Asymptotic unification in supersymmetric E6 theory

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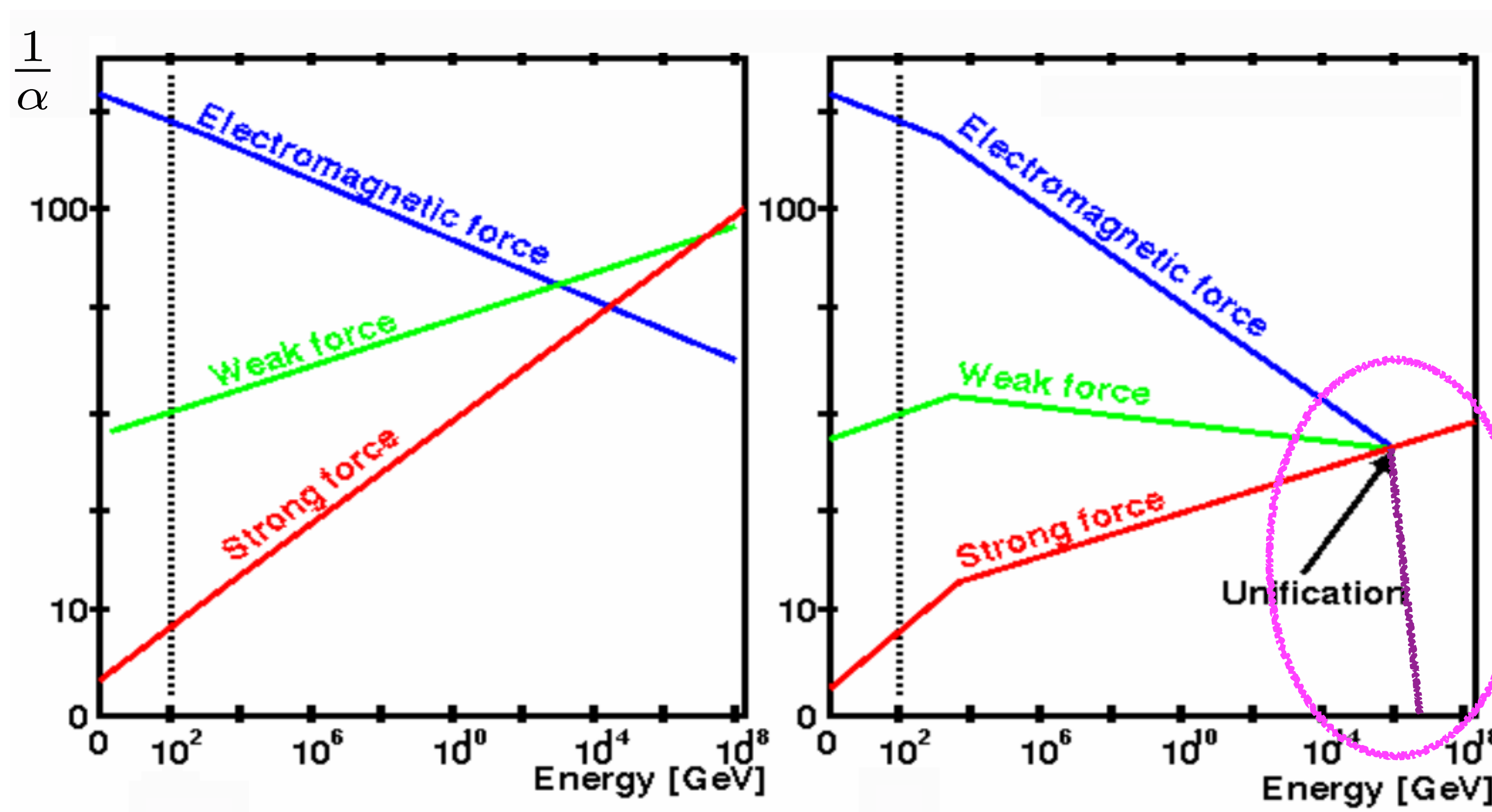
Traditional Grand Unification

- Gauge couplings of the Standard Model (SM) are expected to meet at a some large scale (hence "unification")
- Supersymmetry is often useful to achieve that in particular "realistic" models (such as MSSM, Split-SUSY etc)
- Can we replace the SM symmetry by a simpler one in the UV? e.g. SU(5), SO(10) etc
- Typical drawbacks are

$\Rightarrow$ proton decay hard to avoid

$\Rightarrow$ large matter reps required to break the symmetry leading to a Landau pole!



Asymptotic Grand Unification concept

- Instead of meeting at a fixed scale, the running gauge couplings tend to the same fixed point in the UV

- A simple way to realise aGUTs consists on building theories in 5D

- The common lore is that 5D theories have a natural cut-off due to the linear running of the gauge couplings, which renders them intrinsically non-renormalisable

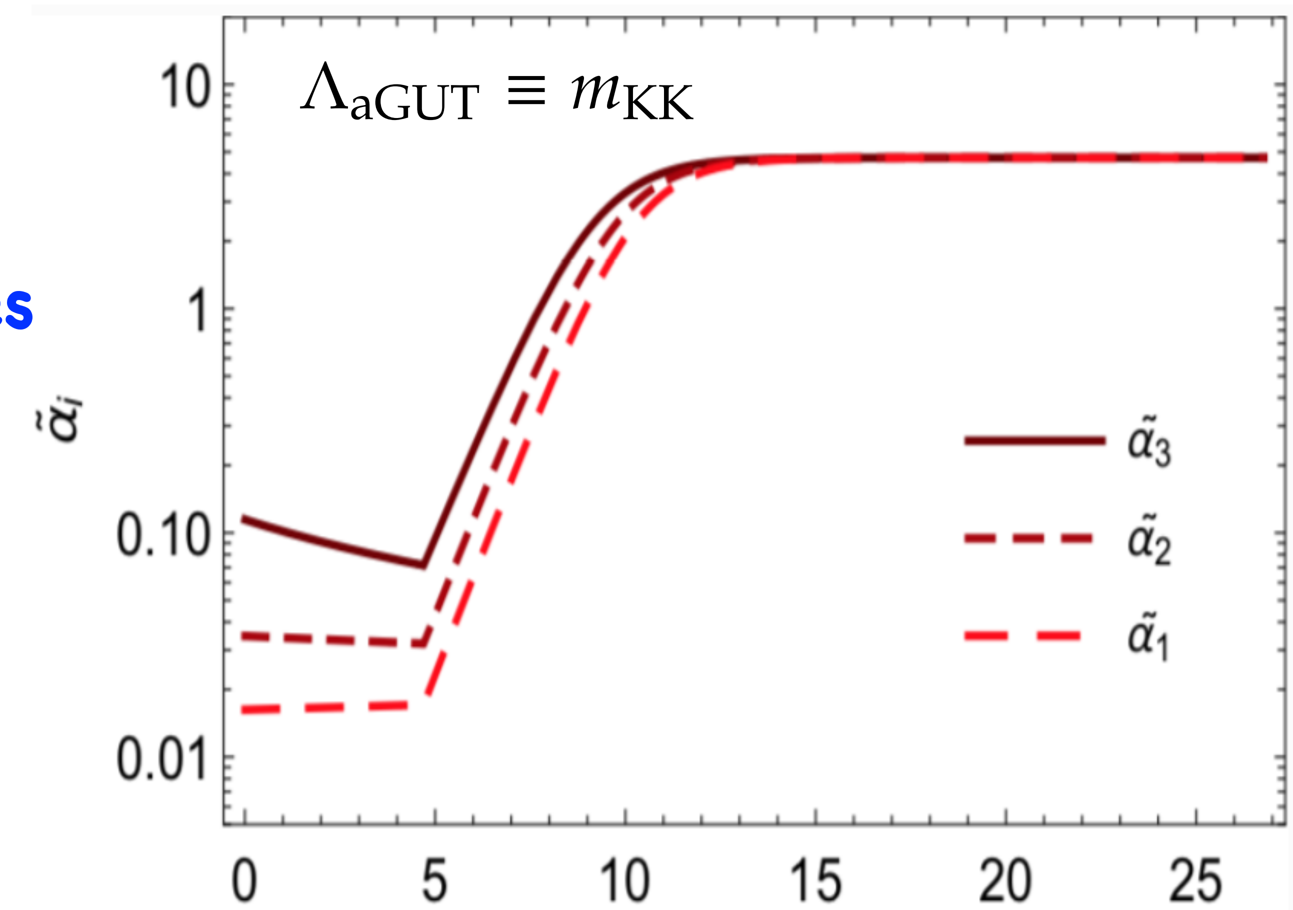
- However, under certain conditions, the gauge running in the UV is tamed by the presence of a fixed point, which renders the theory renormalisable and, therefore, valid up to arbitrary scales

- it suffices that the one-loop beta function is negative, i.e. it would lead to an asymptotically-free theory in 4 dimensions (4D)

Asymptotic Grand Unification concept

Gies, PRD 68 (2003)
Morris, JHEP 01 (2005) 002

- **Gauge couplings are never equal but approach the same UV fixed point**



- **Well known examples are**

⇒ **asymptotically free gauge theories**

via large Nf resummation with intermediate Pati-Salam

Molinaro et al, 1807.03669

via perturbative fixed points and SUSY

Bajc et al, 1610.09681 and 2308.13311

⇒ **Extra compact dimensions (realised e.g. in 5D theories)**

One-loop RGE: $$2\pi \frac{d\alpha}{d\ln\mu} = \mu R b_5 \alpha^2$$

5D 't Hooft coupling: $$\tilde\alpha = \alpha\, \mu R \equiv \alpha\, \frac{\mu}{\mu_{\rm KK}}$$

⇒ $$2\pi\left(\tilde\alpha - \frac{d\tilde\alpha}{d\ln\mu}\right) = b_5 \tilde\alpha^2$$

$$\tilde\alpha_{\rm UV} = \frac{2\pi}{b_5} \qquad b_5 > 0$$

- **Easy for gauge couplings, Yukawa's are tricky –**

bonus: gauge-Higgs unification!

Asymptotic Grand Unified Theories (aGUTs)

A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

- Grand Unified Theories formulated in 5 or more space-time dimensions

⇓

defined on $\mathbb{R}^4 \times K$, where $\mathbb{R}^4$ is the usual 4-dimensional Minkowski space and K defines δ *compact* extra dimensions.

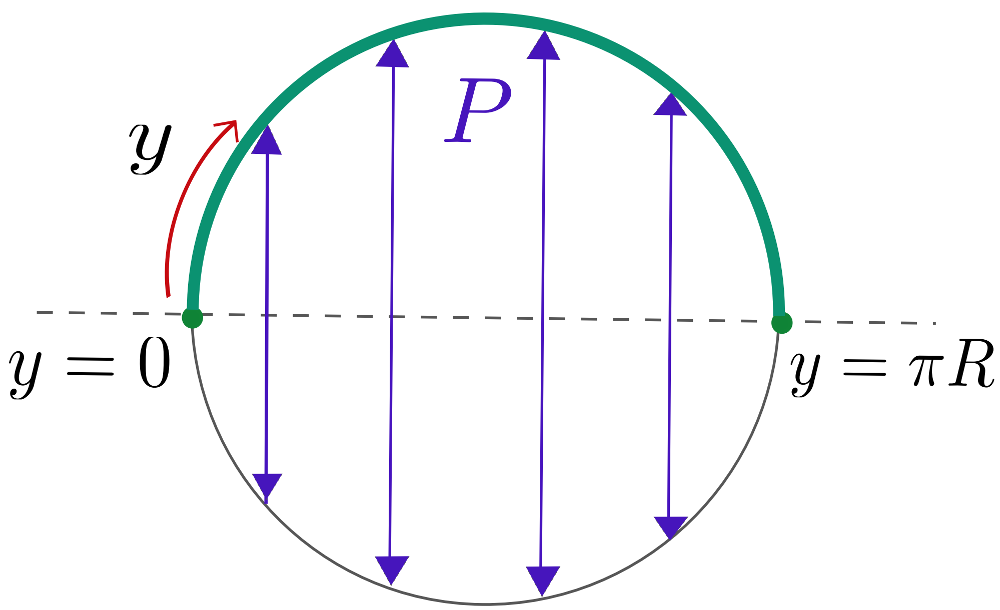
- Gauge symmetry is broken by boundary conditions

$\Rightarrow$ different from the usual Higgs mechanism

- Motivation: solution to hierarchy problem, lower GUT scale, smaller representations...

5D aGUT formulation: orbifolding

- One extra dimension (δ = 1) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$



- The inverse radius R^{-1} sets the scale of compactification

- Each intrinsic $\mathbb{Z}_2$ transformation is specified by a parity matrix acting on the fields

$$\Phi(x^\mu, -y) = P\Phi(x^\mu, y) = \pm\Phi(x^\mu, y)$$

- Each P_i breaks $\mathcal{G} \to \mathcal{H}_i$ on one boundary, such that

$$\mathcal{G}_{\rm 4D} \equiv \mathcal{H}_i \cap \mathcal{H}_j \qquad \mathcal{G}_{\rm 4D} \supset \mathcal{G}_{\rm SM}$$

5D aGUT formulation: 4D EFT

G. Cacciapaglia, arXiv:2309.10098

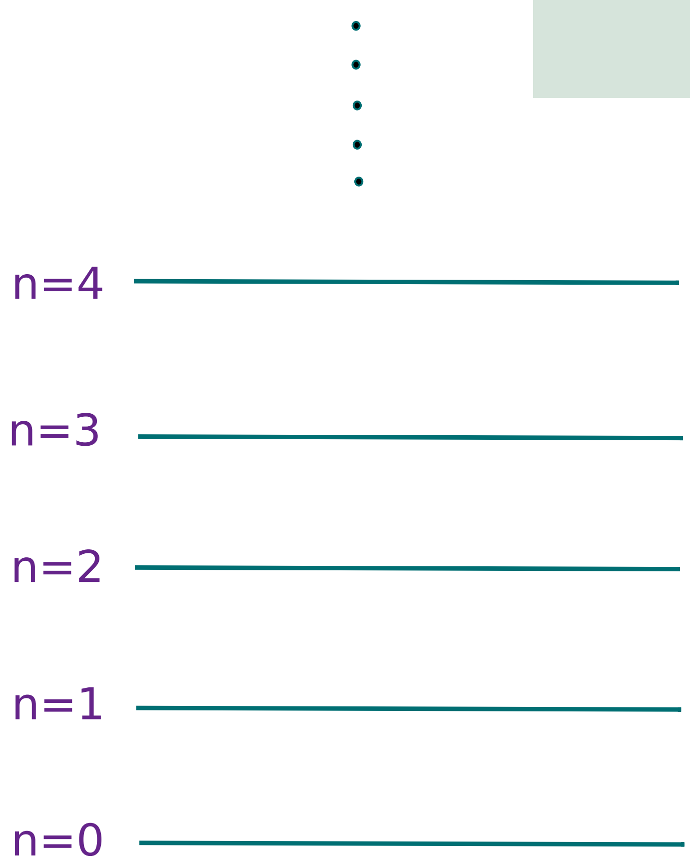
- An aGUT can be fully defined in terms of

Gauge group $\mathcal{G}$ Parity P Parity assignments

- Find minimal matter content that preserves the UV fixed point!

- For a given field, KK decomposition

$$\Phi\left(x^\mu, y\right) = \underbrace{\sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right)}_{\text{parity-even}} + \underbrace{\sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right)}_{\text{parity-odd}}$$



- 4D fields $\phi_\pm^{(n)} \equiv$ KK modes with mass of n/R

- SM fields are the massless zero modes (n=0)

- $E \ll 1/R$ heavy KK towers are integrated out

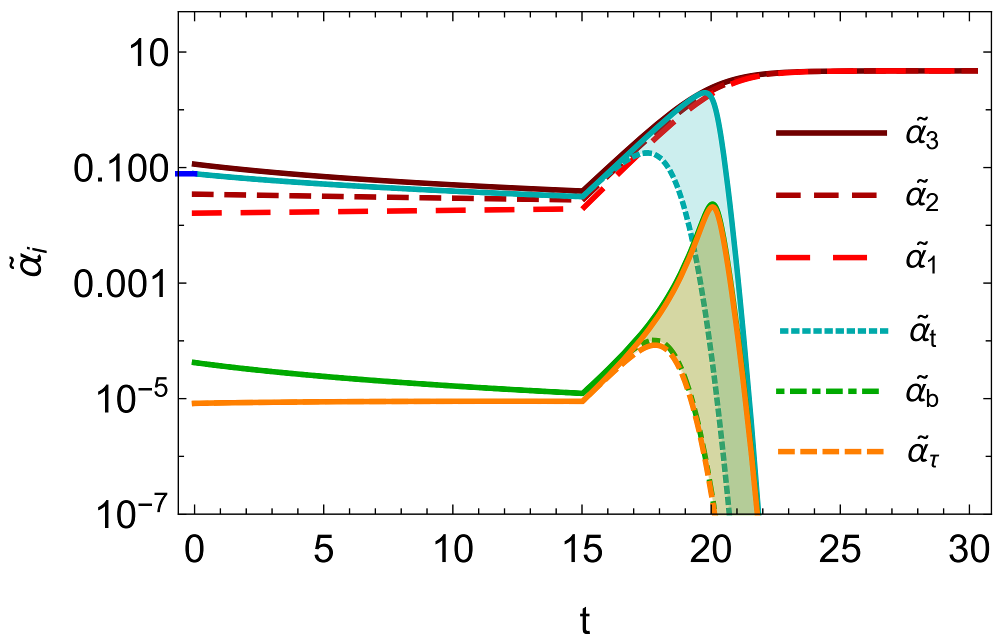
4D effective field theory

Minimal aGUTs: the fate of Yukawa couplings

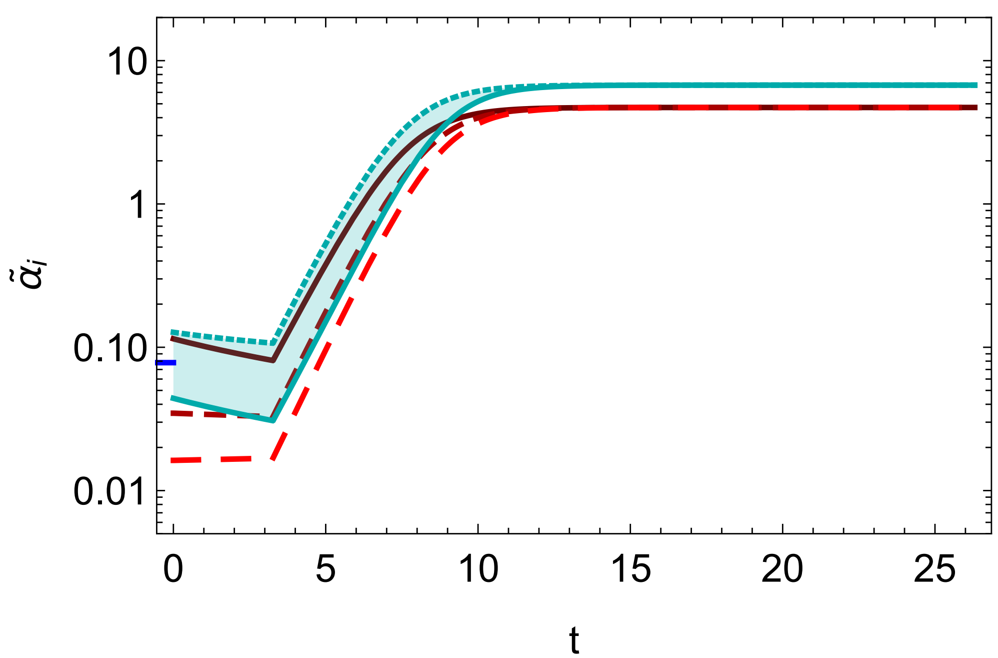
Example: Bulk Yukawas of SU(5) aGUT

Cacciapaglia et al, PRD 104 (2021) 7
Khojali et al, 2210.03596

$R^{-1} = 3.05 \cdot 10^5$ TeV



$R^{-1} = 2.4$ TeV



For SO(10) aGUT see Gao-Xiang's talk!

- ✦ SM fermions cannot be embedded in complete multiplets of SU(5)
- ✦ Yukawas do not unify
- ✦ Baryon/lepton numbers can be defined (no proton decay)
- ✦ For large KK scale, $\frac{1}{R} \gtrsim 3 \cdot 10^5$ TeV all bulk Yukawas run to zero
- ✦ For smaller KK scale, bulk Yukawas run to Landau poles (strong limitation!)
- ✦ Localising all Yukawas except the top, may allow for UV fixed point

The exceptional case: E6 aGUT

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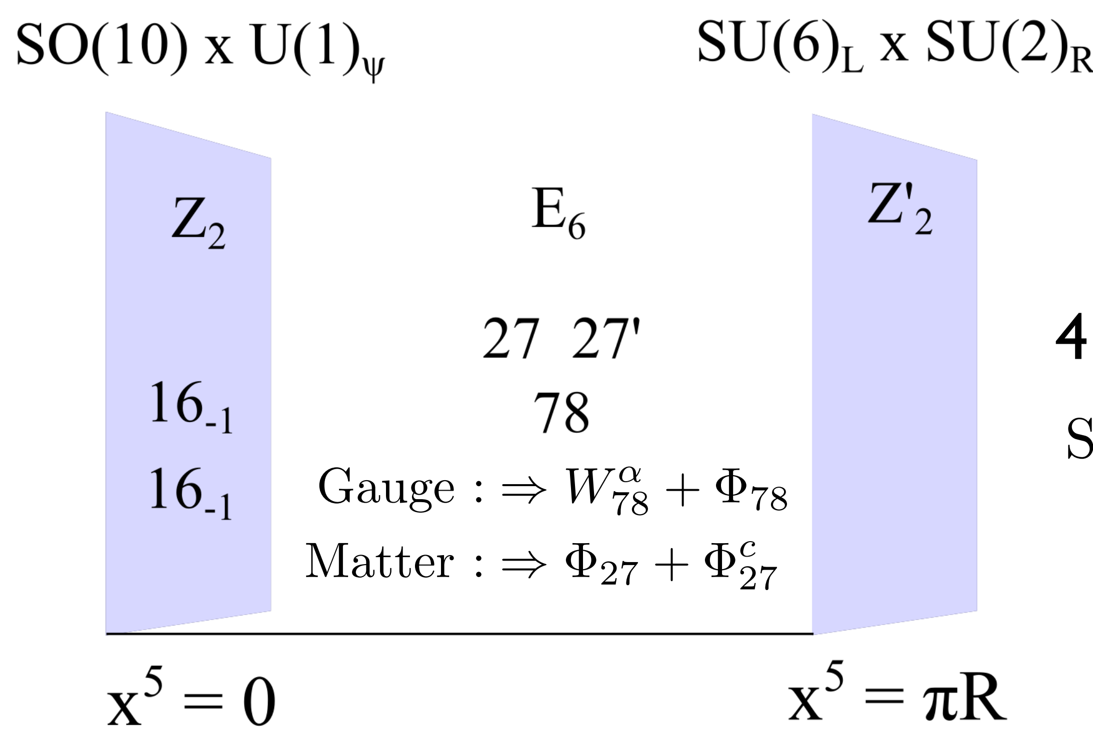
- SUSY allows to generate fermions as gauge fields (gauginos) linking **Yukawa couplings to the bulk gauge coupling!**
- In E6, the adjoint 78 contains right states (in vector-like pairs)

See Kobayashi, Raby, Zhang, Nucl. Phys. B704, 3 (2005)

$S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold

$\mathcal{N}=2$ SUSY in 5D

$\mathcal{N}=1$ SUSY in 4D



4D EFT containing the SM:

$\mathrm{SU}(4) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_\psi$

- The zero modes generate a 4D chiral anomaly for the U(1) gauge symmetry: $A_{4D} = A_{16} + A_{10+1} = 2A_{10+1} = 2A_{16}$
- Add exactly two generations $(16)_{-1}$ on the SO(10) boundary!

The exceptional case: bulk interactions

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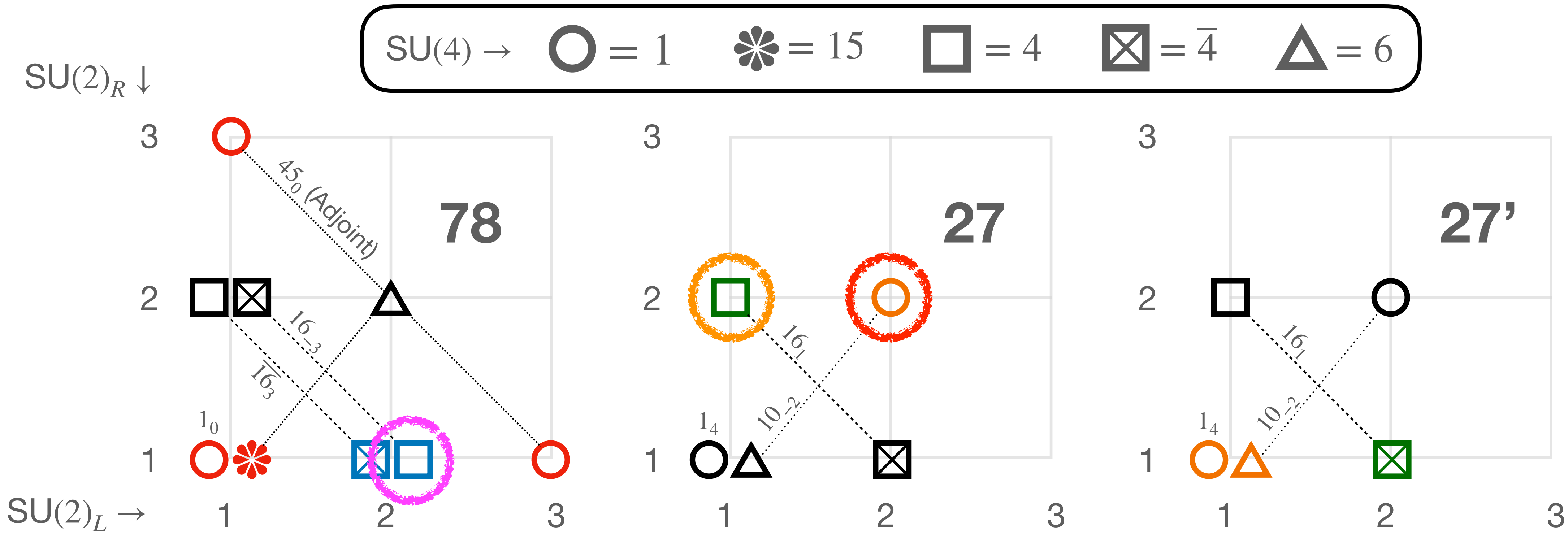
bulk E6 hypermultiplets in terms of $\mathrm{PS}\times\mathrm{U}_\psi$ components



bulk Yukawas:

$$g\,\Phi^c_{27}\Phi_{78}\Phi_{27} \supset \frac{g}{\sqrt{2}}(\mathbf{1},\mathbf{2},\mathbf{2})_2(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-3}(\mathbf{4},\mathbf{2},\mathbf{1})_1$$

⇒ SM Yukawa couplings!

$$g\,\Phi^c_{27'}\Phi_{78}\Phi_{27'} \supset -\frac{g}{\sqrt{2}}(\mathbf{1},\mathbf{1},\mathbf{1})_{-4}\,(\mathbf{4},\mathbf{1},\mathbf{2})_3\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_1 + \frac{g}{\sqrt{2}}(\mathbf{6},\mathbf{1},\mathbf{1})_2\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-3}\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_1$$

⇒ Gives mass to unwanted chiral states via U(1) breaking

Bulk interactions preserve baryon number!

The exceptional case: two pathways

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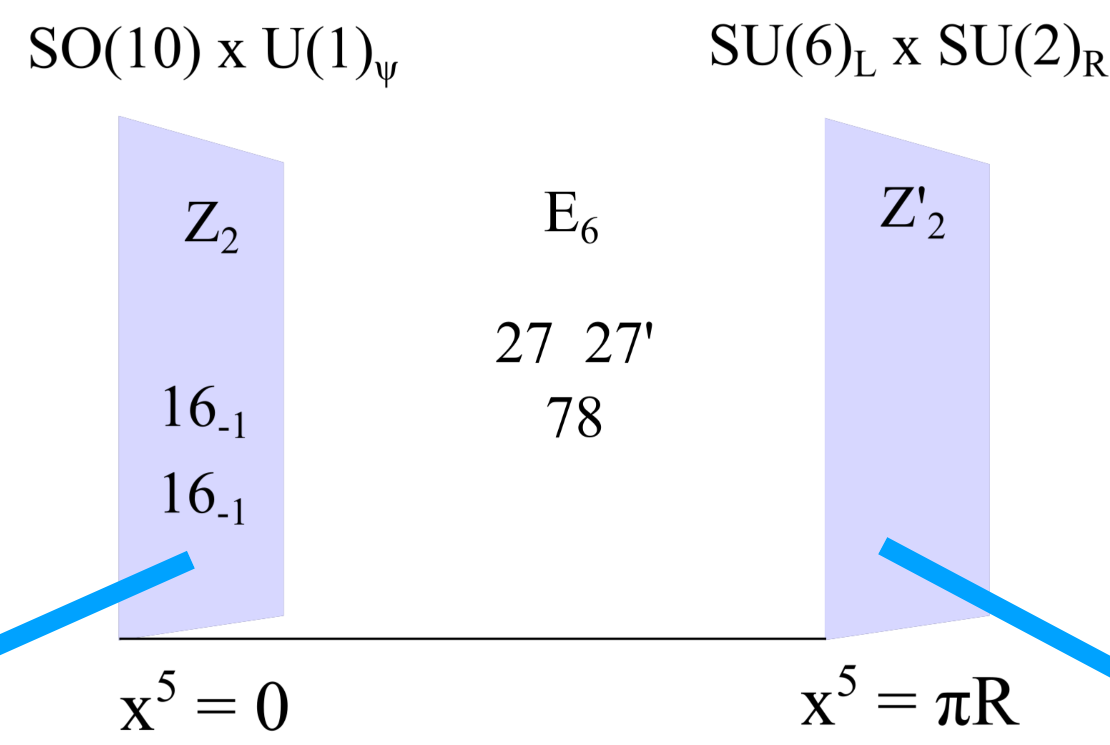
Two $\Phi^i_{16_{-1}}$ generations localised on SO(10) boundary

$y^{ij}\ \Phi^i_{16_{-1}} \Phi^j_{16_{-1}}\ \Phi^c_{27}|_{10_2}$



Two massive

$10_{-2} + 1_4$

and two copies of

$\Phi^j_{(15,1)}$, $\Phi^j_{(\bar{6},2)}$, $\Phi^j_{(20,2)}$

localised on

$\mathrm{SU}(6)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$

boundary

Model I:

- **Predicts total of 3 generations**
- **"Usual" SO(10) model building**
- **Scale pushed high by proton decay**

$m_{\rm KK} \gtrsim 10^{16}\ \mathrm{GeV}$

Model II:

- **Light generations preserve baryon number**
- **Number of gens not predicted**
- **Scale can be lowered (1000's TeV from PS breaking)**

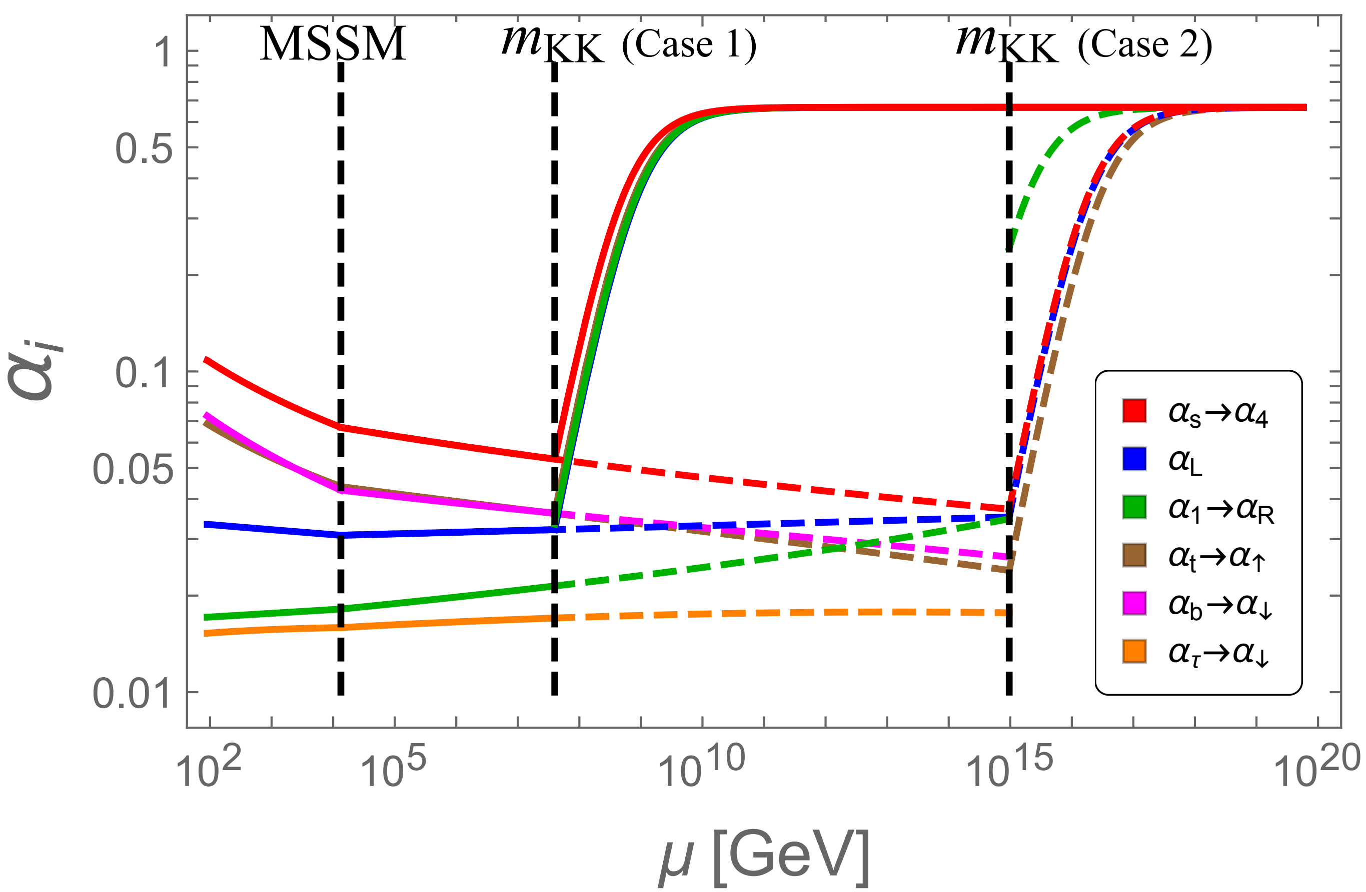
The fixed point

in E6 aGUT:

$$b_5 = \frac{\pi}{2}\left(C(G) - \sum_i T_i(R_i)\right) = 3\pi \qquad C(G) = 12 \qquad T(27) = 3 \qquad \tilde{\alpha}_{\rm UV} = \frac{2}{3}$$

$$\tan\beta = \frac{\langle H_d \rangle}{\langle H_u \rangle} = \frac{m_{\rm t}(m_{\rm KK})}{m_{\rm b}(m_{\rm KK})} \sim 40$$

no more than one extra generation can be added to the bulk!



- ✦ PS breaking: due to a gauge-scalar
- ✦ U(1) breaking by singlet in 27′
- ✦ SUSY breaking - to be studied

Summary

- aGUT is a novel paradigm, avoiding shortcomings of traditional GUTs
- 5D models are very constrained and successful cases can be classified
- A new aGUT based on a SUSY E6 gauge theory is proposed
- It features a single UV fixed point for gauge and Yukawa couplings of the third generation
- The number of SM generations is predicted by gauge anomaly cancellation (Model I)
- A second option (Model II) preserves baryon number and allows to lower the compactification scale
- The model has far reaching implications both for low energy phenomenology (e.g. in the flavour sector) and at high energies, via new UV model building opportunities