E_6

Borut Bajc

J. Stefan Institute, Ljubljana, Slovenia

Babu, BB, Susič, 2305.16398, 2403.20278, in progress

Introduction

You can embed grand unified theories (GUT) along the routes

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SU(6) \subset SU(7) \subset \ldots$$

or

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset SO(18) \subset \ldots$$

These embeddings of SU(5) or SO(10) usually do not bring new insights or features except complications

A completely new path is

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6$$

No further possibility is possible

Renormalisable E_6 will be the hero of this talk

Why E_6?

I will present two reasons:

- Yukawa
- dark matter

Yukawa: bad in SM

In the SM there is no correlation between the charged fermion sector and neutral one. Actually, strictly speaking, in the SM neutrinos are massless.

$$\mathcal{L}_Y = \underbrace{Y_u^{ij} H Q_i u_j^c + Y_d^{ij} H^* Q_i d_j^c + Y_e^{ij} H^* L_i e_j^c}_{\mathcal{L}_Y^{SM}} + \ldots$$

Simplest additions (for example ν^c) which incorporate the nonzero neutrino mass

$$\ldots = Y_\nu^{ij} H L_i \nu_j^c$$

do not connect the two sectors: the Yukawa from the neutrino sector (Y_ν) has nothing in common with the Yukawas from the charged sector (Y_u, Y_d, Y_e)

This however not that surprising: the SM is anyway not a theory of flavour, not even in the charged fermion sector

(no relations among Y_u, Y_d and Y_e)

Let's upgrade the SM embedding it into a GUT

What can we say in GUTs about fermion masses and mixings?

Yukawa: still bad in SU(5)

The SM fermions get unified, instead of 5 irreps of the SM (Q, L, u^c, d^c, e^c) one gets only 2 irreps of SU(5):

$$\bar{5} = (d^c, L) \quad , \quad 10 = (Q, u^c, e^c)$$

The Yukawa sector is more economical than in the SM

$$\mathcal{L}_Y = {Y_{10}}^{ij} 5_H 10_i 10_j + {Y_5}^{ij} 5_H^* 10_i \bar{5}_j + \ldots$$

GOOD: it has only 2 Yukawa matrices (in SM 3)

BAD:

- $M_D = M_E$ approximately correct, but not precise
- no neutrino mass either, or if we add an SU(5) singlet ν^c

$$\ldots = {Y_\nu}^{ij} 5_H \bar{5}_i \nu^c_j$$

the same problem as in the SM: no relation between neutral sector Y_ν and charged sector Y_{10}, Y_5

Yukawa: a bit better in SO(10)

The situation here more promising than in SU(5):

$$16 = (\underbrace{Q, L, u^c, d^c, e^c}_{SU(5)}, \nu^c)$$

ν^c automatically included, so neutrino masses nonzero and related to other fermion masses

However in the minimal model

$$\mathcal{L}_Y = Y_{10}^{ij} 10_H 16_i 16_j + Y_{126}^{ij} 126_H 16_i 16_j$$

the fit turns out not to work

The reason is that 10_H is a real representation and so it contains only one Higgs doublet (i.e. one vev)

Possible solutions are for example

1. add another (real) $10'_H$ with extra Yukawa

$$\delta\mathcal{L}_Y = Y^{ij}_{10'} 10'_H 16_i 16_j$$

   But now 3 Yukawa matrices (Y^{ij}_{10}, Y^{ij}_{126}, $Y^{ij}_{10'}$), not predictive

2. add an extra U(1) symmetry (for example a Peccei-Quinn global) so that 10_H is now automatically complex

   But now the symmetry is not SO(10) but instead SO(10)×U(1), i.e. it is not minimal

3. Another possibility is to supersymmetrise: 10_H is then automatically complex; but again non minimal, $SO(10)\times$ supersymmetry

Fits of fermion masses and mixings work well for complex $10 + 126$

However the minimal theory does not have a complex 10

Yukawa: good in E_6

The fundamental representation is the complex 27

In the decomposition $E_6 \to SO(10) \times U(1)$ we have

$$27 = 16(1) + 10(-2) + 1(4)$$

So 10 in SO(10) coming from 27 of E_6 is automatically complex

E_6 automatically contains the extra U(1) that was needed (but missing) in the minimal SO(10)

This is the first motivation for using E_6

Why E_6? Second reason: dark matter

It is well known that R-parity is a Z_2 symmetry which in MSSM

- forbids dangerous baryon (and lepton) number violation in operators $d = 3$
- the lightest particle odd under it is stable; and thus a dark matter (DM) candidate (neutralino)

It is a bit less known that such a symmetry has actually nothing really to do with supersymmetry

SO(10)

Here matter symmetry (M)

$$
\begin{aligned}
16 &\to -16 \\
10 &\to 10
\end{aligned}
$$

is an SO(10) group element (center of SO(10)). If only Higgses with even matter parity gets VEV, this Z_2 remains exact and the lightest parity odd state is stable.

In the previous example

$$R = M(-1)^S$$

where S is spin

If the lightest scalar is the weak doublet from the parity odd 16, then this inert Higgs doublet is the dark matter providing its mass is

$$m \sim 500 \text{ GeV}$$

In SO(10) we do not necessarily have this 16_H. In fact even if we have it, it usually gets a vev (this is the reason for having it). So in SO(10), although possible, this dark matter candidate looks a bit ad hoc, it is added just for that, not automatic.

It is somehow like addition of right-handed neutrinos in SM or SU(5) to get neutrino masses: possible but not automatic like in SO(10) or E_6

E_6

But this same symmetry is obviously present also in E_6. In fact

$$E_6 \to SO(10) \times U(1)$$

and SO(10) irreps with even (odd) $U(1)$ charge are even (odd) under M-parity

E_6 irreps with dimension ≤ 1000, branching rules:

$$
\begin{aligned}
27 &= 10(2) + 16(-1) + 1(-4) \\
78 &= 1(0) + 45(0) + 16(3) + \overline{16}(-3) \\
351 &= 10(2) + \overline{16}(5) + 16(-1) + 45(-4) + 120(2) + 144(-1) \\
351' &= 1(8) + 10(2) + \overline{16}(5) + 54(-4) + 126(2) + 144(-1) \\
650 &= 1(0) + 10(6) + 10(-6) + 16(3) + \overline{16}(-3) + 45(0) + 54(0) \\
&+ 144(3) + \overline{144}(-3) + 210(0)
\end{aligned}
$$

Spinorial irreps are M-parity odd

If 16 and 144 do not get non-zero VEV

$\rightarrow$ a Z_2 symmetry remains exact and the lightest odd guy is a dark matter candidate. In our case this will be an inert Higgs doublet from 16 or 144 with mass ~ 500 GeV.

In E_6 these spinorial irreps automatically included already in 27_H which is there because of the Yukawa.

This as another motivation for using E_6.

The Yukawa sector

What are the possible Yukawas in E_6?

$$27 \times 27 = \overline{27} + 351 + 351'$$

The minimal Yukawa thus seems to be

$$\mathcal{L}_Y = Y_{27}^{ij}\, 27_i\, 27_H\, 27_j + Y_{351'}^{ij}\, 27_i\, 351'^*_H\, 27_j$$

Y_{27}^{ij}, $Y_{351'}^{ij}$ ... 3×3 symmetric Yukawa matrices

351 seems less promising since the Yukawa matrix is antisymmetric

E_6 compared to SO(10): $27 \leftrightarrow 10 \quad , \quad 351' \leftrightarrow 126 \quad , \quad 351 \leftrightarrow 120$

On top of the usual SM model particle we have an extra $5 + \bar{5}$ plus two SM singlets:

$$27 = \underbrace{16}_{10+\bar{5}+1} + \underbrace{10}_{5+\bar{5}} + 1$$

$$\begin{aligned}
10 &= \begin{pmatrix} u & u^c & d & e^c \end{pmatrix} \\
\bar{5} &= \begin{pmatrix} d^c & e & \nu \end{pmatrix} \\
1 &= \begin{pmatrix} \nu^c \end{pmatrix} \\
5 &= \begin{pmatrix} d' & e'^c & \nu'^c \end{pmatrix} \\
\bar{5} &= \begin{pmatrix} d'^c & e' & \nu' \end{pmatrix} \\
1 &= \begin{pmatrix} n \end{pmatrix}
\end{aligned}$$

Under decomposition $E_6 \to SO(10) \times U(1)$

$$\begin{aligned} 27 &= 1(-4) + 10(2) + 16(-1) \\ 351' &= 1(8) + 10(2) + \overline{16}(5) + 54(-4) + 126(2) + 144(-1) \end{aligned}$$

If spinorial vevs are zero, nonzero vevs have only even U(1) charges

$$U(1) \to Z_2$$

Then the lightest scalar from spinorial Higgses is odd under Z_2 and thus stable

We arrange it to be an inert Higgs doublet $(1, 2, 1/2)$: a fine-tuning in the odd doublet matrix is needed (this is on top of the usual one to get a light SM Higgs)

$Z_2 \rightarrow$

- no mixing between the $\bar{5}$ of 16 and the $\bar{5}$ of 10 in 27

- the extra singlets decouple from the usual ν^c from 16

Only the SO(10) degrees of freedom remain

Due to extra doublet vevs the relations in E_6 are a bit less constrained than in SO(10)

Since in SO(10) there is a solution which fits data, so it is in E_6

The Higgs sector

Only 27_H and $351'_H$ needed for Yukawas enough also in the Higgs sector? A good solution not known

We need to add another Higgs E_6 multiplet:

- try first with 78: problems similar to 45 in SO(10), it contains pseudo-Goldstones at tree level, so 1-loop is needed
- we sill instead use 650_H, which vev $\langle 650_H \rangle \neq 0$ can bring the theory to interesting intermediate symmetries:

$$E_6 \to SO(10) \times U(1) \quad \text{or} \quad SU^3(3) \quad \text{or} \quad SU(6) \times SU(2)$$

The role of 27_H and $351'_H$ is then to break these intermediate symmetries down to the SM (on top of contributing to Yukawas)

The RGE

Once we found the symmetries of the intermediate scale we want to check which of them are realistic.

We consider

1. a single intermediate scale
2. the extended survival hypothesis: all multiplets which can be heavy are heavy except those which will take part to symmetry breaking; threshold corrections will slightly change this pattern:

$$\eta_{cr} \quad , \quad \eta_{fr}$$

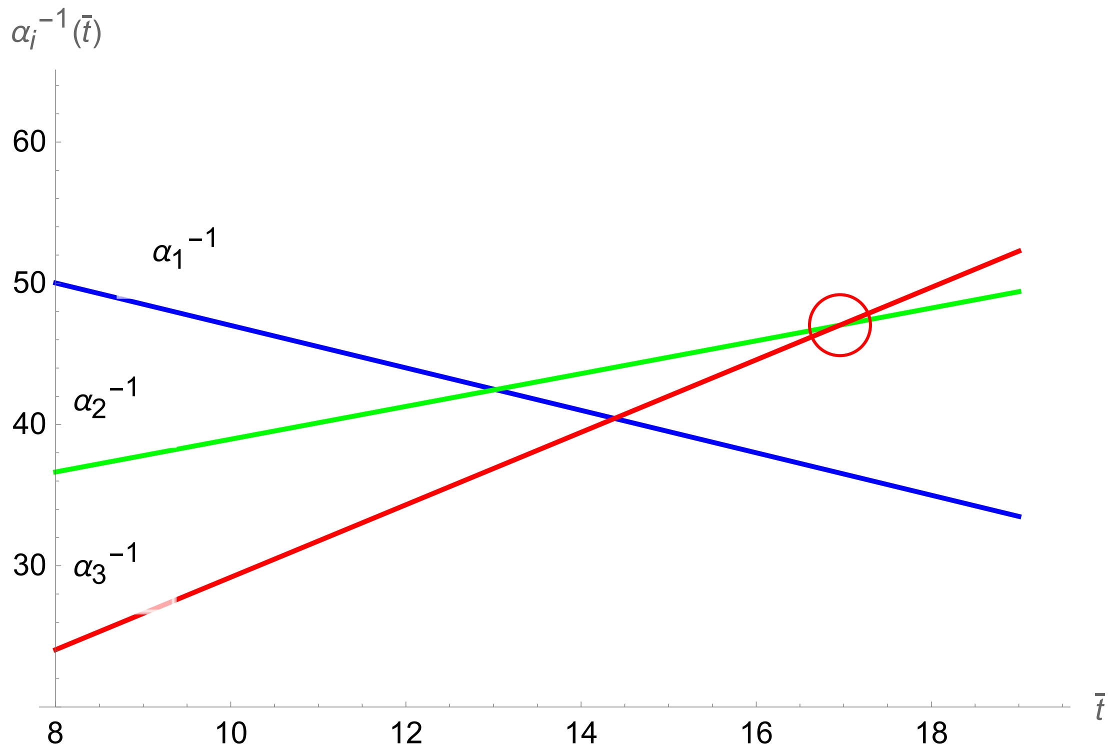
Intermediate symmetries considered (successfull, unsuccessfull):

1. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{LR}$
2. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CR}$
3. $SU(6)_{CR} \times SU(2)_L$
4. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CL}$
5. $SU(6)_{CL} \times SU(2)_R$
6. $SO(10)' \times U(1)'$

- the extra Z_2 parities above are automatic from 650_H, nothing to do with dark matter Z_2 mentioned before
- $SO(10)'$ is flipped SO(10) ($n \leftrightarrow e^c, d'^c \leftrightarrow u^c, \nu'^c \leftrightarrow \nu, e'^c \leftrightarrow e$)

The different behaviour is due to different conditions at intermediate scales ($\mu \to \bar{t} = \log_{10}\left(\frac{\mu}{1\text{ GeV}}\right)$)

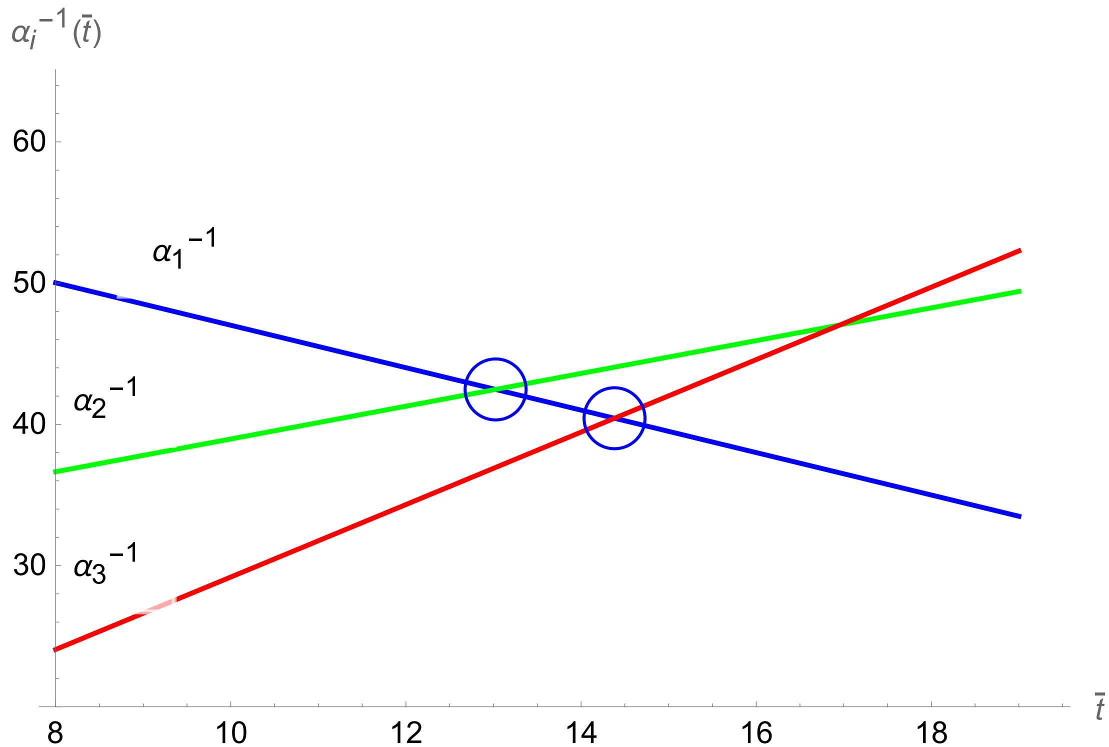
In the RED (unsuccessful) case this condition is $\alpha_2 = \alpha_3$



This happens quite high in energy so that in the meantime α_1^{-1} has decreased too much.

On the contrary the BLUE cases which work need unification

$\alpha_1 = c\,\alpha_2 + (1-c)\,\alpha_3 \quad , \quad 0 \le c \le 1$



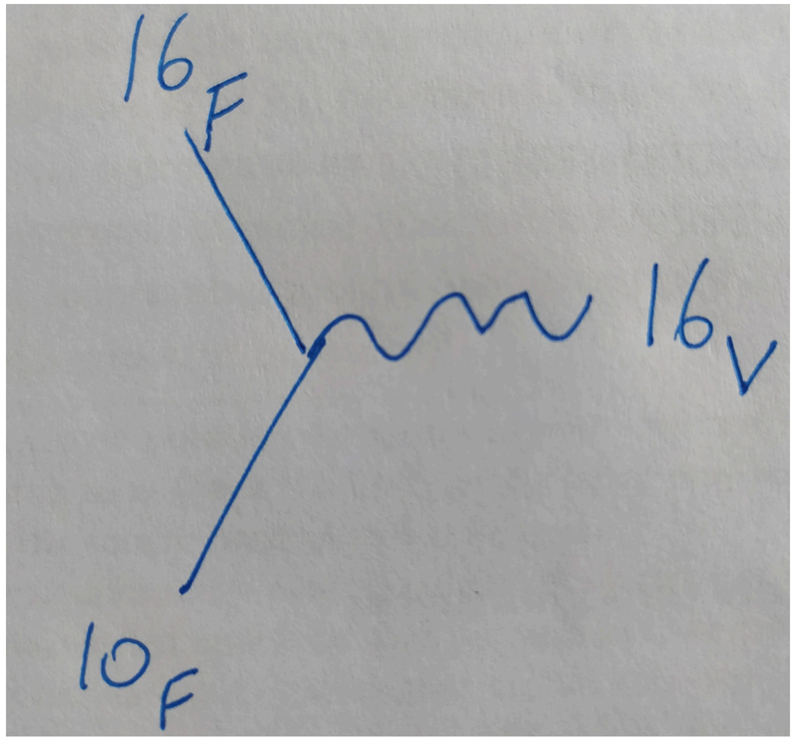
This happens much below the scale of α_2 and α_3 unification.

Proton decay

The dominant operator is $d = 6$ gauge mediated contribution

label	$3_C\, 2_L\, 1_Y$	SU(5)	SO(10)	E_6	ψ	comment
X	$\sim (\mathbf{3}, \mathbf{2}, -5/6)$	**24**	**45**	**78**	0	the SU(5) leptoquark
X'	$\sim (\mathbf{3}, \mathbf{2}, +1/6)$	**10**	**45**	**78**	0	the SO(10) leptoquark
X''	$\sim (\mathbf{3}, \mathbf{2}, +1/6)$	**10**	**16**	**78**	−3	the E_6 leptoquark

However X'' does not contribute if spinorial Z_2 vacuum is taken



The 10_F is heavy so 16_V does not contribute to proton decay

In SU(5) there is no X':

$$\frac{\mathcal{B}_{SU(5)}(p^+ \to \pi^0 e^+)}{\mathcal{B}_{SU(5)}(p^+ \to \pi^+ \bar{\nu})} \approx \frac{5}{2}$$

In E_6 we find $M_X = M_{X'}$:

$$\frac{\mathcal{B}_{E_6}(p^+ \to \pi^0 e^+)}{\mathcal{B}_{E_6}(p^+ \to \pi^+ \bar{\nu})} \approx 1$$

This differentiates between the minimal $SU(5)$ and E_6 scenario

Conclusions

- the minimal grand unified theory of the form $GUT \times$ nothing for fermion masses and dark matter is E_6
- assuming extended survival hypothesis we found 3 possible realistic intermediate symmetries by $\langle 650_H \rangle$:
  1. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{LR}$
  2. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CR}$
  3. $SU(6)_{CR} \times SU(2)_L$
- quark and lepton masses and mixings can be properly described on the same footing with only 2 Yukawa matrices
- the dark matter candidate is the inert Higgs doublet
- proton decay differentiates between E_6 and $SU(5)$

Backup slides

Few facts about E_6

- it is a rank 6 Lie group
- the algebra has 78 generators (78 is the adjoint representation)
- the fundamental representation is 27
- each irreducible representation can be denoted in tensor notation as

$$\phi^{\alpha_1\alpha_2\ldots}_{\beta_1\beta_2\beta_3\ldots} \quad , \quad \alpha_i, \beta_i = 1, \ldots, 27$$

- invariant tensors

  $d^{\alpha\beta\gamma}$, $d_{\alpha\beta\gamma}$ ... completely symmetric made out of 0, ± 1, and $= 0$ if any two indices the same

- Irreducible representations we will need:

  27^α

  $351'^{\alpha\beta}$ … two-index symmetric with $d_{\alpha\beta\gamma}351'^{\beta\gamma}=0$

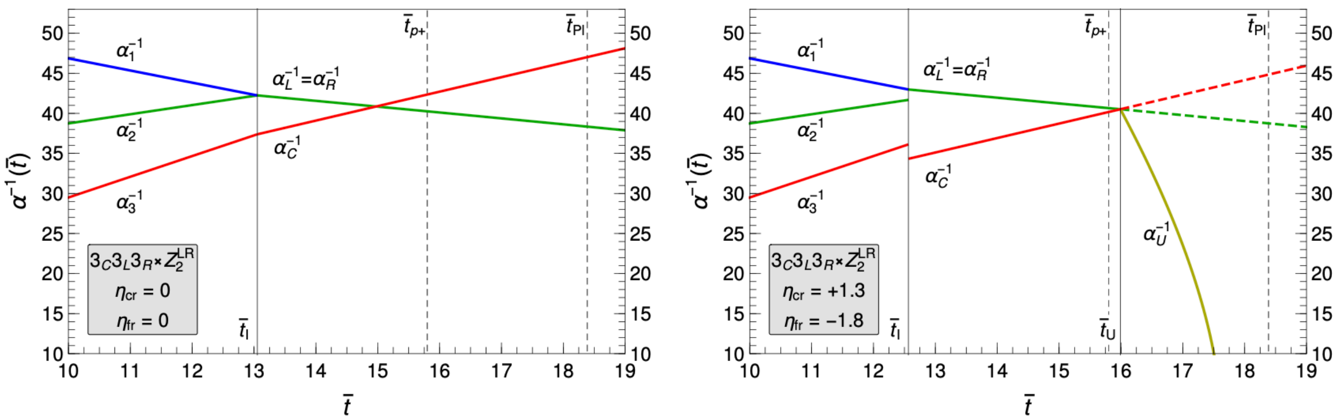
  $650^\alpha{}_\beta$ … two indices with $650^\alpha{}_\beta\left(T^A\right)^\beta{}_\alpha=0 \quad , \quad A=1,\ldots,78$

- invariants are made out of products of irreducible representations $\phi^{\alpha_1\alpha_2\ldots}_{\beta_1\beta_2\beta_3\ldots}$ and $d^{\alpha\beta\gamma}$, $d_{\alpha\beta\gamma}$ with each index up is paired with an index down (and implicitly summed over)

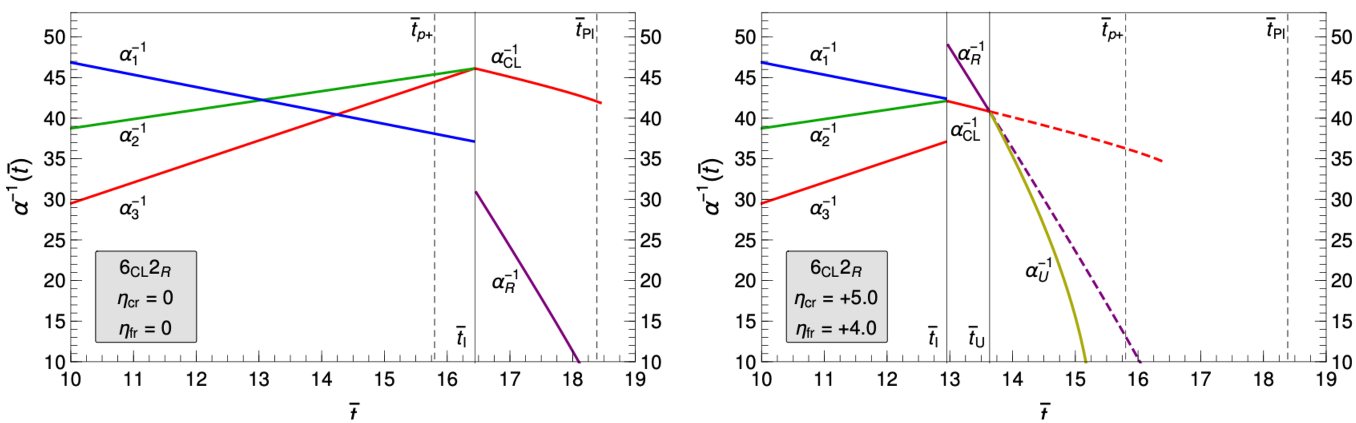
  Example:

$$d_{\alpha\beta\gamma}27^\alpha 27^\beta 27^\gamma \quad , \quad 27^\alpha 27^\beta 351'^*_{\alpha\beta} \quad , \quad \ldots$$

GOOD:


α_1^{-1}
α_2^{-1}
α_3^{-1}
$\alpha_L^{-1}=\alpha_R^{-1}$
α_C^{-1}
α_U^{-1}
$\bar{t}_{p+}$
$\bar{t}_{\rm Pl}$
$\bar{t}_{\rm I}$
$\bar{t}_{\rm U}$
$3_C 3_L 3_R \times Z_2^{\rm LR}$
$\eta_{\rm cr} = 0$
$\eta_{\rm fr} = 0$
$\eta_{\rm cr} = +1.3$
$\eta_{\rm fr} = -1.8$
$\alpha^{-1}(\bar{t})$
$\bar{t}$

BAD:


α_1^{-1}
α_2^{-1}
α_3^{-1}
$\alpha_{\rm CL}^{-1}$
α_R^{-1}
α_U^{-1}
$\bar{t}_{p+}$
$\bar{t}_{\rm Pl}$
$\bar{t}_{\rm I}$
$\bar{t}_{\rm U}$
$6_{\rm CL} 2_R$
$\eta_{\rm cr} = 0$
$\eta_{\rm fr} = 0$
$\eta_{\rm cr} = +5.0$
$\eta_{\rm fr} = +4.0$
$\alpha^{-1}(\bar{t})$
$\bar{t}$

Other vacua?

Yes, in principle possible

- $SU(3) \times G_2$
- F_4
- $\dots$

Possible to find benchmark points in the parameter space in which each of these minima becomes ”global”

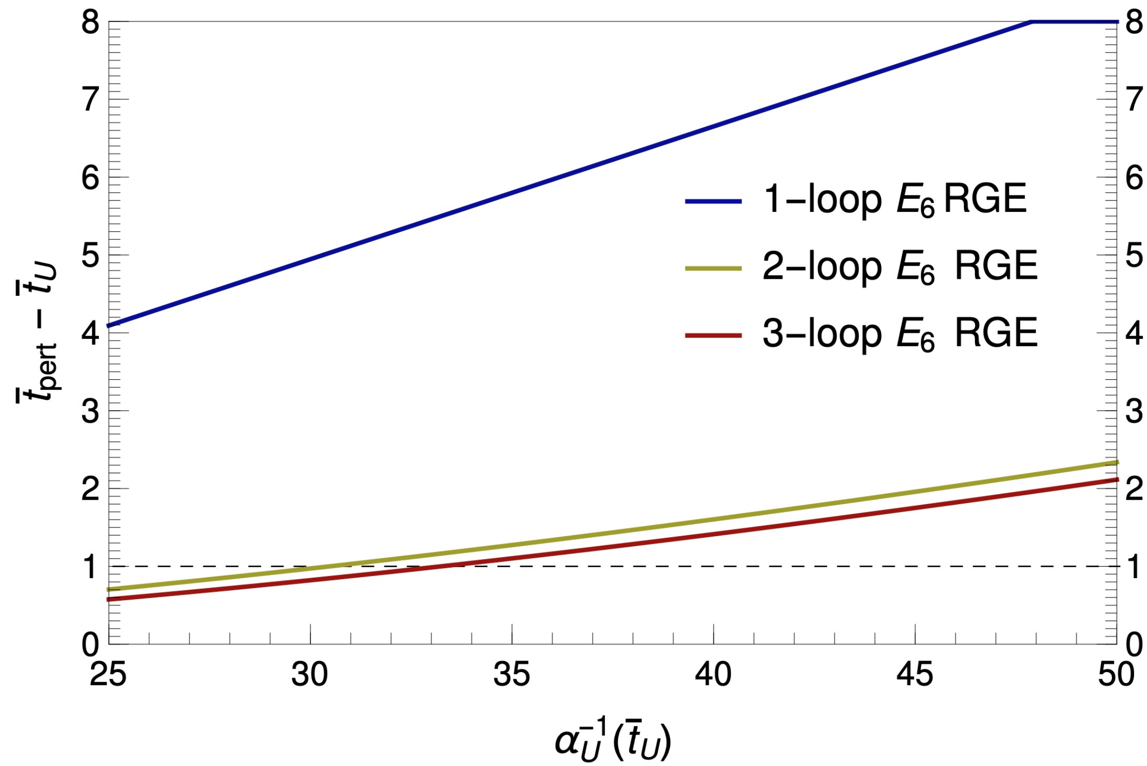
Perturbativity

large representations $\rightarrow$ possible problems

$$\frac{d}{dt}\alpha^{-1} = -\frac{1}{2\pi}\left(a + b\left(\frac{\alpha}{4\pi}\right) + c\left(\frac{\alpha}{4\pi}\right)^2 + \ldots\right)$$

$$a = 16 \quad , \quad b = 11956 \quad , \quad c = 560730$$

a anomalously small $\rightarrow$ 2-loop important, higher loops less



typically $\alpha_U^{-1} \approx 40$

$\to$ Landau pole at 1 to 2 orders of magnitude above GUT

- perturbative unitarity: $Re(a_0) \leq 0.5$

largest-magnitude eigenvalue for the partial-wave coefficient

$$(a_0)^{max} = \frac{\alpha_U}{2\sqrt{78}}\sqrt{\frac{3\pi^2}{4} 27C(27)^2 + \sum_R \zeta(R) dim(R) C(R)^2}$$

$C(R) \ldots$ Casimir of R

$\zeta(R) = 1/2$ (real) or 1 (complex) irrep R

In our case $(a_0)^{max} = 0.69$ but with the approximation of massless particles

$\rightarrow \approx$ 1 order of magnitude above M_{GUT} the theory is non-unitary