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Kähler Moduli Inflation in Large Volume Compactifications

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PRELUDE

▲ The main topic of this talk is cosmological inflation in string theory. After introducing key concepts, I will discuss:

- Moduli stabilization
- Inflation in large volume compactifications

▲ Inflation and Moduli Fields in String Theory

In string theory, inflation can be driven by special scalar fields called **moduli fields**, which arise from the compactification of extra dimensions.

▲ The Need for Moduli Stabilization

For low-energy effective models to match observations, these moduli must be stabilized. Otherwise, fundamental parameters, such as gauge couplings and masses, would remain undetermined.

▲ Key Challenges To clarify why this is a critical issue, I will first provide some essential background in the following slides.

Key Facts About Cosmology and de Sitter Vacua

▲ Major Observational Discovery (Late 1990s)

The universe's expansion is accelerating!

This phenomenon is best explained by the existence of **Dark Energy** - a dominant component of the universe's energy density.

▲ General Relativity Interpretation

Dark Energy enters Einstein's equations as a positive cosmological constant Λ:

$$\Lambda \approx 10^{-122} \ \text{(in } M_{\text{Planck}}^4 \text{ units)}$$

Physical Meaning:

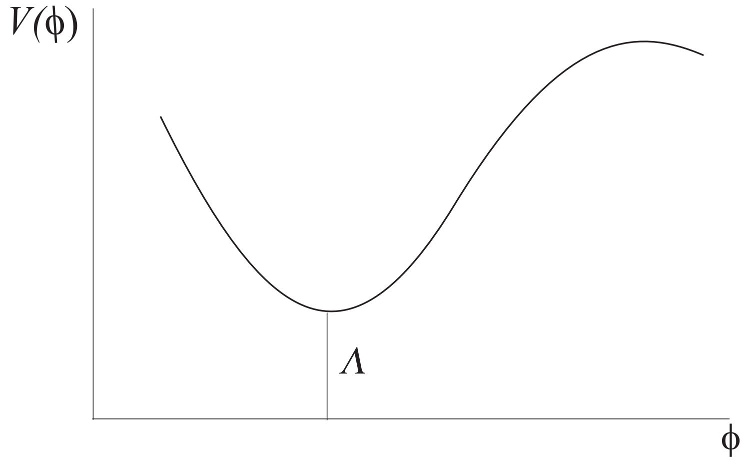
Represents vacuum energy

Exerts negative pressure, driving cosmic acceleration

▲ From the Effective Field Theory point of view:

▲ $\exists$ a simple description in terms of:

Potential Energy $V(\phi)$ of a scalar field, ϕ

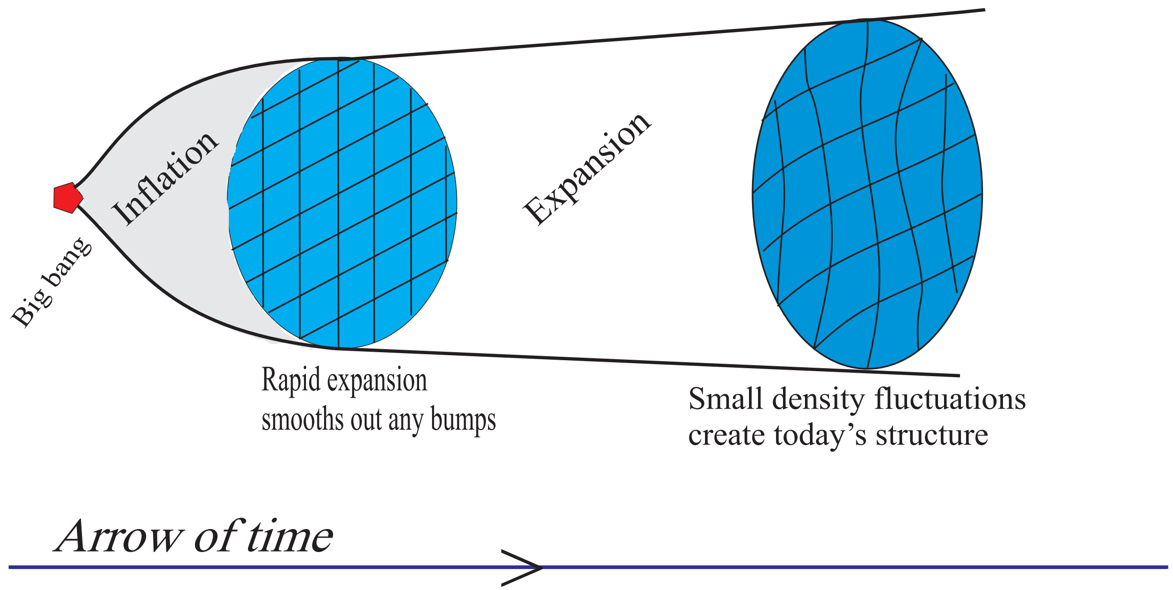


▲ $V(\phi)$ exhibits a (*possibly metastable*) positive minimum

corresponding to a so called:

▲ **de Sitter vacuum** ▲

▲▲ Scalar Potential & Inflation: With a few additional conditions, the scalar potential $V(\phi)$ can support cosmological inflation.



▲ Many effective field theory (EFT) models already satisfy these constraints

▲▲ The Key Challenge: Successfully embedding inflation in a consistent string theory framework.

▲ String theory naturally provides scalar fields (moduli) which arise from compactifications. These fields offer a pathway to model inflation and other phenomena.

However, string compactifications introduce new complications:

• They typically produce a large number of massless moduli in the effective 4D theory.

• These moduli correspond to **geometric deformations** of the compact space

▲ Why is this problematic?
At the EFT level:

• Massless scalars can mediate long-range fifth forces (ruled out by experiments).

• They can also lead to cosmological inconsistencies (e.g., unobserved light particles, moduli overproduction).

▲ **Tasks** ▲

▲ Generate a potential and assure positive mass-squared for all moduli fields, a project usually refer to as:

$\Rightarrow$ $\mathcal{M}$*oduli* $\mathcal{S}$*tabilisation* $\Leftarrow$

▲▲ Look for possible **Inflaton** candidates among the **moduli**

I will present viable scenarios for

★ **moduli stabilisation** and **inflation** ★

in **Large Volume Compactifications**

within **Type-IIB String Theory** framework

▲▲ Some Moduli in Type IIB String Theory

1. ▲ Dilaton $e^{\phi} = \frac{1}{g_s}$, (g_s : *string coupling*)
   Controls the worldsheet perturbative expansion of the theory

2. ▲ p-potentials
   C_p: p-form potentials which define the field strengths:

$$F_{p+1} = dC_p,$$

   ▲ Scalars C_0, ϕ $\rightarrow$ *combined to* **axio-dilaton** *modulus:*

$$S = C_0 + i\, e^{\phi} \rightarrow C_0 + i{g_s}^{-1}$$

3. U^i, *Complex Structure (CS)* moduli$\cdots$ *related to shape* $\rightarrow$
   ... analogous to the complex structure τ of the 2-torus $\mathcal{T}^2$

4. T_i : Kähler (*size*) moduli analogous to the overall size of $\mathcal{T}^2$.

$$T_k = c_k - i\tau_k$$

The Potentials

▲ Low energy dynamics can be captured by a holomorphic superpotential W,

$$\mathcal{W}_0 \quad = \quad \int \mathbf{G_3} \wedge \Omega(U_a), \qquad (\mathbf{G_3} := F_3 - S H_3) \tag{1}$$

and a real Kähler potential K

$$\mathcal{K}_0 \quad = \quad -\log[-i(S - \bar{S})] - 2\log\left(\mathcal{V}(\tau_k)\right) - \log[-i\int \Omega \wedge \bar{\Omega}] \tag{2}$$

▲ The F-term contributions to the scalar potential of 4D $\mathcal{N} = 1$ from the type IIB are encoded in

$$\boxed{V = e^{\mathcal{K}}(K^{\mathcal{A}\overline{\mathcal{B}}}(D_{\mathcal{A}}W)(D_{\overline{\mathcal{B}}}\overline{W}) - 3|W|^2)}$$

,

★ Large Volume Scenarios (LVS) ★
in Type-IIB String Theory

$$\downarrow\downarrow$$

Volume of compactified dimensions defined through Kähler moduli:

$$\mathcal{V} = \frac{1}{3!} k_{ijk} t^i t^k t^k, \; t^k = -\mathrm{Im} T^k$$

LVS lowers the string scale since the following relation holds:

$$M_s^2 = \frac{g_s^2 M_{Pl}^2}{\mathcal{V}}$$

(in string units $\ell_s = 2\pi\sqrt{\alpha'}$). For example, if

$$\mathcal{V} \sim 10^5, \quad \text{then } M_s \sim 10^{16}\mathrm{GeV}$$

$$\mathcal{V} \sim 10^{15}, \quad \text{then } M_s \sim 10^{10}\mathrm{GeV}$$

Moduli Stabilisation

Moduli stabilisation in 4D type IIB effective supergravity models follows a **two-step procedure**.

▲ First, one fixes the CS moduli U_a and the axio-dilaton S by the leading order $W_0 \equiv W_{\rm flux}$ induced by the 3-form fluxes (F_3, H_3)

▲ $\mathcal{W}$-Flatness conditions:

$$\mathcal{D}_{U_a}\mathcal{W} = 0, \quad \mathcal{D}_S\mathcal{W} = 0 \ :$$

$\Rightarrow$ U_a *and* S **stabilised**$\Leftarrow$

but!

▲ Kähler moduli $\notin \mathcal{W}_0$ $\Rightarrow$ remain unfixed! ▲

$\star$ **No-scale** structure protects the Kähler moduli T_α $\rightarrow$ remain massless at tree-level.

Quantum Corrections

▲▲ At a second step the Kähler moduli, T_α , can be stabilised by non-perturbative corrections in W and α' and string-loop (g_s) corrections in K:

$$W = W_0 + W_{\rm np}(S, T_\alpha),$$
$$K = K_{\rm cs} - \ln\left[-i\,(S-\bar{S})\right] - 2\ln\mathcal{U}, \quad (\mathcal{U} = \mathcal{U}(\mathcal{V}, \alpha', \cdots)) \quad (3)$$

$\mathcal{U}$ is a function of $\mathcal{V}$, α' and string-loop corrections.

The GVW superpotential:

$$\mathcal{W}_0 = \int G_3 \wedge \Omega(U_a)\,, \tag{4}$$

$\mathcal{W}_0$ is corrected by **non-perturbative** (NP) contributions.

▲ NP contributions can be generated by divisors[a],which are stable under perturbations and have fixed complex structures, i.e., **rigid** ones, such as del Pezzo (dP) divisors. Generically

$$\mathcal{W} = \mathcal{W}_0 + \sum_k \mathcal{A}_k e^{-a_k T_k} \tag{5}$$

generated by D-brane instantons and gaugino condensation.

The coefficients $\mathcal{A}_k$ may depend on complex structure moduli, which are already fixed at the first step.

[a] i.e. special types of submanifolds, associated with moduli, where D7 branes can be wrapped

The Kähler potential:

Mainly two types of corrections:

A) Leading ${\alpha'}^3$ corrections in the Kähler potential depend on Euler Characteristic χ: (*Becker et al, hep-th/0204254*)

$$\xi = -\frac{\zeta(3)}{4(2\pi)^3}\chi$$

The ${\alpha'}^3$ corrections are incorporated into the Kähler potential through the shift (*Einstein frame*):

$$\hat{\mathcal{V}} \to \mathcal{U} = \hat{\mathcal{V}} + \frac{\xi}{2}\frac{1}{{g_s}^{3/2}} .$$

Then, the α' corrected Kähler potential acquires the form:

$$\mathcal{K}_{\alpha'} = -\log(-i(S-\bar{S})) - 2\log(\mathcal{U}) - \log(-i\int \Omega \wedge \bar{\Omega}), \quad (6)$$

B) multigraviton scattering

generates higher derivative couplings in curvature $\propto \mathcal{R}^4$ in *10-d string IIB action.*
Upon compactification, $\mathcal{R}^4$ leads to a new localised Einstein Hilbert ($\mathcal{EH}$) term in the bulk,

$$\mathcal{R}^4 \to \mathcal{R}_{\mathcal{EH}}$$

In the presence of $D7$-brane stacks they contribute to the Kähler potential (Antoniadis, Chen, GKL: JHEP 01 (2020) 149)

$$\mathcal{K} = -2\log\left(\mathcal{V} + \xi/2 + \gamma \log \mathcal{V}\right)$$

γ calculable coefficient:

$$\gamma = -\frac{1}{2} g_s T_0 \xi \tag{7}$$

▲ The Inflaton in String Compactifications

▲ 1. **Candidate Inflaton Fields**

In string theory, the inflaton can be identified with various moduli fields:

- Volume modulus (controls overall compactification size)
- Kähler moduli (govern complex structure deformations)
- Complex Structure (CS) moduli
- Axionic partners (natural candidates for slow-roll inflation)

★ ★

↓

FIBRE INFLATION (FI)

▲ Two basic approaches will be analyzed:

Non Perturbative

&

Perturbative

★ $\mathcal{A}$ ★

↓

Short description of:

Non-Perturbative Moduli Stabilisation

& Slow Roll Inflation

★ LVS dominant models are based on the following type of volume:

$$\mathcal{V} = f_{\frac{3}{2}}(\tau_i) - \sum_{j=1}^{N_s} \lambda_j {\tau_j}^{3/2}$$

<u>A simple example with three Kähler moduli $h^{1,1} = 3$</u>

(see e.g. 1801.05434, Cicoli, Ciupke, Mayrhofer, Shukla)

In suitable divisor basis $\hat{D}_b, \hat{D}_f, \hat{D}_s$, the internal volume is:

$$\mathcal{V} = \lambda_1 \tau_b \sqrt{\tau_f} - \lambda_j {\tau_s}^{3/2}$$

▲ Assume only ${\alpha'}^3$ corrections in Kähler potential and

$$W = W_0 + A_s e^{-i a_s T_s},$$

Step 1:The **overall Volume** $\mathcal{V}$, and the volume of the small blow-up divisor τ_s are stabilised by corrections described above.

Then $\exists$ $h^{1,1} - 2 = 1$ direction remains flat which means that there is a unique inflaton **candidate**!

Step 2: Subleading $\mathcal{O}(g_s)$ corrections due to KK exchange and winding modes fix the remaining d.o.f.

★ The **potential** for leftover modulus τ_f is **flatter** ★

and thus

★ suitable for **slow roll inflation**. ★

Kähler Cone Constraints

★ The Kähler moduli space must be such that ensures a positive definite Kähler form: ⋆

$$\int_{C_i} J > 0, \qquad (J \sim t^i \hat{D}_i)$$

This Kähler Cone Condition (KCC) concerns all topologically non-trivial effective curves C_i in the internal manifold (*Mori Cone*).

⋆ Thus, whilst at leading order the would be inflaton τ_f remains flat, fixing of $\mathcal{V}$ and τ_s puts bounds on the field range of τ_f.

however, there is an issue here:
For the canonical field $\varphi \sim \sqrt{2}/3 \log(\tau_f)$, these bounds imply:

$$\varphi \lesssim 2.5$$

Notice however, that for a successful slow roll inflation we need

$$\varphi \sim \mathcal{O}(10) M_{Pl}$$

★ $\mathcal{B}$ ★

↓

$\mathcal{PERTURBATIVE}$ $\mathcal{FIBRE}$ $\mathcal{INFLATION}$

in collaboration with

S. Bera, D. Chakraborty, P. Shukla,

Phys.Rev.D 110 (2024) 10, 106009, 2406.01694

The perturbative Large Volume Scenario (LVS) [Antoniadis, Chen, GKL, JHEP 01 (2020) 149] provides a new framework for implementing Fibre Inflation without invoking non-perturbative effects - a significant departure from conventional approaches.

▲ **Key Theoretical Advancements**

• Circumvents the need for rigid divisors (namely τ_s associated with NP-corrections), and thus:

• Removes associated constraints on moduli stabilization

• The inflaton field range is no longer strongly bounded by geometric conditions

• Enables larger field excursions critical for sustained inflation

Global Model :

We consider a CY_3 with $h^{1,1} = 3$
(*polytope Id: 249 in the CY database of KS/hep-th 0002240*)

- Hodge numbers $(h^{2,1}, h^{1,1}) = (115, 3)$,
- Euler number $\chi = -224$.
- In the divisor basis $\{\hat{D}_1, \hat{D}_2, \hat{D}_3\}$, the Kähler form is

$$J = t^1 \hat{D}_1 + t^2 \hat{D}_2 + t^3 \hat{D}_3$$

- The only non-zero intersection is $k_{123} = 2$ leading to

$$\mathcal{V} = 2\, t^1\, t^2\, t^3 = \frac{1}{\sqrt{2}} \sqrt{\tau_1\, \tau_2\, \tau_3}$$

- The Kähler cone conditions are:

$$\text{KCC:} \qquad t^1 > 0, \quad t^2 > 0, \quad t^3 > 0. \tag{8}$$

Global Model: **Subleading Corrections**

▲ Geometry of internal space.

Assuming a configuration of:$3 \times D7$ brane-stacks, each one spans 4 compact dimensions while localised at the remaining 2-d.

D7s	Minkowski				Compact Dimensions					
	0	1	2	3	4	5	6	7	8	9
$D7_a$		$*$	$*$	$*$	$*$	$*$	$*$	$*$	.	.
$D7_b$		$*$	$*$	$*$	$*$	$*$	.	.	$*$	$*$
$D7_c$		$*$	$*$	$*$	.	.	$*$	$*$	$*$	$*$

Among other things, the divisor intersection analysis shows

- ▲ The three $D7$-brane stacks which intersect at $\mathbb{T}^2$
- ▲ Because $D7$-brane stacks intersect on non-shrinkable two-tori $\Rightarrow$

$\exists$ string-loop effects of the winding-type:

$$\boxed{V_{g_s}^{\mathrm{W}} = -\frac{\kappa |W|^2}{\mathcal{V}^3} \sum_a \frac{C_a^w}{t^a}}$$

- ▲ The model does not induce KK-type string-loop corrections to the Kähler potential.

All contributions give rise to the following scalar potential:

$$V_{\rm eff} \approx V_{\rm up} + \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi} - 4\,\hat{\eta} + 2\,\hat{\eta}\,\ln\mathcal{V}\right) \tag{9}$$

$$+\frac{\mathcal{C}_2}{\mathcal{V}^4}\left(\mathcal{C}_{w_1}\tau_1 + \mathcal{C}_{w_2}\tau_2 + \mathcal{C}_{w_3}\tau_3 + \frac{\mathcal{C}_{w_4}\,\tau_1\tau_2}{2(\tau_1+\tau_2)}\right. \tag{10}$$

$$\left.+\frac{\mathcal{C}_{w_5}\,\tau_2\tau_3}{2(\tau_2+\tau_3)} + \frac{\mathcal{C}_{w_6}\,\tau_3\tau_1}{2(\tau_3+\tau_1)}\right) + \frac{\mathcal{C}_3}{\mathcal{V}^3}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right) \tag{11}$$

- Part (9) fixes the volume $\mathcal{V}$.
- Parts (10) and (11) fix one more modulus τ_k.
  Hence:
- two τ_i are integrated out, and $V_{\rm eff}$ only depends on one modulus, $V_{\rm eff} = V(\tau_f) \Rightarrow \tau_f$ drives inflation

▲ Uplifting Methods in Type IIB: From AdS to dS ▲

In Type IIB flux compactifications, the scalar potential typically stabilizes moduli in an AdS vacuum. To achieve a dS vacuum the following uplifting mechanisms could be employed:

▲ Anti-D3-Branes (KKLT Scenario arXiv:hep-th/0301240)
Introduces $\overline{D3}$-branes at the tip of a warped throat.
- Breaks SUSY explicitly.
- Requires tuning to avoid runaway decompactification.

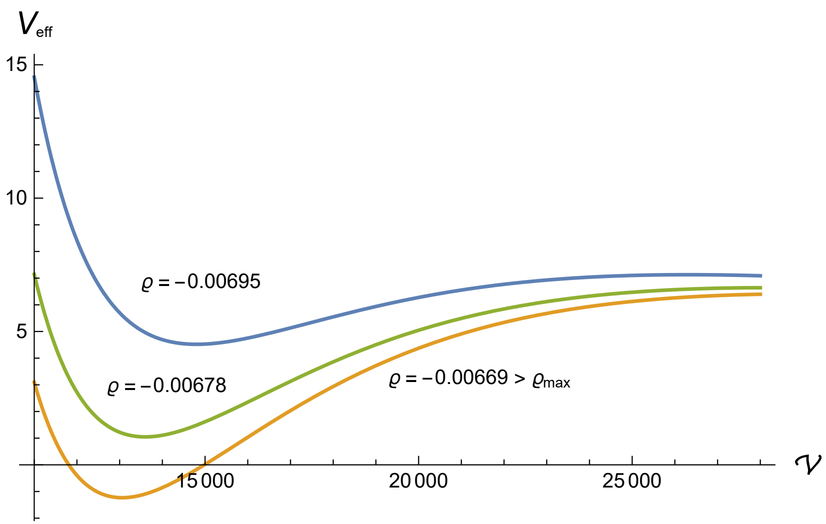
▲ Non-Perturbative Effects (Kähler Uplifting)

▲ D-Term Uplifting: Uses anomalous $U(1)$ gauge symmetries with Fayet-Iliopoulos (FI) terms.

In the present geometric setup: D-Terms are utilised, related to universal $U(1)$'s of D7-stacks:

$$V_{\mathcal{D}} = \frac{g_{D7_i}^2}{2}\left(Q_i \partial_{T_i} K + \sum_j q_j |\Phi_j|^2\right)^2 , \quad \frac{1}{{g_{D7_i}}^2} = \mathrm{Re}T_i + \cdots$$

Sketch of $V_{\rm eff}$ vs $\mathcal{V}$ (volume) for values of an uplift parameter



Inflationary dynamics:

Define the canonically normalized fields,

$$\varphi^{\alpha} = \frac{1}{\sqrt{2}} \ln \tau_{\alpha}, \; \alpha \in \{1,2,3\}, \quad \text{so that}$$

$$\mathcal{V} \propto e^{\frac{1}{\sqrt{2}}(\varphi^1+\varphi^2+\varphi^3)}$$

The scalar potential takes the form $(\varphi \to \langle\varphi\rangle + \phi)$

$$V = \mathcal{C}_0 \left(\mathcal{C}_{\mathrm{up}} + \mathcal{R}_0 e^{-\gamma\phi} - e^{-\frac{\gamma}{2}\phi} + \mathcal{R}_1 e^{\frac{\gamma}{2}\phi} + \mathcal{R}_2 e^{\gamma\phi} \right), \qquad (12)$$

- The size of up-lift required for dS vacuum is $C_{up} = 1 - \mathcal{R}_0 - \mathcal{R}_1 - \mathcal{R}_2$
- D7-brane or T-uplift (1512.04558) can be implemented.

To examine the predictions for inflation we compute the quantities:

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \ \eta = \left(\frac{V''}{V}\right)2, \ \xi = \left(\frac{V'V'''}{V}\right)$$

These determine the following observables:

▲ density fluctuations

▲ exponential expansion for cosmological curvature and anisotropy

▲ spectral tilt of the microwave background anisotropy spectrum

▲ ratio of the amplitudes of tensor and scalar perturbations

which should be in accordance with experimental values at the pivot scale, $k^* = 0.05 Mpc^{-1}$ (setting $M_P = 1$):

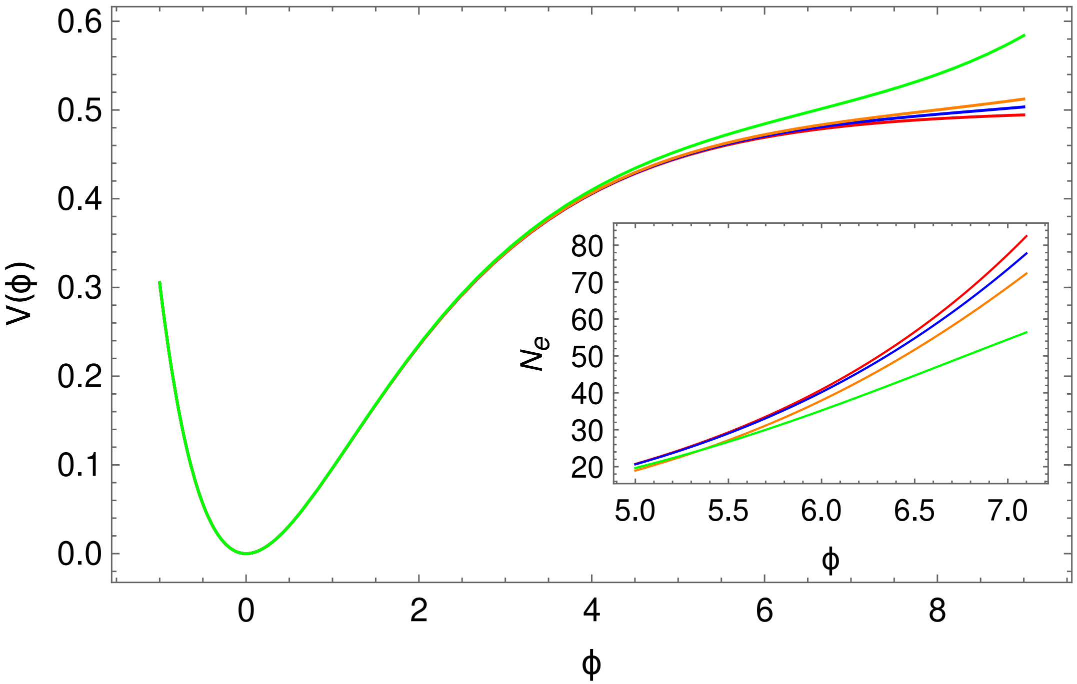
$n_s = 1 - 6\epsilon^* + 2\eta^* = 0.9649 \pm 0.004$, $r \approx 6\epsilon^* \leq 0.034$, $A_s = \frac{V_*}{24\pi\epsilon^*}$

A benchmark model:

$$\mathcal{C}_0 \sim 4 \times 10^{-10}, \quad \mathcal{R}_1 \sim 10^{-6}, \quad \mathcal{R}_2 \sim \times 10^{-7}$$

which correspond to string parameters:

$$|W_0| = 6, \quad g_s = 0.28, \quad \langle \mathcal{V} \rangle = 6 \times 10^3$$



Efolds, scalar perturbation amplitude, spectral index:

$$N_e{}^* = 58, \quad P_s = 2.1 \times 10^{-9}, \quad n_s{}^* = 0.966$$

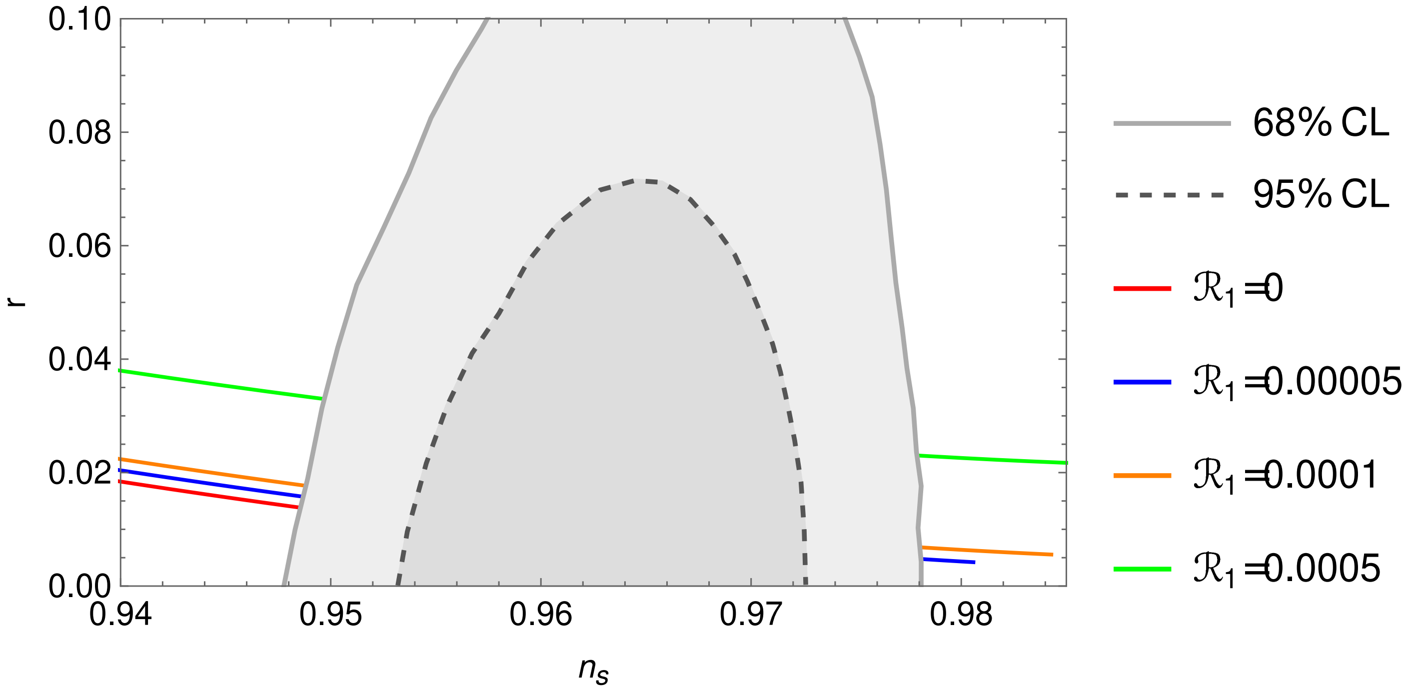


Figure 1: Plot of spectral index n_s vs tensor-to scalar ratio r.

CONCLUSIONS

In this talk, I have presented :

▲ Fibre (Kähler) Inflation

▲ in Large Volume Compactifications

▲ with **Perturbative Corrections** (PLVS)

• It was shown that Kähler Cone Conditions are milder and easy to satisfy in PLVS.

• This was instrumental for a string scenario with Fibre Inflation

▲ The model has Global Embedding within simple CYs having:

• Minimal number of Kähler moduli to accommodate inflation

▲ We explicitly demonstrated this mechanism in a well-controlled compactification:

• Manifold Type: Smooth K3-fibred Calabi-Yau orientifold

• Volume Form: Toroidal-like structure with specific fibration properties

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}$$

▲ Key Features:

• Maintains perturbative control throughout inflationary trajectory

• Preserves consistent effective field theory description

Further investigations:

▲PLVS beyond toroidal volume

▲ **Multifield** inflation for CY_3s with $h^{1,1} > 3$

THANK YOU FOR YOUR ATTENTION

APPENDIX

Non-Perturbative approach String Loop Effects

(*hep-th/0507131,...,0704.0737*)

String-loop effects known as KK and winding types generate new $V_{g_s}^{KK} + V_{g_s}^{W}$ subleading potential terms for τ_f.

Scalar potential to leading order in minimal FI model:

$$V_{\mathrm{LVS}} \approx \frac{|W_0|^2}{\mathcal{V}^2}\left(\frac{\beta_1}{{\tau_f}^2} - \frac{\beta_2}{\mathcal{V}\sqrt{\tau_f}} + \frac{\beta_3 \tau_f}{\mathcal{V}^2}\right) + V_{up}$$

- V_{up} uplift term required to achieve dS minimum.

Such a term is possible by virtue of the presence of suitable D-branes.

Logarithmic Corrections

In String Theory:

multigraviton scattering generates higher derivative couplings in curvature *(see Green et al, hep-th/9704145; Antoniadis, et al hep-th/9707013, Kiritsis, et al hep-th/9707018)*

Type II 10-d effective action with $\mathcal{EH}$ & R^4 terms:

$$\mathcal{S} \quad \supset \quad \frac{c}{l_s^8}\int_{M_{10}} e^{-2\phi}\mathcal{R}_{(10)} + \frac{d}{l_s^2}\int_{M_{10}} (-2\zeta(3)e^{-2\phi} + 4\zeta(2))R^4 \wedge e^2$$

Leading correction term in type II-B action:

$$\propto R^4$$

Reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) induces:

$$\Rightarrow \quad \frac{c}{l_s^8} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)} + \underbrace{2d \frac{\chi}{l_s^2} \int_{M_4} (\zeta(2) - \zeta(3) e^{-2\phi}) \mathcal{R}_{(4)}}_{\textit{induced } \mathcal{EH} \textit{ term}},$$

localised Einstein Hilbert ($\mathcal{EH}$) term $\propto$ Euler characteristic

$$\frac{1}{3!(2\pi)^3} \chi = \int R \wedge R \wedge R$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

$$\Downarrow$$

$\chi \neq 0 \Rightarrow$ localised graviton kinetic terms: $\cdots (\mathcal{V} + \beta \chi) \mathcal{R} \cdots \Rightarrow$

▲▲ Introducing 7-branes ▲▲

Localised vertices can emit gravitons and KK-excitations in 6*d*

$\Rightarrow$ KK-exchange between graviton vertices and $D7$-branes

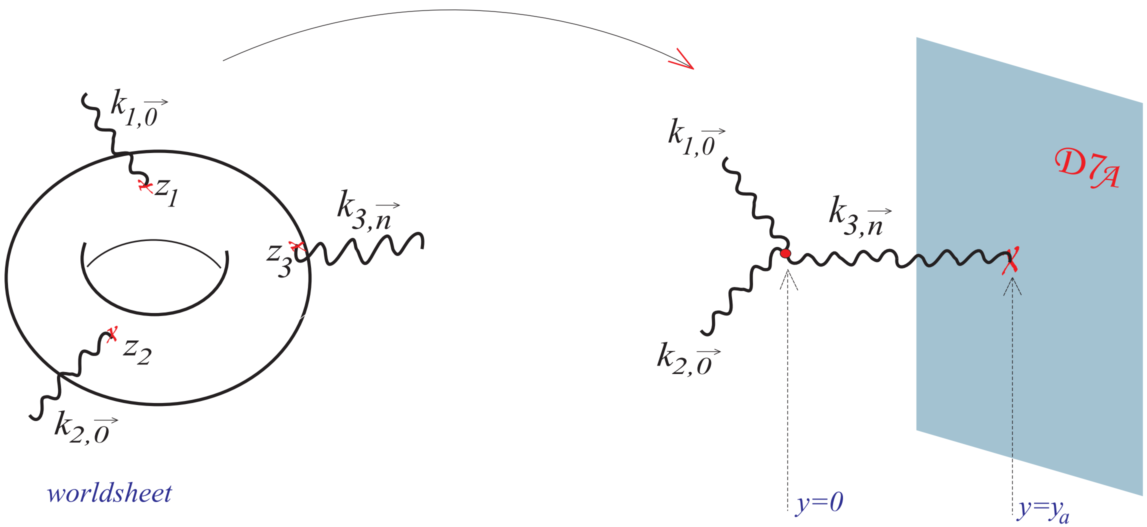


Figure: non-zero contribution from 1-loop; 3-graviton scattering amplitude 2 massless 1 KK Graviton scattering $\langle V^2_{(0,0)} V_{(-1,-1)} \rangle$ & KK-propagating in 2-d towards $D7$

Corrections

Final result:

$$\frac{1}{(2\pi)^3}\int_{M_4\times\mathcal{X}_6} e^{-2\phi}\mathcal{R}_{(10)} + \frac{4\zeta(2)\chi}{(2\pi)^3}\int_{M_4}(1+\sum_{i=1,2,3} e^{2\phi}T_i\log(R^i_\perp)\mathcal{R}_{(4)} \ ,$$