Flavoured GUTs

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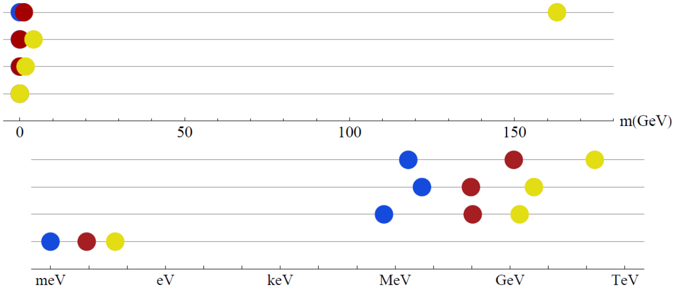
The Standard Model is very successful but...

- Neutrinos have masses (νSM)
- Dark matter (no viable explanation)
- Matter / antimatter asymmetry (no viable explanation)
- Hierarchy problem (fine-tuning between parameters)
- Strong CP problem (fine-tuning between parameters)
- Gauge couplings (additional free parameters) - GUT?
- Flavour problem (many additional free parameters) - FS?

BSM solutions involve additional fields and symmetries

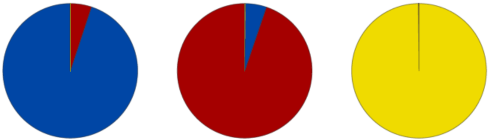
The Standard Model flavour problem: masses

3 fermion generations? Masses span orders of magnitude?



The Standard Model flavour problem: mixing

3 generations of quarks, small mixing



3 generations of leptons, large and peculiar mixing

(mixing between weak and mass eigenstates)

Summary of data: quark mixing

Wolfenstein parametrisation

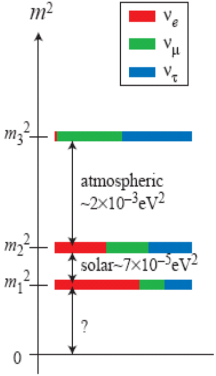
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.23$ (Sine of the Cabibbo angle)

Summary of data: lepton mixing

Tri-bi-maximal (TBM) mixing

$$V_{PMNS} \simeq \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



Beyond the Standard Model with family symmetries

Without $y_f H F f_R$, $\mathcal{L}_{\nu SM}$ has accidental symmetry $SU(3)^6$

FS: upgrade subgroup of $SU(3)^6$ to actual symmetry of $\mathcal{L}$

1. Generations charged differently under FS
2. Yukawa couplings no longer invariant
3. FS must be broken somehow...

Abelian?

A reason

- Simpler

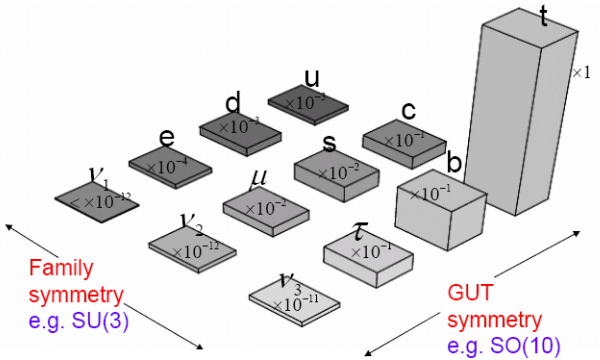
See talk by Calibbi.

Non-Abelian?

3 reasons

- 3 generations explained naturally
- νSM: FS $\subset SU(3)^6$; $SO(10)$ GUT: FS $\subset SU(3)$
- Lepton mixing strongly suggests non-Abelian FS

$SO(10) \times SU(3)$?



Unification with family symmetry

All fermions can have the same Dirac mass structure!

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

$\varepsilon_d = 0.15, \quad a^d = -2/3$

$\varepsilon_l = 0.15, \quad a^e = -3$

$\varepsilon_u = 0.05, \quad a^u = 4/3$

$\varepsilon_\nu = 0.05, \quad a^\nu = 0$

Seesaw and Georgi-Jarlskog (GJ) factors distinguish quarks and leptons

Texture Zero for quarks

$$M_{11}^{LR} = 0 \quad (1)$$

Texture Zero for up and down quarks gives the Gatto-Sartori-Tonin (GST) relation:

$$\sin\theta_c = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right| \quad (2)$$

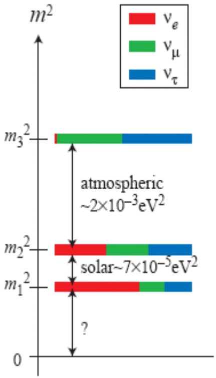
But how to get $M_{11}^{LR} = 0$... And what about the leptons?

Mass matrices from aligned VEVs

In this talk, directions:

$\langle \bar{\phi}_{\rm atm} \rangle \propto (0, 1, -1)$
$\langle \bar{\phi}_{\rm sol} \rangle \propto (1, 1, 1)$

FS invariants $(\bar{\phi}^i_{\rm atm} F_i)$, $(\bar{\phi}^i_{\rm sol} F_i)$



Mass matrices example columns (R)

Term sol L / atm R

$$+y_{\odot}(\bar{\phi}^{i}_{\mathrm{sol}} F_i)(\bar{\phi}^{j}_{\mathrm{atm}} f_{Rj})H$$

Respective mass matrix

$$+y_{\odot}\begin{pmatrix} 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \end{pmatrix}$$

Mass matrices example rows (L)

Term atm L / sol R

$$+y_{@}(\bar{\phi}^{i}_{\mathrm{atm}}F_i)(\bar{\phi}^{j}_{\mathrm{sol}}f_{Rj})H$$

Respective mass matrix

$$+y_{@}\begin{pmatrix} 0 & 0 & 0 \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \\ -\epsilon^3 & -\epsilon^3 & -\epsilon^3 \end{pmatrix}$$

$SO(10) \times \Delta(27)$

Directions $\langle\bar{\phi}_{\rm sol}\rangle = (1,1,1)$ and $\langle\bar{\phi}_{\rm atm}\rangle = (0,1,-1)$
Easy to align in $\Delta(27)$ (discrete) family symmetry
IdMV, S. F. King, G. G. Ross
https://arxiv.org/abs/hep-ph/0607045

Effective Majorana mass terms

$$-\frac{(\kappa^\nu)^2}{\kappa_2^M}(\bar{\phi}_{\rm atm}\nu)(\bar{\phi}_{\rm atm}\nu) - \frac{(\kappa^\nu)^2}{\kappa_1^M}(\bar{\phi}_{\rm sol}\nu)(\bar{\phi}_{\rm sol}\nu) \quad (3)$$

Democratic contribution fills all entries

$$M_{11}^{LR} = 0; M_{11}^{RR} \neq 0; M_{11}^{LL} \neq 0 \quad (4)$$

TBM in neutrino sector, modified slightly by charged lepton matrix which is not diagonal in this basis: θ_{13} too small!

Universal Texture Zero

IdMV, G. G. Ross, J. Talbert

https://arxiv.org/abs/1710.01741

Preserve the M_{11} texture zero in the Majorana mass matrix and into the effective neutrino mass matrix after seesaw:

$$M_{11}^{LR} = M_{11}^{RR} = M_{11}^{LL} = 0 \tag{5}$$

Not TBM in neutrino sector.

Large θ_{13} (correlated with other angles).

Results (2017) summary

At L.O., with 9 (low energy) parameters we fit
all SM fermion mass and mixing
(with tension only in CKM elements 13 and 31)
At H.O. this gets fixed by 2 additional parameters

Universal Texture Zero works very well!

Predictive family symmetry GUT model with UTZ
Important postdictions:
Cabibbo angle (GST),
charged lepton masses (GJ),
reactor angle (TM)

Results (2022) summary

Bernigaud, IdMV, Levy, Talbert

https://arxiv.org/abs/2211.15700

UTZ remains a minimal, viable, and appealing theory of flavour

Demonstrate the potential of examining multi-parameter flavour models with MCMC routines

Multiple Modular?

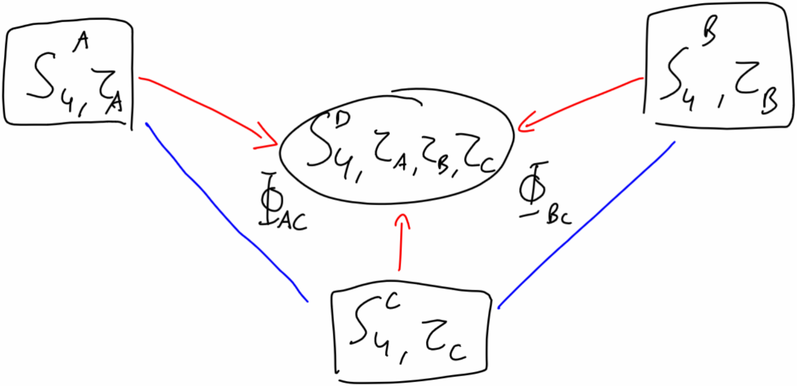
Multiple Modulus enables

- Multiple stabilizers / fixed points
- Stabilizers outside fundamental domain

Modular Models with residual symmetries!

The framework

IdMV, King, Zhou `https://arxiv.org/abs/1906.02208`
Basically, break multiples to diagonal subgroup.



The framework (paper version)

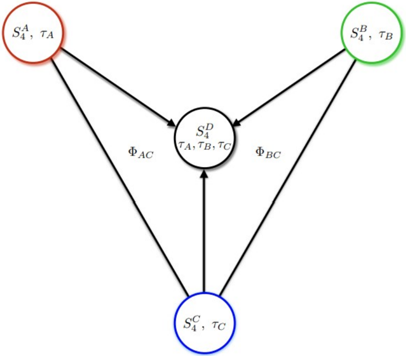


Figure 1: Illustration of the breaking of $S_4^A \times S_4^B \times S_4^C \to S_4^D$, identified as the diagonal subgroup, via the VEVs of Φ_{AC} and Φ_{BC}.

Example superpotential

$$
\begin{aligned}
w_\ell \quad = \quad & \frac{1}{\Lambda}\left[L\Phi_{AC}Y_A(\tau_A)N^c_A + L\Phi_{BC}Y_B(\tau_B)N^c_B\right]H_u \\
& + \left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right]H_d \\
& + \frac{1}{2}M_A(\tau_A)N^c_A N^c_A + \frac{1}{2}M_B(\tau_B)N^c_B N^c_B + M_{AB}(\tau_A,\tau_B)N^c_A N^c_B
\end{aligned}
$$

Example effective superpotential

$$
\begin{aligned}
w_\ell^{\mathrm{eff}} &= \left[\frac{v_{AC}}{\Lambda} L Y_A(\tau_A) N_A^c + \frac{v_{BC}}{\Lambda} L Y_B(\tau_B) N_B^c\right] H_u \\
&\quad +(\ldots)
\end{aligned}
$$

Multiple Multiple Modular Models

King, Zhou
`https://arxiv.org/abs/1908.02770` (2 S_4)
King, Zhou
`https://arxiv.org/abs/2103.02633` ($SU(5)$)

More Multiple Multiple Modular Models

IdMV, Lourenço
https://arxiv.org/abs/2107.04042 (2 A_4)
IdMV, Lourenço
https://arxiv.org/abs/2206.14869 (2 A_5)
IdMV, King, Levy
https://arxiv.org/abs/2211.00654 (Littlest Modular)
IdMV, King, Levy
https://arxiv.org/abs/2309.15901 ($SU(5)$ Littlest Modular)

Littlest Modular Seesaw

$$\tau_A = \frac{1}{2} + \frac{i}{2} : \qquad Y_{\mathbf{3'}}^{(4)}(\tau_A) = (0, -1, 1)\,, \tag{6}$$

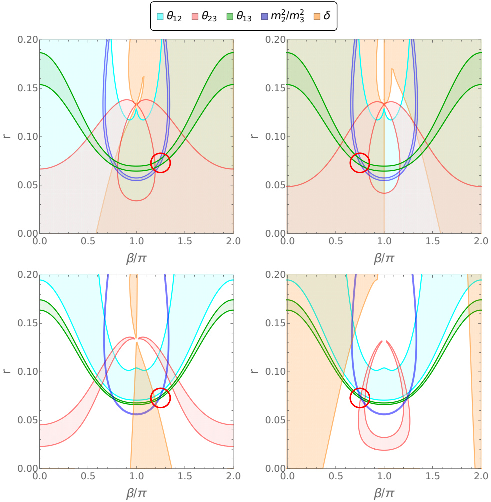
for one of the Dirac mass matrix columns, and

$$\tau_B = \frac{3}{2} + \frac{i}{2} : \qquad Y_{\mathbf{3'}}^{(2)}(\tau_B) = (1, 1-\sqrt{6}, 1+\sqrt{6})\,, \tag{7}$$

or

$$\tau_B = -\frac{1}{2} + \frac{i}{2} : \qquad Y_{\mathbf{3'}}^{(2)}(\tau_B) = (1, 1+\sqrt{6}, 1-\sqrt{6})\,, \tag{8}$$

LMS Results



Flavoured GUTs

- Interesting and ambitious to build Flavoured GUTs.
- Mediators, Seesaw mechanism play key roles.
- Viable structures that are Universal.

Thank you

谢谢

Thank you!