General chiral structures for BNV nucleon decays and some phenomenological applications

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Baryon number violation

- One of the three Sahkarov conditions to explain matter-antimatter asymmetry in the Universe

- BNV is commonly predicted in BSM theories

- Due to the fundamental importance, there are a lot of experimental efforts, e.g., IMB, SNO+, KamLAND, Kamiokande, etc. → Null result

- All possible decay channels should be attempted, **exotic** as well as conventional → complete analysis of relavant interactions

General $|\Delta B| = 1$ triple-quark interactions

- If BNV exists → High energy scale → EFT-based framework
- Triple-quark operators without derivative acting on quark fields in LEFT or xLEFT

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q^\alpha_{\mathrm{L},y})(\overline{q^{\beta C}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q^\alpha_{\mathrm{R},y})(\overline{q^{\beta C}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta C}_{\mathrm{L},z\}}} \gamma^\mu q^\gamma_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta C}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L,w}\}})\epsilon_{\alpha\beta\gamma}$$

$+\mathrm{L} \leftrightarrow \mathrm{R}$ → Chiral partners

Combination of non-QCD fields

$x, y, z = 1,2,3$

$q_{1,2,3} = u, d, s$

$$A_{\{y}B_{z\}} \equiv \frac{1}{2}(A_y B_z + A_z B_y)$$

$$A_{\{y}B_z C_{w\}} \equiv \frac{1}{6}[A_y B_z C_w + 5 \text{ permutations of } (y,z,w)]$$

Completeness and independence of them can be proved using Fierz identities:

$$(\overline{\Psi}_{c,\mu} q^\alpha_{\mathrm{L},[y})(\overline{q^{\beta C}_{\mathrm{L},z]}} \gamma^\mu q^\gamma_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} = \frac{1}{2}(\overline{\Psi}_{c,\mu}\gamma^\mu q^\alpha_{\mathrm{R,w}})(\overline{q^{\beta C}_{\mathrm{L,z}}} q^\gamma_{\mathrm{L,y}})\epsilon_{\alpha\beta\gamma} \sim \mathcal{O}_b^{yzw}$$

$$(\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{R,y}})(\overline{q^{\beta C}_{\mathrm{L,z}}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} = -2i(\overline{\Psi}_{d,\mu\nu}\gamma^\mu q^\alpha_{\mathrm{L},\{z})(\overline{q^{\beta C}_{\mathrm{L,w}\}}}\gamma^\nu q^\gamma_{\mathrm{R,y}})\epsilon_{\alpha\beta\gamma} \sim \mathcal{O}_c^{yzw}$$

$$(\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{L},[y})(\overline{q^{\beta C}_{\mathrm{L},z]}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} = -\frac{1}{2}(\overline{\Psi}_{d,\mu\nu}\sigma^{\mu\nu} q^\alpha_{\mathrm{L,w}})(\overline{q^{\beta C}_{\mathrm{L,y}}} q^\gamma_{\mathrm{L,z}})\epsilon_{\alpha\beta\gamma} \sim \mathcal{O}_a^{yzw}$$

$$A_{[y}B_{z]} \equiv \frac{1}{2}(A_y B_z - A_z B_y)$$

*Y. Liao, X.-D. Ma, and **HLW**, arXiv:2504.xxxxx*

General $|\Delta B| = 1$ triple-quark interactions

- EFT-based framework

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- Triple-quark operators without derivative acting on quark fields in LEFT or xLEFT

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q^\alpha_{\mathrm{L},y})(\overline{q^{\beta \mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q^\alpha_{\mathrm{R},y})(\overline{q^{\beta \mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma} \qquad +\mathrm{L} \leftrightarrow \mathrm{R}$$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta \mathrm{C}}_{\mathrm{L},z\}}} \gamma^\mu q^\gamma_{\mathrm{R},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta \mathrm{C}}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},w\}})\epsilon_{\alpha\beta\gamma}$$

Combination of non-QCD fields

E.g. For dim-6 LEFT operators

$\overline{\Psi}_{a,b} = \bar{\ell},\, \bar{\nu}$ Well-known in the 1980s in the form of dim-6 LEFT operators

F. Wilczek and A. Zee, PRL 43 (1979)

J. R. Ellis, M. k. Gaillard, and D. V. Nanopoulos, PLB 88 (1979)

S.Weinberg, PRL 43 (1979) & PRD 22 (1980)

L. F. Abbott and M. B. Wise, PRD 22 (1980)

O. Kaymakcalan, C.-H. Lo, and K. C. Wali, PRD 29 (1984)

M. Claudson, M. B. Wise, and L. J. Hall, NPB 195 (1982)

$\overline{\Psi}_{c,\mu} = \bar{\ell}\gamma_\mu \qquad \overline{\Psi}_{d,\mu\nu} = \bar{\ell}\sigma_{\mu\nu}$

$$\gamma^\mu P_\pm \otimes \gamma_\mu P_\mp = -2P_\mp \odot P_\pm$$

$$\sigma^{\mu\nu} P_\pm \otimes \sigma_{\mu\nu} P_\pm = -4P_\pm \otimes P_\pm - 8P_\pm \odot P_\pm$$

Anti-symmetric in flavor index

The operators corresponding to $\mathcal{O}_{c,d}^{yzw}$ vanish for dim-6 LEFT operators

When $\mathcal{O}_c$ and $\mathcal{O}_d$ enter?

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q^\alpha_{\mathtt{L},\{y}})(\overline{q^{\beta C}_{\mathtt{L},z\}}} \gamma^\mu q^\gamma_{\mathtt{R},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathtt{L},\{y}})(\overline{q^{\beta C}_{\mathtt{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathtt{L},w\}})\epsilon_{\alpha\beta\gamma}$$

$+\mathtt{L} \leftrightarrow \mathtt{R}$

- **Identified for the first time**
- **When will they become relevant or even important?**

- **Important for *exotic* nucleon decays:**
  - Forbidden processes at dim-6: e.g. $\Delta I = 3/2$ BNV nucleon decays, $n \to \ell^- \pi^+$
  - BNV nucleon decays through higher dimensional operators
  - BNV nucleon decays involving new light particles. e.g. aLEFT, DSEFT et al.
- **We will show the new structures have nontrivial chiral realizations in ChPT**

Chiral and Lorentz properties

- Explore the chiral realization within ChPT
- Decompose each interaction into non-quark and quark factors

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q_{L,y}^{\alpha})(\overline{q_{L,z}^{\beta C}} q_{L,w}^{\gamma})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q_{R,y}^{\alpha})(\overline{q_{L,z}^{\beta C}} q_{L,w}^{\gamma})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q_{L,\{y}^{\alpha})(\overline{q_{L,z\}}^{\beta C}} \gamma^{\mu} q_{R,w}^{\gamma})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q_{L,\{y}^{\alpha})(\overline{q_{L,z}^{\beta C}} \sigma^{\mu\nu} q_{L,w\}}^{\gamma})\epsilon_{\alpha\beta\gamma}$$

$+ L \leftrightarrow R$

$$\mathcal{L}_{q^3}^{\not B} = \sum_i C_i^{yzw} \mathcal{O}_i^{yzw}$$

Non-quark factor

$$C_i^{yzw}\overline{\Psi} \equiv \mathcal{P}_{yzw}^{i}$$

Quark factor

$$\Gamma_1 q(\overline{q^C}\Gamma_2 q) \equiv \mathcal{N}_{yzw}^{i}$$

$$\mathcal{L}_{q^3}^{\not B} = \Big\{ \Big[(\mathcal{P}_{yzw}^{LL} q_{L,y}^{\alpha})(\overline{q_{L,z}^{\beta C}} q_{L,w}^{\gamma})\epsilon_{\alpha\beta\gamma} + (\mathcal{P}_{yzw}^{RL} q_{R,y}^{\alpha})(\overline{q_{L,z}^{\beta C}} q_{L,w}^{\gamma})\epsilon_{\alpha\beta\gamma}$$
$$+ (\mathcal{P}_{yzw}^{LR,\mu} q_{L,\{y}^{\alpha})(\overline{q_{L,z\}}^{\beta C}} \gamma_{\mu} q_{R,w}^{\gamma})\epsilon_{\alpha\beta\gamma} + (\mathcal{P}_{yzw}^{LL,\mu\nu} q_{L,\{y}^{\alpha})(\overline{q_{L,z}^{\beta C}} \sigma_{\mu\nu} q_{L,w\}}^{\gamma})\epsilon_{\alpha\beta\gamma} \Big] + L \leftrightarrow R \Big\} + h.c.$$

Chiral and Lorentz properties

Triple-quark sector:

Chiral group: $G \equiv SU(3)_{\mathrm{L}} \otimes SU(3)_{\mathrm{R}}$

- Under the chiral group, each structure belongs an irreducible representation

$$\mathcal{N}^{\mathrm{LL}}_{yzw} \equiv q^{\alpha}_{\mathrm{L,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{8}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{RL}}_{yzw} \equiv q^{\alpha}_{\mathrm{R,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \bar{\mathbf{3}}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}}$$

May be organized in the matrix form

$$\mathcal{N}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}} \xrightarrow{G} \hat{L}\mathcal{N}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}}\hat{L}^{\dagger} \qquad \mathcal{N}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}} \xrightarrow{G} \hat{R}\mathcal{N}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}}\hat{L}^{\dagger}$$

$$\mathcal{N}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}} = \begin{pmatrix} \mathcal{N}^{\mathrm{LL}}_{uds} & \mathcal{N}^{\mathrm{LL}}_{usu} & \mathcal{N}^{\mathrm{LL}}_{uud} \\ \mathcal{N}^{\mathrm{LL}}_{dds} & \mathcal{N}^{\mathrm{LL}}_{dsu} & \mathcal{N}^{\mathrm{LL}}_{dud} \\ \mathcal{N}^{\mathrm{LL}}_{sds} & \mathcal{N}^{\mathrm{LL}}_{ssu} & \mathcal{N}^{\mathrm{LL}}_{sud} \end{pmatrix} \qquad \mathcal{N}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}} = \begin{pmatrix} \mathcal{N}^{\mathrm{RL}}_{uds} & \mathcal{N}^{\mathrm{RL}}_{usu} & \mathcal{N}^{\mathrm{RL}}_{uud} \\ \mathcal{N}^{\mathrm{RL}}_{dds} & \mathcal{N}^{\mathrm{RL}}_{dsu} & \mathcal{N}^{\mathrm{RL}}_{dud} \\ \mathcal{N}^{\mathrm{RL}}_{sds} & \mathcal{N}^{\mathrm{RL}}_{ssu} & \mathcal{N}^{\mathrm{RL}}_{sud} \end{pmatrix}$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta C}_{\mathrm{L,z}\}}}\gamma^{\mu}q^{\gamma}_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{6}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}} \qquad \mathcal{N}^{\mathrm{LR},\mu}_{yzw} \xrightarrow{G} \hat{L}_{yy'}\hat{L}_{zz'}\hat{R}_{ww'}\mathcal{N}^{\{y'z'\}w',\mu}_{\mathbf{6}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}}$$

$$\mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}\sigma^{\mu\nu}q^{\gamma}_{\mathrm{L,w}\}})\epsilon_{\alpha\beta\gamma} \in \mathbf{10}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}} \qquad \mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \xrightarrow{G} \hat{L}_{yy'}\hat{L}_{zz'}\hat{L}_{ww'}\mathcal{N}^{\{y'z'w'\},\mu\nu}_{\mathbf{10}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}}$$

Chiral and Lorentz properties

Triple-quark sector:

- It important to incorporate the correct Lorentz properties for chiral matching

$$\mathcal{N}^{\mathrm{LL}}_{yzw} \equiv q^{\alpha}_{\mathrm{L,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}} q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{8}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{RL}}_{yzw} \equiv q^{\alpha}_{\mathrm{R,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}} q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \bar{\mathbf{3}}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}}$$

$$\mathcal{N}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}} \in (\frac{1}{2},0) \qquad \mathcal{N}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}} \in (0,\frac{1}{2})$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta C}_{\mathrm{L,z}\}}}\gamma^{\mu} q^{\gamma}_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{6}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}\sigma^{\mu\nu} q^{\gamma}_{\mathrm{L,w}\}})\epsilon_{\alpha\beta\gamma} \in \mathbf{10}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \in \left(1,\frac{1}{2}\right) \qquad \mathcal{N}^{\mathrm{LL},\mu}_{yzw} \in \left(\frac{3}{2},0\right)$$

These complicates the chiral matching

$$\gamma_\mu \mathcal{N}^{\mathrm{LR},\mu}_{yzw} = \gamma_\mu \mathcal{N}^{\mathrm{RL},\mu}_{yzw} = 0 \qquad \gamma_\mu \mathcal{N}^{\mathrm{LL},\mu}_{yzw} = \gamma_\mu \mathcal{N}^{\mathrm{RR},\mu}_{yzw} = 0$$

Chiral and Lorentz properties

Non-quark sector:

Chiral group: $SU(3)_{\rm L}\otimes SU(3)_{\rm R}$

- We treat non-quark factors as a spurion field.
- We assign transformation rules to the spurion fields so that the interactions are invariant

$$\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}\to \hat{L}\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}\hat{L}^\dagger \qquad \mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\to \hat{L}\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\hat{R}^\dagger \qquad \mathscr{P}^{\rm LR,\mu}_{yzw}\to \hat{L}^*_{yy'}\hat{L}^*_{zz'}\hat{R}^*_{ww'}\mathscr{P}^{\rm LR,\mu}_{y'z'w'} \qquad \mathscr{P}^{\rm LL,\mu\nu}_{yzw}\to \hat{L}^*_{yy'}\hat{L}^*_{zz'}\hat{L}^*_{ww'}\mathscr{P}^{\rm LL,\mu\nu}_{y'z'w'}$$

- This is a convenient way to organize terms with definite chiral properties
- Organize $\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}$ and $\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}$ in matrix form similarly

$$\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}} = \begin{pmatrix} 0 & \mathscr{P}^{\rm LL}_{dds} & \mathscr{P}^{\rm LL}_{sds} \\ \mathscr{P}^{\rm LL}_{usu} & \mathscr{P}^{\rm LL}_{dsu} & \mathscr{P}^{\rm LL}_{ssu} \\ \mathscr{P}^{\rm LL}_{uud} & \mathscr{P}^{\rm LL}_{dud} & \mathscr{P}^{\rm LL}_{sud}\end{pmatrix} \qquad \mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}} = \begin{pmatrix} \mathscr{P}^{\rm RL}_{uds} & \mathscr{P}^{\rm RL}_{dds} & \mathscr{P}^{\rm RL}_{sds} \\ \mathscr{P}^{\rm RL}_{usu} & \mathscr{P}^{\rm RL}_{dsu} & \mathscr{P}^{\rm RL}_{ssu} \\ \mathscr{P}^{\rm RL}_{uud} & \mathscr{P}^{\rm RL}_{dud} & \mathscr{P}^{\rm RL}_{sud}\end{pmatrix}$$

$$\mathscr{P}^{\rm LR,\mu}_{yzw} \qquad \mathscr{P}^{\rm LL,\mu\nu}_{yzw}$$

$$\begin{aligned}\mathscr{L}^{\not B}_{q^3} = {} & {\rm Tr}\left[\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}\mathscr{N}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}} + \mathscr{P}_{\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}}\mathscr{N}_{\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}}\right] \\ & + {\rm Tr}\left[\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\mathscr{N}_{\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}} + \mathscr{P}_{\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}}\mathscr{N}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\right] \\ & + \left[\mathscr{P}^{\{yz\}w,\mu}_{\bar{\mathbf{6}}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\mathscr{N}^{\{yz\}w}_{\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R},\mu} + \mathscr{P}^{\{yz\}w,\mu}_{\bar{\mathbf{3}}_{\rm L}\otimes\bar{\mathbf{6}}_{\rm R}}\mathscr{N}^{\{yz\}w}_{\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R},\mu}\right] \\ & + \left[\mathscr{P}^{\{yzw\},\mu\nu}_{\bar{\mathbf{10}}_{\rm L}\otimes\mathbf{1}_{\rm R}}\mathscr{N}^{\{yzw\}}_{\mathbf{10}_{\rm L}\otimes\mathbf{1}_{\rm R},\mu\nu} + \mathscr{P}^{\{yzw\},\mu\nu}_{\mathbf{1}_{\rm L}\otimes\bar{\mathbf{10}}_{\rm R}}\mathscr{N}^{\{yzw\}}_{\mathbf{1}_{\rm L}\otimes\mathbf{10}_{\rm R},\mu\nu}\right] \\ & + {\rm h.c.}\end{aligned}$$

Chiral matching

- To build the interactions for baryons and pseudosaclars $\rightarrow$

ChPT is a systematic and consistent approach

Building blocks in ChPT:

$$\Sigma(x) = \xi^2(x) = \exp\left(\frac{i\sqrt{2}\Pi(x)}{F_0}\right)$$

$$\Pi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

$$B(x) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}$$

Chiral group: $SU(3)_{\mathrm{L}} \otimes SU(3)_{\mathrm{R}}$

$$\Sigma \rightarrow \hat{L}\Sigma\hat{R}^\dagger \quad \xi \rightarrow \hat{L}\xi\hat{h}^\dagger = \hat{h}\xi\hat{R}^\dagger \quad B \rightarrow \hat{h}B\hat{h}^\dagger$$

+ Spurion fields

- Chiral Lagrangian is ordered by the typical momentum transfer p

$$\{\Sigma, \xi, B, D_\mu B\} \sim \mathcal{O}(p^0) \qquad D_\mu\Sigma \sim \mathcal{O}(p^1)$$

$$D_\mu\Sigma = \partial_\mu\Sigma - il_\mu\Sigma + i\Sigma r_\mu \qquad D_\mu B = \partial_\mu B + [\Gamma_\mu, B] \qquad \Gamma_\mu = \frac{1}{2}\left[\xi(\partial_\mu - ir_\mu)\xi^\dagger + \xi^\dagger(\partial_\mu - il_\mu)\xi\right]$$

- The higher the chiral order, more suppressed the term

Chiral and Lorentz properties

E. Delgado et al., EPJA 51 (2015)

- Project the approprociate irreducible Lorentz rep. from ***vector-spinor*** and ***tensor-spinor***:

$$\Gamma^{\mathrm{L,R}}_{\mu\nu} \equiv (g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu)P_{\mathrm{L,R}} \qquad \left(1,\frac{1}{2}\right) \quad \left(\frac{1}{2},1\right)$$

$$\Gamma^{\mathrm{L,R}}_{\mu\rho}\Gamma^{\mathrm{L,R}\,\rho}_{\nu} = \Gamma^{\mathrm{L,R}}_{\mu\nu} \qquad \gamma^\mu\Gamma^{\mathrm{L,R}}_{\mu\nu} = 0$$

$$\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\rho\sigma}\hat{\Gamma}^{\mathrm{L,R}\,\rho\sigma}_{\alpha\beta} = \hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} \qquad \gamma^\mu\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} = 0$$

$$\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} \equiv \frac{1}{24}\Big(2[\sigma_{\mu\nu},\sigma_{\alpha\beta}] - \{\sigma_{\mu\nu},\sigma_{\alpha\beta}\}\Big)P_{\mathrm{L,R}} \qquad \left(\frac{3}{2},0\right) \quad \left(0,\frac{3}{2}\right)$$

- Construct the LO chiral-invariant Lagrangian:

*Y. Liao, X.-D. Ma, and **HLW**, arXiv:2504.xxxxx*

$$\mathscr{L}^{\not B,0}_{B} = c_1\mathrm{Tr}\left[\mathscr{P}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}}\xi B_{\mathrm{L}}\xi - \mathscr{P}_{\mathbf{3}_{\mathrm{L}}\otimes\bar{\mathbf{3}}_{\mathrm{R}}}\xi^\dagger B_{\mathrm{R}}\xi^\dagger\right] + c_2\mathrm{Tr}\left[\mathscr{P}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}}\xi B_{\mathrm{L}}\xi^\dagger - \mathscr{P}_{\mathbf{1}_{\mathrm{L}}\otimes\mathbf{8}_{\mathrm{R}}}\xi^\dagger B_{\mathrm{R}}\xi\right]$$
$$+\frac{c_3}{\Lambda_\chi}\left[\mathscr{P}^{\mathrm{LR},\mu}_{yzi}\Gamma^{\mathrm{L}}_{\mu\nu}(\xi iD^\nu B_{\mathrm{L}}\xi)_{yj}\Sigma_{zk}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RL},\mu}_{yzi}\Gamma^{\mathrm{R}}_{\mu\nu}(\xi^\dagger iD^\nu B_{\mathrm{R}}\xi^\dagger)_{yj}\Sigma^*_{kz}\epsilon_{ijk}\right] + \text{h.c.},$$

$$\mathscr{L}^{\not B,1}_{B} = \frac{c_4}{\Lambda_\chi^2}\left[\mathscr{P}^{\mathrm{LL},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{L}}_{\mu\nu\alpha\beta}(\xi D^\alpha B_{\mathrm{L}}\xi)_{yi}\Sigma_{zj}(D^\beta\Sigma)_{wk}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RR},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{R}}_{\mu\nu\alpha\beta}(\xi^\dagger D^\alpha B_{\mathrm{R}}\xi^\dagger)_{iy}\Sigma^*_{jz}(D^\beta\Sigma)^*_{kw}\epsilon_{ijk}\right] + \text{h.c.}\,.$$

Involve at least one pesudoscalar

Determination of the LECs

- When matching the quark interactions with hadron interactions, there arise low energy constants due to strong QCD.

$$\mathcal{L}_B^{\not{B},0} = c_1 \mathrm{Tr}\left[\mathcal{P}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}}\xi B_\mathrm{L}\xi - \mathcal{P}_{\mathbf{3}_\mathrm{L}\otimes\bar{\mathbf{3}}_\mathrm{R}}\xi^\dagger B_\mathrm{R}\xi^\dagger\right] + c_2 \mathrm{Tr}\left[\mathcal{P}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}}\xi B_\mathrm{L}\xi^\dagger - \mathcal{P}_{\mathbf{1}_\mathrm{L}\otimes\mathbf{8}_\mathrm{R}}\xi^\dagger B_\mathrm{R}\xi\right]$$
$$+\frac{c_3}{\Lambda_\chi}\left[\mathcal{P}^{\mathrm{LR},\mu}_{yzi}\Gamma^\mathrm{L}_{\mu\nu}(\xi i D^\nu B_\mathrm{L}\xi)_{yj}\Sigma_{zk}\epsilon_{ijk} - \mathcal{P}^{\mathrm{RL},\mu}_{yzi}\Gamma^\mathrm{R}_{\mu\nu}(\xi^\dagger i D^\nu B_\mathrm{R}\xi^\dagger)_{yj}\Sigma^*_{kz}\epsilon_{ijk}\right] + \text{h.c.},$$

$$\mathcal{L}_B^{\not{B},1} = \frac{c_4}{\Lambda_\chi^2}\left[\mathcal{P}^{\mathrm{LL},\mu\nu}_{yzw}\hat{\Gamma}^\mathrm{L}_{\mu\nu\alpha\beta}(\xi D^\alpha B_\mathrm{L}\xi)_{yi}\Sigma_{zj}(D^\beta\Sigma)_{wk}\epsilon_{ijk} - \mathcal{P}^{\mathrm{RR},\mu\nu}_{yzw}\hat{\Gamma}^\mathrm{R}_{\mu\nu\alpha\beta}(\xi^\dagger D^\alpha B_\mathrm{R}\xi^\dagger)_{iy}\Sigma^*_{jz}(D^\beta\Sigma)^*_{kw}\epsilon_{ijk}\right] + \text{h.c.}\,.$$

- LQCD results: $c_1 = \alpha = -0.01257(111)\,\mathrm{GeV}^3$ $c_2 = \beta = 0.01269(107)\,\mathrm{GeV}^3$ *J.-S. Yoo, et al., PRD 105 (2022)*

- NDA for the unknown LECs:

$$C_{q,\mathrm{had}} = g(4\pi)^{2-m}\Lambda_\chi^{D-4} \Rightarrow \boxed{c_{1,2,3} \sim \Lambda_\chi^3/(4\pi)^2 \approx 0.011\,\mathrm{GeV}^3} \qquad c_4 \sim \Lambda_\chi^2 F_0/(4\pi\sqrt{2}) \approx 0.007\,\mathrm{GeV}^3$$

Reduced coupling

g: coupling constant

m: minimal # of physical fields

D: dim. of op.

*Y. Liao, X.-D. Ma, and **HLW**, arXiv:2504.xxxxx*

S. Weinberg, PRL 63 (1989)

A. Manohar and H. Georgi, NPB 234 (1984)

B. M. Gavela et al. EPJC 76 (2016)

The reliability of NDA can be verified with $c_{1,2}$

- **We expect NDA really works, but still we encourage calculations on the two new LECs c_3 and c_4 by LQCD community**

Phenomenological applications

$\mathscr{L} \supset M^n(\mathscr{P}B)$

- There are many interesting applications
- Restricted to nucleon decays induced by the new chiral structrures
- Two types of BNV nucleon decays: (1). Involves only SM particles; (2). Contains a new light particle

A. The $\Delta I = 3/2$ processes: $n \to \ell^- \pi^+$ with $\ell = e, \mu$:

leading contribution from irreps $\mathbf{6}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$

$$\mathscr{L}_{\mathscr{P}\pi n} = \frac{i\sqrt{2}c_3}{(F_0\Lambda_\chi)} \pi^- \mathscr{P}^{\mathrm{LR},\mu}_{ddd}\, i\tilde{\partial}_\mu n_{\mathrm{L}} + \mathrm{L} \leftrightarrow \mathrm{R}$$

$$\tilde{\partial}_\mu = \Gamma_{\mu\nu}\partial^\nu = \partial_\mu - \frac{1}{4}\gamma_\mu \partial\!\!\!/$$

Can be induced from LEFT dim-7 operators

Descend from dim-7 SMEFT operators:

$$\mathcal{O}^{prst}_{DLddQ} = (\overline{iD_\mu L_p} d^\alpha_{\{r})(\overline{d^{\beta C}_{s\}}} \gamma^\mu Q^\gamma_t)\epsilon_{\alpha\beta\gamma}$$

$$\mathscr{P}^{\mathrm{RL},\mu}_{ddd} = -0.91 C^{\ell 111}_{DLddQ} i\partial^\mu \overline{\ell_{\mathrm{L}}}$$

RGE effects $\sim \Lambda_\ell^{-3}$

$$\Gamma(n \to \ell^- \pi^+) \sim \left(\frac{7 \times 10^9\,\mathrm{GeV}}{\Lambda_\ell}\right)^6 / (5 \times 10^{31}\,\mathrm{yr})$$

Phenomenological applications

B. The process: $p \to \ell_\alpha^+ \ell_\alpha^+ \ell_\beta^-$ **with** $\ell_\alpha = e, \quad \ell_\beta = \mu$ **or vice versa**

- Among the most experimentally constrained proton decay processes
- Can be only mediated by dim-9 and higher LEFT operators
- Associated with irreps $\mathbf{6}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$

$$\mathcal{O}_{\ell\ell'} = (\overline{\ell^{\mathrm{C}}_{\mathrm{L}}}\gamma_\mu \ell_{\mathrm{R}})(\overline{\ell'_{\mathrm{R}}} u^\alpha_{\mathrm{L}})(\overline{u^{\beta\mathrm{C}}_{\mathrm{L}}}\gamma^\mu d^\gamma_{\mathrm{R}})\epsilon_{\alpha\beta\gamma}$$

$$\mathscr{P}^{\mathrm{LR},\mu}_{uud} = \Lambda^{-5}_{\ell\ell'}(\overline{\ell^{\mathrm{C}}_{\mathrm{L}}}\gamma^\mu \ell_{\mathrm{R}})\overline{\ell'_{\mathrm{R}}}$$

$$\Gamma(p \to e^+ e^+ \mu^-) \sim \Gamma(p \to \mu^+ \mu^+ e^-)$$

$$\sim \left(\frac{4 \times 10^5 \mathrm{GeV}}{\Lambda_{\mu e}}\right)^{10} / (10^{34}\ \mathrm{yr})$$

Phenomenological applications

- Now we turn to the examples involving a new light particle in the final states.

C. The process $p \to e^+ a$ in aLEFT

$$[\mathcal{O}^{VL,SR}_{\partial audue}]^\dagger = -\partial_\mu a \epsilon^{\alpha\beta\gamma}(\overline{e^{\mathrm{C}}_{\mathrm{L}}} u^\alpha_{\mathrm{L}})(\overline{u^\beta_{\mathrm{L}}}\gamma^\mu d^\gamma_{\mathrm{R}})$$

Belongs to irreps $\mathbf{6}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$

$p \to e^+ a$

$p \to e^+ a \pi^0$

$p \to e^+ a K^0$ (With d_{R} replaced by s_{R})

$$\mathcal{L}_{pea} \supset -ic_3 \Lambda_\chi^{-1} \Lambda_d^{-4} \partial^\mu a \overline{e^{\mathrm{C}}_{\mathrm{L}}} \tilde{\partial}_\mu p_{\mathrm{L}}$$

$$\mathcal{P}^{\mathrm{LR},\mu}_{uud} = -\Lambda_d^{-4} \partial^\mu a \overline{e^{\mathrm{C}}_{\mathrm{L}}}$$

$$\Gamma(p \to e^+ a) \sim \frac{(1.4 \times 10^7 \mathrm{GeV}/\Lambda_d)^8}{8 \times 10^{32}\ \mathrm{yr}}$$

- The new structures were overlooked or simply discarded as terms of a higher chiral order than the known ones in the previous work
- The constraint is comparable to the value obtained from operators in the $\mathbf{8}_{\mathrm{L(R)}} \otimes \mathbf{1}_{\mathrm{R(L)}}$ and $\bar{\mathbf{3}}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$ representations

T. Li, M. A. Schmidt, and C.-Y. Yao, JHEP 08 (2024)

Phenomenological applications

D. Nucleon decay involving a light dark photon (X^μ)

E.g., $\Delta I = 3/2$ process: $n \to \ell^- \pi^+ X^\mu$ with $\ell = e, \mu$

$$\mathcal{O}_{X\ell} = X_{\mu\nu}(\overline{\ell_{\mathrm{R}}} d_{\mathrm{L}}^{\alpha})(\overline{d_{\mathrm{L}}^{\beta \mathrm{C}}} \sigma^{\mu\nu} d_{\mathrm{L}}^{\gamma})\epsilon_{\alpha\beta\sigma} \quad \in \mathbf{10}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathscr{P}_{ddd}^{\mathrm{LL},\mu\nu} = \Lambda_{X\ell}^{-4} X^{\mu\nu} \overline{\ell_{\mathrm{R}}}$$

$$\mathscr{L}_{n\ell\pi X} = \frac{-i\sqrt{2}c_4}{(F_0 \Lambda_\chi^2 \Lambda_{X\ell}^4)} X^{\mu\nu} (\overline{\ell_{\mathrm{R}}} \hat{\Gamma}^{\mathrm{L}}_{\mu\nu\alpha\beta} \partial^\alpha n) \partial^\beta \pi^-$$

$$\Gamma(n \to \ell^- \pi^+ X) \sim \left(\frac{3 \times 10^6\,\mathrm{GeV}}{\Lambda_{X\ell}}\right)^8 / (10^{30}\,\mathrm{yr})$$

BNV hydrogen decays

- Hydrogen is the most abundant atom in the universe, which can be used to probe BNV interactions
- We estimated the two-body decay widths of hydrogen atom into SM particles by following a series of EFTs

SMEFT ⬅ LEFT ➡ ChPT

- We work with dim-6 and dim-7 operators in SMEFT according to the violation pattern of baryon and lepton numbers.

dim-6: violate B and L but conserve B-L

dim-7: violate B and L but conserve B+L

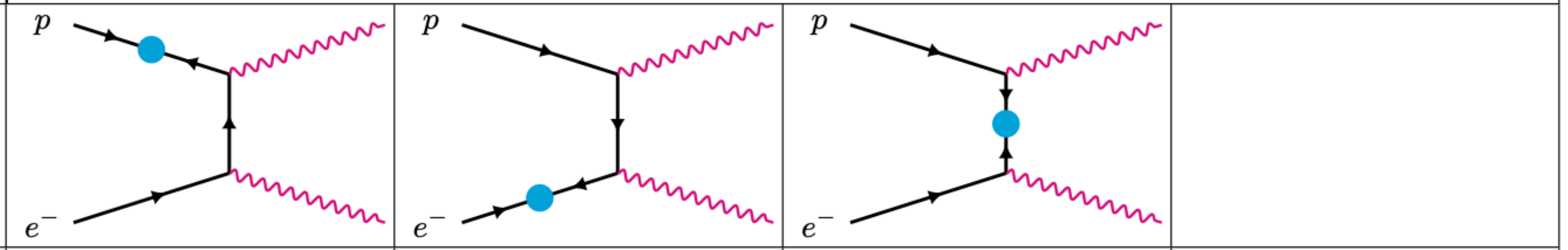
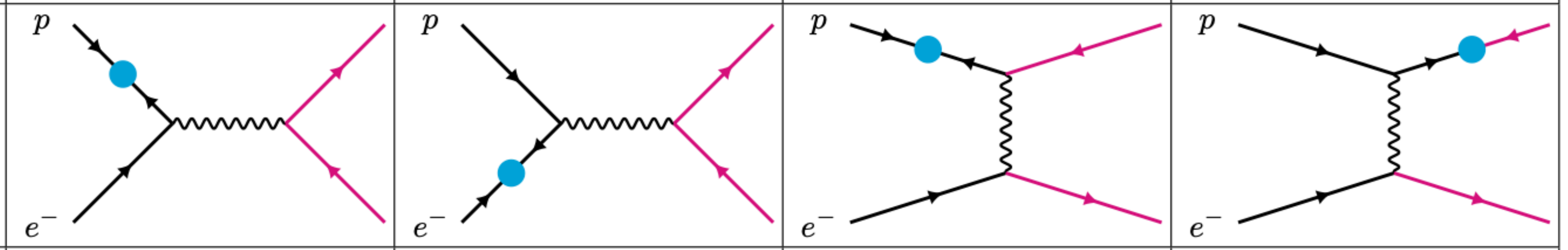
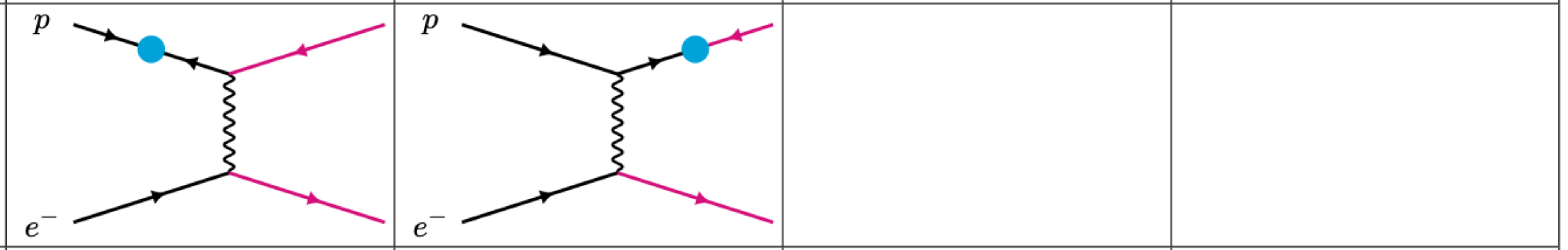
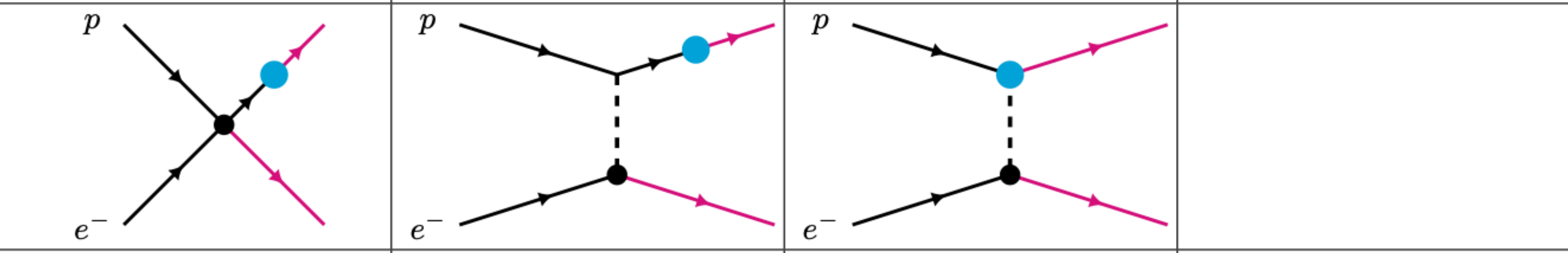
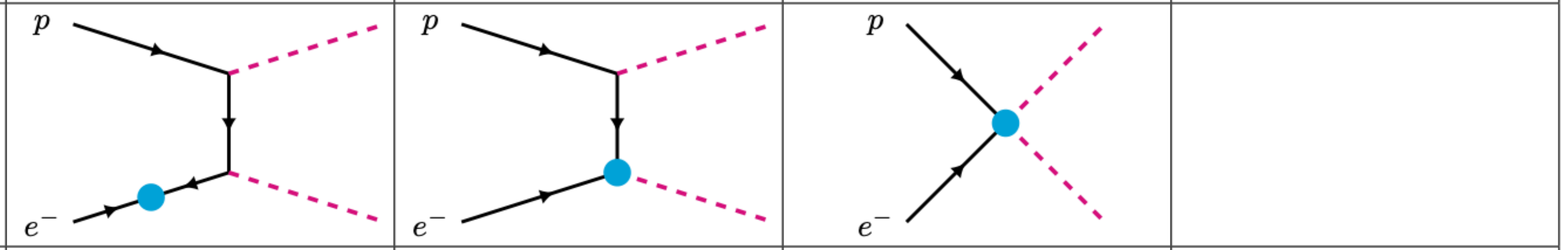
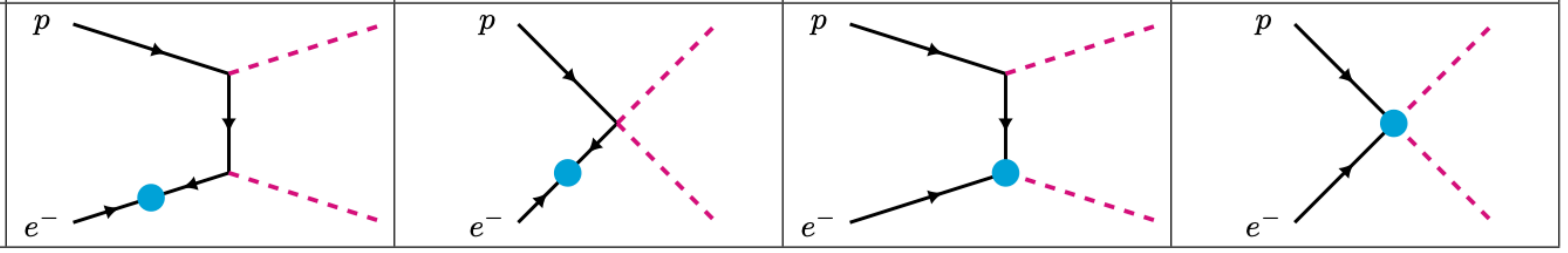
$$\mathcal{M}_{\mathrm{H}\to a+b} = \sqrt{\frac{m_{\mathrm{H}}}{2m_e m_p}} \int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3} \tilde{\psi}(\boldsymbol{p}_e) \mathcal{M}_{e+p\to a+b}.$$

Amplitude for free particles

Electron's wavefunction

W.Q. Fan, Y. Liao, X.-D. Ma, and HLW, PLB 862 (2025) 139335

BNV hydrogen decays

Mode	LO Feynman diagrams
$\mathrm{H}\to\gamma\gamma$ ($\not{B}\otimes$QED)	
$\mathrm{H}\to e^+e^-$ ($\not{B}\otimes$QED)	
$\mathrm{H}\to\mu^+e^-$ ($\not{B}\otimes$QED)	
$\mathrm{H}\to\nu_e\nu_{e,\mu,\tau}$ ($\not{B}\otimes$Weak)	
$\mathrm{H}\to\pi^0\pi^0$ $\mathrm{H}\to\pi^0\eta$ ($\not{B}\otimes$QCD)	
$\mathrm{H}\to\pi^+\pi^-$ $\mathrm{H}\to\pi^0K^0$ $\mathrm{H}\to\pi^-K^+$ ($\not{B}\otimes$QCD)	

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$\Lambda^{(6)}\sim 10^{15}\,\mathrm{GeV}\qquad \Lambda^{(7)}\sim 10^{10}\,\mathrm{GeV}$

Derived bound on hydrogen 2-body decay			
Mode	Γ^{-1}(yr)	Mode	Γ^{-1}(yr)
$\mathrm{H}\to\gamma\gamma$	7.6×10^{44}	$\mathrm{H}\to\pi^0\pi^0$	1.1×10^{48}
$\mathrm{H}\to e^+e^-$	1.5×10^{45}	$\mathrm{H}\to\pi^0\eta$	9.6×10^{46}
$\mathrm{H}\to e^-\mu^+$	1.5×10^{45}	$\mathrm{H}\to\pi^+\pi^-$	6.0×10^{47}
$\mathrm{H}\to e^+\mu^-$	✗(LEFT@dim9)	$\mathrm{H}\to\pi^0K^0$	4.5×10^{46}
$\mathrm{H}\to\mu^+\mu^-$	✗(QED@loop)	$\mathrm{H}\to\pi^-K^+$	5.0×10^{46}
$\mathrm{H}\to\nu_i\bar{\nu}_j$	✗(m_ν)	$\mathrm{H}\to\pi^+K^-$	✗(Weak)
$\mathrm{H}\to\nu_e\nu_e$	9.1×10^{55}	$\mathrm{H}\to\pi^0\bar{K}^0$	✗(Weak)
$\mathrm{H}\to\nu_e\nu_{\mu,\tau}$	1.8×10^{56}		
$\mathrm{H}\to\bar{\nu}_i\bar{\nu}_j$	✗(LEFT@dim9)		
$\mathrm{H}\to\pi^0\gamma$	–	$\mathrm{H}\to K^0\gamma$	–
$\mathrm{H}\to\eta\gamma$	–	$\mathrm{H}\to\bar{K}^0\gamma$	–

Summary

- We classified general triple-quark interactions and carried out their chiral matching.
- We identified new chiral structures that induce exotic nucleon decays beyond the conventional ones.
- We estimated all of four LECs by NDA, which show consistency with lattice results on two known LECs. We encourage the lattice community to compute the new LECs.
- We discussed some phenomenological applications, with more to be done.

Thank You!