Understanding Standard Model Gauge Group From SMEFT

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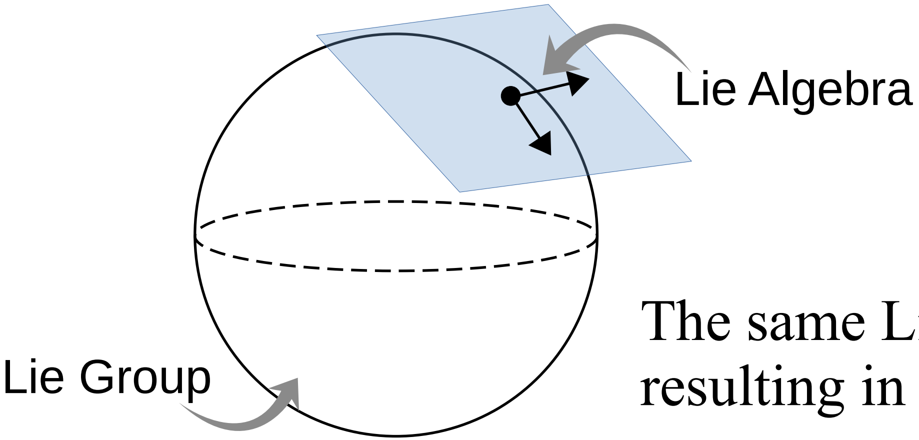

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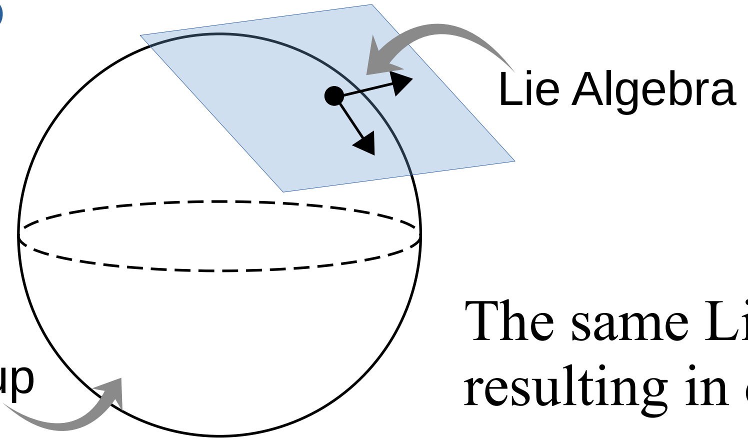

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Example: SU(2) versus SO(3) gauge theory

- They have the same Lie algebra
- But they are not exactly the same: $SO(3) \cong SU(2)/Z_2$

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- The consequence of the quotient:
  $SO(3)$ only has the integer representations of the Lie algebra
  $SU(2)$ has both the integer and half integer representations of the Lie algebra
- In general, $G = \tilde{G}/H$ has the same Lie algebra as $\tilde{G}$,
  where H is a subgroup of the center.
  The Lie algebra representation R is a representation of $G \Leftrightarrow H$ on R trivial.

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- When it's $SO(3)$, $\Gamma = Z_2$ should act trivially in the full theory, i.e. only heavy fields of integer representations can present.
- Distinguish $SU(2)$ and $SO(3)$ needs the discovery of at least one heavy particle of half integer representation.

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$$Z_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\} \qquad \alpha = \{e^{\frac{2\pi i}{3}} \mathbb{1}_{3\times 3}, e^{\pi i} \mathbb{1}_{2\times 2}, e^{\frac{2\pi i}{6}}\}$$

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- In the Standard Model this generator acts as

$$U_\alpha(R_3, R_2, Q_Y) = e^{\frac{2\pi i}{3}\mathcal{N}(R_3) + i\pi\mathcal{N}(R_2) + \frac{2\pi i}{6}(6Q_y)} = e^{2\pi i\left(\frac{\mathcal{N}(R_3)}{3} + \frac{\mathcal{N}(R_2)}{2} + Q_Y\right)}$$

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- Therefore Z_6 trivial indicates that

$$\mathcal{N}(R_3) = 6Q_Y \mod 3 \quad \text{and} \quad \mathcal{N}(R_2) = 6Q_Y \mod 2$$

- One can verify all SM matter fields satisfy this constraint

$q_L : (\mathbf{3}, \mathbf{2}, 1/6)$ $\qquad \mathcal{N}(\mathbf{3}) = \mathcal{N}(\mathbf{2}) = 6Q_Y = 1$

Which Standard Model

- There are four Standard Model

[Group Structure of Gauge Theories O'Raifeartaigh, 1986]
[D. Tong 1705.01853]

$$G = \frac{SU(3) \times SU(2) \times U(1)}{\Gamma} \qquad \Gamma = \{1, Z_2, Z_3, Z_6\}$$

- Here Z_2 and Z_3 are the two non-trivial subgroup of Z_6, which are generated by α^3 and α^2 respectively. They act trivially when

$$Z_2 : \mathcal{N}(R_2) = 6Q_Y \mod 2$$

$$Z_3 : \mathcal{N}(R_3) = 6Q_Y \mod 3$$

- This tells different Γ have different constraints on the representation of the heavy particles, which may indicates different correlations on the Wilson coefficients in the low energy EFT.

Heavy Particles and SMEFT

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- Z_6 exotics cannot trigger the electroweak symmetry breaking

– It must be color singlet since color is not broken: $\mathcal{N}(R_3) = 0$

– (j, Q_Y) to have charge zero component

$$\mathcal{N}(R_2) = 2j \mod 2$$

$$-j \leq Q_Y \leq j \quad \text{and} \quad j - Q_Y \in \mathbb{Z}$$

– $\mathcal{N}(R_3) = 6Q_Y \mod 3 = 0 \to Z_3$ trivial

– $\mathcal{N}(R_2) - 6Q_Y = (\mathcal{N}(R_2) - 2Q_Y) - 4Q_Y = 0 \mod 2 \to Z_2$ trivial

Heavy Particles and SMEFT

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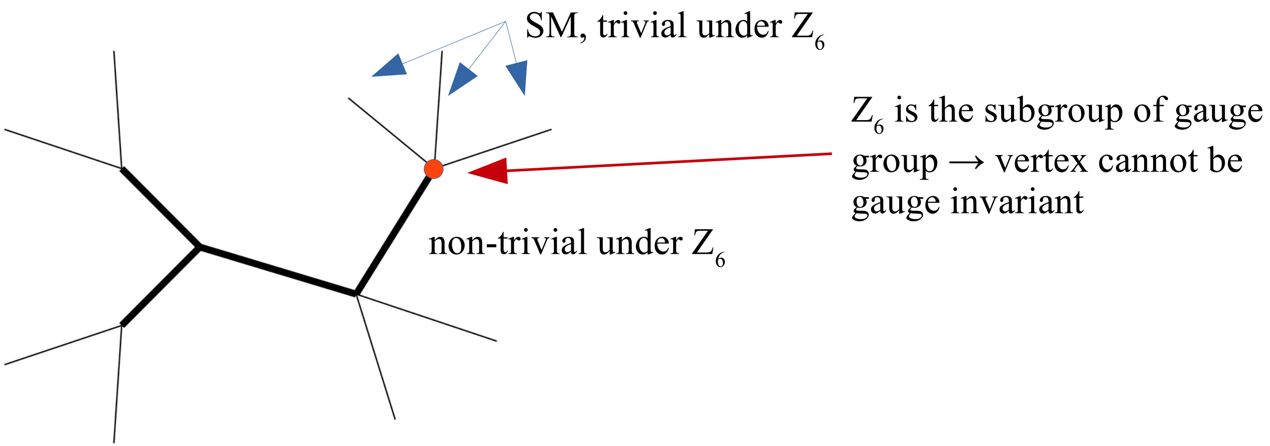
- It is easy to see that no SMEFT operator can be generated at tree level



Therefore loop generation must be considered

Heavy Particles and SMEFT

- Benchmark minimal model: adding one complex scalar:

$$\mathcal{L}_\phi \supset (D_\mu \phi^\dagger)(D^\mu \phi) - M^2 \phi^\dagger \phi - \lambda_{\mathbf{3}}(H^\dagger \sigma^I H)(\phi^\dagger T^I \phi) - \lambda_{\mathbf{1}}(H^\dagger H)(\phi^\dagger \phi)$$

- Integrate out the heavy complex scalar gives:

$$c_{3G} = \frac{g_3^3}{(4\pi)^2 180 M^2}\,\mu(R_3)\,d(R_2)\,,\quad c_{3W} = \frac{g_2^3}{(4\pi)^2 180 M^2}\,\mu(R_2)\,d(R_3)\,,$$

$$c_{HG} = \frac{g_3^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\,\mu(R_3)\,d(R_2)\,,\quad c_{HW} = \frac{g_2^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\,\mu(R_2)\,d(R_3)\,,$$

$$c_{HB} = \frac{g_1^2 Y_\phi^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\,d(R_2)\,d(R_3)\,,\quad c_{HWB} = \frac{g_1 g_2 Y_\phi \lambda_{\mathbf{3}}}{(4\pi)^2 6 M^2}\,\mu(R_2)\,d(R_3)\,,$$

$$c_{H\Box} = -\frac{1}{(4\pi)^2 12 M^2}\left[d(R_2)d(R_3)\left(\lambda_{\mathbf{1}}^2 + \frac{g_1^4 Y_\phi^2}{20}\right) + \mu(R_2)d(R_3)\left(\frac{3g_2^2}{80} - \lambda_{\mathbf{3}}^2\right)\right]$$

$$c_{HD} = -\frac{1}{(4\pi)^2 3 M^2}\lambda_{\mathbf{3}}^2 \mu(R_2)d(R_3) - \frac{g_1^4}{(4\pi)^2 60 M^2} Y_\phi^2 d(R_3)d(R_2),$$

$$c_{ee} \equiv c_{ee}^{pppp} = -\frac{g_1^4 Y_\phi^2}{(4\pi)^2 60 M^2} d(R_2)d(R_3),$$

$$c_{ll} \equiv c_{ll}^{pppp} = -\frac{1}{(4\pi)^2 240 M^2}\left[g_2^4 \mu(R_2)d(R_3) + g_1^4 Y_\phi^2 d(R_3)d(R_2)\right]$$

$\mu(R)$: Dynkin index

$d(R)$: dimension

Heavy Particles and SMEFT

- From these relations one can solve for the quantum numbers

R_3, R_2, Q_Y all non-trivial

$$Y_\phi^2 = \frac{g_2 c_{HWB}^2}{15 g_1^2 c_{3W}(c_{ee} - c_{HD})},$$
$$\frac{\mu(R_3)}{d(R_3)} = \frac{g_2 c_{HG} c_{HWB}^2}{15 g_3^2 c_{HB} c_{3W}(c_{ee} - c_{HD})},$$
$$\frac{\mu(R_2)}{d(R_2)} = \frac{c_{HW} c_{HWB}^2}{15 g_2 c_{HB} c_{3W}(c_{ee} - c_{HD})}.$$

at least one of R_3, R_2, Q_Y trivial

Representation	Solution
$\phi\ (\cdot, \cdot, Y_\phi)$	$Y_\phi^2 = \frac{4c_{HB}^2}{5(4c_{H\Box} - c_{HD})c_{HD}}$
$\phi(R_3, \cdot, 0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15 g_3 c_{H\Box} c_{3G}}$
$\phi(\cdot, R_2, 0)$	$\frac{\mu(R_2)}{d(R_2)} = \frac{80 c_{HW}^2 c_{ll}}{225 c_{3W}^2 \left[g_2^2(4c_{H\Box} + c_{HD}) - 3c_{ll}\right]}$
$\phi\ (R_3, \cdot, Y_\phi)$	$Y_\phi^2 = \frac{4 g_3^2 c_{HG}^2 c_{HD}}{45 g_1^4 c_{3G}^2 (4c_{H\Box} - c_{HD})}$ $\quad \frac{\mu(R_3)}{d(R_3)} = \frac{4 c_{HG}^3 c_{HD}}{45 g_1^2 c_{3G}^2 c_{HB}(4c_{H\Box} - c_{HD})}$
$\phi\ (\cdot, R_2, Y_\phi)$	$Y_\phi^2 = \frac{g_2 c_{HWB}^2}{15 g_1^2 c_{3W}(c_{ee} - c_{HD})}$ $\quad \frac{\mu(R_2)}{d(R_2)} = -\frac{g_1^2 c_{HWB}^2}{5 g_2^2 c_{ee}(c_{ee} - c_{HD})}$
$\phi\ (R_3, R_2, 0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15 g_3 c_{3G}(c_{H\Box} - 3c_{ll}/(4g_2^2) + c_{HD}/4)}$ $\quad \frac{\mu(R_2)}{d(R_2)} = -\frac{c_{HW}^2}{15 g_2 c_{3W}(c_{H\Box} - 3c_{ll}/(4g_2^2) + c_{HD}/4)}$

Group theoretical data

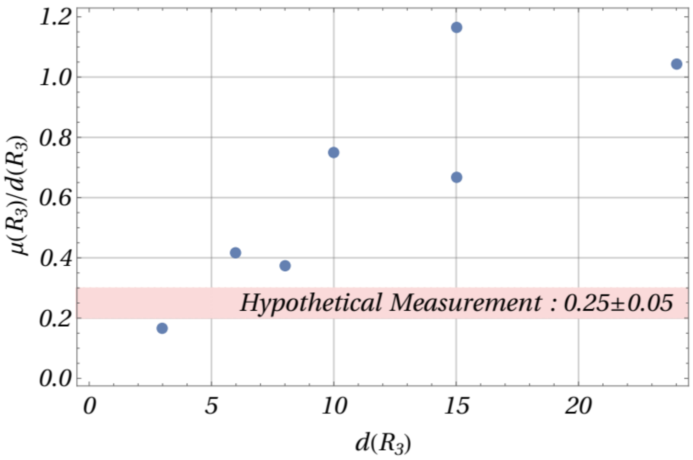
Measurable Wilson coefficients

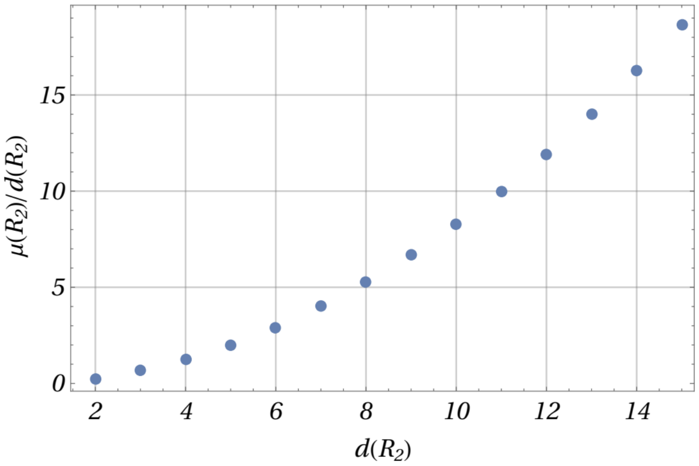
Heavy Particles and SMEFT

- If the U(1) hypercharge $6Q_Y$ is measured to be fractional, then neither of below constraint can be satisfied, thus $\Gamma = 1$, and resulting in stable fractionally charged particle

$$\mathcal{N}(R_3) = 6Q_Y \mod 3 \qquad \mathcal{N}(R_2) = 6Q_Y \mod 2$$

- μ(R)/d(R), discrete values, can be used to falsify this minimal model, but does not say anything conclusive on the global structure of gauge groups.





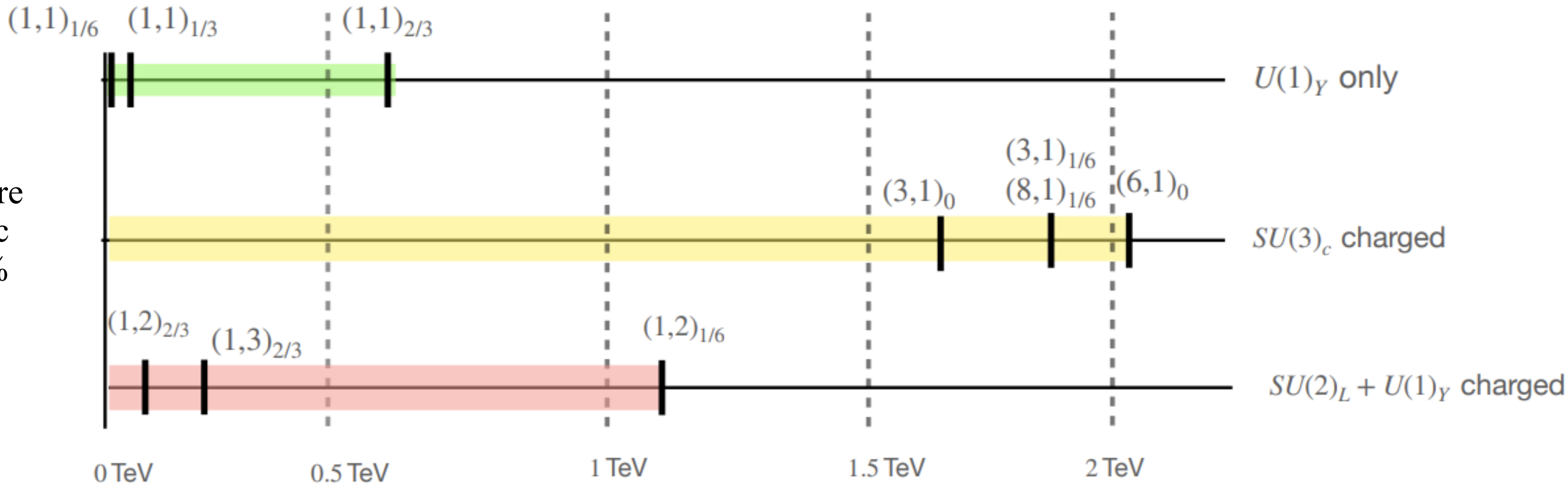
Heavy Particle Phenomenology

- Searching for fractionally charged particle at collider, based on unique dE/dx

[CMS, 2402.09932]

Stable fractionally charged particle → Z_6 non-trivial

Dirac Fermions
Bounds on Complex scalar are weaker than Dirac fermion by ~ 20%



[S. Koren, A. Martin 2406.178503]

- Cosmology constraint on relic density → constraint on T_{reheat}

SM Gauge Group and UV

- Different SM gauge group corresponds to different UV embedding

$$SU(5), SO(10), E_6 \qquad \Gamma = Z_6$$

Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R \qquad \Gamma = Z_3$

Trinification $SU(3)_C \times SU(3)_L \times SU(3)_R \qquad \Gamma = Z_2$

[C.Cordova, S. Hong, L.-T. Wang 2309.05636]

[Y. Choi, M. Forslund, H.Tam S.-H. Shao 2309.03937]

- Fractionally charged particle help to classify UV embedding

$e/6$	Non-trivial Z_6
$e/3$	Non-trivial $Z_{3,6}$ allow Trinification
$e/2$	Non-trivial $Z_{2,6}$ allow Pati-Salam

Summary

- Global structure of the Standard Model gauge group is unknown
- Global structure of the Standard Model can be used to determine the UV embedding of the gauge group
- Discovering new particle and determine its quantum number helps discern different Standard Model gauge groups.
- If new particle is heavy correlation between SMEFT Wilson coefficients can be used to infer the gauge quantum number of Z6 exotics.
- Strong motivation to study the phenomenology of fractionally charged particle.

Some example of the Z6 exotics

(fund, fund, 0), allowed Γ=1 forbidden Γ $Z_{2,3,6}$

(fund, fund, 2/3), allowed Γ=1 or Z_3, forbidden Γ=$Z_{2,6}$

(fund,fund, ½), allowed Γ=1 or Z_2, forbidden Γ=$Z_{3,6}$

- All particles are invariant under Z6, Γ remains undetermined as in the SM.
- At least one heavy particle is not invariant under Z3 but invariant under Z2 (hence not invariant under Z6), Γ can be either 1 or Z3.
- At least one heavy particle is not invariant under Z2 but invariant under Z3 (hence not invariant under Z6), Γ can be either 1 or Z2.
- At least one heavy particle is invariant under neither Z2 nor Z3 (hence not invariant under Z6), Γ is uniquely determined to be 1.

Relation to 1-form symmetry

- Free Maxwell theory with no matter:
  the Gauss law is understood as electric 1-form symmetry
- Pure gauge theory with no matter:
  the center of the gauge group measures the N-ality of a Wilson line, which is understood as electric 1-form symmetry
- Adding matter fields breaks the electric 1-form symmetry explicitly, i.e. Wilson lines can be screened/trivialized by particles.
- Nevertheless, the notions of electric 1-form symmetry and Wilson lines are still valid below the mass scale of the heavy particles that screen the Wilson lines. As such, the 1-form symmetry is viewed as accidental at low energy.

Singlet in SU(2) and
SU(3), charged under U(1)