基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

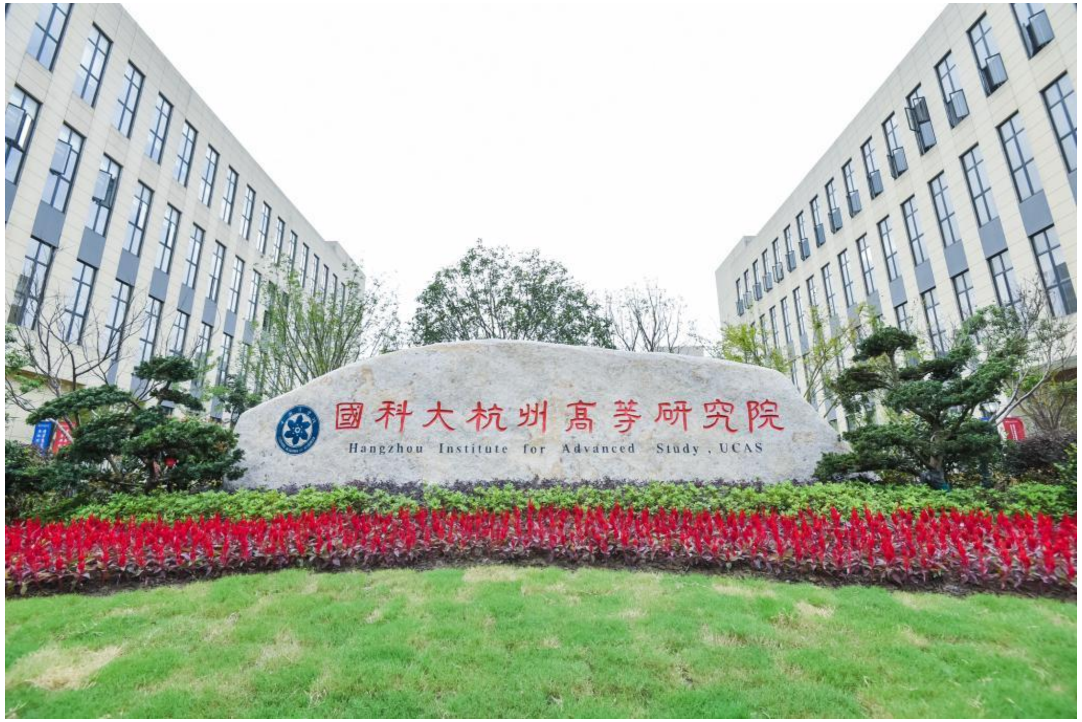
國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

Asymptotic grand unification in SO(10) with one extra dimension

Gao-Xiang Fang (HIAS)
2025-04-19

Based on **GXF**, Z.W. Wang, Y.L. Zhou,
arXiv:2504.xxxxx

Contents



基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

國科大杭州髙等研究院
Hangzhou Institute for Advanced Study,UCAS

Introduction

Electromagnetic force

Weak force

Strong force

Grand unified theory (GUT)

Unification of matter

Unification of symmetries

Unification of gauge couplings

Particles in GUT $\cup$ Particles in SM

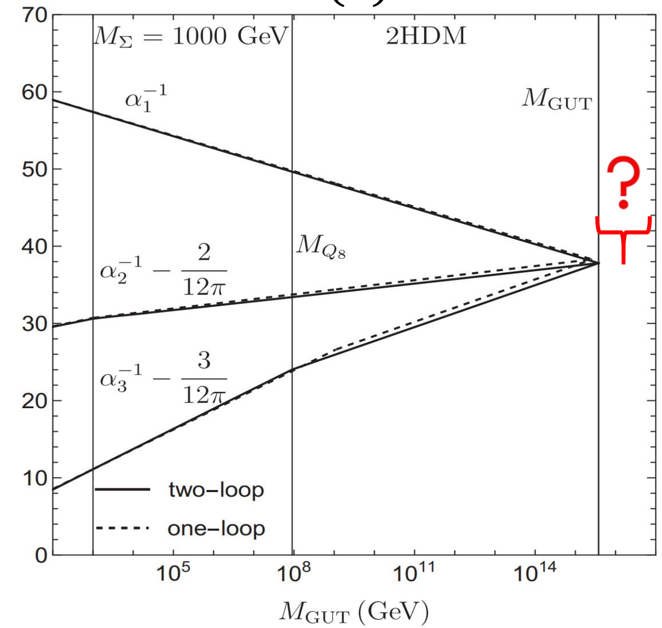
$G_{\rm GUT} \supset G_{\rm SM}$

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$g_{\rm GUT} = g_3 = g_2 = g_1$

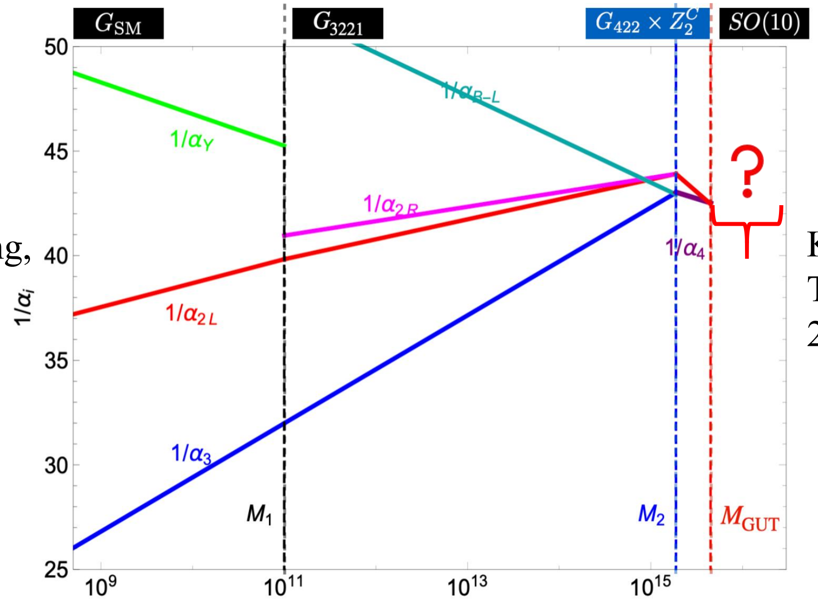
Running behavior above $M_{\rm GUT}$ is blank

$SU(5)$ GUT



Gao-Xiang Fang, Ye-Ling Zhou, 2406.06861

$SO(10)$ GUT



King, Pascoli, Turner, Zhou, 2106.15634

GUT scale: The scale where three gauge couplings are unified, denoted as $M_{\rm GUT}$

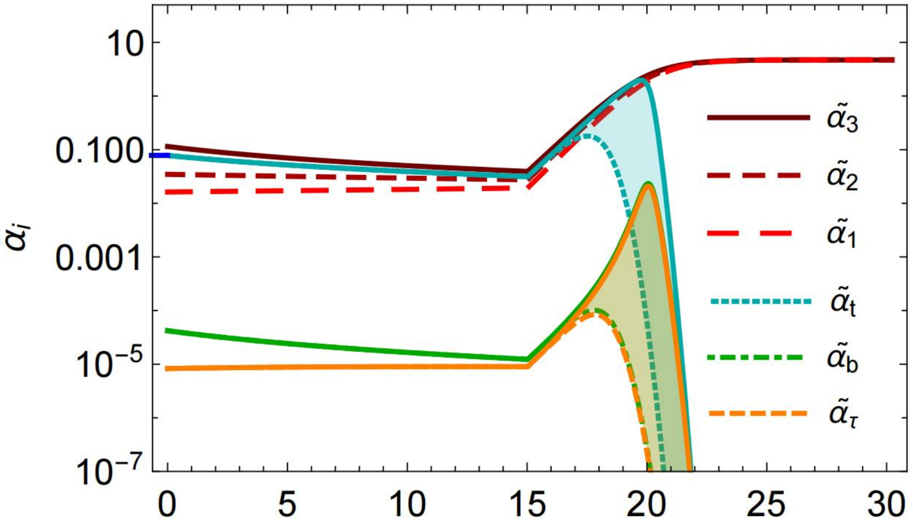
How about UV behavior of GUTs?

Introduction

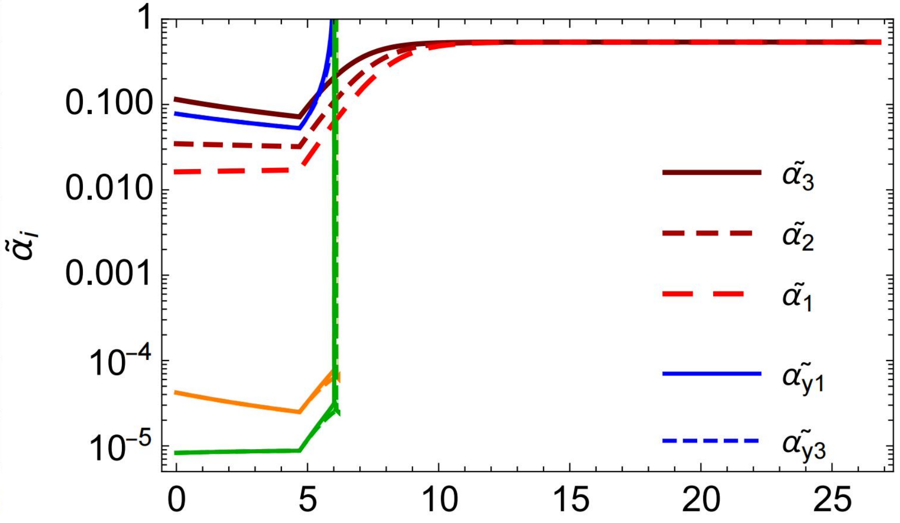
Asymptotic unification in 5-dimensional spacetime of gauge couplings: these gauge couplings gradually approach the same value in the deep UV limit.

Asymptotic freedom in 5-dimensional spacetime of Yukawa couplings: these Yukawa couplings sharply approach zero in the deep UV limit.

5D $SU(5)$



5D $SO(10)$ with only H_{10}



1. Yukawa couplings flow to Landau pole
2. Cannot explain fermion masses

Not realistic!

Cacciapaglia, et al, 2012.14732, 2210.03596, 2302.11671, 2309.10098

Whether there exist a realistic 5D $SO(10)$ GUT with more than one Higgs field?

Framework

Breaking chain: 5D $SO(10) \xrightarrow[\text{BC}]{M_{\text{KK}}}$ 4D Pati-Salam$(SU(4)_c \times SU(2)_L \times SU(2)_R) \xrightarrow[H_{\mathbf{16}}]{M_{\text{PS}}}$ SM

Energy scale	Symmetry	Fermion	Higgs
$\mu > M_{\text{KK}}$	$SO(10)$	$\Psi_{\mathbf{16}} \sim \mathbf{16}$ $\Psi_{\overline{\mathbf{16}}} \sim \overline{\mathbf{16}}$ $\nu_{\text{S}} \sim \mathbf{1}$	$H_{\mathbf{10}} \sim \mathbf{10}$, complex $H_{\mathbf{120}} \sim \mathbf{120}$, real $H_{\mathbf{16}} \sim \mathbf{16}$
$M_{\text{PS}} < \mu < M_{\text{KK}}$	G_{422}	$\psi_L \sim (4,2,1)$ $\psi_R \sim (4,1,2)$ $\nu_{\text{S}} \sim (1,1,1)$	$h_1, h_1' \sim (1,2,2), h_{15} \sim (15,2,2)$ $h_{\bar{4}} \sim (\bar{4},1,2)$
$M_Z \ll \mu < M_{\text{PS}}$	G_{SM}	q_L, d_R, u_R $l_L, \nu_R, e_R, \nu_{\text{S}}$	h_{SM}

Table 1. Gauge symmetries and particle contents remnant of the model at different energy scales.

Economical choice to achieve fermion mass splitting

Zero mode No zero mode

$$\Psi_{\mathbf{16}} = \psi_L + \Psi_R^c$$
$$\Psi_{\overline{\mathbf{16}}} = \Psi_L^c + \psi_R$$

$$\psi_L = (q_L, l_L) \sim (4,2,1)\,, \Psi_R^c = (Q_R^c, L_R^c) \sim (\bar{4},1,2)$$
$$\Psi_L^c = (Q_L^c, L_L^c) \sim (\bar{4},2,1)\,, \psi_R = (q_R, l_R) \sim (4,1,2)$$

Boundary conditions (BC) for fields of zero modes in UV and IR branes:

$$\text{left-handed fermions, scalars} \;\Rightarrow\; (+,+)$$
$$\text{right-handed fermions} \;\Rightarrow\; (-,-)$$

Framework

Yukawa coupling terms in 5D $SO(10)$:

$$-\mathcal{L}_Y = y_{\mathbf{10_1}} \overline{\Psi_{\mathbf{16}}} H_{\mathbf{10}} \Psi_{\overline{\mathbf{16}}} + y_{\mathbf{10_2}} \overline{\Psi_{\mathbf{16}}} H^*_{\mathbf{10}} \Psi_{\overline{\mathbf{16}}} + y_{\mathbf{120}} \overline{\Psi_{\mathbf{16}}} H_{\mathbf{120}} \Psi_{\overline{\mathbf{16}}} + y_{\mathbf{16}} \overline{\nu_{\mathrm{S}}} H_{\mathbf{16}} \Psi_{\overline{\mathbf{16}}} + \frac{L}{2} \mu_{\mathrm{M}} \, \overline{\nu_{\mathrm{S}}} \nu^c_{\mathrm{S}} \, \delta(y - \pi R) + \mathrm{h.c.}$$

Yukawa coupling terms in 4D PS:

$$-\mathcal{L}_y \supset y_{1_1} \overline{\psi_L} h_1 \psi_R + y_{1_2} \overline{\psi_L} h_1^* \psi_R + \overline{\psi_L} (y_1' h_1' + y_{15} h_{15}) \psi_R + y_4 \overline{\nu_{\mathrm{S}}} h_{\bar{4}} \psi_R + \frac{1}{2} \mu_{\mathrm{M}} \overline{\nu_{\mathrm{S}}} \nu^c_{\mathrm{S}} + \mathrm{h.c.}$$

Dirac mass matrices of fermions: $c = v/v_{\mathrm{EW}}$

$$\begin{aligned}
y_t &= \sqrt{2}\, y_{\mathbf{10}} c^u_{\mathbf{10}} + \sqrt{2}\, y_{\mathbf{120}} \left(c^{d'}_{\mathbf{120}} + \frac{1}{\sqrt{3}} c^d_{\mathbf{120}}\right), \\
y_b &= \sqrt{2}\, y_{\mathbf{10}} c^d_{\mathbf{10}} + \sqrt{2}\, y_{\mathbf{120}} \left(c^{d'}_{\mathbf{120}} + \frac{1}{\sqrt{3}} c^d_{\mathbf{120}}\right), \\
y_\tau &= \sqrt{2}\, y_{\mathbf{10}} c^d_{\mathbf{10}} + \sqrt{2}\, y_{\mathbf{120}} \left(c^{d'}_{\mathbf{120}} - \sqrt{3} c^d_{\mathbf{120}}\right), \\
y_\nu &= \sqrt{2}\, y_{\mathbf{10}} c^u_{\mathbf{10}} + \sqrt{2}\, y_{\mathbf{120}} \left(c^{d'}_{\mathbf{120}} - \sqrt{3} c^d_{\mathbf{120}}\right).
\end{aligned}$$

$SO(10)$

Yukawa matching relation from $SO(10)$ to PS:

$$y_{\mathbf{10}} = \frac{1}{\sqrt{2}} y_1, \; y_{\mathbf{16}} = y_4, \qquad y_{\mathbf{120}} = \frac{1}{\sqrt{2}} y_1' = \frac{1}{2\sqrt{2}} y_{15}$$

$$\begin{aligned}
y_t &= y_1 c^u_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} + \frac{1}{2\sqrt{3}} y_{15} c^d_{\mathbf{120}}, \\
y_b &= y_1 c^d_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} + \frac{1}{2\sqrt{3}} y_{15} c^d_{\mathbf{120}}, \\
y_\tau &= y_1 c^d_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} - \frac{\sqrt{3}}{2} y_{15} c^d_{\mathbf{120}}, \\
y_\nu &= y_1 c^u_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} - \frac{\sqrt{3}}{2} y_{15} c^d_{\mathbf{120}}.
\end{aligned}$$

PS

Inverse seesaw: $\begin{pmatrix} 0 & m_{\mathrm{D}} & 0 \\ m_{\mathrm{D}} & 0 & m_{\mathrm{S}} \\ 0 & m_{\mathrm{S}} & \mu_{\mathrm{M}} \end{pmatrix} \longrightarrow m_\nu = \mu_{\mathrm{M}} \frac{m^2_{\mathrm{D}}}{m^2_{\mathrm{S}}}, \; m_{\mathrm{S}} = y_{16} M_{\mathrm{PS}}$

Considering global $U(1)$ symmetry, terms of $y_{\mathbf{10_2}}$ and y_{1_2} are forbidden

Gauge running and asymptotic unification

RG running for gauge couplings at different energy scale:

$$M_Z \xrightarrow{2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = b_i^{\mathrm{SM}}\alpha_i^2} M_{\mathrm{PS}} \xrightarrow{2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = b_i^{\mathrm{PS}}\alpha_i^2} M_{\mathrm{KK}} \xrightarrow{2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = b_i^{\mathrm{PS}}\alpha_i^2 + (S(t)-1)b_{\mathbf{10}}\alpha_i}$$

K.R. Dienes, et al, hep-ph/9803466, hep-th/0210294
T.R. Morris, hep-ph/0410142

Express KK states contribution in a continuous approximation:

Define effective 't Hooft coupling with respect to KK excitations:

$$S(t) = \begin{cases} 1 & \text{for } \mu < M_{\mathrm{KK}}\,, \\ \mu/M_{\mathrm{KK}} = M_Z e^t/M_{\mathrm{KK}} & \text{for } \mu > M_{\mathrm{KK}}\,. \end{cases} \longrightarrow \tilde{\alpha}_i(t) = \alpha_i(t)S(t)$$

$$2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = b_i^{\mathrm{PS}}\alpha_i^2 + (S(t)-1)b_{\mathbf{10}}\alpha_i^2$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_i}{\mathrm{d}t} = 2\pi\tilde{\alpha}_i + b_{\mathbf{10}}\tilde{\alpha}_i^2$$

$$\tilde{\alpha}_i = \frac{2\pi}{e^{-t+c_i}-b_{\mathbf{10}}}$$

$$\tilde{\alpha}_4, \tilde{\alpha}_{2L}, \tilde{\alpha}_{2R} \xrightarrow{\mathrm{UV}} \tilde{\alpha}_{\mathbf{10}}^{\mathrm{UV}} = -\frac{2\pi}{b_{\mathbf{10}}}$$

$b_{\mathbf{10}} < 0$ is crucial for gauge couplings existing asymptotically safe fixed point

Gauge running and asymptotic unification

β-coefficient of $SO(10)$ gauge group above the compactification scale M_{KK}:

$$b_{\mathbf{10}} = -(\tfrac{11}{3} - \tfrac{1}{6})C_2(SO(10)) + \tfrac{4}{3}\sum_F T(F_i) + \tfrac{1}{6}\sum_S T(S_i)$$

$H_{\mathbf{10}} \sim \mathbf{10}$, real
$H_{\mathbf{126}} \sim \mathbf{126}$, complex
Too large
$\tilde{\alpha}^{\text{UV}}_{\mathbf{10}} = 12\pi$

$H_{\mathbf{10}} \sim \mathbf{10}$, complex
$H_{\mathbf{120}} \sim \mathbf{120}$, real
$H_{\mathbf{16}} \sim \mathbf{16}$

$$\longrightarrow b_{\mathbf{10}} = -\frac{19}{3} \longrightarrow \tilde{\alpha}_4, \tilde{\alpha}_{2L}, \tilde{\alpha}_{2R} \xrightarrow{\text{UV}} \tilde{\alpha}^{\text{UV}}_{\mathbf{10}} = \frac{6\pi}{19}$$

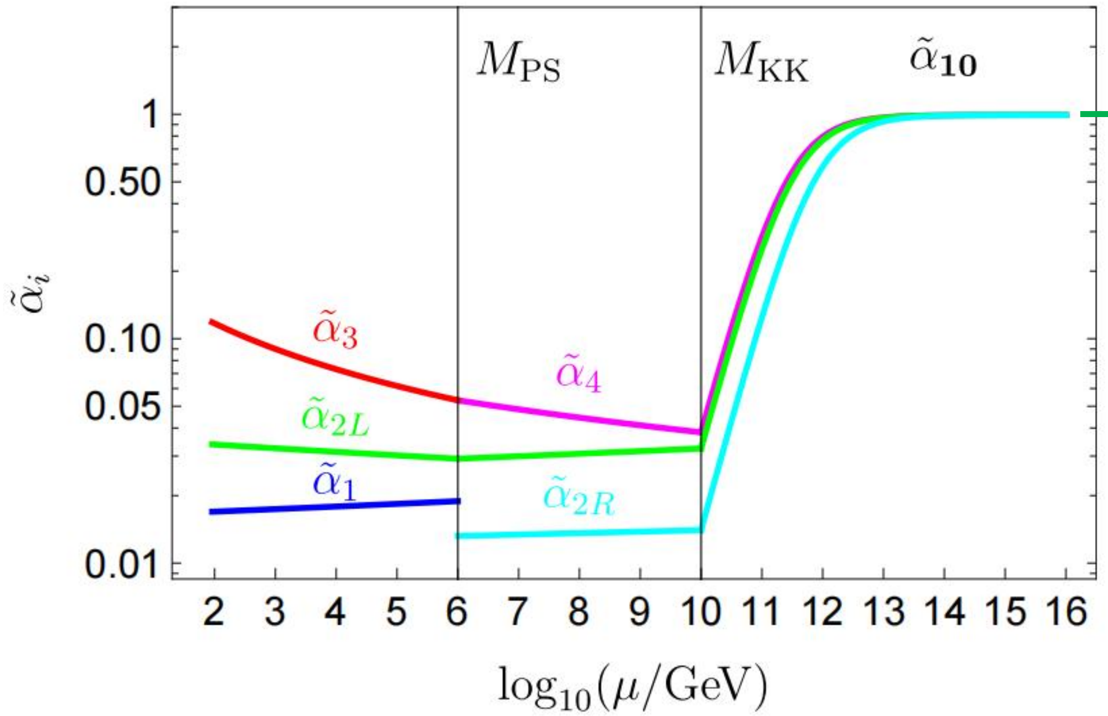


Figure 1. Running of the gauge couplings with $M_{\text{PS}} = 10^6$ GeV and $M_{\text{KK}} = 10^{10}$ GeV.

Asymptotically safe

5D effective gauge coupling:

$$\left.\frac{\Omega(d)}{(2\pi)^d} 4\pi\tilde{\alpha}\right|_{d=5} = \frac{8\pi^2}{3(2\pi)^5} 4\pi\tilde{\alpha} = \frac{2}{19\pi} \sim 0.033 \ll 1$$

Well under perturbative control

RGEs of Yukawa couplings

RG running of Yukawa couplings:

Only consider Yukawa couplings of the third generation fermions for simplicity

$$\begin{array}{lcccc} \mu: & M_Z \longrightarrow & M_{\mathrm{PS}} \longrightarrow & M_{\mathrm{KK}} \longrightarrow & \mathrm{UV} \\ \mathrm{Yukawas}: & y_t, y_b, y_\tau & y_1, y_1', y_{15} & y_1, y_1', y_{15} & y_{\mathbf{10}}, y_{\mathbf{120}} \end{array}$$

$$2\pi\frac{\mathrm{d}\alpha_t}{\mathrm{d}t} = \left[\frac{9}{2}\alpha_t + \frac{3}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{17}{20}\alpha_1 - 8\alpha_3\right]\alpha_t$$

$$2\pi\frac{d\alpha_b}{\mathrm{d}t} = \left[\frac{3}{2}\alpha_t + \frac{9}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{1}{4}\alpha_1 - 8\alpha_3\right]\alpha_b$$

$$2\pi\frac{\mathrm{d}\alpha_\tau}{\mathrm{d}t} = \left[3\alpha_t + 3\alpha_b + \frac{5}{2}\alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{9}{4}\alpha_1\right]\alpha_\tau$$

$M_{\mathrm{Z}} < \mu < M_{\mathrm{PS}}$, one-loop Yukawa RGEs in SM

$$2\pi\frac{\mathrm{d}\alpha_{y1}}{\mathrm{d}t} = \left[6\alpha_{y1} + 4\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1}$$

$$2\pi\frac{\mathrm{d}\alpha_{y1'}}{\mathrm{d}t} = \left[2\alpha_{y1} + 8\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1'}$$

$$2\pi\frac{\mathrm{d}\alpha_{y15}}{\mathrm{d}t} = \left[8\alpha_{y15} + 2\alpha_{y1} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y15}$$

$M_{\mathrm{PS}} < \mu < M_{\mathrm{KK}}$, one-loop Yukawa RGEs in PS

$$-\mathcal{L}_Y = y_{\mathbf{10}}\overline{\Psi_{\mathbf{16}}}H_{\mathbf{10}}\Psi_{\overline{\mathbf{16}}} + y_{\mathbf{120}}\overline{\Psi_{\mathbf{16}}}H_{\mathbf{120}}\Psi_{\overline{\mathbf{16}}} + y_{\mathbf{16}}\overline{\nu_{\mathrm{S}}}H_{\mathbf{16}}\Psi_{\overline{\mathbf{16}}} + \mathrm{h.c.}$$

$$y_6(\overline{\psi_L}H_6\Psi_L^c + \overline{\Psi_R^c}H_6\psi_R) + y_6'(\overline{\psi_L}H_{6L}\Psi_L^c + \overline{\Psi_R^c}H_{6R}\psi_R) + y_{10}(\overline{\psi_L}H_{10}\Psi_L^c + \overline{\Psi_R^c}H_{\overline{10}}\psi_R) + \mathrm{h.c.}$$

$+$

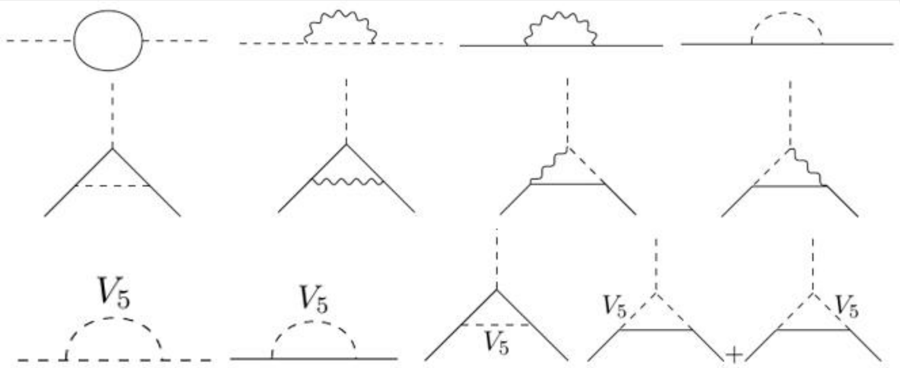
$$y_1\overline{\psi_L}h_1\psi_R + \overline{\psi_L}(y_1'h_1' + y_{15}h_{15})\psi_R + y_4\overline{\nu_{\mathrm{S}}}h_{\bar{4}}\psi_R + \mathrm{h.c.}$$

$SO(10)$ Yukawa couplings $\Longrightarrow$ PS decomposition

$$2\pi\frac{\mathrm{d}\alpha_{yr}}{\mathrm{d}t} = 2\pi\frac{\mathrm{d}\alpha_{yr}}{\mathrm{d}t}\bigg|_{\mathrm{PS}} + (S(t) - 1)\, 2\pi\frac{\mathrm{d}\alpha_{yr}}{\mathrm{d}t}\bigg|_{\mathrm{KK}}$$

$M_{\mathrm{KK}} < \mu$, $SO(10)$

RGEs of Yukawa couplings



$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t}=\left[2\pi+26\tilde{\alpha}_{y\mathbf{10}}+24\tilde{\alpha}_{y\mathbf{120}}-\frac{171}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{10}}$$
$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t}=\left[2\pi+10\tilde{\alpha}_{y\mathbf{10}}+120\tilde{\alpha}_{y\mathbf{120}}-\frac{219}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{120}}$$

One-loop Yukawa RGEs in 5D $SO(10)$

$$2\pi\frac{\mathrm{d}\alpha_{y1}}{\mathrm{d}t}\Big|_{\mathrm{KK}}=\Big[10\alpha_{y1}+\frac{3}{2}\alpha_{y6}+4\alpha_{y1'}-\frac{5}{2}\alpha_{y10}+\frac{9}{4}\alpha_{y6'}-\frac{81}{8}\alpha_4-\frac{45}{8}(\alpha_{2L}+\alpha_{2R})\Big]\alpha_{y1}\,,$$
$$2\pi\frac{\mathrm{d}\alpha_{y1'}}{\mathrm{d}t}\Big|_{\mathrm{KK}}=\Big[2\alpha_{y1}+\frac{3}{2}\alpha_{y6}+12\alpha_{y1'}+\frac{15}{2}\alpha_{y10}+\frac{9}{4}\alpha_{y6'}-\frac{129}{8}\alpha_4-\frac{45}{8}(\alpha_{2L}+\alpha_{2R})\Big]\alpha_{y1'}\,,$$
$$2\pi\frac{\mathrm{d}\alpha_{y15}}{\mathrm{d}t}\Big|_{\mathrm{KK}}=\Big[2\alpha_{y1}+\frac{3}{2}\alpha_{y6}+9\alpha_{y15}+\frac{3}{2}\alpha_{y10}+\frac{9}{4}\alpha_{y6'}-\frac{129}{8}\alpha_4-\frac{45}{8}(\alpha_{2L}+\alpha_{2R})\Big]\alpha_{y15}\,.$$

PS decomposition

Explicit unification: all Yukawa couplings have already been fully unified into their $SO(10)$ values at KK scale

$$\begin{aligned}\frac{1}{2}\alpha_{y1},\ \frac{1}{4}\alpha_{y6} &= \alpha_{y\mathbf{10}}\,,\\ \frac{1}{2}\alpha_{y1'},\ \frac{1}{8}\alpha_{y15},\ \frac{1}{8}\alpha_{y10},\ \frac{1}{16}\alpha_{y6'} &= \alpha_{y\mathbf{120}}\,,\end{aligned}\qquad \text{at } \mu=M_{\mathrm{KK}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t}=\left[2\pi+26\tilde{\alpha}_{y\mathbf{10}}+24\tilde{\alpha}_{y\mathbf{120}}-\frac{81}{8}\tilde{\alpha}_4-\frac{45}{8}(\tilde{\alpha}_{2L}+\tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{10}}$$
$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t}=\left[2\pi+10\tilde{\alpha}_{y\mathbf{10}}+120\tilde{\alpha}_{y\mathbf{120}}-\frac{129}{8}\tilde{\alpha}_4-\frac{45}{8}(\tilde{\alpha}_{2L}+\tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{120}}$$

RGEs of Yukawa couplings

One-loop Yukawa RGEs in 5D $SO(10)$:

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t}=\left[2\pi+26\tilde{\alpha}_{y\mathbf{10}}+24\tilde{\alpha}_{y\mathbf{120}}-\frac{171}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{10}}$$
$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t}=\left[2\pi+10\tilde{\alpha}_{y\mathbf{10}}+120\tilde{\alpha}_{y\mathbf{120}}-\frac{219}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{120}}$$

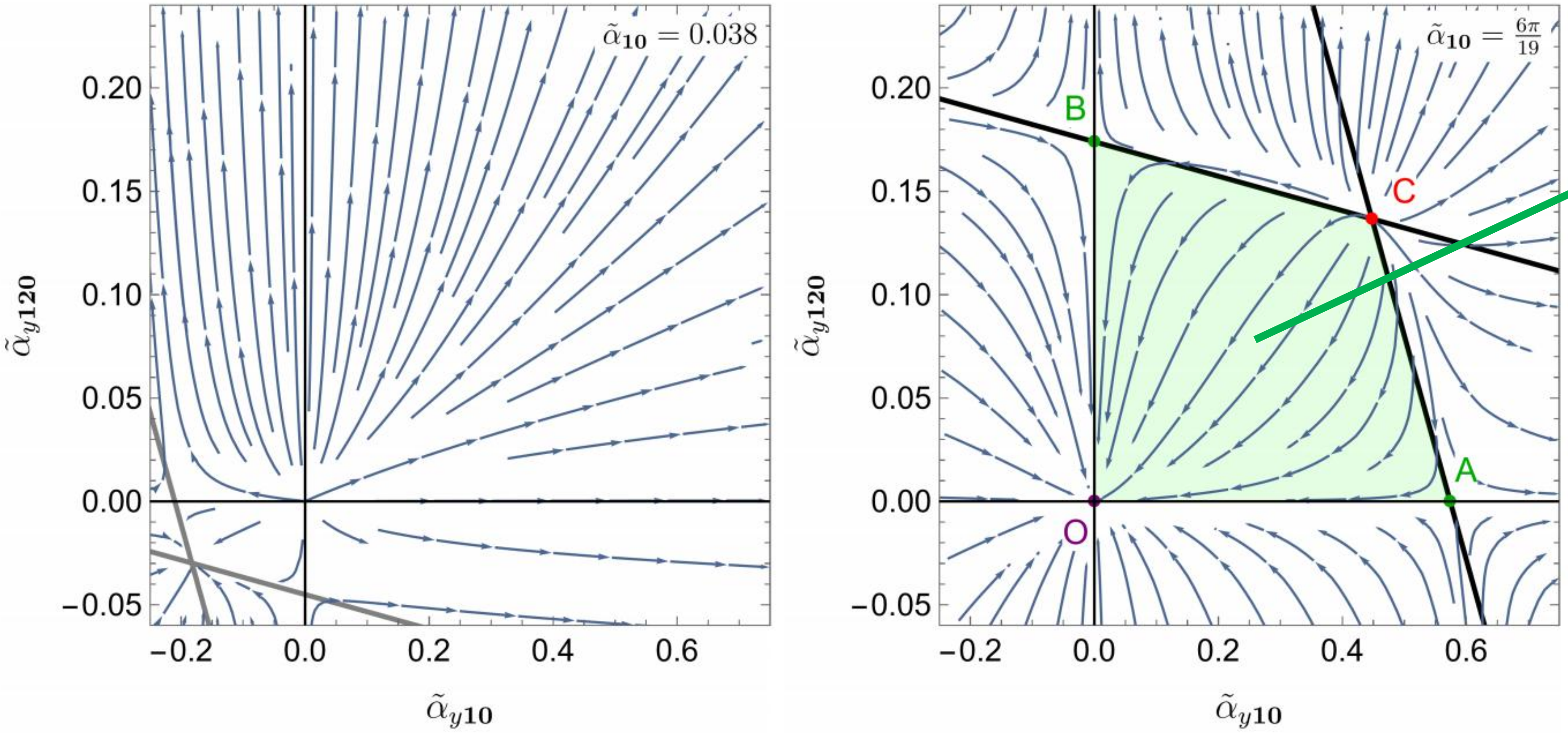


Figure 2. Stream plot of Yukawa couplings $\tilde{\alpha}_{y\mathbf{10}}, \tilde{\alpha}_{y\mathbf{120}}$ in 5D $SO(10)$ GUT.

Asymptotically free region

As $\tilde{\alpha}_{10}$ gets larger, asymptotically free region will gets bigger, the negative contribution of gauge couplings can suppress the positive contribution of Yukawa couplings, thereby achieving the asymptotic freedom of Yukawa couplings.

Scan Yukawa couplings

Free parameters: $\{y_{\mathbf{10}}(M_{\rm KK}), y_{\mathbf{120}}(M_{\rm KK})\}$

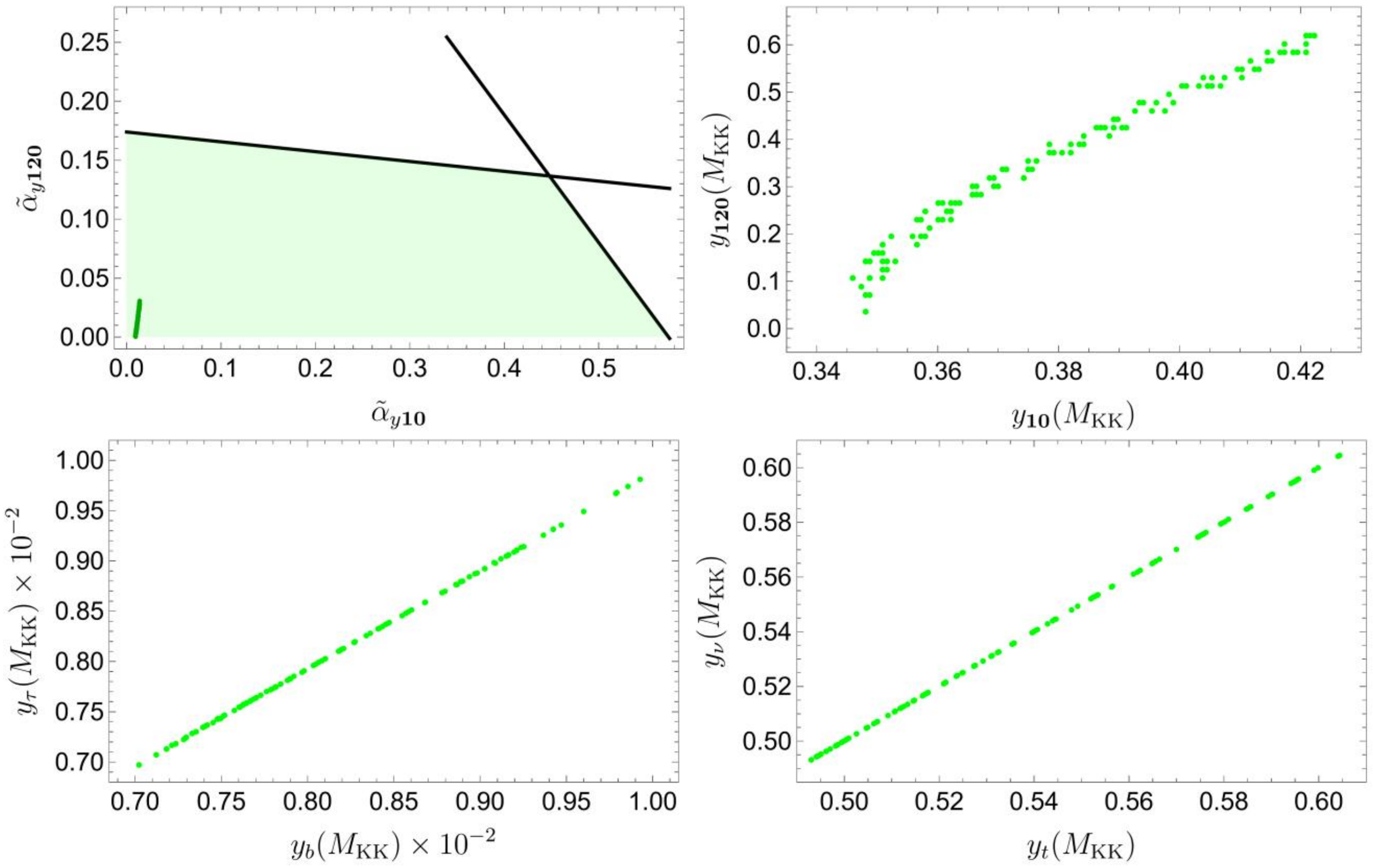
evolve couplings from $M_{\rm KK}$ to $M_{\rm PS}$ through one-loop Yukawa RGE in $SO(10)$

VEV constraint: $(c^u_{\mathbf{10}})^2 + (c^d_{\mathbf{10}})^2 + 2(c^d_{\mathbf{120}})^2 + 2(c^{d'}_{\mathbf{120}})^2 = 1$

Initial values for one-loop Yukawa RGEs in SM to evolve couplings from the EW scale to $M_{\rm PS}$:

$$\{y_t, y_b, y_\tau\} = \{0.97, 0.016, 0.010\} \text{ at } \mu = M_Z$$

G.Y. Huang, S. Zhou, 2009.04851



$y_{\mathbf{10}}(M_{\rm KK}) \in (0.3460, 0.4223)$
$y_{\mathbf{120}}(M_{\rm KK}) \in (0.0353, 0.6187)$
$y_b(M_{\rm KK}) \in (0.0070, 0.0099)$
$y_\tau(M_{\rm KK}) \in (0.0070, 0.0098)$
$y_t(M_{\rm KK}) \in (0.4931, 0.6044)$
$y_\nu(M_{\rm KK}) \in (0.4930, 0.6043)$

There are some parameter space that can explain fermion masses

Benchmark point

$$\mu > M_{\rm KK} \quad \begin{aligned} 2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t} &= \left[2\pi + 26\tilde{\alpha}_{y\mathbf{10}} + 24\tilde{\alpha}_{y\mathbf{120}} - \frac{81}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L}+\tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{10}} \\ 2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t} &= \left[2\pi + 10\tilde{\alpha}_{y\mathbf{10}} + 120\tilde{\alpha}_{y\mathbf{120}} - \frac{129}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L}+\tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{120}} \end{aligned}$$

Negative gauge contribution ultimately suppressing the positive Yukawa contribution

Asymptotically free

Inputs	$y_{\mathbf{10}}(M_{\rm KK})$ 0.348	$y_{\mathbf{120}}(M_{\rm KK})$ 0.035		
	$M_{\rm PS}$ 10^6 GeV	$M_{\rm KK}$ 10^{10} GeV	$\mu_{\rm M}$ 1 keV	$y_{\mathbf{16}}(M_{\rm KK})$ 10^{-2}
Outputs	$c^u_{\mathbf{10}}$ 0.999	$c^d_{\mathbf{10}}$ 0.013	$c^d_{\mathbf{120}}$ 0.0004	$c^{d'}_{\mathbf{120}}$ 0.0217
	$y_1(M_{\rm PS})$ 0.681	$y'_1(M_{\rm PS})$ 0.075	$y_{15}(M_{\rm PS})$ 0.149	m_ν 0.07 eV
	$y_b(M_{\rm KK})$ 0.0074	$y_\tau(M_{\rm KK})$ 0.0073	$y_t(M_{\rm KK})$ 0.534	$y_\nu(M_{\rm KK})$ 0.493

Table 2. Inputs and predictions of VEVs, Yukawa couplings, charged fermion masses and neutrino masses of one point.

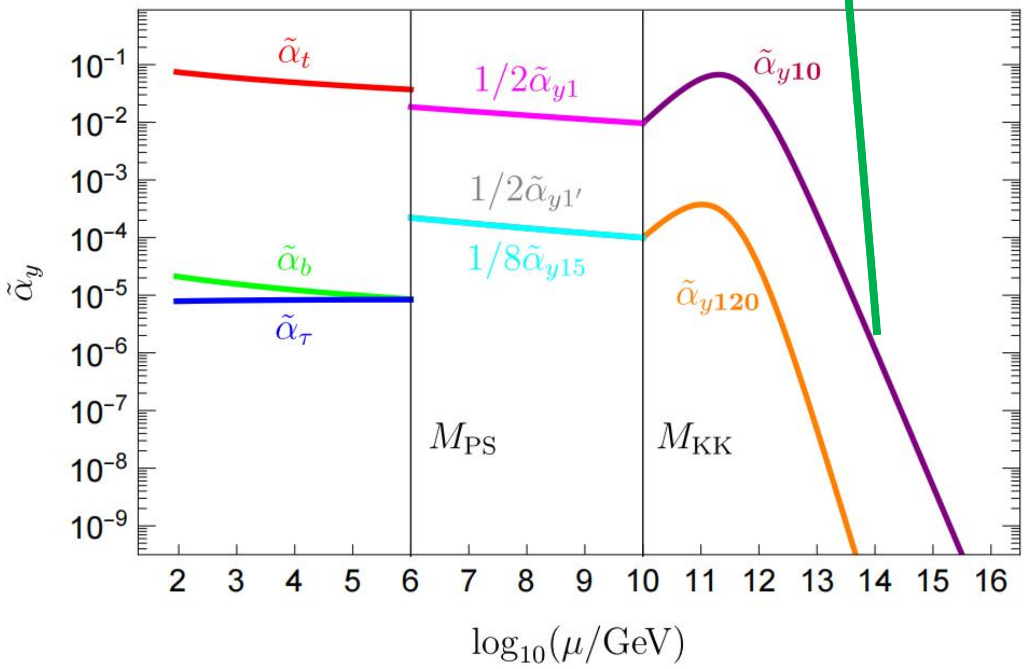


Figure 4. Running of the Yukawa couplings for the benchmark point with $M_{\rm PS} = 10^6$ GeV and $M_{\rm KK} = 10^{10}$ GeV.

Conclusion

1. 5D $SO(10)$ GUT with PS as an intermediate scale can realize asymptotic unification of gauge couplings and asymptotic freedom of Yukawa couplings.
2. We recover experimental data on the masses of third generation quarks and leptons.
3. We have derived one-loop Yukawa RGEs for 5-dimensional $SO(10)$ group and its decomposition to PS group.
4. As the energy scale runs toward the UV limit, the ‘t Hooft couplings of all gauge couplings approach the UV fixed point $-2\pi/b_{10}$ regardless of their initial values. A negative b_{10} is crucial for realizing asymptotic unification of gauge couplings, which limits the redundancy of Higgs content in $SO(10)$ GUTs.
5. Separate left-handed and right-handed chiral fields into $\Psi_{\mathbf{16}}$ and $\Psi_{\overline{\mathbf{16}}}$, proton decay is naturally forbidden.

Thanks！