathbf{2}\otimes 2\otimes 0\otimes\frac{1}{2},\ \bar{\mathbf{3}}\otimes\mathbf{1}\otimes-2\otimes 0\otimes\frac{1}{3},\ \bar{\mathbf{3}}\otimes\mathbf{1}\otimes-\frac{1}{3}\otimes-7\otimes\frac{1}{3},\ \bar{\mathbf{3}}\otimes\mathbf{2}\otimes 0\otimes 0\otimes\frac{5}{6}\}$
$\mathbf{84}$	$\{\mathbf{10}\otimes\mathbf{1}\otimes 1\otimes 3\otimes-1,\ \bar{\mathbf{6}}\otimes\mathbf{1}\otimes-1\otimes 3\otimes-\frac{2}{3},\ \bar{\mathbf{6}}\otimes\mathbf{1}\otimes\frac{2}{3}\otimes-4\otimes-\frac{2}{3},\ \bar{\mathbf{6}}\otimes\mathbf{2}\otimes 1\otimes 3\otimes-\frac{1}{6},$ $\mathbf{3}\otimes\mathbf{1}\otimes-3\otimes 3\otimes-\frac{1}{3},\ \mathbf{3}\otimes\mathbf{1}\otimes-\frac{4}{3}\otimes-4\otimes-\frac{1}{3},\ \mathbf{3}\otimes\mathbf{2}\otimes-1\otimes 3\otimes\frac{1}{6},\ \mathbf{3}\otimes\mathbf{1}\otimes\frac{1}{3}\otimes-11\otimes-\frac{1}{3},$ $\mathbf{3}\otimes\mathbf{2}\otimes\frac{2}{3}\otimes-4\otimes\frac{1}{6},\ \mathbf{3}\otimes\mathbf{3}\otimes 1\otimes 3\otimes\frac{2}{3},\ \mathbf{1}\otimes\mathbf{1}\otimes-5\otimes 3\otimes 0,\ \mathbf{1}\otimes\mathbf{1}\otimes-\frac{10}{3}\otimes-4\otimes 0,$ $\mathbf{1}\otimes\mathbf{2}\otimes-3\otimes 3\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{1}\otimes-\frac{5}{3}\otimes-11\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes-\frac{4}{3}\otimes-4\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{3}\otimes-1\otimes 3\otimes 1,$ $\mathbf{1}\otimes\mathbf{1}\otimes 0\otimes-18\otimes 0,\ \mathbf{1}\otimes\mathbf{2}\otimes\frac{1}{3}\otimes-11\otimes\frac{1}{2},\ \mathbf{1}\otimes\mathbf{3}\otimes\frac{2}{3}\otimes-4\otimes 1,\ \mathbf{1}\otimes\mathbf{4}\otimes 1\otimes 3\otimes\frac{3}{2}\}$

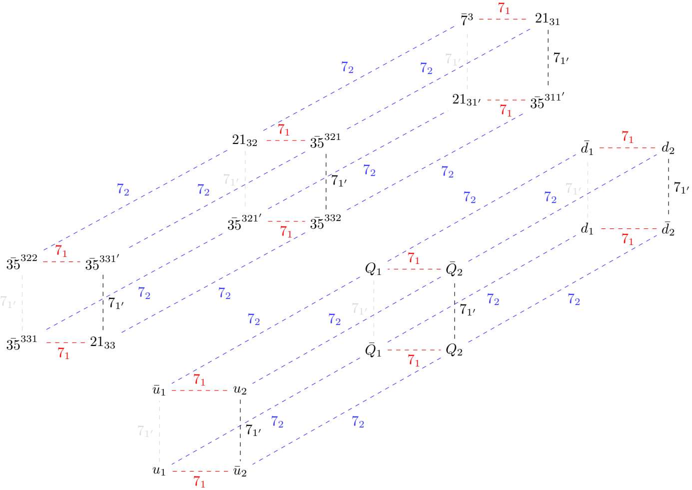
$SU(7)$ Decomposition

SU(7)	$SU(3)_C$ x $SU(2)_L$ x $U(1)_A$ x $U(1)_B$ x $U(1)_C$ content
$\overline{\mathbf{196}}$	$\{\mathbf{6}\otimes\mathbf{1}\otimes\frac{4}{3}\otimes 4\otimes -\frac{4}{3}, \mathbf{8}\otimes\mathbf{1}\otimes -\frac{2}{3}\otimes 4\otimes -1, \mathbf{8}\otimes\mathbf{1}\otimes 1\otimes -3\otimes -1,$ $\mathbf{8}\otimes\mathbf{2}\otimes\frac{4}{3}\otimes 4\otimes -\frac{1}{2}, \overline{\mathbf{6}}\otimes\mathbf{1}\otimes -\frac{8}{3}\otimes 4\otimes -\frac{2}{3}, \overline{\mathbf{6}}\otimes\mathbf{1}\otimes -1\otimes -3\otimes -\frac{2}{3}, \overline{\mathbf{3}}\otimes\mathbf{1}\otimes -1\otimes -3\otimes -\frac{2}{3},$ $\overline{\mathbf{6}}\otimes\mathbf{2}\otimes -\frac{2}{3}\otimes 4\otimes -\frac{1}{6}, \overline{\mathbf{3}}\otimes\mathbf{2}\otimes -\frac{2}{3}\otimes 4\otimes -\frac{1}{6}, \overline{\mathbf{6}}\otimes\mathbf{1}\otimes\frac{2}{3}\otimes -10\otimes -\frac{2}{3}, \overline{\mathbf{6}}\otimes\mathbf{2}\otimes 1\otimes -3\otimes -\frac{1}{6},$ $\overline{\mathbf{3}}\otimes\mathbf{2}\otimes 1\otimes -3\otimes -\frac{1}{6}, \overline{\mathbf{6}}\otimes\mathbf{3}\otimes\frac{4}{3}\otimes 4\otimes\frac{1}{3}, \overline{\mathbf{3}}\otimes\mathbf{1}\otimes\frac{4}{3}\otimes 4\otimes\frac{1}{3}, \mathbf{3}\otimes\mathbf{1}\otimes -3\otimes -3\otimes -\frac{1}{3},$ $\mathbf{3}\otimes\mathbf{2}\otimes -\frac{8}{3}\otimes 4\otimes\frac{1}{6}, \mathbf{3}\otimes\mathbf{1}\otimes -\frac{4}{3}\otimes -10\otimes -\frac{1}{3}, \mathbf{3}\otimes\mathbf{2}\otimes -1\otimes -3\otimes\frac{1}{6}, \mathbf{3}\otimes\mathbf{2}\otimes -1\otimes -3\otimes\frac{1}{6},$ $\mathbf{3}\otimes\mathbf{3}\otimes -\frac{2}{3}\otimes 4\otimes\frac{2}{3}, \mathbf{3}\otimes\mathbf{1}\otimes -\frac{2}{3}\otimes 4\otimes\frac{2}{3}, \mathbf{3}\otimes\mathbf{2}\otimes\frac{2}{3}\otimes -10\otimes\frac{1}{6}, \mathbf{3}\otimes\mathbf{3}\otimes 1\otimes -3\otimes\frac{2}{3},$ $\mathbf{3}\otimes\mathbf{1}\otimes 1\otimes -3\otimes\frac{2}{3}, \mathbf{3}\otimes\mathbf{2}\otimes\frac{4}{3}\otimes 4\otimes\frac{7}{6}, \mathbf{1}\otimes\mathbf{1}\otimes -\frac{10}{3}\otimes -10\otimes 0, \mathbf{1}\otimes\mathbf{2}\otimes -3\otimes -3\otimes\frac{1}{2},$ $\mathbf{1}\otimes\mathbf{3}\otimes -\frac{8}{3}\otimes 4\otimes 1, \mathbf{1}\otimes\mathbf{2}\otimes -\frac{4}{3}\otimes -10\otimes\frac{1}{2}, \mathbf{1}\otimes\mathbf{3}\otimes -1\otimes -3\otimes 1, \mathbf{1}\otimes\mathbf{1}\otimes -1\otimes -3\otimes 1,$ $\mathbf{1}\otimes\mathbf{2}\otimes -\frac{2}{3}\otimes 4\otimes\frac{3}{2}, \mathbf{1}\otimes\mathbf{3}\otimes\frac{2}{3}\otimes -10\otimes 1, \mathbf{1}\otimes\mathbf{2}\otimes 1\otimes -3\otimes\frac{3}{2}, \mathbf{1}\otimes\mathbf{1}\otimes\frac{4}{3}\otimes 4\otimes 2\}$
$\overline{\mathbf{210'}}$	$\{\mathbf{15'}\otimes\mathbf{1}\otimes\frac{4}{3}\otimes 4\otimes -\frac{4}{3}, \mathbf{10}\otimes\mathbf{1}\otimes -\frac{2}{3}\otimes 4\otimes -1, \mathbf{10}\otimes\mathbf{1}\otimes 1\otimes -3\otimes -1,$ $\mathbf{10}\otimes\mathbf{2}\otimes\frac{4}{3}\otimes 4\otimes -\frac{1}{2}, \overline{\mathbf{6}}\otimes\mathbf{1}\otimes -\frac{8}{3}\otimes 4\otimes -\frac{2}{3}, \overline{\mathbf{6}}\otimes\mathbf{1}\otimes -1\otimes -3\otimes -\frac{2}{3}, \overline{\mathbf{6}}\otimes\mathbf{2}\otimes -\frac{2}{3}\otimes 4\otimes -\frac{1}{6},$ $\overline{\mathbf{6}}\otimes\mathbf{1}\otimes\frac{2}{3}\otimes -10\otimes -\frac{2}{3}, \overline{\mathbf{6}}\otimes\mathbf{2}\otimes 1\otimes -3\otimes -\frac{1}{6}, \overline{\mathbf{6}}\otimes\mathbf{3}\otimes\frac{4}{3}\otimes 4\otimes\frac{1}{3}, \mathbf{3}\otimes\mathbf{1}\otimes -\frac{14}{3}\otimes 4\otimes -\frac{1}{3},$ $\mathbf{3}\otimes\mathbf{1}\otimes -3\otimes -3\otimes -\frac{1}{3}, \mathbf{3}\otimes\mathbf{2}\otimes -\frac{8}{3}\otimes 4\otimes\frac{1}{6}, \mathbf{3}\otimes\mathbf{1}\otimes -\frac{4}{3}\otimes -10\otimes -\frac{1}{3}, \mathbf{3}\otimes\mathbf{2}\otimes -1\otimes -3\otimes\frac{1}{6},$ $\mathbf{3}\otimes\mathbf{3}\otimes -\frac{2}{3}\otimes 4\otimes\frac{2}{3}, \mathbf{3}\otimes\mathbf{1}\otimes\frac{1}{3}\otimes -17\otimes -\frac{1}{3}, \mathbf{3}\otimes\mathbf{2}\otimes\frac{2}{3}\otimes -10\otimes\frac{1}{6}, \mathbf{3}\otimes\mathbf{3}\otimes 1\otimes -3\otimes\frac{2}{3},$ $\mathbf{3}\otimes\mathbf{4}\otimes\frac{4}{3}\otimes 4\otimes\frac{7}{6}, \mathbf{1}\otimes\mathbf{1}\otimes -\frac{20}{3}\otimes 4\otimes 0, \mathbf{1}\otimes\mathbf{1}\otimes -5\otimes -3\otimes 0, \mathbf{1}\otimes\mathbf{2}\otimes -\frac{14}{3}\otimes 4\otimes\frac{1}{2},$ $\mathbf{1}\otimes\mathbf{1}\otimes -\frac{10}{3}\otimes -10\otimes 0, \mathbf{1}\otimes\mathbf{2}\otimes -3\otimes -3\otimes\frac{1}{2}, \mathbf{1}\otimes\mathbf{3}\otimes -\frac{8}{3}\otimes 4\otimes 1, \mathbf{1}\otimes\mathbf{1}\otimes -\frac{5}{3}\otimes -17\otimes 0,$ $\mathbf{1}\otimes\mathbf{2}\otimes -\frac{4}{3}\otimes -10\otimes\frac{1}{2}, \mathbf{1}\otimes\mathbf{3}\otimes -1\otimes -3\otimes 1, \mathbf{1}\otimes\mathbf{4}\otimes -\frac{2}{3}\otimes 4\otimes\frac{3}{2}, \mathbf{1}\otimes\mathbf{1}\otimes 0\otimes -24\otimes 0,$ $\mathbf{1}\otimes\mathbf{2}\otimes\frac{1}{3}\otimes -17\otimes\frac{1}{2}, \mathbf{1}\otimes\mathbf{3}\otimes\frac{2}{3}\otimes -10\otimes 1, \mathbf{1}\otimes\mathbf{4}\otimes 1\otimes -3\otimes\frac{3}{2}, \mathbf{1}\otimes\mathbf{5}\otimes\frac{4}{3}\otimes 4\otimes 2\}$

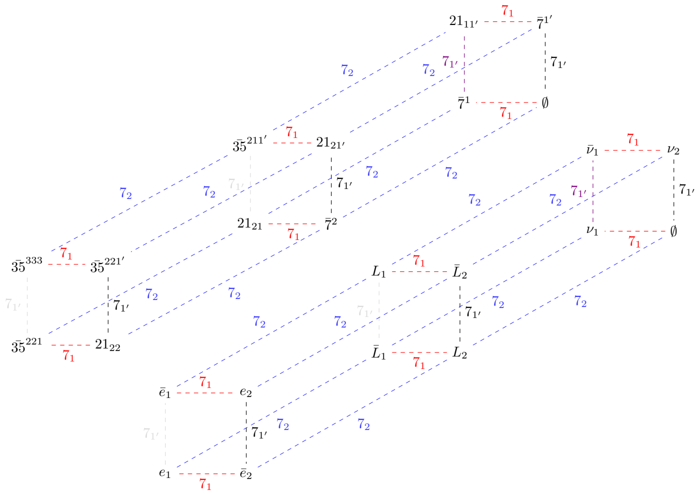
Gauge Bosons' Masses

- $m^2\left(\mathbf{3},\mathbf{1},\frac{1}{3},7,-\frac{1}{3}\right) \propto g_u^2\left(v_c - v_1\right)^2$
- $m^2\left(\mathbf{3},\mathbf{1},2,0,-\frac{1}{3}\right) \propto g_u^2\left(v_c - v_{1'}\right)^2$
- $m^2\left(\mathbf{3},\mathbf{2},0,0,-\frac{5}{6}\right) \propto g_u^2\left(v_c - v_l\right)^2$
- $m^2\left(\mathbf{1},\mathbf{2},-2,0,-\frac{1}{2}\right) \propto g_u^2\left(v_l - v_1\right)^2$
- $m^2\left(\mathbf{1},\mathbf{2},-\frac{1}{3},-7,-\frac{1}{2}\right) \propto g_u^2\left(v_l - v_{1'}\right)^2$
- $m^2\left(\mathbf{1},\mathbf{1},-\frac{5}{3},7,0\right) \propto g_u^2\left(v_1 - v_{1'}\right)^2$
- $m^2_{U(1)_{1'}} \propto g_{1'}^2\left\langle 7_{S,1'}\right\rangle^2 + g_1^2\left\langle 210_{1111}\right\rangle^2$
- $m^2_{F,1} \propto g^2_{F,1}\left\langle 210_{1111}\right\rangle^2$
- $m^2_{11'} \propto g_1 g_{1'}\left\langle 210_{1111}\right\rangle^2$
- $m^2_{\mathrm{SM}} \propto g_2^2\left\langle 7_{S,2}\right\rangle^2$

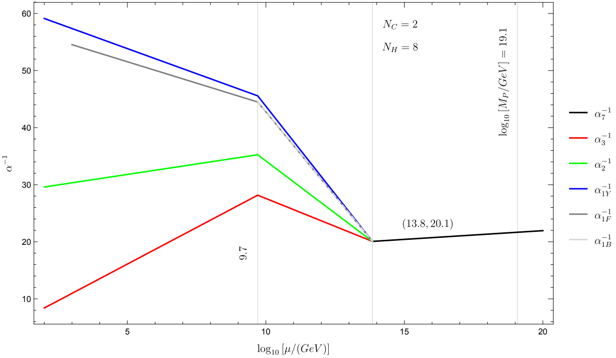
Fermion (Quark) Masses



Fermion (Lepton) Masses



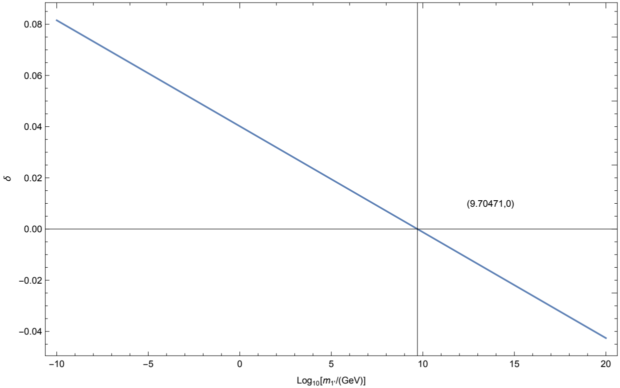
Unification of gauge couplings



$$\alpha_i^{-1}(m_2) = \alpha_i^{-1}(m_1) + \frac{-b_i}{2\pi} \ln\left(\frac{m_2}{m_1}\right)$$

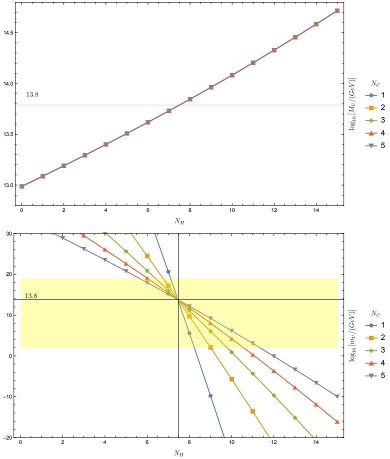
$$\beta_i(g_i) = \frac{-g_i^3}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}\kappa_f S_2(R_F) - \frac{1}{3}\kappa_s S_2(R_S) + \frac{2\kappa_f}{(4\pi)^2}Y_4(R_F)\right]$$

Unification of gauge couplings



$$\delta = \frac{\alpha_3^{-1}(m_z) - \alpha_2^{-1}(m_z) - (b_{3d} - b_{3u} - b_{2d} + b_{2u}) \ln\left(\frac{m_{1'}}{m_z}\right)/2\pi}{\alpha_2^{-1}(m_z) - \alpha_1^{-1}(m_z) - (b_{2d} - b_{2u} - b_{1d} + b_{1u}) \ln\left(\frac{m_{1'}}{m_z}\right)/2\pi} - \frac{b_{3u} - b_{2u}}{b_{2u} - b_{1u}}$$

Unification of gauge couplings



$$b_7 = -\frac{5}{6} \qquad b_{3u} = 5 + \frac{N_c}{6} \quad b_{2u} = \frac{26}{3} + \frac{N_H}{6} \qquad b_{1yu} = 16 + \frac{N_H}{10} \, b_{1Fu} = 16 + \frac{N_H}{90}$$

$$b_{1(1')} = 16 + \frac{N_H}{126} \qquad b_{3d} = -7 \quad b_{2d} = -\frac{10}{3} + \frac{N_H}{6} \qquad b_{1yd} = 4 + \frac{N_H}{10} \, b_{1Fd} = 4 + \frac{N_H}{90}$$

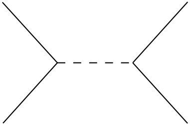
Possible proton decay scenarios for $SU(7)$ model

- Set 1 : $Q = 21_{32}, \bar{u} = \bar{35}^{322}, \bar{d} = \bar{7}^{3}, L = \bar{35}^{211'}, \bar{e} = \bar{35}^{333}, \nu = 21_{11'}$
- Set 2 : $Q = \bar{35}^{332}, \bar{u} = 21_{33}, \bar{d} = \bar{35}^{311'}, L = \bar{35}^{211'}, \bar{e} = \bar{35}^{333}, \nu = 21_{11'}$
- If set 1 is chosen and $\langle 7_1 \rangle = 0 \wedge \langle 84_{111'} \rangle \neq 0$ then proton decays can be categorized into two groups: $B + L$ conserving decays require the presence of 28_S in the model due to presence of accidental global symmetry in the yukawa sector. Therefore all such decays are proportional to the amount of violation of this symmetry, *e.g.* $p \to \nu\pi^{+}$. The other category is $B - L$ conserving decays which are proportional to $\langle 84_{111'} \rangle$, *e.g.* $p \to e^{+}\pi^{0}$.

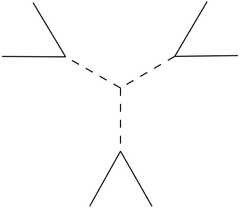
Possible proton decay scenarios for $SU(7)$ model

- If set 1 is chosen and $\langle 7_1 \rangle = 0 \wedge \langle 84_{111'} \rangle = 0 \wedge \langle 210_{1111} \rangle \neq 0$ then proton decays proceed via three body decays with $(B-L) = -4$, *e.g.* $p \to e^+ \bar{\nu}\bar{\nu}$.
- If set 2 is chosen and $\langle 7_1 \rangle = 0 \wedge \langle 84_{111'} \rangle \neq 0$ then proton decays are forbidden due to the combination of the residual symmetry $U(1)_F \to \mathbb{Z}_n$ and Lorentz symmetry.
- Due to $\langle 7_1 \rangle = 0$ fermion sector is split into even (SM) and odd (BSM) under $\mathbb{Z}_2$ residual symmetry. This makes all B violating processes to proceed at loop level with odd fields inside the loop, similar to the scotogenic models.

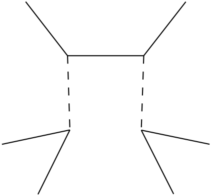
Possible topologies of diagrams for proton decay



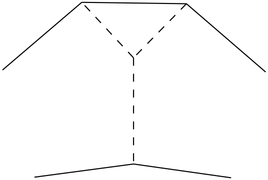
(a) T4001



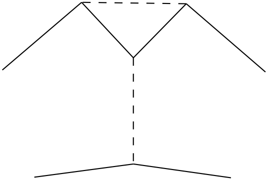
(b) T6001



(c) T6011

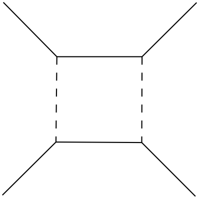


(d) T4111

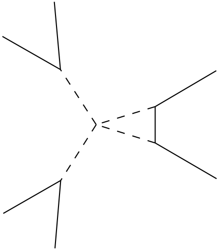


(e) T4121

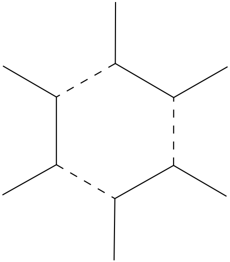
Possible topologies of diagrams for proton decay



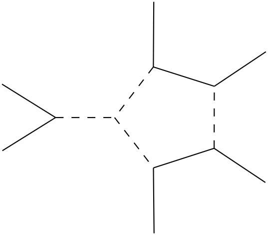
(f) T4122



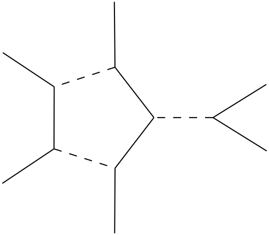
(g) T6111



(h) T6131

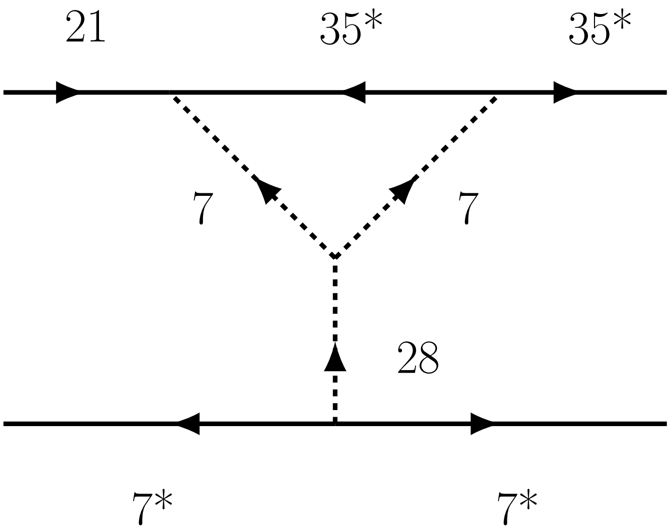


(i) T6121



(j) T6132

Triangular loop diagram example (Penguin)



Triangular loop diagram example (Penguin)

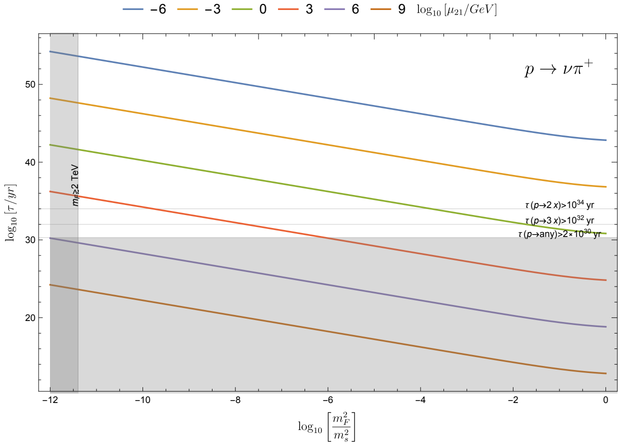
$$O(p \to \nu\pi^+) \sim C_\mu \left(\bar{3}\bar{5}^\dagger_{211'} \bar{\sigma}^\mu 21_{32}\right) \left(\bar{7}^\dagger_3 \bar{7}^\dagger_3\right) \Lambda^{-3}$$

$$\Gamma = \frac{m_p}{32\pi}\left(1 - \frac{m^2_{\pi^+}}{m^2_p}\right)^2 \frac{W_0^2}{m_s^4} \frac{\left|Y_e Y_u Y_{28}\right|^2}{(16\pi^2)^2} \frac{\mu_{28}^2}{m_s^2} x \left[F_0(x) + F_1(x)\frac{q^2}{m_s^2}\right]^2$$

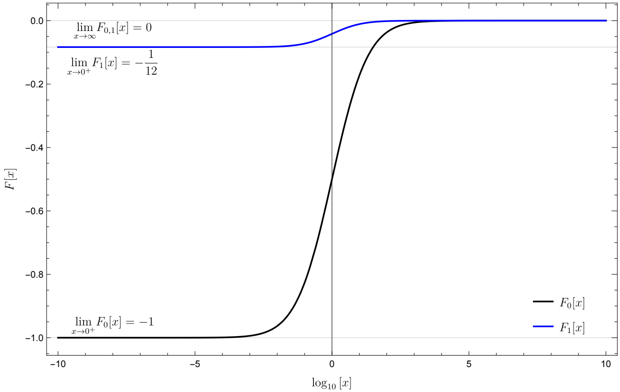
$$F_0(x) = \frac{1}{x-1}\left(1 - \frac{x\ln x}{x-1}\right) \qquad x = \frac{m_F^2}{m_S^2}$$

$$F_1(x) = -\frac{2x^2 + 5x - 1}{12(x-1)^3} + \frac{x^2 \ln x}{2(x-1)^4}$$

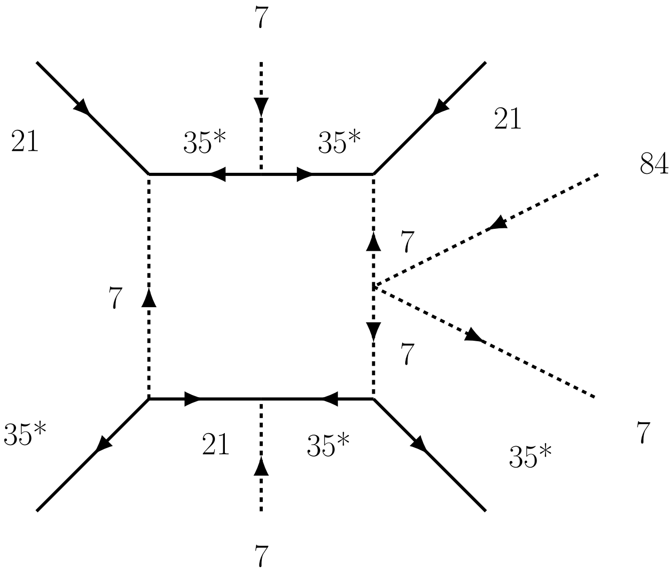
Triangular loop diagram



Triangular loop function



Box diagram example



Box diagram example

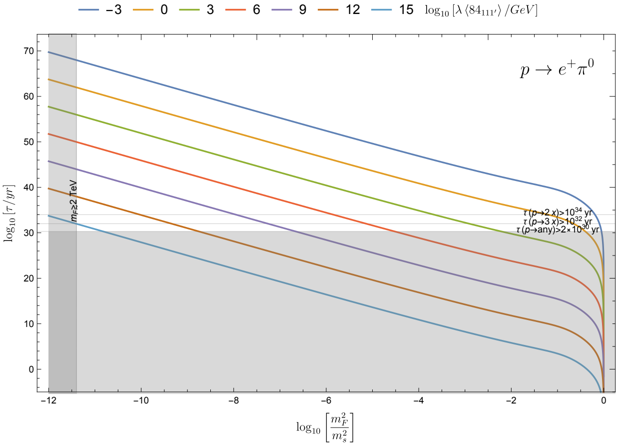
$$O \sim (21_{32}21_{32})\left(\bar{35}^{\dagger}_{322}\bar{35}^{\dagger}_{333}\right)7_{1'}7_{1'}7^{\dagger 1'}84_{111'}\Lambda^{-6}$$

$$\Gamma(p \to e^{+}\pi^{0}) = \frac{m_p}{32\pi}\left(1 - \frac{m^2_{\pi^0}}{m^2_p}\right)^2 \frac{\left|Y^3_u Y_e\right|^2}{(16\pi^2)^2}$$

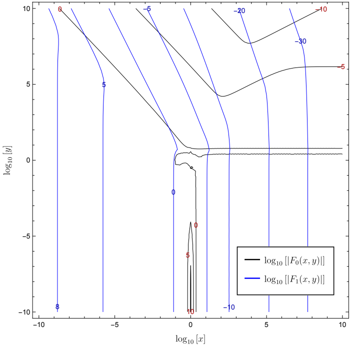
$$\frac{\lambda^2 \langle 7_{1'}\rangle^2 \langle 84_{111'}\rangle^2}{m^4_s} x^2 \left[\frac{W_0}{m^2_s} F_0(x,y) + \frac{W_1}{m^2_s} F_1(x,y) \frac{q^2}{m^2_s}\right]^2$$

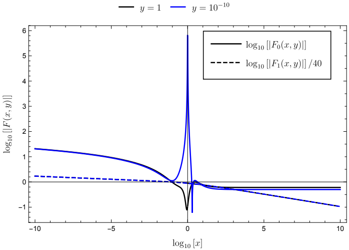
$$x = \frac{m^2_F}{m^2_S} \quad y = \frac{m^2_p}{m^2_S}$$

Box diagram example

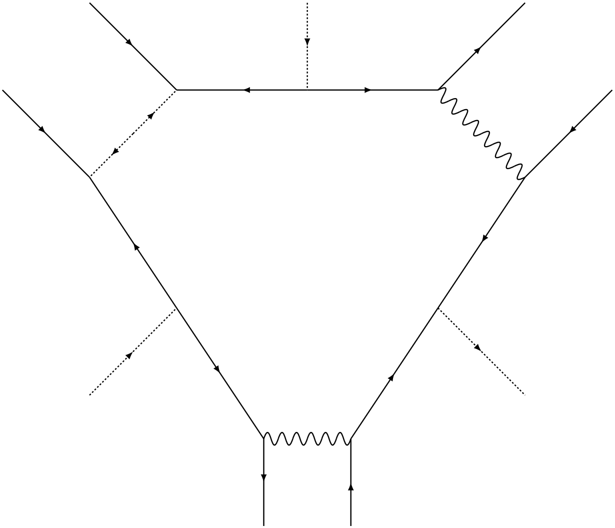


Box loop function





Hexagonal loop diagram



Hexagonal loop function

$$p \to e^+ e^+ e^-$$

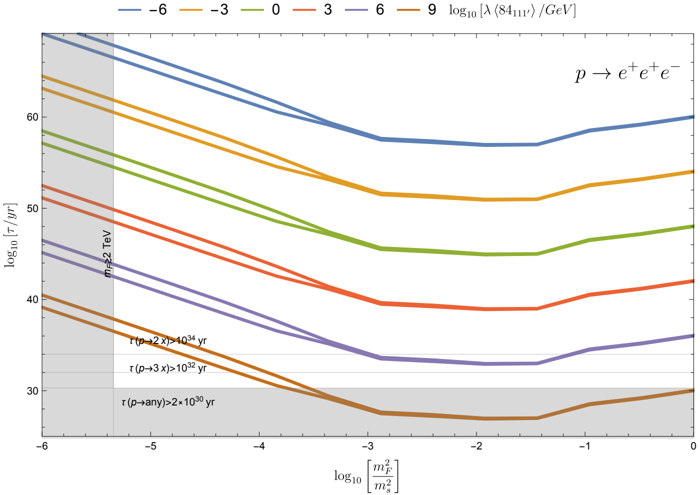
$$O \sim \left(\overline{35}^{\dagger}_{322}\bar{\sigma}^{\mu} 21_{32}\right)\left(\bar{7}^{\dagger}_{3}\bar{\sigma}_{\mu}\overline{35}^{211'}\right)\left(\overline{35}^{211'}\,\overline{35}^{333}\right) 7_{1'} 7_{1'} 7^{\dagger 2} 7^{\dagger 1'} 84_{111'} \Lambda^{-10}$$

$$\Gamma = \frac{m_p}{(8\pi)^3}\frac{1}{(16\pi^2)^2}\left(g_u^4 Y_u^2\right)^2 \left(\frac{\lambda \langle 84_{111'}\rangle}{m_S}\right)^2 \frac{v_2^2}{m_S^2}\frac{\alpha^2}{m_S^6}\frac{f(s,t,u,m_p,m_e)}{m_S^4} x^3 R($$

$$x = \frac{m_F^2}{m_S^2} \quad y = \frac{m_S^2}{m_G^2} \sim 1 \quad z = \frac{m_p^2}{m_S^2} \ll 1$$

$R(x, y, z)$ is a two loop 2×7 propagator loop function and calculated using *pySecDec*.

Hexagonal loop diagram



Dark Matter Candidates

- BSM fermions are naturally lighter than BSM gauge and scalar bosons in the context of the model.
- There are total of 7 colorless and neutral fermions in the model. They are divided into two sectors by Z_2 residual symmetry.
- Neutral sector containing Dirac neutrino pair contains 4 fermions. The other sector consisting of the 3 neutral fermions, one of which is a Majorana fermion will contain a natural dark matter candidate.
- Annihilation cross−section can be obtained via 2−to−2 t−channel with SM fermions (neutrinos) with $\mathbb{Z}_2$ odd scalar or vector boson propagator.

Options for neutrino sector

- Neutral sector, even under $\mathbb{Z}_2$ residual symmetry contains total of 4 fermions, 2 of which are Dirac neutrino pair.
- Depending on the relative size of the Yukawa couplings involved this can lead to the following neutrino scenarios: inverse seesaw, double seesaw, lopsided seesaw, Dirac seesaw.
- For the discussion of the proton decay mentioned earlier neutrinos are naturally pseudo−Dirac fermions.

Advantages and Disadvantages

- Rich options for proton decay scenarios, which include: 2−body $B + L$ and/or $B - L$ conserving proton decays, no 2−body, *i.e.* 3−body leading order proton decays, no proton decay (proton stability at the GUT level)
- Proton decays at one−loop leading order which leads to suppressed decay widths.
- Naturally accommodates dark matter candidates.
- Several possibilities for the neutrino mass sector.
- Testable.
- Multistep phase transitions. (Gravitational waves).
- Splitting of scales in the fermion sector.

Future Prospects

- Many options for stochastic gravitational waves sources from multistep first order phase transitions at low scale.
- Rich scalar sector structure.
- Dark matter candidates analysis.
- Possibilities for inverse seesaw, double seesaw, lopsided seesaw, and their extensions in the neutrino sector.
- Prospects at future e^+e^- and pp colliders.

Conclusion

- Exotic proton decay channels
- Natural stability of proton
- Dark matter candidates
- Unification of gauge forces
- Testability

Thanks for your attention!