Workshop on Grand Unified Theory, Phenomenology and Cosmology

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Testing Froggatt-Nielsen flavour models with gravitational waves

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mainly based on arXiv:2410.08668
with S. Blasi, A. Mariotti, K. Turbang

Warning

I won't actually mention the word "GUT" in the following (sorry!)

but

- Flavour charge assignments compatible with a GUT embedding (e.g. SU(5)-invariant) are possible in the FN models I will discuss

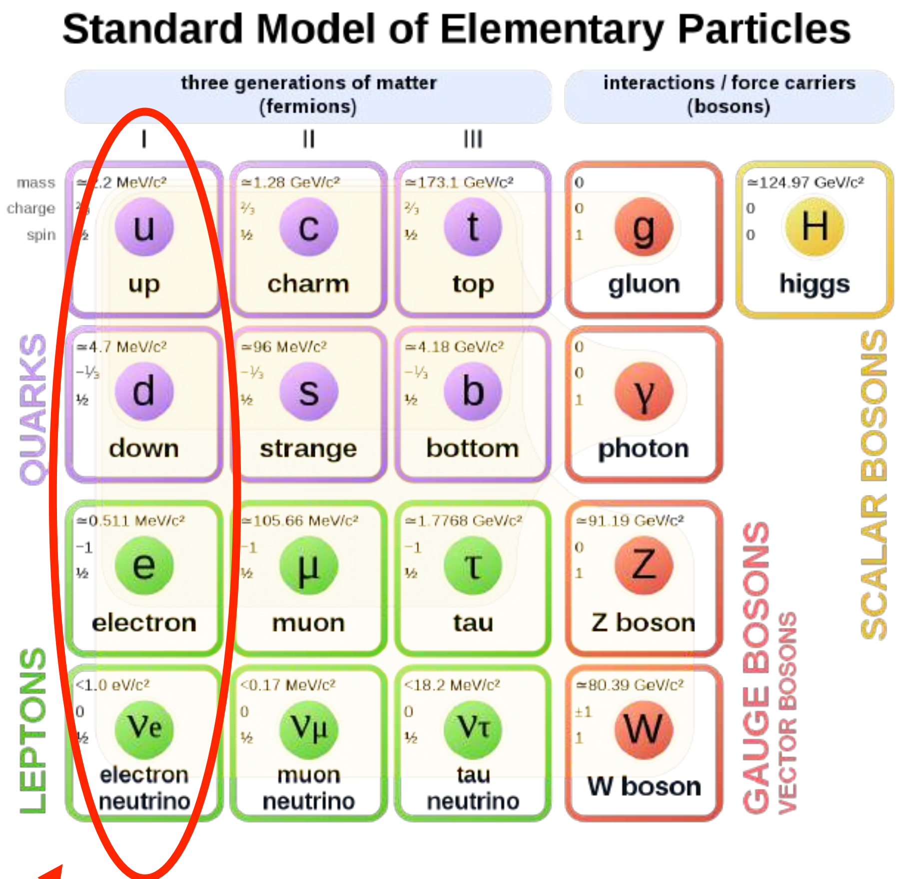
  e.g. Chankowski et al '05

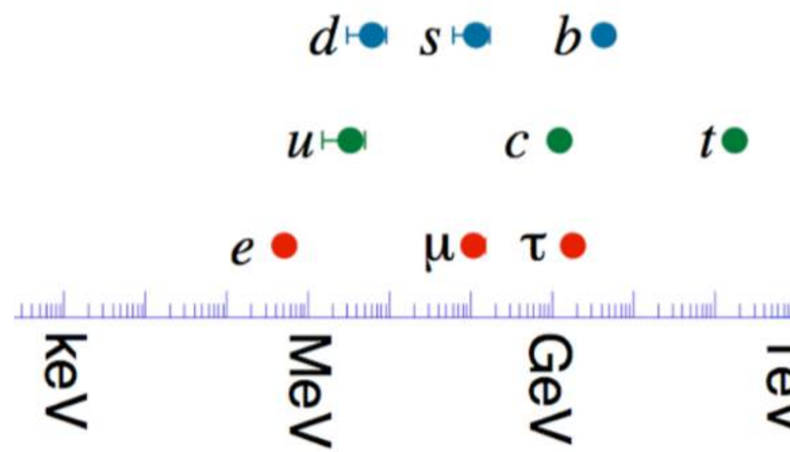
- A similar analysis could be done and similar results are expected for non-abelian flavour symmetries explicitly compatible with GUT

  e.g. Linster Ziegler '18

- The following is another illustration of the capability of GW searches to test very large scales including those related to GUT breaking

  see for instance Dunsky et al '21, Zhou Ye-Ling and collaborators '20, '21, '23 see e.g. J. Zupan's review arXiv:1903.05062





3 fermion generations (or families)

Hierarchical fermion masses

You are here (why?)

(why?)

Flavor in the SM

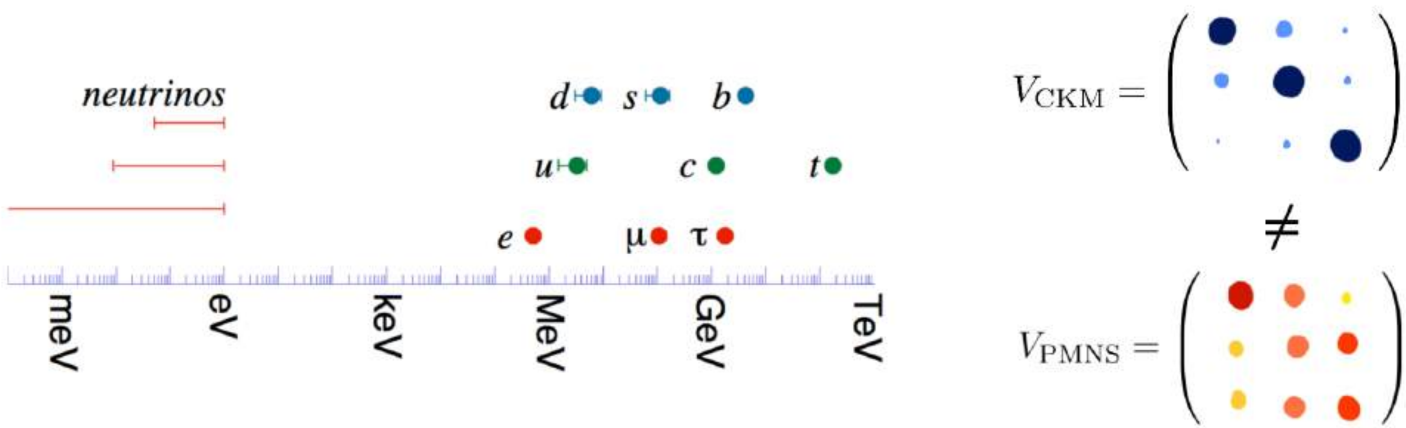
courtesy of O. Sumensari

- The SM **flavor sector** is **loose**: (*even w/o considering neutrinos*)

⇒ 13 free parameters (masses and quark mixing) — *fixed by data.*

$$\mathcal{L}_{\text{Yuk}} = -Y_d^{ij}\,\overline{Q}_i d_{Rj}\,H - Y_u^{ij}\,\overline{Q}_i u_{Rj}\,\widetilde{H} - Y_\ell^{ij}\,\overline{L}_i e_{Rj}\,H + \text{h.c.}$$

⇒ These (many) parameters exhibit a hierarchical structure which we do not understand.



How to **explain the observed patterns** in terms of **less** and **more fundamental parameters**?

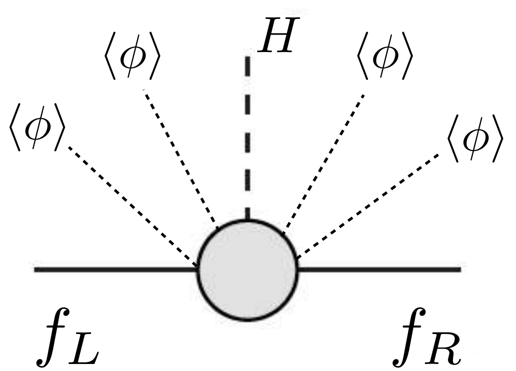
A possible solution: Froggatt-Nielsen flavour models

- SM fermions charged under a new horizontal symmetry G_F
- G_F forbids Yukawa couplings at the renormalisable level
- G_F spontaneously broken by the vev(s) of one or more scalars (the “flavons”)
- Yukawas arise as higher dimensional operators

Froggatt Nielsen ‘79
Leurer Seiberg Nir ’92, ’93
...

$$-\mathcal{L} = a^u_{ij}\left(\frac{\phi}{\Lambda}\right)^{n^u_{ij}} \overline{Q}_i u_j\, \tilde{H} + a^d_{ij}\left(\frac{\phi}{\Lambda}\right)^{n^d_{ij}} \overline{Q}_i d_j\, H$$

flavour-anarchical O(1) coefficients



$\langle\phi\rangle < \Lambda \;\Rightarrow\; \epsilon \equiv \langle\phi\rangle/\Lambda$ small expansion parameter (Λ=UV scale)

n^f_{ij} dictated by the symmetry

G_F could abelian or non-abelian, continuous or discrete, local or global

The simplest option: Froggatt-Nielsen U(1)

Quark sector

FN charges

	ϕ	Q_i	u_i	d_i	H
U(1)	1	$\mathcal{Q}_{Q_i}$	$\mathcal{Q}_{u_i}$	$\mathcal{Q}_{d_i}$	0

$\Rightarrow$

$$Y^u_{ij} = a^u_{ij}\,\epsilon^{\mathcal{Q}_{Q_i}-\mathcal{Q}_{u_j}}$$

$$Y^d_{ij} = a^d_{ij}\,\epsilon^{\mathcal{Q}_{Q_i}-\mathcal{Q}_{d_j}}$$

Rotation matrices $Y^f = V^{f\dagger}\hat{Y}^f W^f$ $\Rightarrow$ $V^{u,d}_{ij} \sim \epsilon^{|\mathcal{Q}_{Q_i}-\mathcal{Q}_{Q_j}|}$ $W^{u,d}_{ij} \sim \epsilon^{|\mathcal{Q}_{u_i,d_i}-\mathcal{Q}_{u_j,d_j}|}$

Successful predictions for $V_{\rm CKM} = V^u\,V^{d\,\dagger}$:

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx 1 \qquad V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

(independent of charge assignment)

Example:

$$(\mathcal{Q}_{Q_1},\,\mathcal{Q}_{Q_2},\,\mathcal{Q}_{Q_3}) = (3,\,2,\,0),\;\; (\mathcal{Q}_{u_1},\,\mathcal{Q}_{u_2},\,\mathcal{Q}_{u_3}) = (-4,\,-2,\,0),\;\; (\mathcal{Q}_{d_1},\,\mathcal{Q}_{d_2},\,\mathcal{Q}_{d_3}) = (-4,\,-2,\,-2)$$

$$Y^u \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^3 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad Y^d \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^5 \\ \epsilon^6 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \end{pmatrix} \quad V_{\rm CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

$\epsilon = \langle\phi\rangle/\Lambda \approx 0.2$

Lepton sector

$$-\mathcal{L} \supset \left[a^{\ell}_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_\ell} \right)^{\mathcal{Q}_{L_i} - \mathcal{Q}_{e_j}} \overline{L}_i e_j H + h.c. \right] + \kappa^{\nu}_{ij} \left(\frac{\langle \phi^* \rangle}{\Lambda_\ell} \right)^{\mathcal{Q}_{L_i} + \mathcal{Q}_{L_j}} \frac{(\overline{L^c_i}\, \tilde{H})(\tilde{H}^T L_j)}{\Lambda_N}$$

⇨ $Y^\ell = V^\ell \hat{Y}^\ell W^{\ell\dagger}, \qquad m^\nu = V^\nu \hat{m}^\nu V^{\nu T} \qquad V^{\ell,\nu}_{ij} \sim \epsilon_\ell^{\left|\mathcal{Q}_{L_i} - \mathcal{Q}_{L_j}\right|}, \qquad W^\ell_{ij} \sim \epsilon_\ell^{\left|\mathcal{Q}_{e_i} - \mathcal{Q}_{e_j}\right|}$

LH charges can chosen to give a (quasi-)anarchical $U_{\text{PMNS}} = V^\nu V^{\ell\,\dagger}$
RH charges then responsible for charged leptons hierarchy

Examples: Altarelli Feruglio Masina Merlo '12

- Anarchy $(\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L, \mathcal{Q}_L, \mathcal{Q}_L)$
- Mu-tau anarchy $(\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L + 1, \mathcal{Q}_L, \mathcal{Q}_L)$
- Hierarchy $(\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L + 2, \mathcal{Q}_L + 1, \mathcal{Q}_L)$

Charged lepton hierarchy, e.g. : $(\mathcal{Q}_{e_1}, \mathcal{Q}_{e_2}, \mathcal{Q}_{e_3}) = (\mathcal{Q}_L - 4, \mathcal{Q}_L - 2, \mathcal{Q}_L - 1)$
(with $\epsilon_\ell \approx \epsilon^2 \approx 0.04$)

Flavour non-universal **local** U(1) symmetry generating the hierarchies of fermion masses and mixing through the Froggatt-Nielsen mechanism

(anomalies cancelled by suitable UV completions Smolkovič Tammaro Zupan '19 Bonnefoy Dudas Pokorski '19)

Below the cutoff Λ, only **two** new particles:

$$\phi = \frac{v_\phi + \varphi}{\sqrt{2}} e^{ia/v_\phi}$$

longitudinal component of

Physical flavon

$$m_\varphi^2 = \frac{1}{2}\lambda_\phi v_\phi^2$$

$$\mathcal{L} = n_{ij}^f \frac{m_{ij}^f}{v_\phi} \overline{f}_i P_R f_j \varphi$$

U(1) gauge boson, *Z'*

$$m_{Z'} = \sqrt{2}\, g_F \langle\phi\rangle = g_F\, v_\phi$$

$$\mathcal{L} \supset g_F\, \overline{f} \gamma^\mu (\mathcal{Q}_{f_L} P_L + \mathcal{Q}_{f_R} P_R) f\, Z'_\mu$$

→ both fields decay into SM fermions and are produced in the early universe by thermal interactions (O(1) couplings with the fields at Λ)

→ we have to require their lifetime < 0.1 s in order not to affect **BBN**

Flavour-violating FN Z'

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Interactions of the new gauge boson Z' **flavour-violating** by construction:

$$\mathcal{L} \; = \; g_F \, Z'_\mu \, \Big[\overline{u}_\alpha \gamma^\mu (C^u_{L\,\alpha\beta} \, P_L + C^u_{R\,\alpha\beta} \, P_R) u_\beta \, + \overline{d}_\alpha \gamma^\mu (C^d_{L\,\alpha\beta} \, P_L + C^d_{R\,\alpha\beta} \, P_R) d_\beta \, + \overline{\ell}_\alpha \gamma^\mu (C^\ell_{L\,\alpha\beta} \, P_L + C^\ell_{R\,\alpha\beta} \, P_R) \ell_\beta \, + \bar{\nu}_\alpha \gamma^\mu \, C^\nu_{L\,\alpha\beta} \, P_L \nu_\beta \Big] \; ,$$

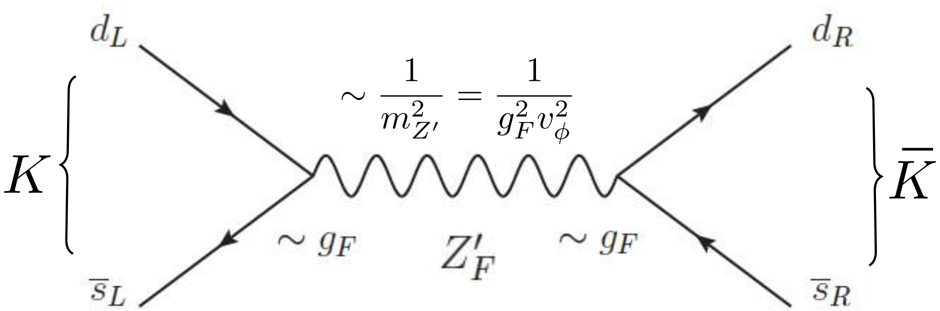
new U(1) gauge coupling

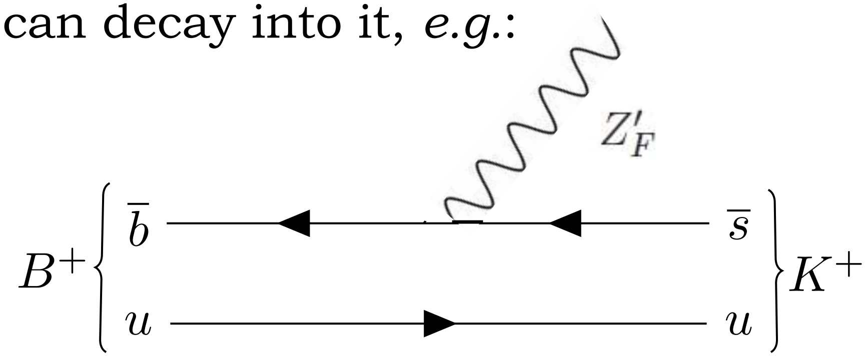
$$C^f_{L\,\alpha\beta} \equiv V^f_{\alpha i} \mathcal{Q}_{f_{L\,i}} V^{f\,*}_{\beta i} \qquad C^f_{R\,\alpha\beta} \equiv W^f_{\alpha i} \mathcal{Q}_{f_{R\,i}} W^{f\,*}_{\beta i} \qquad C^f_{V,A} = \frac{C^f_R \pm C^f_L}{2}$$

unitary rotations to the fermion mass basis

matrices of U(1) charges

⇨ Z' mediates flavour-violating processes and, if light, mesons and leptons can decay into it, *e.g.*:





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$$\mathcal{L} = g_F\, Z'_\mu \left[\overline{u}_\alpha \gamma^\mu (C^u_{L\,\alpha\beta} P_L + C^u_{R\,\alpha\beta} P_R) u_\beta + \overline{d}_\alpha \gamma^\mu (C^d_{L\,\alpha\beta} P_L + C^d_{R\,\alpha\beta} P_R) d_\beta + \overline{\ell}_\alpha \gamma^\mu (C^\ell_{L\,\alpha\beta} P_L + C^\ell_{R\,\alpha\beta} P_R) \ell_\beta + \bar{\nu}_\alpha \gamma^\mu C^\nu_{L\,\alpha\beta} P_L \nu_\beta \right],$$

new U(1) gauge coupling

$$C^f_{L\,\alpha\beta} \equiv V^f_{\alpha i} \mathcal{Q}_{f_{L\,i}} V^{f\,*}_{\beta i} \qquad C^f_{R\,\alpha\beta} \equiv W^f_{\alpha i} \mathcal{Q}_{f_{R\,i}} W^{f\,*}_{\beta i} \qquad C^f_{V,A} = \frac{C^f_R \pm C^f_L}{2}$$

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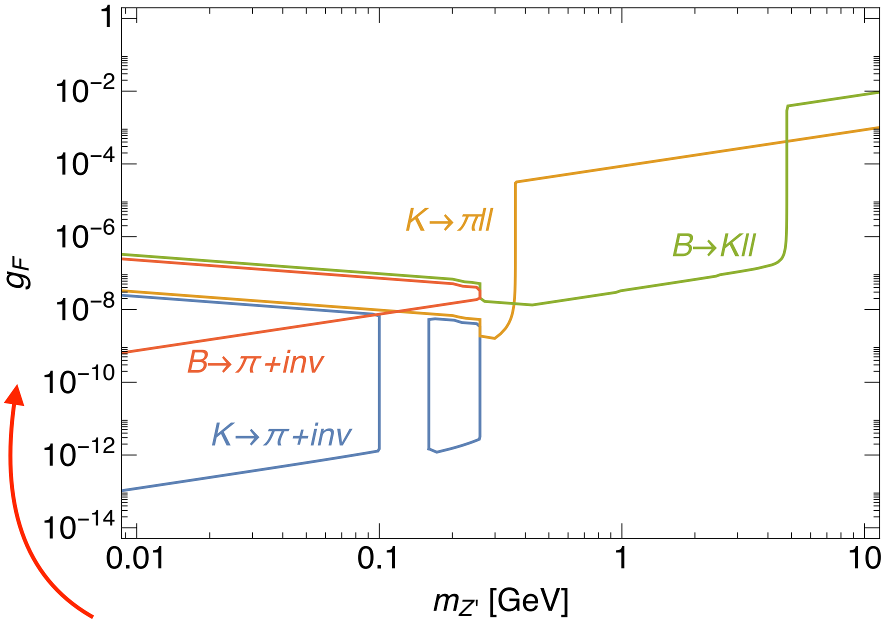
$$\mathrm{BR}(K^+ \to \pi^+ Z') = \frac{g_F^2}{16\pi\,\Gamma_K} \frac{m_K^3}{m_{Z'}^2} \left[\lambda\left(1, \frac{m_\pi^2}{m_K^2}, \frac{m_{Z'}^2}{m_K^2}\right)\right]^{\frac{3}{2}} [f_+(m_{Z'}^2)]^2 |C^d_{V\,sd}|^2$$

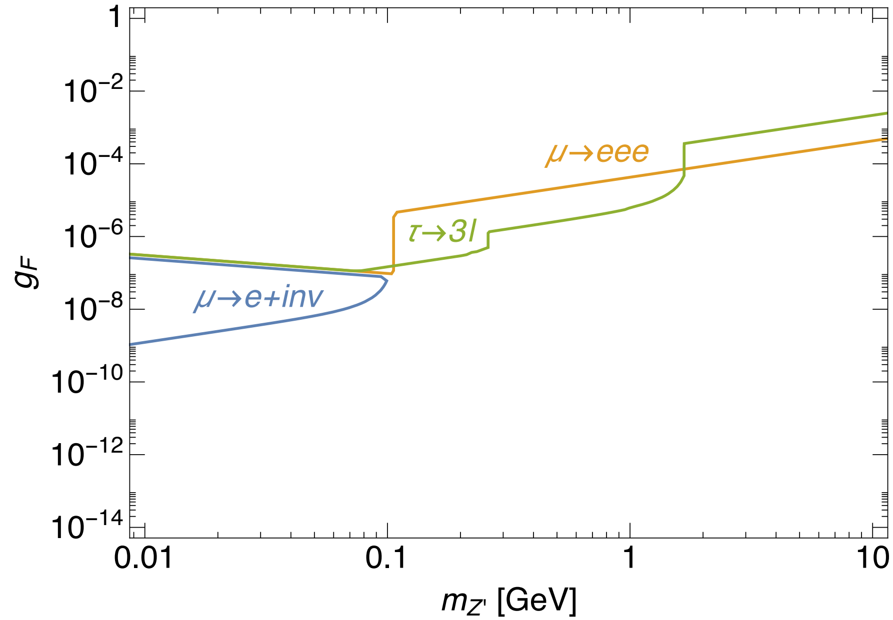
$$\mathrm{BR}(\ell_\alpha \to \ell_\beta Z') = \frac{g_F^2}{16\pi\,\Gamma_{\ell_\alpha}} \frac{m_{\ell_\alpha}^3}{m_{Z'}^2} \left(|C^\ell_{V\,\alpha\beta}|^2 + |C^\ell_{A\,\alpha\beta}|^2\right) \left(1 + 2\frac{m_{Z'}^2}{m_{\ell_\alpha}^2}\right) \left(1 - \frac{m_{Z'}^2}{m_{\ell_\alpha}^2}\right)^2$$

Flavour-violating FN Z' : flavour bounds

Meson decays into Z'



Lepton decays into Z'



U(1) coupling

$$m_{Z'} \;=\; \sqrt{2}\, g_F \langle\phi\rangle = g_F\, v_\phi$$

Z' boson mass

Flavour processes set stringent **lower bounds** on the U(1) breaking scale

$$K^+ \to \pi^+ Z' :\; v_\phi \gtrsim\; 8.3\times 10^{10}\ \text{GeV}\,, \qquad B^+ \to K^+ Z' :\; v_\phi \gtrsim\; 3.0\times 10^{7}\ \text{GeV}$$

$$\mu \to e Z' :\; v_\phi \gtrsim\; 1.3\times 10^{7}\ \text{GeV}\,, \qquad \tau \to \ell Z' :\; v_\phi \gtrsim\; 7.6\times 10^{5}\ \text{GeV}$$

$$K - \bar{K}\ \text{mix.} \;:\; v_\phi \gtrsim\; 6.5\times 10^{5}\ \text{GeV}$$

see also Smolkovič Tammaro Zupan '19

What if the U(1) breaking occurs at higher energies?

A new promising direction: **gravitational waves** (GW)

U(1) breaking → cosmic strings → emission of a GW background!

Kibble '76 (for a review: Vilenkin Shellard '00)

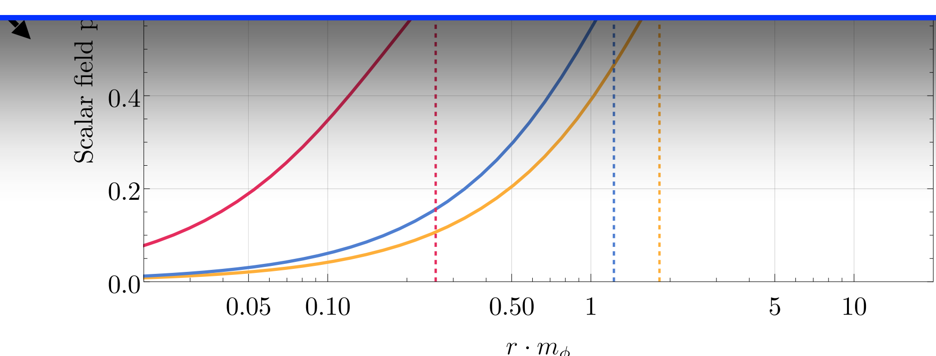
Key assumptions:

- After inflation, the universe reheats with $T_{\rm RH} > v_\phi$

  ⇨ FN U(1) unbroken in the early universe

- At $T \sim v_\phi$ the universe undergoes a 2nd order phase transition

  ⇨ gauge strings form



flavon/Z' mass ratio squared

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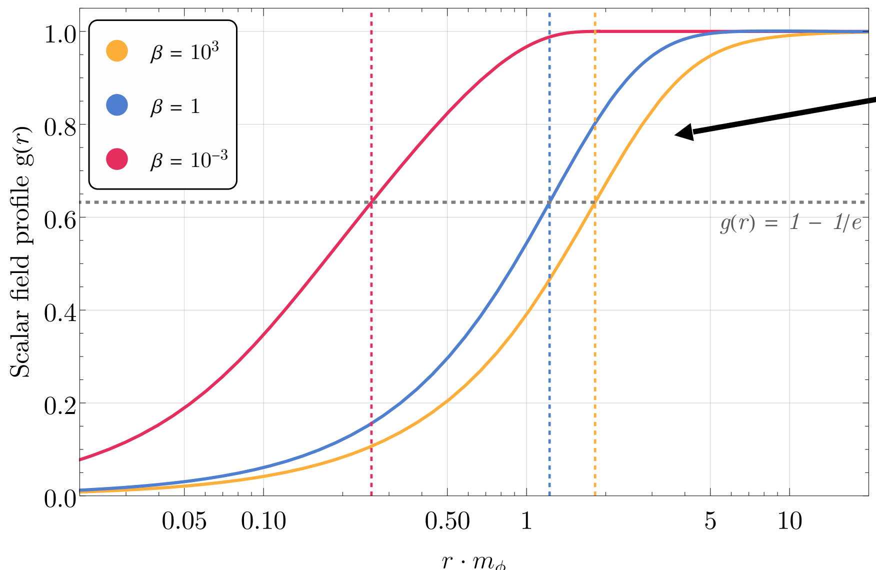
EoM: $D_\mu D^\mu \phi + \frac{\lambda_\phi}{2}\phi\left(\phi\phi^* - \eta^2\right) = 0\,, \qquad \partial_\mu F'^{\,\mu\nu} = 2g_F\;\mathrm{Im}\left(\phi^* D^\nu \phi\right)$

static, cylindrically symmetric solutions (strings):

$$\phi_s(\mathbf{r}) = e^{in\theta} g(r)\,, \qquad Z'_{s,\theta}(\mathbf{r}) = -\frac{n}{g_F r}\alpha(r)$$

string profile ($\sim\phi/v_\phi$)



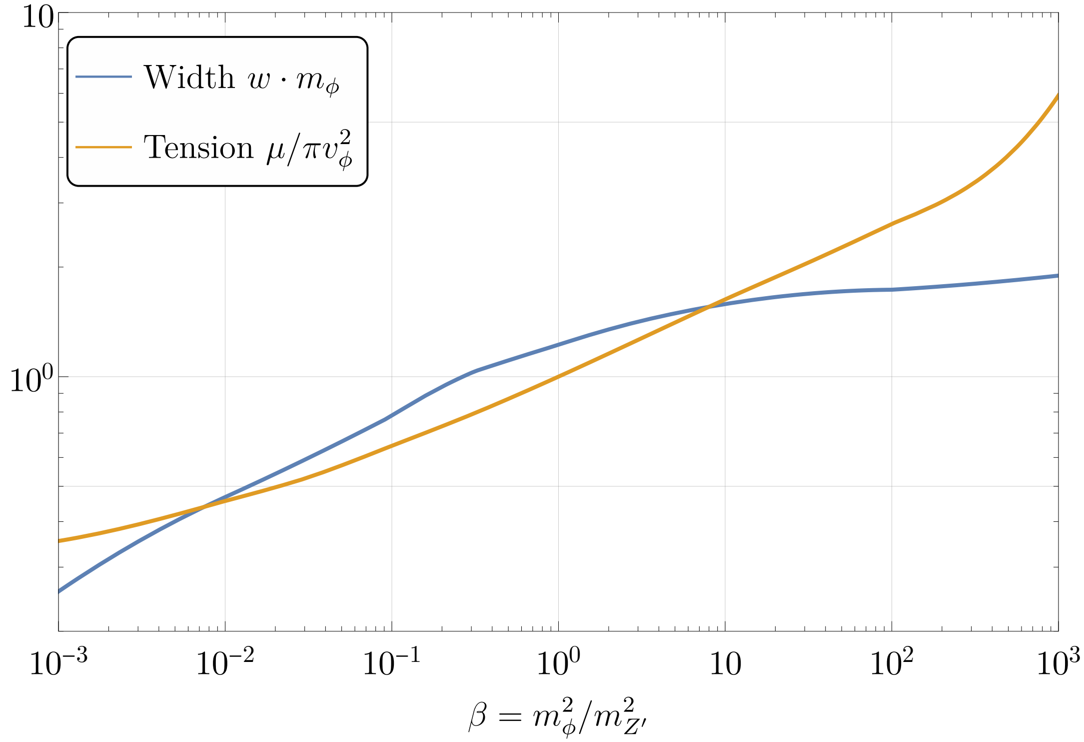
string width depends on

$$\beta \equiv \frac{m_\phi^2}{m_{Z'}^2} = \frac{\lambda_\phi}{2g_F^2}$$

flavon/Z' mass ratio squared

Numerical solutions for the string **width** and **tension**:

$$w = \frac{1}{m_\phi} W(\beta) \qquad\qquad G\mu = \frac{\pi v_\phi^2}{8\pi M_p^2} B(\beta)$$



$$\beta \equiv \frac{m_\phi^2}{m_{Z'}^2} = \frac{\lambda_\phi}{2g_F^2}$$

Numerical solutions for the string **width** and **tension**:

String tension (energy per unit length): $G\mu = \frac{\pi v_\phi^2}{8\pi M_p^2} B(\beta)$

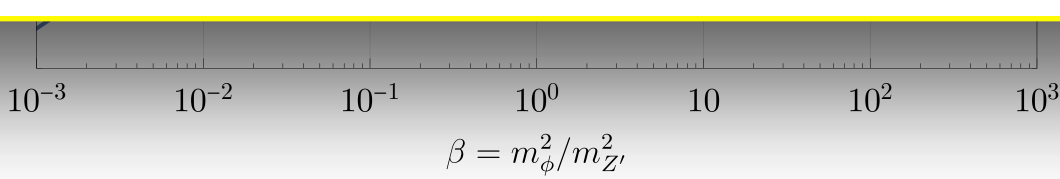
it grows quadratically with the U(1) breaking scale

String loops and string network collisions emit GWs

⇨ stochastic GW background with frequency spectrum

$$\Omega_{\rm GW}(f) = \sum_{k=1}^{\infty} \Omega_{\rm GW}^{(k)}(f) = \frac{8\pi}{3H_0^2}(G\mu)^2 f \sum_{k=1}^{\infty} C_k(f) P_k$$

Larger signal for larger tension (higher U(1) breaking scales)

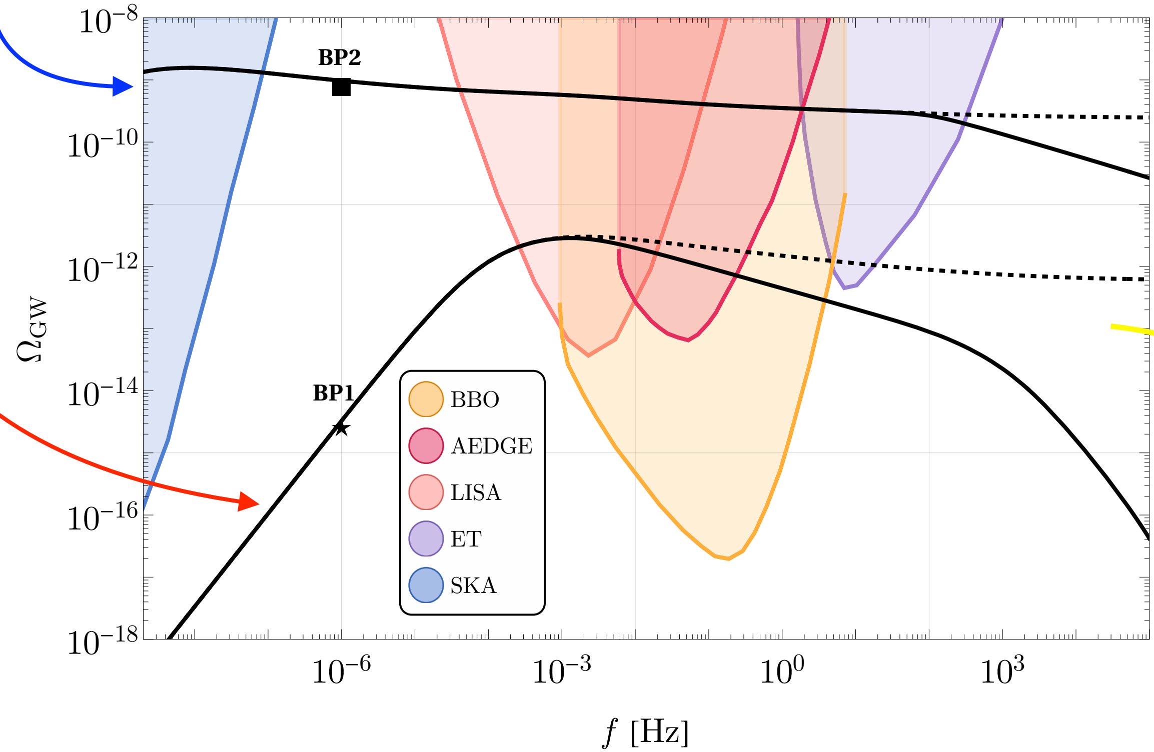


$$= \frac{\lambda_\phi}{2g_F^2}$$

Illustrative GW spectra

BP1 $m_{Z'} = 2 \cdot 10^2$ GeV, $g_F = 10^{-9}$, $\beta = 1$, $v_\phi = \frac{m_{Z'}}{g_F} = 2 \cdot 10^{11}$ GeV

BP2 $m_{Z'} = 10^7$ GeV, $g_F = 10^{-7}$, $\beta = 1$, $v_\phi = \frac{m_{Z'}}{g_F} = 10^{14}$ GeV



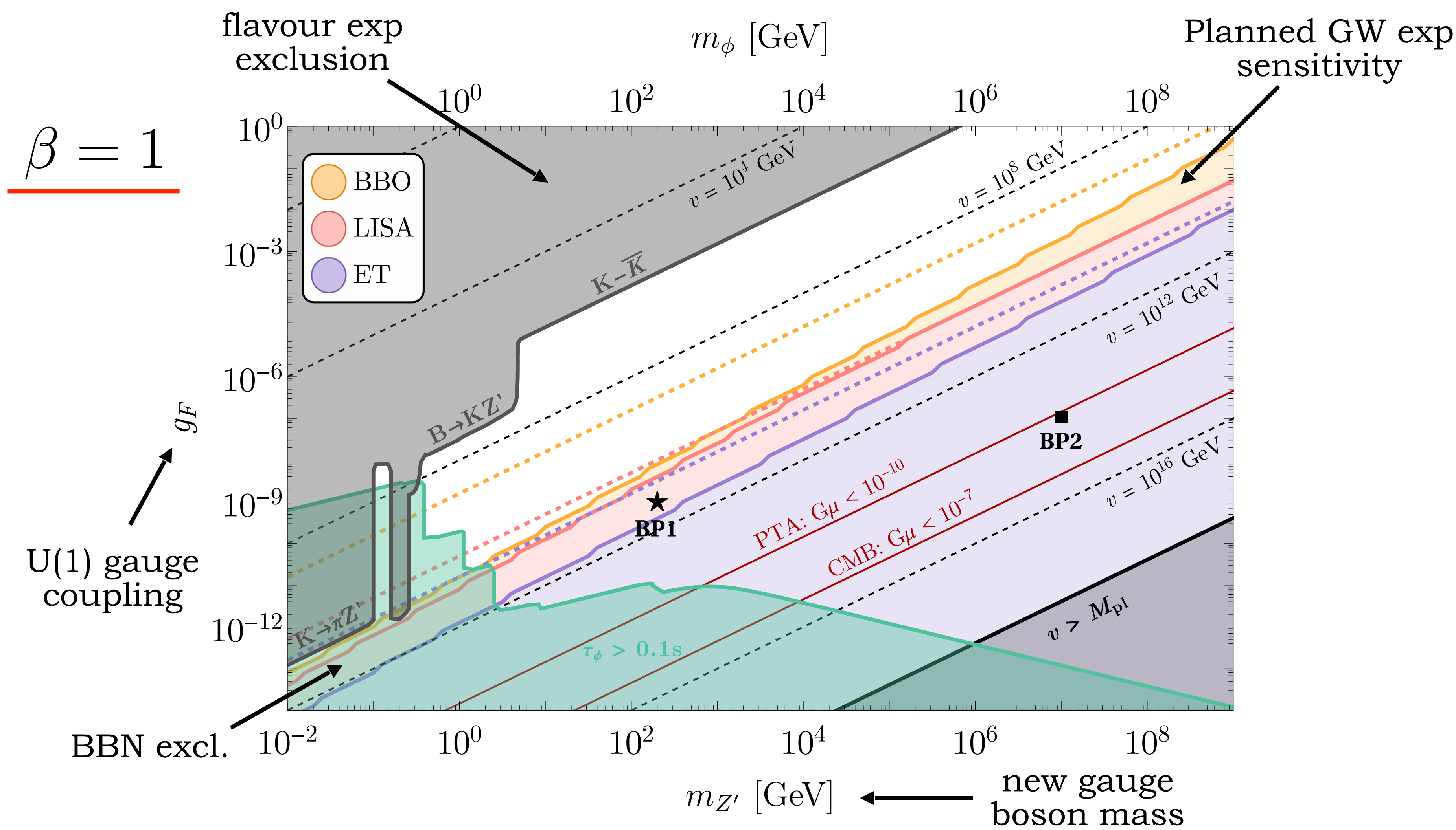
high-frequency cut-off due to large width effects

string loops lose energy mostly through particle (Z') emission below the critical size: $\ell < \ell_c \sim \frac{w}{(\Gamma G\mu)^2}$

Matsunami et al '19

Flavour limits vs future GW sensitivities

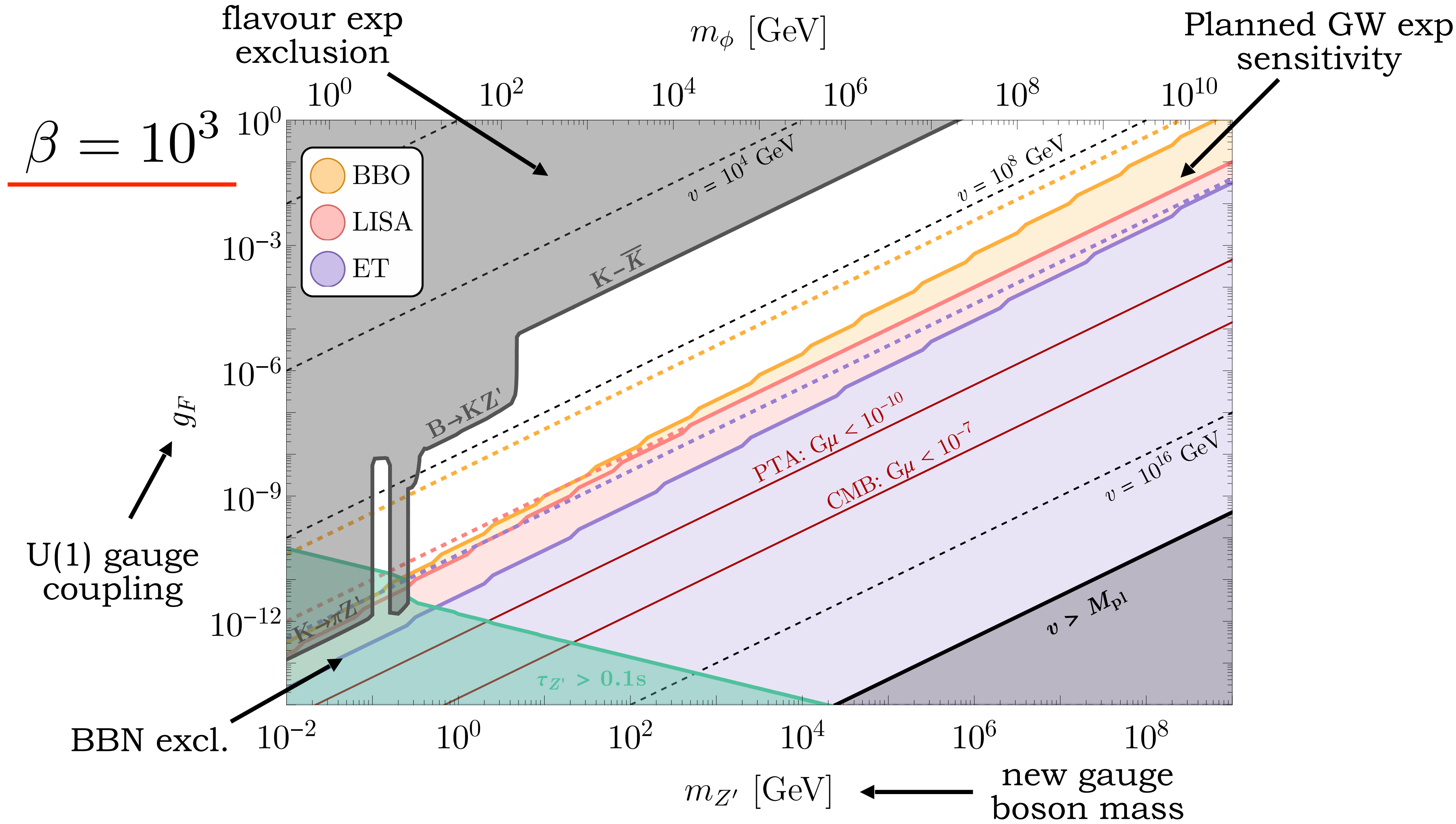
$\beta = 1$



GW and flavour exps. interplay can (almost) close the parameter space!

Flavour limits vs future GW sensitivities

$\beta = 10^3$



GW and flavour exps. interplay can (almost) close the parameter space!

Summary and conclusions

We don't know the origin of the SM flavour sector (dynamical?)

It may involve energy scales unaccessible at lab experiments

Stochastic GW from cosmic strings might open a window on that

The example we discussed shows an interesting interplay:
Flavour processes probe low to intermediate scales, $\lesssim 10^6$-10^{11} GeV

Future GW observatories will test high to intermediate scales
$\gtrsim 10^{11}$ GeV (ET), $\gtrsim 10^9$ GeV (BBO)

Thank you!

大家谢谢!