Studies on Domain Walls, Cosmic Strings, and Their Gravitational Wave Signatures

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https://yzhxxzxy.github.io

Based on Qing-Quan Zeng, Xi He, ZHY, Jiaming Zheng, arXiv:2501.10059

Shi-Qi Ling, ZHY, arXiv:2502.16576

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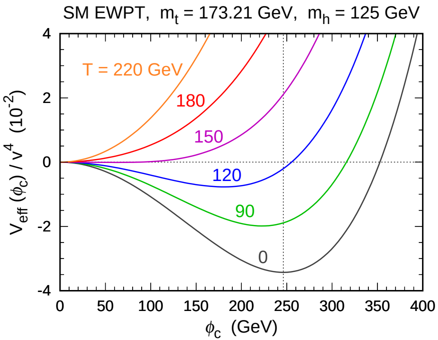
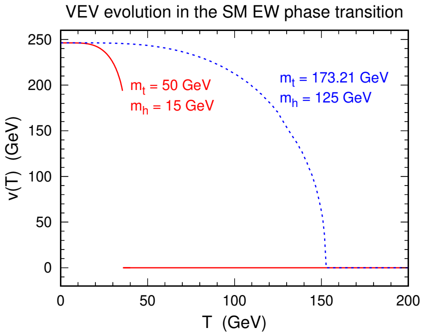
Cosmological Phase Transition

🔥 **Spontaneously broken symmetries** in field theories can be **restored** at **sufficiently high temperatures** due to **thermal corrections** to the **effective potential**

🌨️ In the history of the Universe, **spontaneous symmetry breaking** manifests itself as a **cosmological phase transition**





Topological Defects

✂️ Consider that **some scalar fields** acquire nonzero **vacuum expectation values** (VEVs), which **break** a **symmetry group** G to a **subgroup** H

⚖️ The **manifold** consisting of all **degenerate vacua** is the **coset space** G/H

🧵 The **topology** of the **vacuum manifold** G/H can be characterized by its **n-th homotopy group** $\pi_n(G/H)$, which are formed by the homotopy classes of the mappings from an **n-dimensional sphere** S^n into G/H

🪢 A **nontrivial** $\pi_n(G/H)$ leads **topological defects** [Kibble, J. Phys. A9 (1976) 1387], as commonly predicted in **grand unified theories**

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🟥 **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

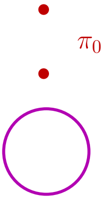
- 👉 **Domain walls** (2-dim topological defects)

♑ **Nontrivial** $\pi_1(G/H)$: incontractable closed paths

- 👉 **Cosmic strings** (1-dim topological defects)

🟠 **Nontrivial** $\pi_2(G/H)$: incontractable spheres

- 👉 **Monopoles** (0-dim topological defects)



$\pi_0(G/H) = Z_2$

$\pi_1(G/H) = \mathbb{Z}$

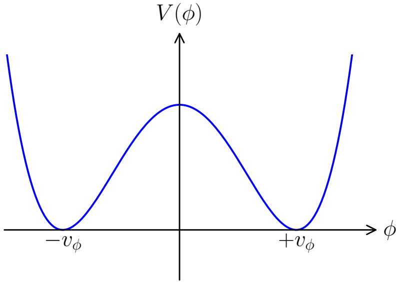
Domain Walls

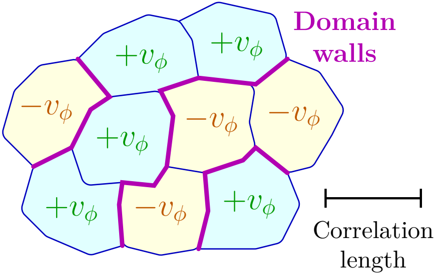
☯ **Domain walls (DWs)** are **two-dimensional topological defects** which could be formed when a **discrete symmetry** of the **scalar potential** is **spontaneously broken** in the early Universe

♊ They are **boundaries** separating spatial regions with different **degenerate vacua**

🚫 **Stable DWs** are thought to be a **cosmological problem** [Zeldovich, Kobzarev, Okun, Zh.Eksp.Teor.Fiz. **67** (1974) 3]

⚠️ As the Universe expands, the **DW energy density** decreases **slower** than radiation and matter, and would soon **dominate** the total energy density

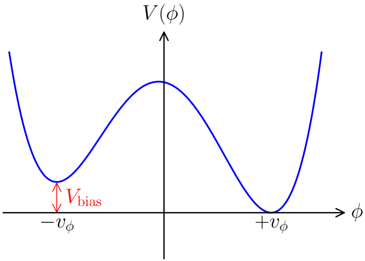




Collapsing Domain Walls

It is **allowed** if **DWs collapse** at a very early epoch [Vilenkin, PRD **23** (1981) 852; Gelmini, Gleiser, Kolb, PRD **39** (1989) 1558; Larsson, Sarkar, White, hep-ph/9608319, PRD]

Such **unstable DWs** can be realized if the **discrete symmetry** is **explicitly broken** by a **small potential term** that gives an **energy bias** V_{bias} among the minima of the potential



The bias induces a **volume pressure force** acting on the DWs that leads to their collapse

Collapsing DWs can produce significant **GWs** [Preskill *et al.*, NPB 363 (1991) 207; Gleiser, Roberts, astro-ph/9807260, PRL; Hiramatsu, Kawasaki, Saikawa, 1002.1555, JCAP]

A **stochastic gravitational wave background (SGWB)** would be formed and remain to the present time

Spontaneously Broken Z_2 Symmetry

We study the dynamics of **DWs** formed through the **spontaneous breaking** of an **approximate $\mathbb{Z}_2$ symmetry** in a **scalar field** ϕ, focusing on the influence of **quantum** and **thermal corrections** induced by a **$\mathbb{Z}_2$-violating Yukawa coupling** to **Dirac fermions** f in the **thermal bath** [QQ Zeng, X He, **ZHY**, JM Zheng, 2501.10059]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \mathrm{i}\bar{f}\gamma^\mu\partial_\mu f - m_f\bar{f}f - y\phi\bar{f}f - V_0(\phi)$$

$$V_0(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{3}\mu_3\phi^3 + \frac{1}{4}\lambda_\phi\phi^4, \qquad \mu_\phi^2 > 0, \quad \lambda_\phi > 0$$

The small **couplings** y and μ_3 **explicitly violate** the **$\mathbb{Z}_2$ symmetry** $\phi \to -\phi$

Considering the **Coleman-Weinberg correction** $V_{\mathrm{CW}}(\phi)$ and **finite-temperature correction** $V_{\mathrm{T}}(\phi, T)$ at one-loop level, the **effective potential** becomes

$$V(\phi, T) = V_0(\phi) + V_{\mathrm{CW}}(\phi) + V_{\mathrm{T}}(\phi, T)$$

The **vacuum expectation value (VEV)** v_ϕ of ϕ corresponds to

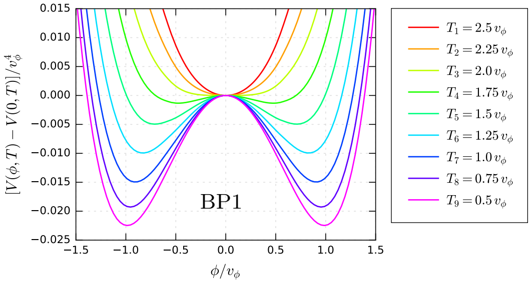
$$\frac{\partial}{\partial\phi}\left[V_0(\phi) + V_{\mathrm{CW}}(\phi)\right]\Big|_{\phi=v_\phi} = 0$$

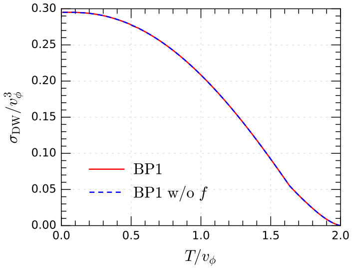
Potential Evolution and DW Tension

☂️ By solving the equation of motion for the **DW solution** and integrating the energy density, the **DW tension** σ_{DW}, i.e., **energy per unit area**, can be obtained

🌧️ We choose **three benchmark points** (BPs) to highlight remarkable features

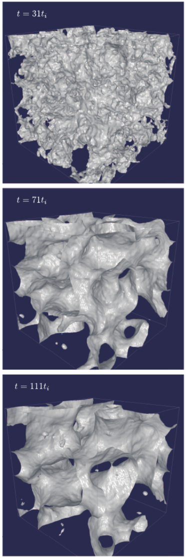
	v_ϕ [GeV]	μ_3/v_ϕ	λ_ϕ	y	m_f/v_ϕ
BP1	2×10^{9}	-10^{-17}	0.1	2.5×10^{-5}	4×10^{-5}
BP2	5×10^{4}	-10^{-27}	0.1	-9×10^{-8}	10^{-7}
BP3	1.5×10^{11}	-1.2148×10^{-13}	0.1	3×10^{-4}	4×10^{-4}





Evolution of Domain Walls

- After DWs are created, the **tension** $\sigma_{\rm DW}$ acts to **stretch** them up to the **horizon size** if the **friction** F_f is **small**, and they would enter the **scaling regime** with **energy density** $\rho_{\rm DW} = \dfrac{\mathcal{A}\sigma_{\rm DW}}{t}$
- $\mathcal{A} \approx 0.8 \pm 0.1$ is a numerical factor given by lattice simulation
- $\rho_{\rm DW} \propto t^{-1}$ implies that DWs are **diluted more slowly** than **radiation** and **matter** as the Universe expands
- If DWs are **stable**, they would soon **dominate** the evolution of the Universe, **conflicting** with cosmological observations



[Hiramatsu *et al.*, 1002.1555]

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However, the **potential bias** $V_{\rm bias}(T) = V(\phi_-, T) - V(\phi_+, T)$ between the false and true vacua ϕ_- and ϕ_+ provides a **pressure** $p_{\rm V}(T) \sim V_{\rm bias}(T)$ acting on the DWs, against the **tension force per unit area** $p_{\rm T}(T) \sim \rho_{\rm DW}(T)$

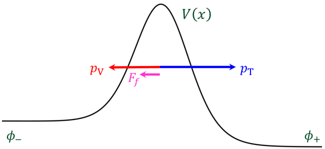
This makes the **DWs collapse** and the **false vacuum domains shrink**

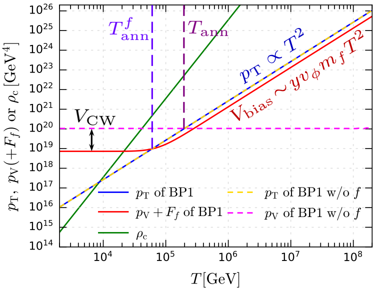
[Hiramatsu *et al.*, 1002.1555]

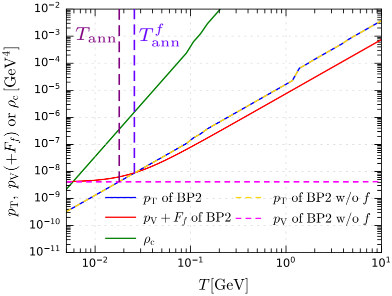
Annihilation Temperature

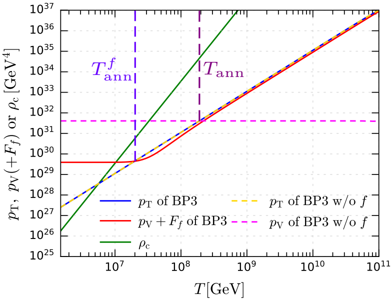
The **domain walls collapse** at the **annihilation temperature** T_{ann} when

$$p_{\text{V}}(T_{\text{ann}}) + F_f(T_{\text{ann}}) \simeq p_{\text{T}}(T_{\text{ann}})$$









SGWB Spectrum from Collapsing DWs

- The **SGWB spectrum** is commonly characterized by $\Omega_{\mathrm{GW}}(f) = \dfrac{f}{\rho_{\mathrm{c}}}\dfrac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$
- ρ_{GW} is the **GW energy density**, and ρ_{c} is the critical energy density
- The SGWB from **collapsing DWs** can be estimated by **numerical simulations**

[Hiramatsu, Kawasaki, Saikawa, 1002.1555, 1309.5001, JCAP]

- The **present SGWB spectrum** induced by collapsing DWs can be evaluated by

$$\Omega_{\mathrm{GW}}(f)h^2 = \Omega_{\mathrm{GW}}^{\mathrm{peak}}h^2 \times \begin{cases} \left(\dfrac{f}{f_{\mathrm{peak}}}\right)^3, & f < f_{\mathrm{peak}} \\ \dfrac{f_{\mathrm{peak}}}{f}, & f > f_{\mathrm{peak}} \end{cases}$$

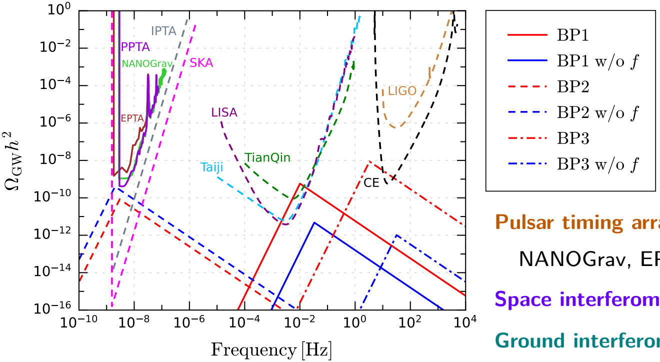
$$\Omega_{\mathrm{GW}}^{\mathrm{peak}}h^2 = 7.2\times 10^{-18}\ \tilde{\epsilon}_{\mathrm{GW}}\mathcal{A}^2 \left[\frac{g_{*s}(T_{\mathrm{ann}})}{10}\right]^{-4/3} \left[\frac{\sigma_{\mathrm{DW}}(T_{\mathrm{ann}})}{1\ \mathrm{TeV}^3}\right]^2 \left(\frac{T_{\mathrm{ann}}}{10\ \mathrm{MeV}}\right)^{-4}$$

$$f_{\mathrm{peak}} = 1.1\times 10^{-9}\ \mathrm{Hz}\ \left[\frac{g_{*}(T_{\mathrm{ann}})}{10}\right]^{1/2} \left[\frac{g_{*s}(T_{\mathrm{ann}})}{10}\right]^{-1/3} \frac{T_{\mathrm{ann}}}{10\ \mathrm{MeV}}$$

- $\tilde{\epsilon}_{\mathrm{GW}} = 0.7 \pm 0.4$ is derived from numerical simulation

SGWB Spectra with and without the Fermion

- A **decrease** of T_{ann} by **one** order of magnitude would **increase** $\Omega_{\mathrm{GW}}^{\mathrm{peak}} h^2$ by **four** orders of magnitude and **decrease** f_{peak} by **one** order of magnitude
- The **differences** between the **SGWB spectra** predicted by the scenarios **with** and **without the fermion** could potentially be verified by **future GW experiments**



Pulsar timing arrays (PTAs):

NANOGrav, EPTA, PPTA, IPTA, SKA

Space interferometers: LISA, TianQin, Taiji

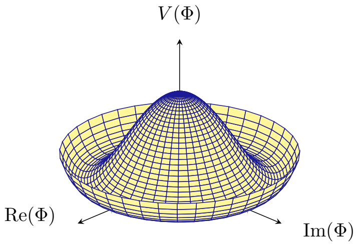
Ground interferometers: LIGO, CE

Cosmic Strings from $\mathrm{U}(1)$ Gauge Symmetry Breaking

- Consider the **Abelian Higgs model** with a **complex scalar field** Φ

$$\mathcal{L} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) - \frac{1}{4} X^{\mu\nu} X_{\mu\nu}, \quad V(\Phi) = -\mu_\phi^2 |\Phi|^2 + \frac{\lambda_\Phi}{2} |\Phi|^4$$

- The covariant derivative of Φ is $D_\mu \Phi = (\partial_\mu - \mathrm{i} q_\Phi g_X X_\mu)\Phi$
- The field strength tensor of the $\mathrm{U}(1)_\mathrm{X}$ **gauge field** X^μ is $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$
- Assume a **Mexican-hat potential** $V(\Phi)$ with **degenerate vacua** $\langle \Phi \rangle = v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$



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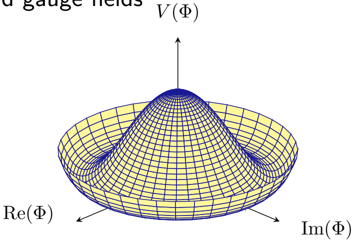
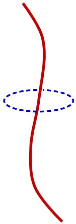
- Assume a **Mexican-hat potential** $V(\Phi)$ with **degenerate vacua** $\langle\Phi\rangle = v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$
- The **spontaneous breaking** of the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** in the early Universe would induce **cosmic strings (CSs)**, which are concentrated with energies of the scalar and gauge fields





Degenerate vacua

$v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$

$\varphi = \varphi + 2\pi n$

$n \neq 0$ leads to

cosmic stirngs

Cosmic String Tension

A **network** of **cosmic strings** would be formed in the early Universe after the spontaneous breaking of the $\mathrm{U}(1)_\mathrm{X}$ gauge symmetry

The **tension** of **cosmic string** μ (energy per unit length) can be estimated as

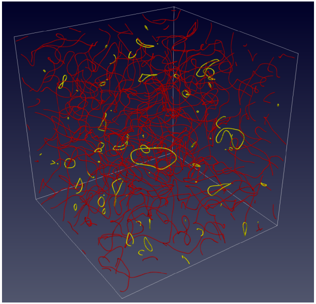
$$\mu \simeq \begin{cases} 1.19\pi v_\Phi^2 b^{-0.195}, & 0.01 < b < 100, \\ \dfrac{2.4\pi v_\Phi^2}{\ln b}, & b > 100, \end{cases}$$

$$b \equiv \frac{2q_\Phi^2 g_X^2}{\lambda_\Phi}$$

[Hill, Hodges, Turner, PRD **37**, 263 (1988)]

As $\mu \propto v_\Phi^2$, a **high symmetry-breaking scale** v_Φ would lead to cosmic strings with **high tension**

Denoting G as the **Newtonian constant of gravitation**, the **dimensionless quantity** $G\mu$ is commonly used to describe the **tension** of cosmic strings



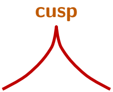
[Kitajima, Nakayama, 2212.13573, JHEP]

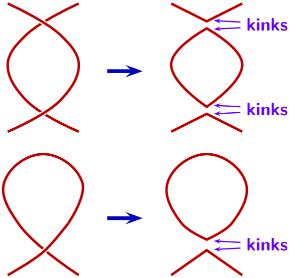
Gravitational Waves from Cosmic Strings

- According to the analysis of string dynamics, the **intersections** of **long strings** could produce **closed loops**, whose size is smaller than the Hubble radius

- **Cosmic string loops** could further fragment into **smaller loops** or reconnect to **long strings**

- Loops typically have localized features called **"cusps"** and **"kinks"**





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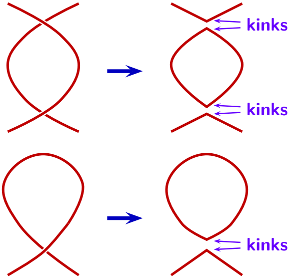

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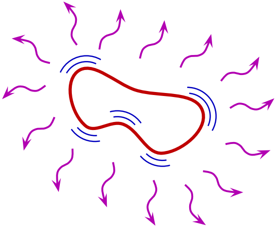
- The **relativistic oscillations** of the **loops** due to their **tension** emit **Gravitational Waves (GWs)**, and the loops would **shrink** because of **energy loss**



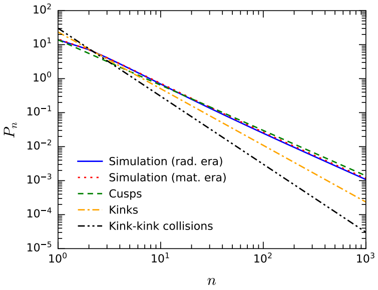
- Moreover, the **cusps** and **kinks** propagating along the loops could produce **GW bursts** [Damour & Vilenkin, gr-qc/0004075, PRL]

Power of Gravitational Radiation

At the **emission time** t_e, a **cosmic string loop** of **length** l emits GWs with **frequencies** $f_\mathrm{e} = \frac{2n}{l}$

$n = 1, 2, 3, \cdots$ denotes the **harmonic modes** of the loop oscillation

Denoting P_n as the **power** of **gravitational radiation** for the harmonic mode n in units of $G\mu^2$, the total power is given by $P = G\mu^2 \sum_n P_n$



According to the **simulation** of **smoothed cosmic string loops** [Blanco-Pillado & Olum, 1709.02693, PRD], P_n for loops in the **radiation** and **matter** eras are obtained

The **total dimensionless power** $\Gamma = \sum_n P_n$ is estimated to be ~ 50

For comparison, analytic studies imply $P_n \simeq \frac{\Gamma}{\zeta(q) n^q}$ with $q = \frac{4}{3}, \frac{5}{3}, 2$ for **cusps**, **kinks**, and **kink-kink collisions**

Stochastic GW Background Induced by Cosmic Strings

- The **energy** of **cosmic strings** is converted into the **energy** of **GWs**, and an **SGWB** is formed due to **incoherent superposition**
- The **SGWB energy density** $\rho_{\rm GW}$ per unit frequency at the present is

$$\frac{d\rho_{\rm GW}}{df} = G\mu^2 \int_{t_{\rm ini}}^{t_0} a^5(t) \sum_n \frac{2nP_n}{f^2}\, n_{\rm CS}\left(\frac{2na(t)}{f}, t\right) dt$$

- $n_{\rm CS}(l,t)$ is the **number density per unit length** of **CS loops** with length l at cosmic time t
- $a(t)$ is the **scale factor** satisfying $\dfrac{da(t)}{dt} = a(t)H(t)$ and $a(t_0) = 1$
- $H(t)$ is the **Hubble rate** and $t_{\rm ini}$ is the cosmic time when the GW emissions start
- The **SGWB spectrum** is commonly represented by

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df}, \quad \rho_c \equiv \frac{3H_0^2}{8\pi G}$$

Velocity-dependent One-scale Model

🍇 The evolution of the **CS network** can be described using the **velocity-dependent one-scale (VOS) model** [Martins & Shellard, hep-ph/9507335, PRD]

🍋 The parameters are the **correlation length** L and the **root-mean-square velocity** v of string segments; the **energy density** of **long strings** is expressed as $\rho = \mu/L^2$

🍑 Introducing a **dimensionless quantity** $\xi \equiv L/t$, the evolution equations are

$$t\dot{\xi} = H(1+v^2)t\xi - \xi + \frac{1}{2}\tilde{c}v, \quad t\dot{v} = (1-v^2)\left[\frac{k(v)}{\xi} - 2Htv\right]$$

$$\tilde{c} \simeq 0.23, \quad k(v) = \frac{2\sqrt{2}}{\pi}(1-v^2)(1+2\sqrt{2}v^3)\frac{1-8v^6}{1+8v^6}$$

🍉 The solutions converge to **constant values** [Marfatia & YL Zhou, 2312.10455, JHEP]:

$$\xi_{\mathrm{r}} = 0.271, \quad v_{\mathrm{r}} = 0.662, \quad \text{radiation-dominated (RD) era}$$

$$\xi_{\mathrm{m}} = 0.625, \quad v_{\mathrm{m}} = 0.582, \quad \text{matter-dominated (MD) era}$$

🍆 This implies that the CS network quickly evolves into a **linear scaling regime** characterized by $L \propto t$

Loop Production Functions

🍊 The **CS loop number density** is given by $n_{\rm CS}(l,t) = \dfrac{1}{a^3(t)} \displaystyle\int_{t_{\rm ini}}^{t} \mathcal{P}(l',t')\, a^3(t')\, \mathrm{d}t'$

🍏 Motivated by **numerical simulations** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD], the **loop production functions** can be approximated as

$$\mathcal{P}_{\rm r}(l,t) = \frac{\mathcal{F}_{\rm r}\tilde{c}v\,\delta(\alpha_{\rm r}\xi - l/t)}{\gamma_v \alpha_{\rm r}\xi^4 t^5}, \quad \text{RD era}$$

$$\mathcal{P}_{\rm m}(l,t) = \frac{\mathcal{F}_{\rm m}\tilde{c}v\,\Theta(\alpha_{\rm m}\xi - l/t)}{\gamma_v (l/t)^{1.69}\xi^3 t^5}, \quad \text{MD era}$$

🍎 $\gamma_v = (1-v^2)^{-1/2}$ is the Lorentz factor

🍒 At the **loop production time** $t_\star$, we have $l_\star = l + \Gamma G\mu(t - t_\star)$, $\alpha_{\rm r}\xi_\star \simeq 0.1$ and $\alpha_{\rm m}\xi_\star \simeq 0.18$

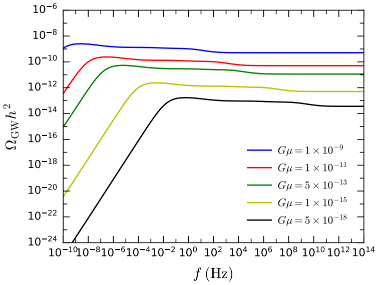
🍐 Adopting $\mathcal{F}_{\rm r} = 0.1$ and $\mathcal{F}_{\rm m} = 0.36$, the obtained **loop number densities** in the **RD** and **MD eras agrees** with the **simulation results** in the **scaling regime**

Loop Production Functions

- The **CS loop number density** is given by $n_{\rm CS}(l,t) = \frac{1}{a^3(t)} \int_{t_{\rm ini}}^{t} \mathcal{P}(l',t')\, a^3(t')\, dt'$
- Motivated by **numerical simulations** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD], the **loop production functions** can be approximated as

$$\mathcal{P}_{\rm r}(l,t) = \frac{\mathcal{F}_{\rm r}\tilde{c}v\,\delta(\alpha_{\rm r}\xi - l/t)}{\gamma_v \alpha_{\rm r}\xi^4 t^5}, \quad \text{RD era}$$

$$\mathcal{P}_{\rm m}(l,t) = \frac{\mathcal{F}_{\rm m}\tilde{c}v\,\Theta(\alpha_{\rm m}\xi - l/t)}{\gamma_v (l/t)^{1.69}\xi^3 t^5}, \quad \text{MD era}$$



- $\gamma_v = (1-v^2)^{-1/2}$ is the Lorentz factor
- At the **loop production time** $t_\star$, we have $l_\star = l + \Gamma G\mu(t - t_\star)$, $\alpha_{\rm r}\xi_\star \simeq 0.1$ and $\alpha_{\rm m}\xi_\star \simeq 0.18$
- Adopting $\mathcal{F}_{\rm r} = 0.1$ and $\mathcal{F}_{\rm m} = 0.36$, the obtained **loop number densities** in the **RD** and **MD eras agrees** with the **simulation results** in the **scaling regime**
- The **SGWB spectra** in the ΛCDM **cosmological model** is further calculated

Early Cosmic History

- Cosmological observations can **hardly** date back to eras **prior to big bang nucleosynthesis** (BBN)
- **Various hypotheses** beyond the standard cosmic history **predating BBN** are possible, such as an **early matter-dominated (EMD) era**, a **kination-dominated era**, and an **intermediate inflationary era**
- Traditional **electromagnetic detection** methods are **ineffective** when the Universe was **opaque** to **photons**

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- Traditional **electromagnetic detection** methods are **ineffective** when the Universe was **opaque** to **photons**
- Nonetheless, **GWs** can **propagate freely** through space, preserving information from the early Universe and reaching us in the present day
- We study how the **SGWB spectrum** originated from a preexisting **CS network** is modified by an **EMD era** [SQ Ling & **ZHY**, 2502.16576]

Origin of the Early Matter-dominated Era

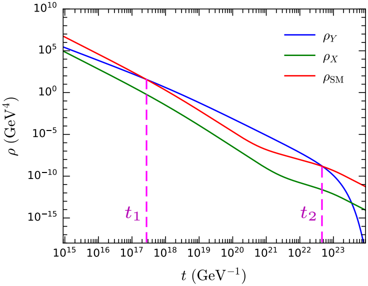
- Consider **dark matter (DM) dilution mechanism** as the origin of the **EMD era**
- Thermal production of a **light DM candidate** X with low annihilation cross sections typically results in an **overproduction problem**
- DM overproduction can be **mitigated** by **entropy injection** from the **decays** of a **dilutor particle** Y, which **dominates** the Universe for a period, inducing an **EMD era**
- Taking the **minimal left-right symmetric model** as an example, where the lightest and next-to-lightest right-handed neutrinos N_1 and N_2 can serve as X and Y
- The related Boltzmann equations are

$$\frac{\mathrm{d}\rho_Y}{\mathrm{d}t} + 3H\rho_Y = -\Gamma_Y \rho_Y$$

$$\frac{\mathrm{d}\rho_X}{\mathrm{d}t} + 4H\rho_X = yB_X\Gamma_Y \rho_Y$$

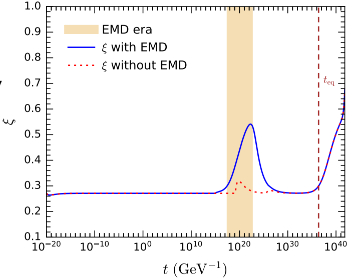
$$\frac{\mathrm{d}\rho_{\mathrm{SM}}}{\mathrm{d}t} + 4H\rho_{\mathrm{SM}} = (1 - yB_X)\Gamma_Y \rho_Y$$

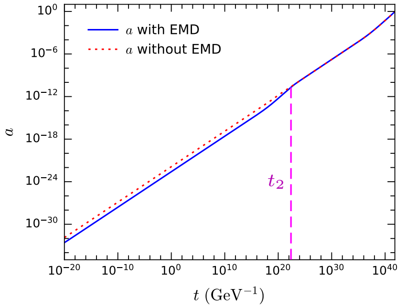
[Nemevšek & Y Zhang, 2206.11293, PRL]

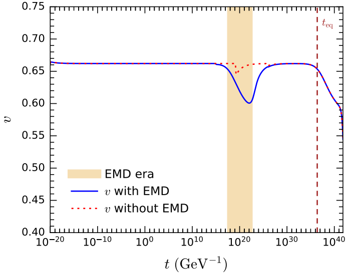


Impact on the Scale Factor and the VOS Parameters

- Compared with the ΛCDM **model**, the presence of the **EMD era reduces** the **scale factor** a **before** t_2
- $a \propto t^{2/3}$ during an **MD era** increases more rapidly than $a \propto t^{1/2}$ during an **RD era**, and a is **smaller** at the **onset** of the **EMD era** to ensure $a(t_0) = 1$
- Moreover, the **EMD era** introduces a **nonscaling effect** to the evolution of the **CS network**

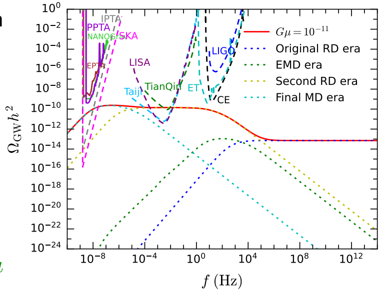






Imprints in the SGWB spectrum

- Affected by the **EMD era**, the SGWB spectrum displays a **suppression** at **high frequencies**
- This corresponds to the contributions from CS loops formed in the **original RD** and **EMD eras**
- The **lengths** of the generated **CS loops** are **positively correlated** with the **scale factor** a
- Since the **EMD era reduces the scale factor** a **before** t_2, the CS loops with a given **initial length** l, which is related to the **GW emission frequency** by $f_e = 2n/l$, are **formed at a later time**, when the **energy densities** of both CS loops and the emitted GWs are **reduced**



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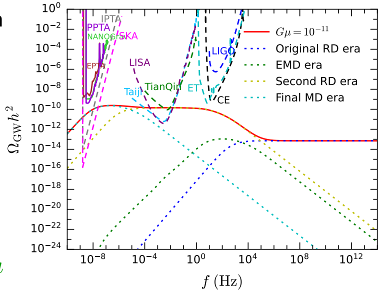
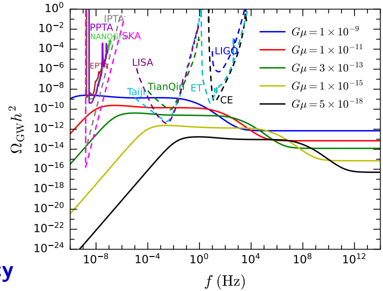
- For a **smaller CS tension**, the **loop lifetime** is **extended** and the **average length** at t_2 is **smaller**, causing **suppression** to begin at a **higher frequency**





Summary

- In the early Universe, the **spontaneous breaking** of **symmetries** could leads to **topological defects**, such as **domain walls** and **cosmic strings**
- **Cosmic strings** or **collapsing domain walls** may results in a **stochastic GW background**, which could be probed in GW experiments
- We consider **quantum** and **thermal corrections** to the **effective potential** and explore their impact on the **dynamics** of **domain walls** and the resulting **GW signatures**
- We investigate how an **early matter-dominated era** in cosmic history influences the **dynamics** of **cosmic strings** and the produced **GW spectrum**

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Thanks for your attention!

Friction on the Domain Walls

The interaction between a **domain wall** and the f **fermions** in the **thermal bath** induces **friction** on the wall as it moves in the plasma

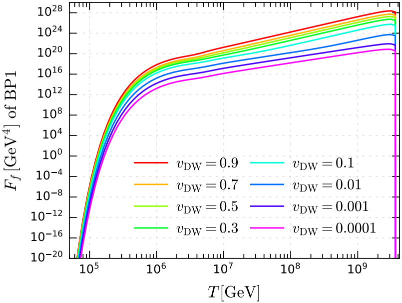
The **friction force** per unit area exerted on the **DW** is

$$F_f = \frac{2}{\pi^2}\frac{1}{1-v_{\mathrm{DW}}^2}\int_0^{+\infty}\int_{-\infty}^{+\infty} R(p_x)\,\frac{(p_x-\omega v_{\mathrm{DW}})^2}{\omega - p_x v_{\mathrm{DW}}}\,\frac{1}{e^{\omega/T}+1}\,p_\perp\,\mathrm{d}p_x\,\mathrm{d}p_\perp$$

The **reflection probability** $R(p_x)$ can be estimated by considering one-dimensional scattering of a free particle off a step potential

F_f **decreases exponentially** due to the **Boltzmann suppression** at **low temperatures**

The friction is **negligible** when evaluating the **annihilation temperature** T_{ann} for the BPs



Values of $T_{\rm ann}$, $f_{\rm peak}$, and $\Omega_{\rm GW}(f_{\rm peak})h^2$ for the BPs

	$T_{\rm ann}$ [GeV]	$f_{\rm peak}$ [Hz]	$\Omega_{\rm GW}(f_{\rm peak})h^2$
BP1	6.02×10^{4}	1.00×10^{-2}	5.77×10^{-10}
BP1 w/o f	2.00×10^{5}	3.32×10^{-2}	4.77×10^{-12}
BP2	2.62×10^{-2}	2.98×10^{-9}	8.36×10^{-11}
BP2 w/o f	1.77×10^{-2}	2.01×10^{-9}	4.01×10^{-10}
BP3	1.98×10^{7}	3.30	8.77×10^{-9}
BP3 w/o f	1.90×10^{8}	3.17×10^{1}	1.02×10^{-12}

DM Dilution Mechanism

A **long-lived dilutor** Y has mass m_Y **much larger** than m_X can effectively address the **overproduction problem** of X **particles**

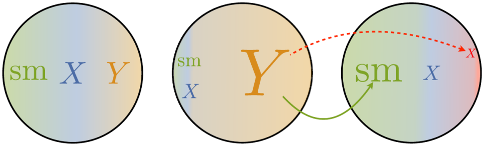
First, during the **RD era**, both Y and X particles **decouple relativistically** at a similar temperature, resulting in comparable yields, $Y_Y \simeq Y_X$

Second, because of $m_Y \gg m_X$, Y **particles** become **nonrelativistic** at a relatively high temperature, while X **particles** remain **relativistic**

Consequently, Y **particles** quickly **dominate** the energy density of the Universe, initiating an **EMD era**

Finally, when the **lifetime** of Y **particles** comes to an end, they **decay** into SM particles and X particles, **injecting entropy** and consequently **diluting** the **energy density** ρ_X of X particles

[Nemevšek & Y Zhang, 2206.11293, PRL]



Minimal Left-right Symmetric Model

- The **DM candidate** is $X = N_1$, and the **dilutor** is $Y = N_2$, which undergoes a three-body decay mediated by a **right-handed gauge boson** $W_{\mathrm{R}}^{\pm}$ into two charged leptons $\ell\ell'$ and one N_1
- The related **right-handed charged current interactions** are described by

$$\mathcal{L}_1 = \frac{g}{\sqrt{2}} W_{\mathrm{R}}^{\mu} \left(\sum_{i=1}^{2} \bar{N}_i \gamma_\mu V_{\mathrm{PMNS}}^{R\dagger} \ell_{\mathrm{R}} + \bar{u}_{\mathrm{R}} \gamma_\mu V_{\mathrm{CKM}}^{\mathrm{R}} d_{\mathrm{R}} \right) + \mathrm{H.c.}$$

- N_2 **decay channels** include $N_2 \to N_1 \ell\ell'$, $N_2 \to \ell q \bar{q}'$, and $N_2 \to \ell W$
- **Benchmark parameters** used in the previous slides:

$$m_{N_2} = 200 \text{ GeV}, \quad m_{N_1} = 6.5 \text{ keV}, \quad m_{W_{\mathrm{R}}} = 5 \times 10^7 \text{ GeV}, \quad \tan\beta = 0.5$$

$$\Gamma_{N_2} = 2.22 \times 10^{-23} \text{ GeV}, \quad B_X = 4.41 \times 10^{-3}, \quad y = 0.35,$$

- B_X is the **branching ratio** of the decay channel $N_2 \to N_1 \ell\ell'$
- y is the **energy fraction** carried away by the X particle from the Y particle

Effects of the Dilutor Decay Width and Mass

- A **smaller dilutor decay width** Γ_Y corresponds to a **longer duration** of the **EMD era**, leading to **stronger suppression** effects at high frequencies
- A **larger dilutor mass** m_Y implies that the **EMD era** occurs **earlier**, and hence a **higher frequency** at which the suppression of the GW spectrum commences