# Cosmological Signatures of Neutrino Seesaw

Chengcheng Han
Sun Yat-sen University

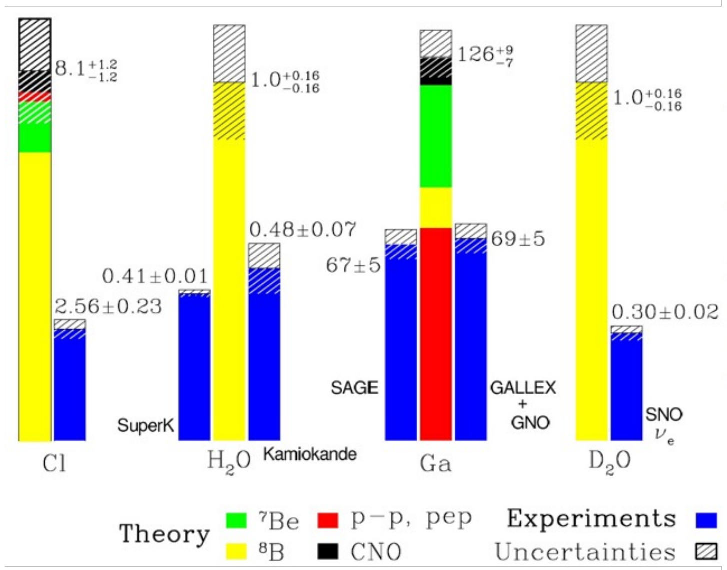
With Hongjian He, Linghao Song, Jintao You, arXiv: 2412.21045，2412.16033

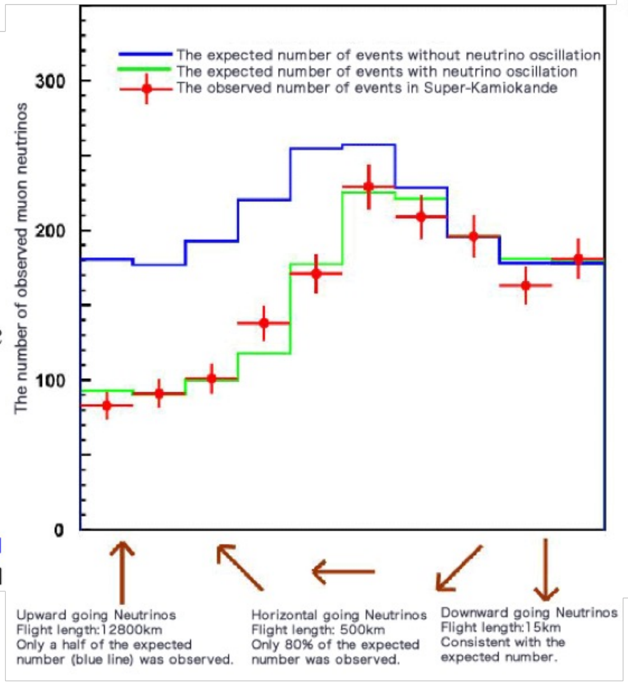
The 2nd Workshop on Grand Unified Theories: Phenomenology and Cosmology (GUTPC 2025)

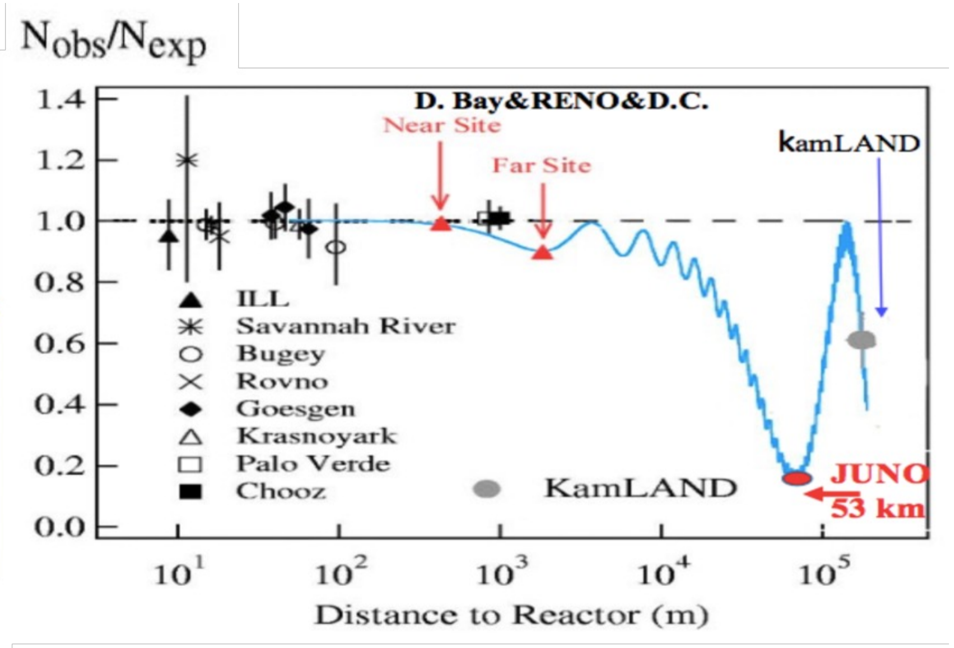
2025.4.21

# Neutrino masses

## Neutrino oscillation indicates massive neutrinos




Solar Neutrino oscillations

$\theta_{12}$

Atmospheric Neutrino Oscillations

$\theta_{23}$

Reactor Neutrino Oscillations

$\theta_{13}$

$$\Delta m^2_{21} \simeq 7.42 \times 10^{-5}\ \mathrm{eV}^2$$

$$|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

# Seesaw mechanism

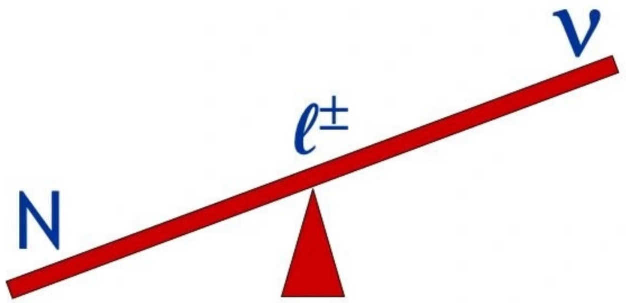
## Origin of neutrino masses: seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\rm SM} + y_\nu \tilde{H} \bar{L} N_R - \frac{1}{2} M_R \bar{N}_R^c N_R + h.c.$$

P. Minkowski ; T. Yanagida; S. L. Glashow; M. Gell-Mann, P. Ramond and R. Slansky

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2M_R}$$




- **Natural prediction of small neutrino masses**
- **Explaining the baryon asymmetry of the universe: leptogenesis**

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′

# Seesaw mechanism

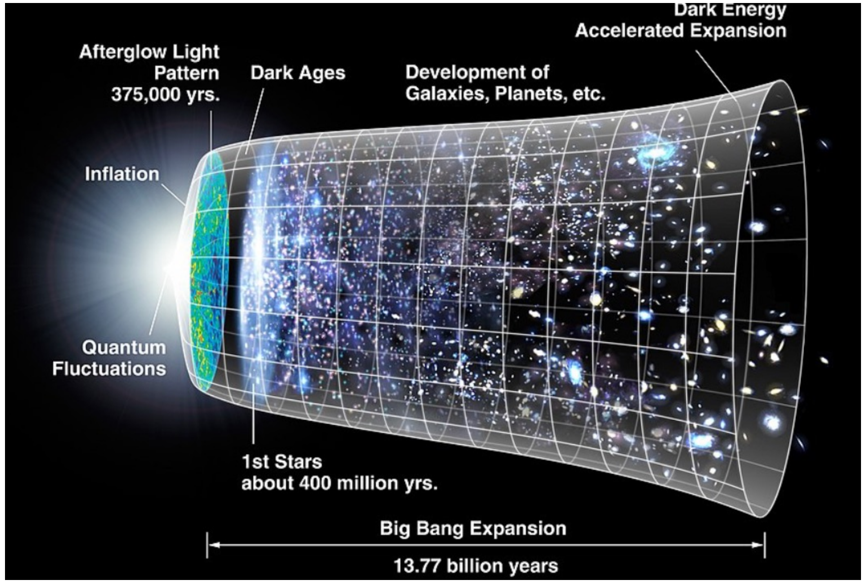
$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2M_R}$$

**If the Yukawa coupling is O(1)(as predicted by the GUT), the seesaw scale $M_R$ should be around $10^{13\text{-}14}$ GeV, which is much beyond the reach of particle experiments.**

**How to test such high scale seesaw?**

# Inflation

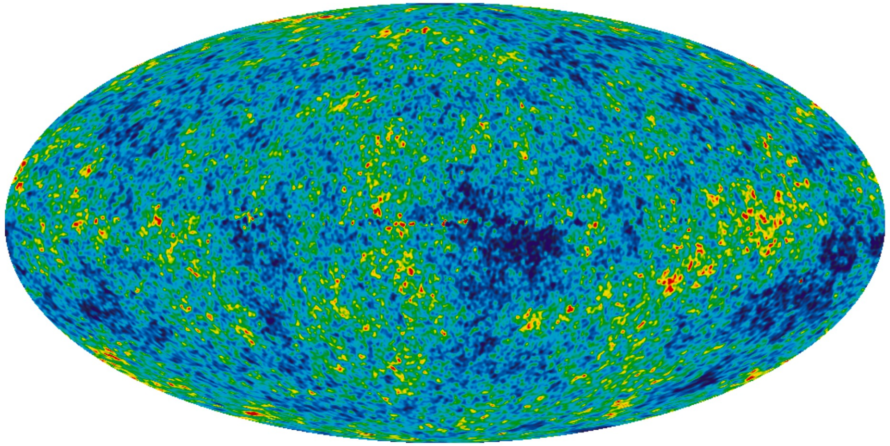
## Rapid expansion of the universe in the early time



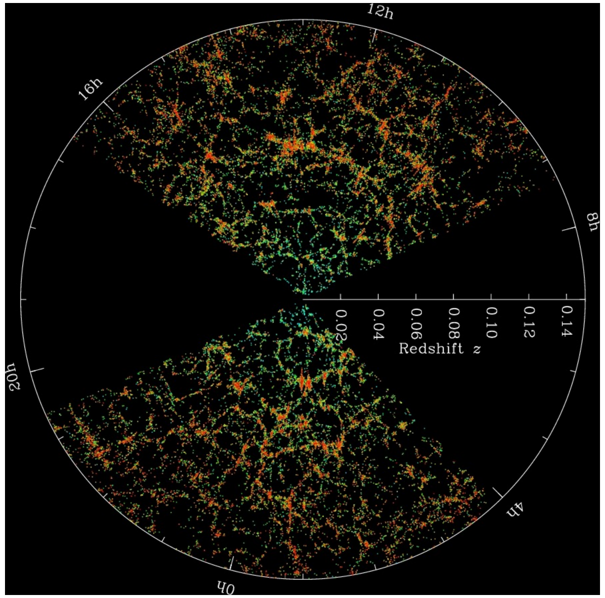

- Flatness problem
- Horizon problem
- Seeding the primordial anisotropies in CMB

# Inflation

**Stretching quantum fluctuations to large scale**



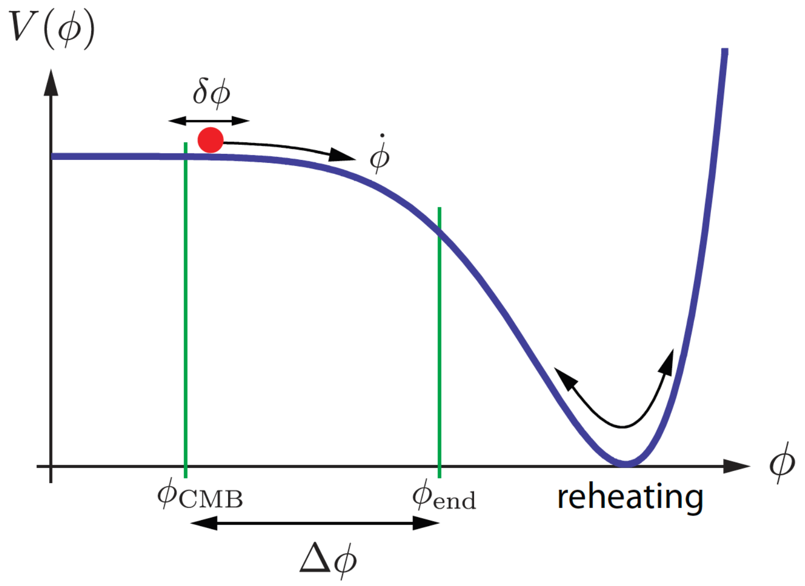
$$\frac{\delta T}{T} \sim 10^{-5}$$




**Such small fluctuations finally develops the large structure of our universe**

# Slow-roll Inflation

## Inflation is driven by a scalar field (inflaton)




$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

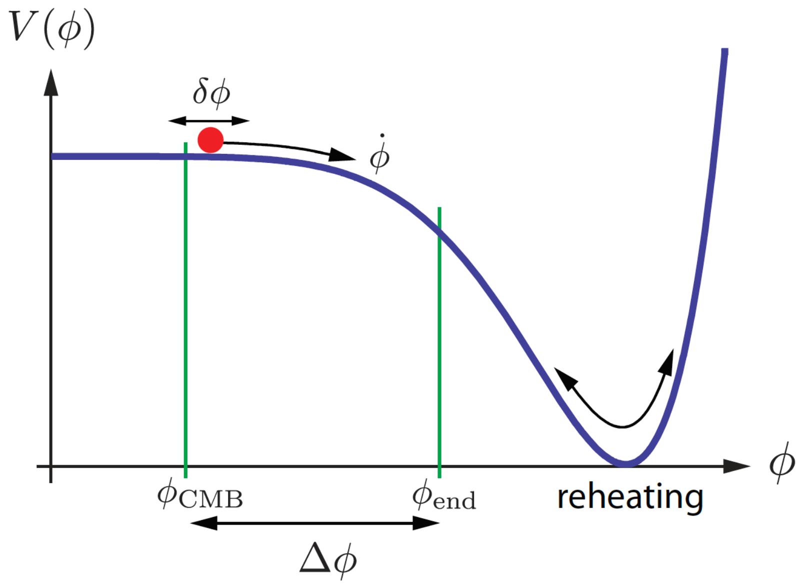
## Slow roll condition

$$\dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|\,, |V_{,\phi}|$$

- **Hubble parameter is nearly constant(de Sitter universe)**
- **After inflation, inflaton oscillates at the bottom of the potential and finally decays into SM particles, then reheats the universe(still no clear how it occurs)**

# Slow-roll Inflation

**In a de Sitter universe, scalar fields get quantum fluctuation(roughly H/2Pi each e-fold)**




$$\delta\phi(x,\tau) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[u_a(\tau,\mathbf{k})b_a(\mathbf{k}) + u_a^*(\tau,-\mathbf{k})b_a^\dagger(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}$$

**Massless limit** $u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}\left[1 + ik\tau\right]e^{-ik\tau}$

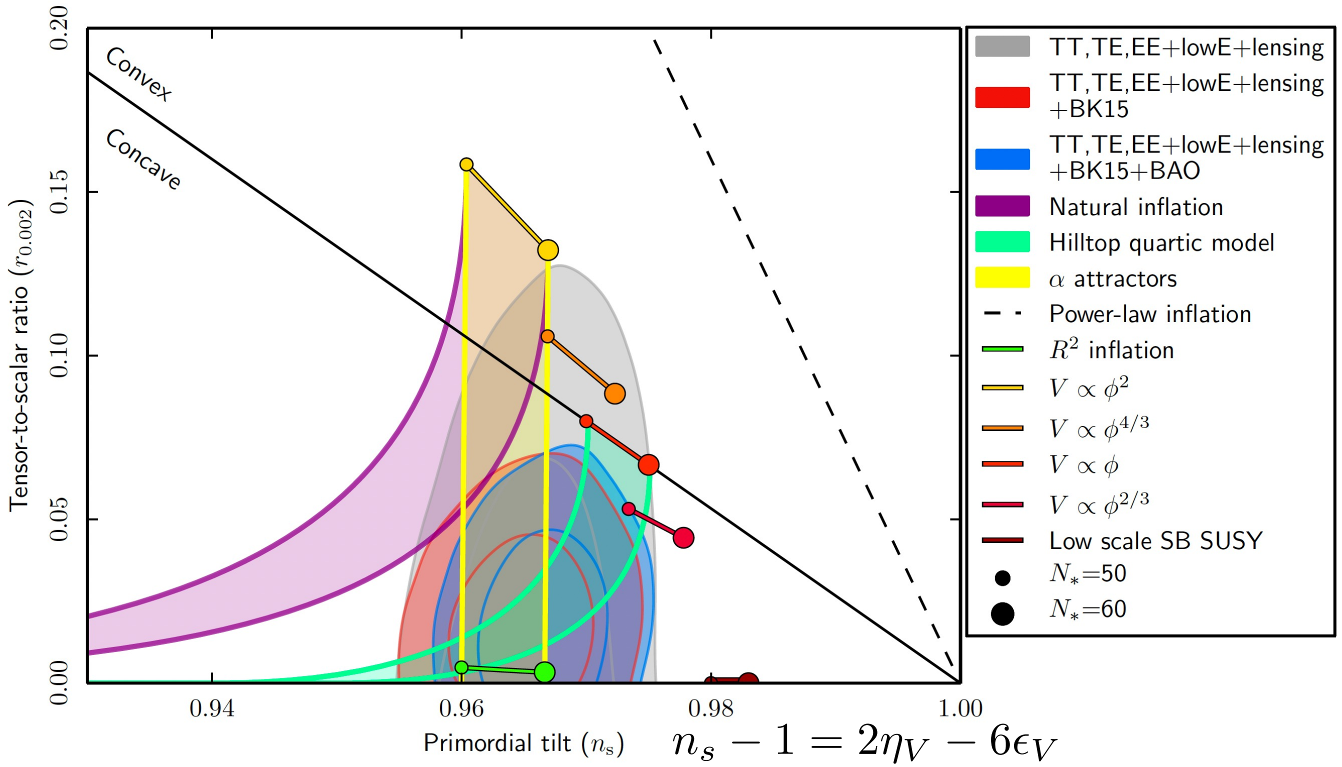
$$\langle \delta\phi_{\mathbf{k}}\,\delta\phi_{\mathbf{k}'}\rangle = (2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}')\,\frac{2\pi^2}{k^3}\left(\frac{H}{2\pi}\right)^2$$

$$\zeta = -\frac{H}{\dot{\phi}}\delta\phi$$

- **Quantum fluctuation of inflaton induces CMB anisotropies(or curvature perturbations)**
- **In the single field inflation, the fluctuations should be nearly gaussian and adiabatic, close to scale invariant**

# Inflation

$$r = 16\epsilon_V$$




$$n_s - 1 = 2\eta_V - 6\epsilon_V$$

$$\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$\eta_V(\phi) \equiv M_{pl}^2 \frac{V_{,\phi\phi}}{V}$$

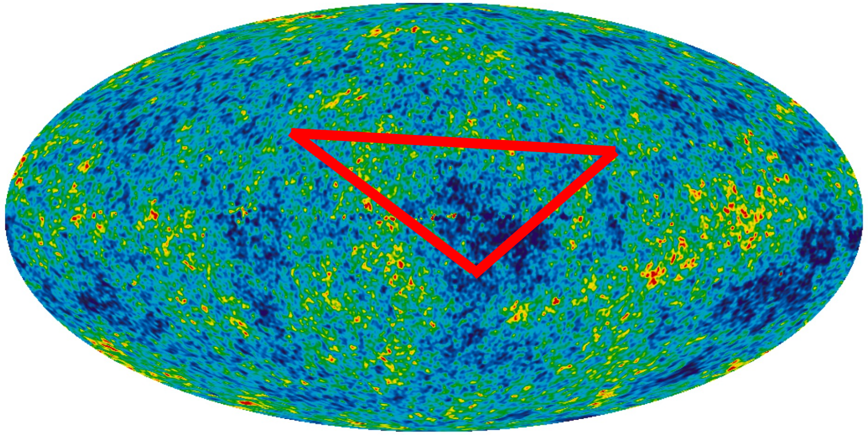
$$\epsilon_V < 0.0097$$

$$\eta_V = -0.010^{+0.007}_{-0.011}$$

$$\frac{H_*}{M_{Pl}} < 2.5 \times 10^{-5}$$

- **Inflaton potential should be flat enough(shift-symmetry?)**
- **Hubble scale could be as high as $6*10^{13}$ GeV(close to seesaw scale), providing access to the high scale physics**

# Non-Gaussianity



## Non-Gaussianity is sensitive to new physics

- **New physics could induce large non-Gaussianity : multi-field inflation models, modulated reheating, curvaton scenario…**
- **Current limit from Planck on local type $f_{NL}$~ O(10), future CMB observations, LiteBIRD O(1), large scale structure observations DESI O(1), 21 cm tomography O(0.01-0.1)**
- **Non-Gaussianity could provide information to the new particle mass, spin, interactions: cosmological collider signals**

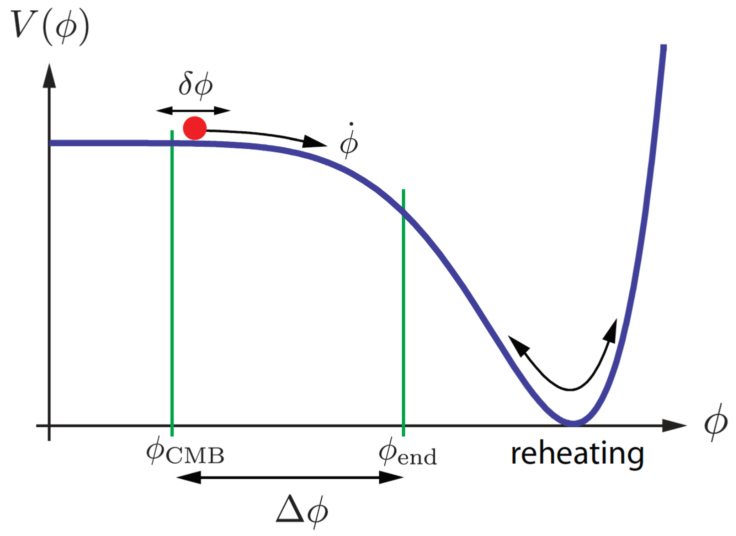
Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043

Xingang Chen, Yi Wang, JCAP 04 (2010) 027

# The model

## Minimal model incorporates inflation and seesaw

$$\Delta\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu - V(\phi) + \overline{N}_{\rm R} i\partial\!\!\!/ N_{\rm R} + \frac{1}{\Lambda}\partial_\mu\phi\,\overline{N}_{\rm R}\gamma^\mu\gamma^5 N_{\rm R} + \left(-\frac{1}{2}M\overline{N^c_{\rm R}}N_{\rm R} - y_\nu\,\bar{\ell}_{\rm L}\tilde{\mathbb{H}}N_{\rm R} + {\rm H.c.}\right)\right],$$




- V(phi) is the potential for inflation is unknown but  denominated by the mass term after inflation
- Derivative coupling to keep the flatness of the inflaton potential(shift-symmetry)
- Lambda > 60 Hubble to keep perturbative unitarity
- After inflation, inflaton oscillates at the bottom of the potential until decays into heavy neutrinos ( mphi > 2 mN). The heavy neutrinos quickly decay into SM particles and reheat the universe.
- The assumption here is that inflaton mostly decays into right-handed neutrinos (the inflaton decay into SM particles can be suppressed)

# The model

## Consequence of the seesaw mechanism

$$\mathcal{L} \supset \frac{1}{2}\bar{\psi}_L \mathbf{M}_\nu \psi_R + \mathrm{h.c.}, \qquad \mathbf{M}_\nu = \begin{pmatrix} 0 & \frac{y_\nu h}{\sqrt{2}} \\ \frac{y_\nu h}{\sqrt{2}} & M \end{pmatrix}$$

$$m_\nu \simeq -\frac{y_\nu^2 h^2}{2M}, \qquad M_N \simeq M + \frac{y_\nu^2 h^2}{2M}$$

- **Light neutrino gets a mass**
- **Heavy neutrino mass are get lifted (h dependent)**

**Decay rate of the inflaton is h dependent:**

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1 + \frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

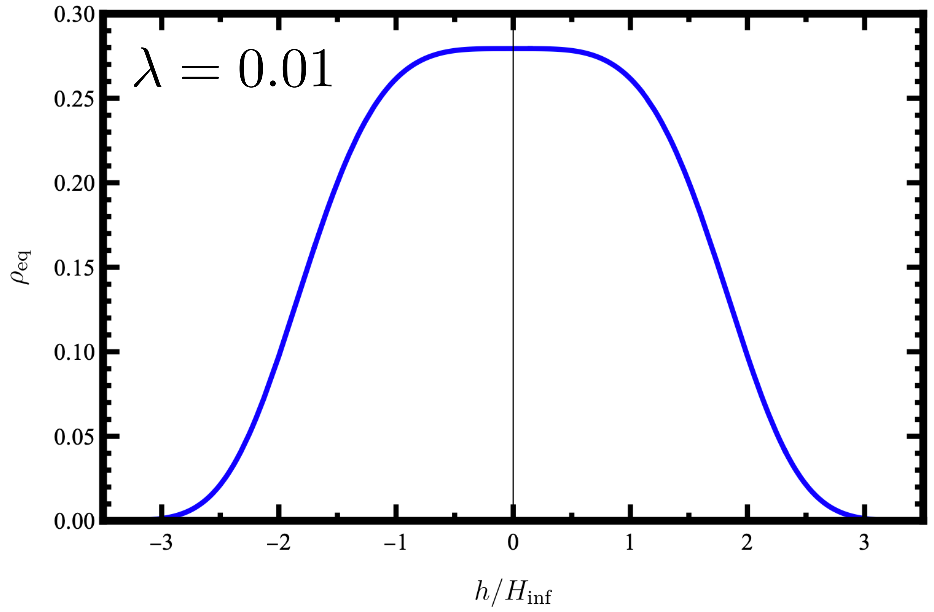
**What happens to h in the early universe?**

# Higgs during inflation

Alexei A. Starobinsky, Jun'ichi Yokoyama, Phys.Rev.D 50 (1994) 6357-6368

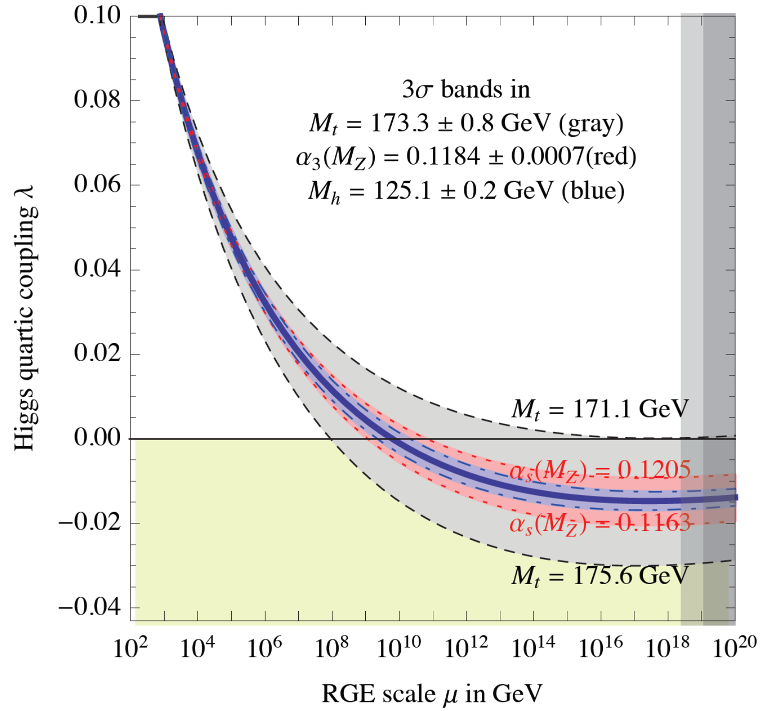
- **During inflation(de-Sitter universe), Higgs also gets quantum fluctuations**
- **Different part of universe Higgs field takes different value**
- **If inflation lasts long enough, the fluctuations reach a equilibrium state**

$$\rho_{\rm eq}(h) = \frac{2\lambda^{1/4}}{\Gamma(1/4)}\left(\frac{2\pi^2}{3}\right)^{1/4}\exp\left(\frac{-2\pi^2\lambda h^4}{3H_{\rm inf}^4}\right)$$




$$\bar{h} = \sqrt{\langle h^2\rangle} = \left[\int_{-\infty}^{+\infty} dh\, h^2\rho_{\rm eq}(h)\right]^{1/2} \simeq 0.363\left(\frac{H_{\rm inf}}{\lambda^{1/4}}\right)$$

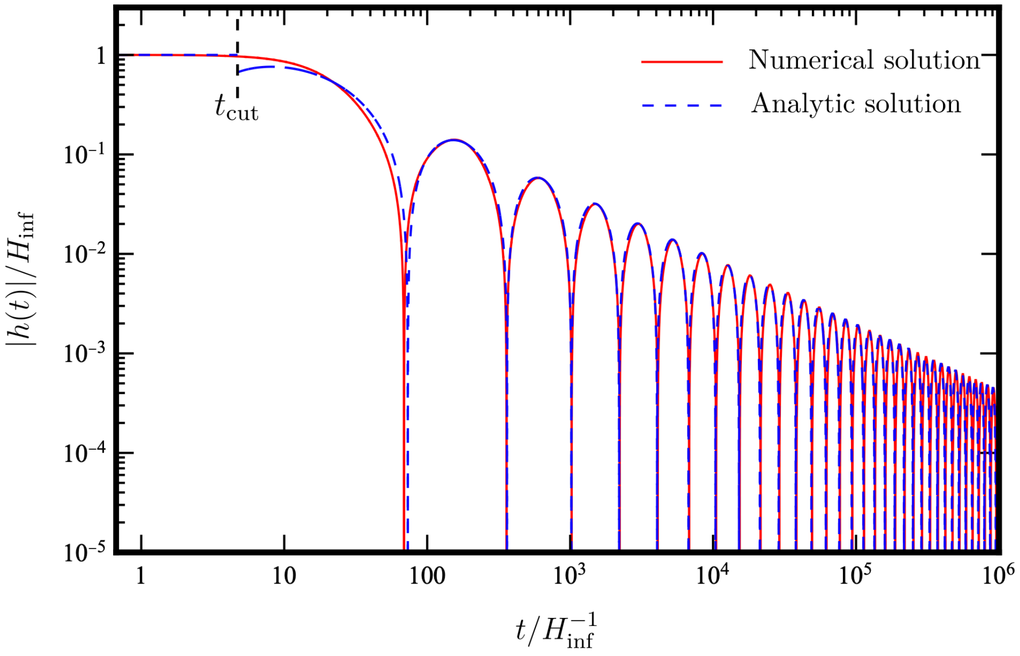
D. Buttazzo, et al arXiv:1307.3536



# Higgs value after inflation

**Inflaton oscillates at the bottom potential. If the inflaton potential is dominated by the mass term, the Universe is matter-dominated**

$$\ddot{h}(t) + \frac{2}{t}\dot{h}(t) + \lambda h^3(t) = 0$$




$$h(t) = \begin{cases} h_{\rm inf}\,, & t \le t_{\rm cut} \\ A H_{\rm inf}\left(\frac{h_{\rm inf}}{H_{\rm inf}\lambda}\right)^{\frac{1}{3}}(H_{\rm inf}\,t)^{-\frac{2}{3}}\cos\left(\lambda^{\frac{1}{6}}|h_{\rm inf}|^{\frac{1}{3}}\,\omega\, t^{\frac{1}{3}} + \theta\right), & t > t_{\rm cut} \end{cases}$$

$$A = \left(\frac{2}{9}\right)^{\frac{1}{3}} 5^{\frac{1}{4}} \simeq 0.9\,, \quad \omega = \frac{\Gamma^2(3/4)}{\sqrt{\pi}} 6^{\frac{1}{3}} 5^{\frac{1}{4}} \simeq 2.3\,,$$

$$\theta = -3^{-\frac{1}{3}} 2^{\frac{1}{6}} \omega - \arctan 2 \simeq -2.9\,.$$

**Higgs value would oscillate and decrease**

# Higgs modulated reheating

**Considering decay rate of the inflaton is h dependent**

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1+\frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga, Phys.Rev. D69 (2004) 023505

- **Different patches of the universe reheat differently (modulated reheating)**
- **The curvature perturbation is generated by Higgs field**
- **Different universe has different e-fold N (from the end of inflation to the time after reheating completed)**
- **Curvature perturbation is delta N= N- <N>**

# Higgs modulated reheating

**Equation of state:** $\dot{\rho} + 3H(1+\omega)\rho = 0$

**From matter-dominated universe to radiation dominated universe**

$$N(\mathbf{x}) = -\frac{1}{3}\ln\frac{\rho_{\mathrm{reh}}\big(h(\mathbf{x})\big)}{\rho_{\mathrm{inf}}} - \frac{1}{4}\ln\frac{\rho_f}{\rho_{\mathrm{reh}}\big(h(\mathbf{x})\big)}$$

**Reheating occurs** $H_{\mathrm{reh}} = \Gamma_{\mathrm{reh}}$ $\quad 3H^2M_p^2 = \rho$

**Curvature perturbation in terms of the decay rate**

$$\begin{aligned}\zeta_h(t > t_{\mathrm{reh}}, \mathbf{x}) &= \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x})\rangle \\ &= -\frac{1}{6}\left[\ln(\Gamma_{\mathrm{reh}}) - \langle \ln(\Gamma_{\mathrm{reh}})\rangle\right]\end{aligned}$$

# Higgs modulated reheating

## Curvature perturbation contains two parts

$$\zeta = \zeta_\phi + \zeta_h$$

$$\mathcal{P}_\zeta^{(\phi)} = \left(\frac{H}{\dot{\phi}}\right)^2 \mathcal{P}_\phi = \left(\frac{H}{\dot{\phi}}\right)^2 \frac{H^2}{4\pi^2}$$

## Taylor expansion of the curvature perturbations

$$\zeta_h(\mathbf{x}) = -\frac{1}{6}\left[\frac{\Gamma_0'}{\Gamma_0}\delta h_{\rm inf}(\mathbf{x}) + \frac{\Gamma_0\Gamma_0'' - \Gamma_0'\Gamma_0'}{2\Gamma_0^2}\delta h_{\rm inf}^2(\mathbf{x})\right] \equiv z_1 \delta h_{\rm inf}(\mathbf{x}) + \frac{1}{2} z_2 \delta h_{\rm inf}^2(\mathbf{x})$$

$$\mathcal{P}_\zeta^{(h)} = z_1^2 \mathcal{P}_{\delta h} = z_1^2 \frac{H^2}{4\pi^2}$$

$$R = \left(\frac{\mathcal{P}_\zeta^{(h)}}{\mathcal{P}_\zeta}\right)^{1/2} = |z_1| \left(\frac{\mathcal{P}_{\delta h}}{\mathcal{P}_\zeta}\right)^{1/2}$$

**R should be less than 1**

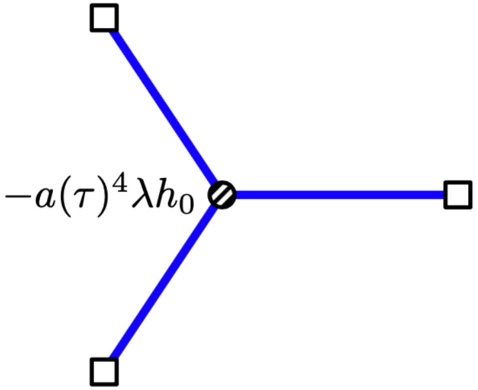
# Bispectrum

## Considering the three point correlation function

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle_h = z_1^3 \langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle + z_1^2 z_2 \langle \delta h^4 \rangle_{\mathrm{2nd}} (\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$$

## First term is from Higgs self-coupling

$$z_1^3 \langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle$$




## Calculated by in-in formalism/Schwinger-Keldysh formalism

Steven Weinberg, Phys.Rev.D 72 (2005) 043514, Phys.Rev.D 74 (2006) 023508

Xingang Chen, Yi Wang, Zhong-Zhi Xianyu, JCAP 1712 (2017) 006

# Bispectrum

$$\langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle' = 12\lambda\bar{h}\mathrm{Im}\left( \int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+\left(\mathbf{k}_i, \tau\right) d\tau \right)$$

$$\mathrm{Im}\left( \int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+\left(\mathbf{k}_i, \tau\right) d\tau \right)$$

$$N_e = \log(\frac{a_{\mathrm{end}}}{a_k}) = \log(\frac{-\frac{1}{H\tau_f}}{\frac{k_t}{H}}) = -\log(k_t|\tau_f|) \sim 60$$

$$= \mathrm{Im} \int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^4} \cdot \frac{H^6}{8k_1^3k_2^3k_3^3} \left( \prod_{i=1}^{3} (1 - ik_i\tau) \right) e^{i(k_1+k_2+k_3)\tau}$$

$$= \frac{H^2}{24k_1^3k_2^3k_3^3} \cdot \left\{ (k_1^3 + k_2^3 + k_3^3)[\log(k_t|\tau_f|) + \gamma - \frac{4}{3}] + k_1k_2k_3 - \sum_{a\neq b} k_a^2 k_b \right\}$$

# Bispectrum

## Second term is from non-linear evolution of the Higgs

$$
\begin{aligned}
& z_1^2 z_2 \langle \delta h^4 \rangle_{2\text{nd}}(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) \\
&= \frac{1}{2} z_1^2 z_2 \int \frac{\mathrm{d}^3\mathbf{k_0}}{(2\pi)^3} \langle \delta h(\mathbf{k_1}) \delta h(\mathbf{k_2}) \delta h(\mathbf{k_0}) \delta h(\mathbf{k_3} - \mathbf{k_0}) \rangle + 2 \quad \text{perm} \\
&= \frac{1}{2} z_1^2 z_2 \left[ \int \frac{\mathrm{d}^3\mathbf{k_0}}{(2\pi)^3} \langle \delta h(\mathbf{k_1}) \delta h(\mathbf{k_0}) \rangle \langle \delta h(\mathbf{k_2}) \delta h(\mathbf{k_3} - \mathbf{k_0}) \rangle + (\mathbf{k_1} \leftrightarrow \mathbf{k_2}) \right] + 2 \text{ perm} \\
&= \frac{1}{2} z_1^2 z_2 \left[ \int \frac{\mathrm{d}^3\mathbf{k_0}}{(2\pi)^3} (2\pi)^3 \, \delta^3 (\mathbf{k_1} + \mathbf{k_0}) \, \frac{H^2}{2k_1^3} \right. \\
&\qquad\qquad \left. \times (2\pi)^3 \, \delta^3 (\mathbf{k_2} + \mathbf{k_3} - \mathbf{k_0}) \, \frac{H^2}{2k_2^3} + (\mathbf{k_1} \leftrightarrow \mathbf{k_2}) \right] + 2 \text{ perm} \\
&= (2\pi)^3 \, \delta^3 (\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \, z_1^2 z_2 \left( \frac{H^2}{2k_1^3} \cdot \frac{H^2}{2k_2^3} + 2 \text{ perm} \right) .
\end{aligned}
$$

# Local type non-gaussianity

**The local type non-gaussianity which is defined by Bardeen Potential** $\Phi \equiv \frac{3}{5}\zeta$

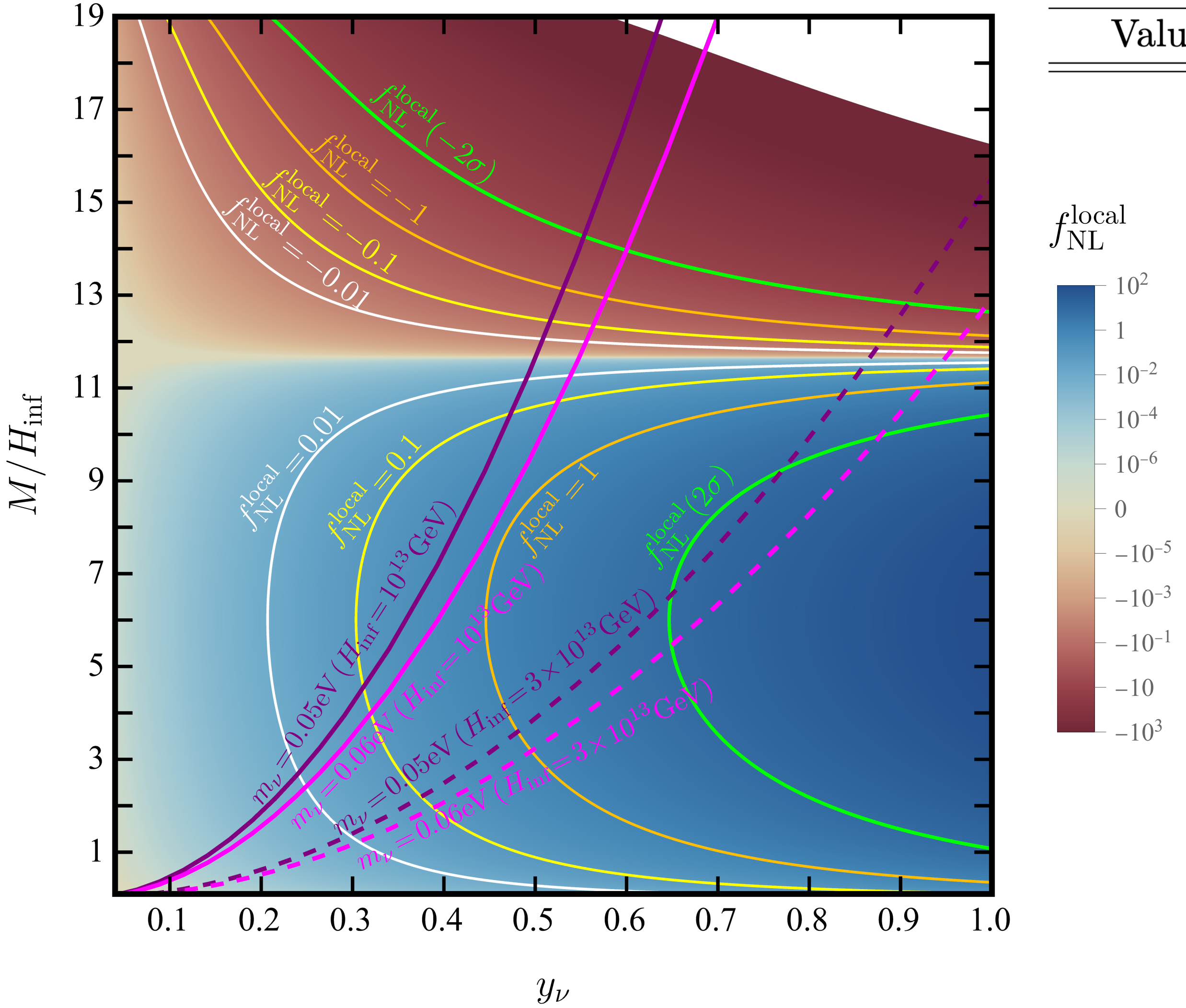
$$\langle \Phi_{\mathbf{k_1}} \Phi_{\mathbf{k_2}} \Phi_{\mathbf{k_3}} \rangle'_{\text{local}} = 2A^2 f_{\text{NL}}^{\text{local}} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$

**In the limit $k_1 \sim k_2 >> k_3$, we find**

$$f_{\text{NL}}^{\text{local}} \sim -\frac{10}{3} \frac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_\zeta^2} \cdot \left( \frac{\lambda \bar{h}}{2H} N_e - \frac{H \cdot z_2}{4 z_1} \right)$$

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \quad (68\% \text{ C.L., Planck 2018})$$

# Local type non-gaussianity




| Parameters | $\mathcal{P}_\zeta$ | $N_e$ | $H_{\rm inf}$ | $m_\phi$ | $\Lambda$ | $\lambda$ |
|---|---|---|---|---|---|---|
| Values | $2.1\times10^{-9}$ | 60 | $(1,3)\times10^{13}\,{\rm GeV}$ | $40H_{\rm inf}$ | $60H_{\rm inf}$ | 0.01 |

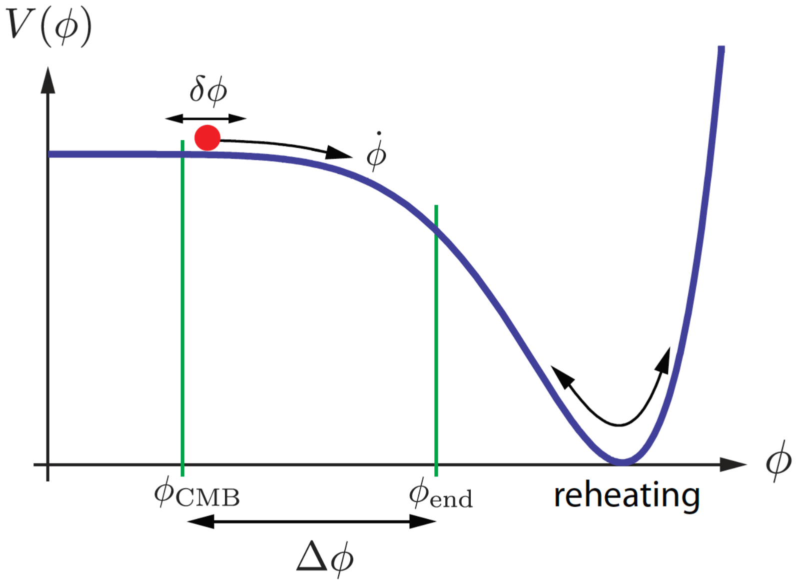
- **Colored curves indicating future searches**
- **Parameter space with Yukawa O(1) could be probed by future observations**
- **The contribution from self-interaction and non-linear term are both important**
- **Interplaying with neutrino experiments(JUNO, DUNE for neutrino ordering)**

# Summary

- **A minimal model incorporates inflation and seesaw**
- **Non-Gaussianity induced by seesaw could be probed in near future CMB or large-scale structure observations**

# Thanks!

# Slow-roll Inflation




$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$\eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V}$$

$$\Delta_{\rm s}^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{\rm pl}^4}\frac{1}{\epsilon_{\rm v}}\bigg|_{k=aH}$$

$$\Delta_{\rm t}^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{\rm pl}^4}\bigg|_{k=aH}$$

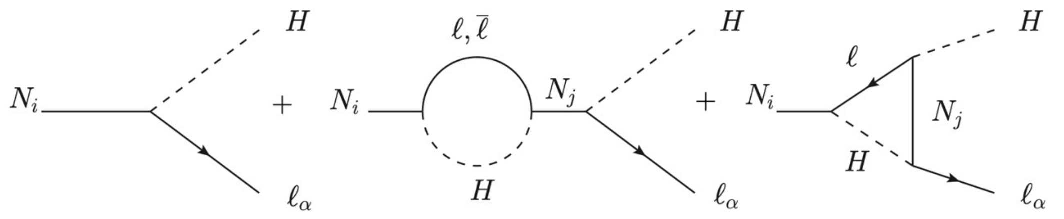
$$r \equiv \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\epsilon_{\rm v}$$

# Leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′

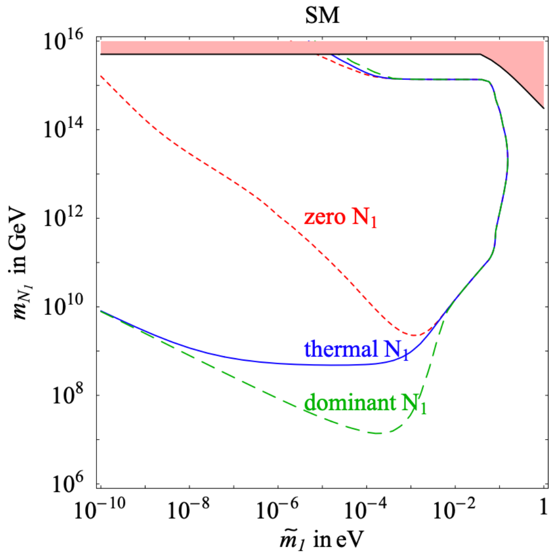
**G.F. Giudice, et al,**
**Nucl.Phys.B 685 (2004) 89-149**

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + h.c.\right)$$




$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}$$

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$




**Mass of the right-handed neutrino should heavier than $10^7$ GeV**

# S-K formalism

$$Q(\tau) \equiv \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N)$$

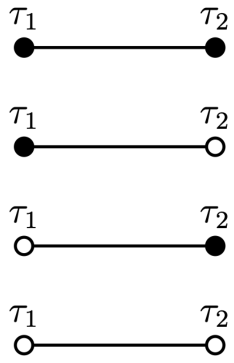
$$\langle Q(\tau) \rangle = \langle \Omega \big| \bar{F}(\tau, \tau_0) Q_I(\tau) F(\tau, \tau_0) \big| \Omega \rangle$$

$$F(\tau, \tau_0) = \mathrm{T} \exp\left( -\mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$

$$\bar{F}(\tau, \tau_0) = \bar{\mathrm{T}} \exp\left( \mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$

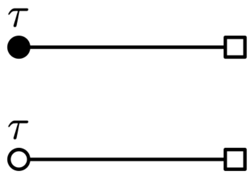
# S-K formalism

$$\begin{cases} G_{++}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_{>}\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_1-\tau_2)+G_{<}\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_2-\tau_1) \\ G_{+-}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_{<}\left(\mathbf{k};\tau_1,\tau_2\right) \\ G_{-+}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_{>}\left(\mathbf{k};\tau_1,\tau_2\right) \\ G_{--}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_{<}\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_1-\tau_2)+G_{>}\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_2-\tau_1) \end{cases}$$




$$= G_{++}(k;\tau_1,\tau_2)$$
$$= G_{+-}(k;\tau_1,\tau_2)$$
$$= G_{-+}(k;\tau_1,\tau_2)$$
$$= G_{--}(k;\tau_1,\tau_2)$$

$$G_{>}(k;\tau_1,\tau_2)\equiv u(\tau_1,k)u^*(\tau_2,k)$$
$$G_{<}(k;\tau_1,\tau_2)\equiv u^*(\tau_1,k)u(\tau_2,k)$$




$$= G_{+}(k;\tau)\equiv G_{++}(k;\tau,\tau_f)$$
$$= G_{-}(k;\tau)\equiv G_{-+}(k;\tau,\tau_f)$$

$$\Box u_{\mathbf{k}} = \ddot{u}_{\mathbf{k}} + 3H\dot{u}_{\mathbf{k}} + \frac{\mathbf{k}^2}{a^2(t)}u_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}\left[1+ik\tau\right]e^{-ik\tau}$$

# S-K formalism

## Bulk-to-Boundary propagator

$$G_{\pm}(\mathbf{k},\tau) \equiv G_{\pm+}\left(\mathbf{k};\tau,\tau_f\right)$$


$$= G_{+}(\mathbf{k},\tau)$$


$$= G_{-}(\mathbf{k},\tau)$$


$$= G_{+}(\mathbf{k},\tau) + G_{-}(\mathbf{k},\tau)$$

$$G_{+}(\mathbf{k},\tau) = \frac{H^2}{2k^3}\left[1 - ik(\tau-\tau_f) + k^2\tau\tau_f\right]e^{ik(\tau-\tau_f)} \simeq \frac{H^2}{2k^3}\left[1 - ik\tau\right]e^{ik\tau}$$

$$G_{-}(\mathbf{k},\tau) \simeq \frac{H^2}{2k^3}\left[1 + ik\tau\right]e^{-ik\tau}$$

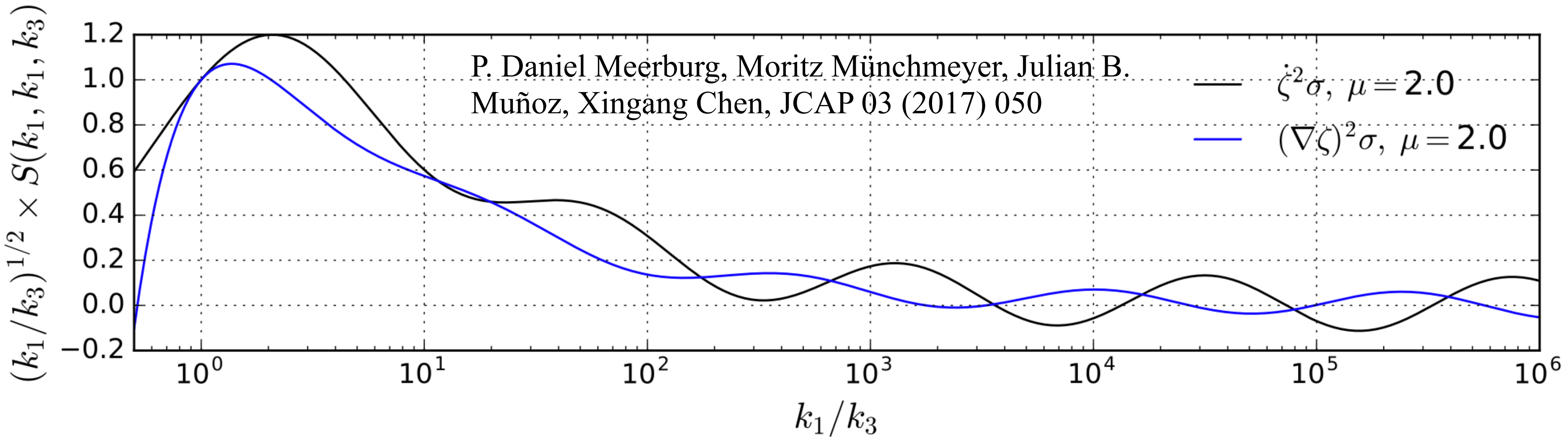
# Cosmological collider signals

**Bispectrum**

$$\langle\zeta^3\rangle \equiv (2\pi)^3\delta_D(\mathbf{k}_{123})\frac{A^2}{(k_1k_2k_3)^2}S(k_1,k_2,k_3) \qquad P_\zeta(k) = A/k^3$$

**Massive particle coupling to the inflaton could induce**

$$S_{\rm squeezed} \propto \left(\frac{k_{\rm long}}{k_{\rm short}}\right)^{1/2\pm i\mu} \qquad \mu = \sqrt{\frac{m^2}{H^2}-\frac{9}{4}}$$



# Slow-roll Inflation

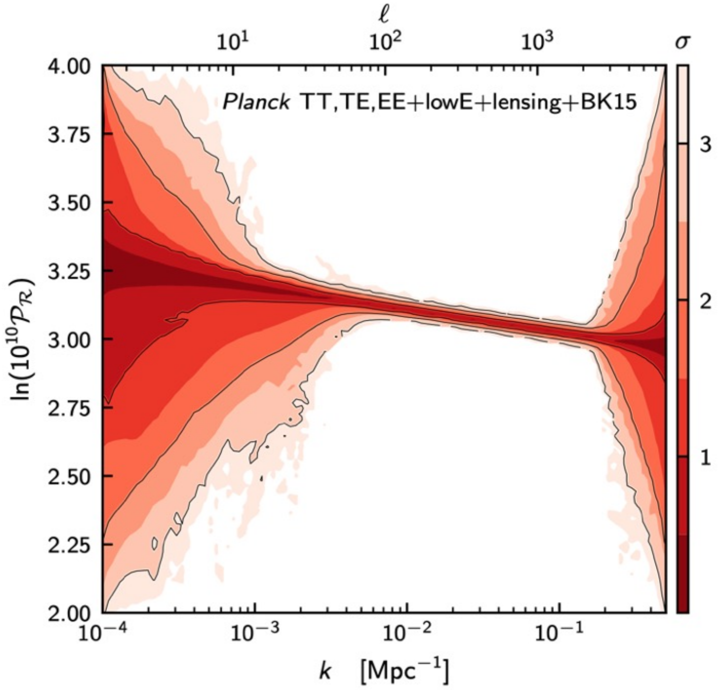
**标量扰动**

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^4}\frac{1}{\epsilon_V}\bigg|_{k=aH}$$

$$\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_V(\phi) \equiv M_{pl}^2\frac{V_{,\phi\phi}}{V}$$

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = 2\eta_V - 6\epsilon_V$$

$n_s \simeq 0.965$ **$n_s$=1 scale invariant**




**张量扰动**

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{pl}^4}\bigg|_{k=aH} \qquad r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

# Higgs modulated reheating

$$\zeta_h = -\frac{1}{6}\left[\ln(\Gamma_{\text{reh}}) - \langle \ln(\Gamma_{\text{reh}}) \rangle\right]$$

$$\uparrow$$

$$\Gamma_{\text{reh}} = \Gamma\big(h(t_{\text{reh}})\big)$$

$$\uparrow$$

$$h(t_{\text{reh}}) = h(h_{\text{inf}}, t_{\text{reh}})$$

$$h(t) = A\left(\frac{h_{\text{inf}}}{\lambda}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}} h_{\text{inf}}^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta\right)$$

**n-point correlation function of zeta changes into n-point correlation function of hinf**