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Chemical potential driven chiral phase transition

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Based on work, Zhaofeng Kang & Jiangzhu, arxiv 2501.15242

Conclusion

A dark-QCD sector is well motivated in new physics models, and it may experience an intrinsic first order PT, thus a source of stochastic GW signal

However, the resulting GW amplitude usually is very small because of the very short PT duration (much shorter than the Hubble time scale), leading to the suppressed GW release

We explore the chiral PT driven by a large chemical potential, and find that it can change the situation and makes chiral PT with T_c~(1 GeV, 100GeV) observable at BBO

Start from a big story

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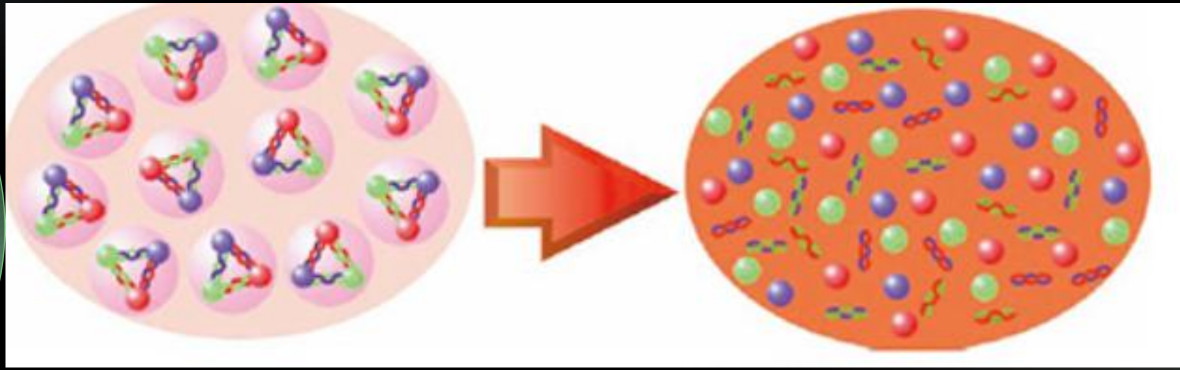
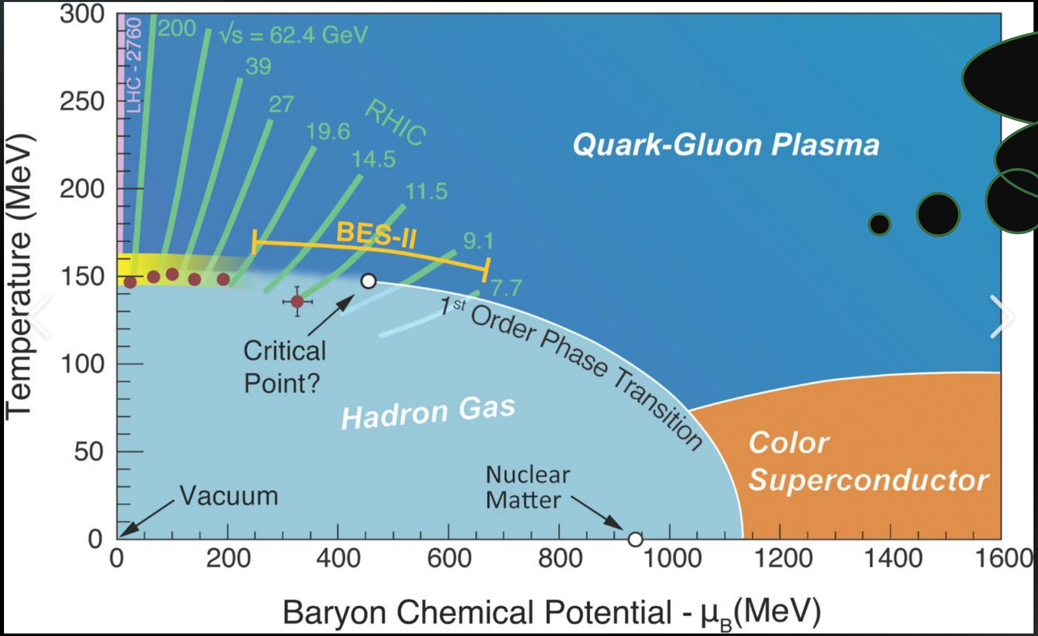
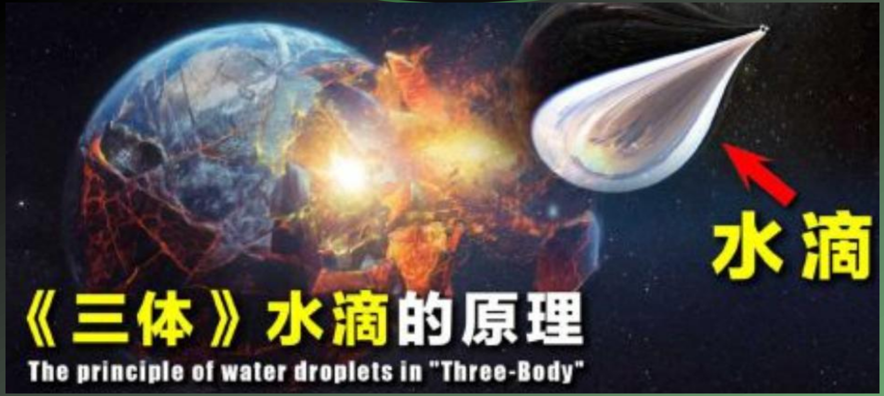
Explore the QCD matter in the nature, really

In 2004, collision of heavy ions $^{197}\mathrm{Au}$ at RHIC created a new state: quark-gluon plasma (QGP, **perfect liquid**), surviving~$10^{-22}s$





QCD matter may have many phases, and the ultimate goal is to determine the QCD phase diagram in the $\mu - T$ plane



To explore dark-QCD matters

Dark colored sector, with or without dynamic dark quarks, widely appeared in the context of new physics

Composite Higgs/dark matter/axion & scale gensis….

System with non-Abelian gauge interaction is plagued by **non-perturbative effects**, but also appears fascinating as a result

monopole/instanton /ADS-CFT….

Catalyzed by the discover of gravitational wave (GW), which may be the faint remnant of intense activities in the early universe

e.g., event like first order phase transition, **intrinsic** for some dark colored sectors

To explore dark-QCD matters

What does intrinsic mean?

e.g., for the pure $SU(N)$ Yang-Mills system with $N > 2$, transition from the gluon (deconfinement) phase to glueball (confinement) phase is proven to be first order by lattice, **no need of tuning parameters as in the EWPT**

Zhaofeng Kang, Jiang Zhu & Shinya Matsuzaki, JHEP 21

W. C. Huang, M. Reichert, F. Sannino & Z. W. Wang, PRD21

Ad: we proposed the quasi-particle approach with **varying $M_g(T)$** to perturbatively deal with this PT

Zhaofeng Kang, Jiang Zhu, and Jun Guo, PRD 23

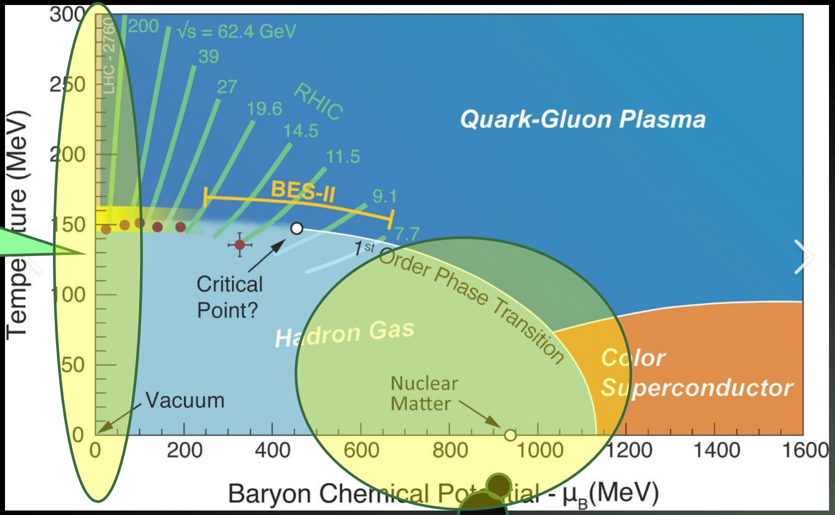
Adding massless quarks may bring a new PT, chiral PT, and its order is also intrinsic

Lattice shows that chiral PT is firt order for $\mathrm{N}_f \geq 3$ and $\mu = 0$

To explore dark-QCD matters

Most studies on QCD-like PT is along the T-axis

our universe is in the **high entropy** state, with $\eta \equiv \frac{n_B}{n_S} \sim 10^{-10}$, so (relativistic) quark chemical potential was tiny in the early universe: $\frac{n_B}{s} = \frac{n_+ - n_-}{s} \propto \frac{\mu}{T}$



A large lepton asymmetry Y_L in the neutrino sector changes it?

Yes, but BBN limits $|Y_L| < 0.01$, and the resulting GW is not detectable

Dominik J. Schwarz and M. Stuke, JCAP 2009
Fei Gao and Isabel M. Oldengott, PRL 2022
H. w. Zheng, F. Gao, L. Bian, S. x. Qin and Y. x. Liu, PRD 2025

However, this is not necessary in the dark-QCD! The goal of this talk is to address the big role of μ

Our dark-QCD is based on three flavor dark quarks in the chiral limit

01

An old QCD story
NJL at $T = 0$

What are the QCD symmetries?

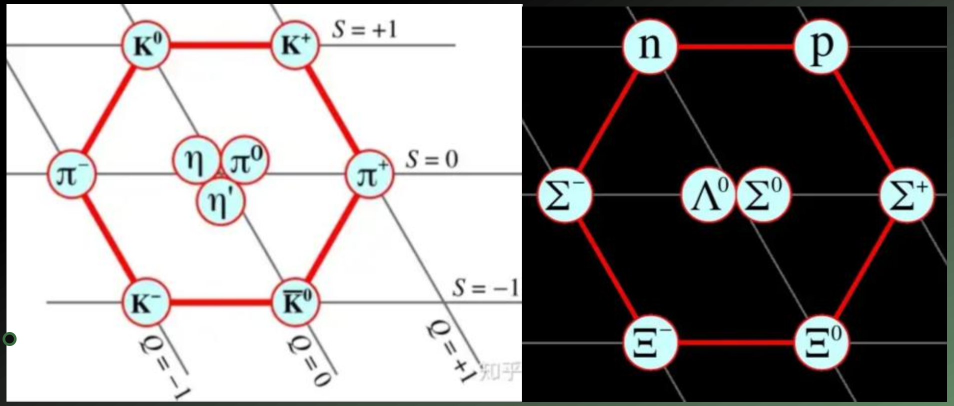
QCD with N_f flavor massless quarks $q^i = q^i_L + q^i_R$ shows chiral symmetries at classical level

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$$

Anomaly at quantum level

Baryon number

However, not all of these symmetries are shown in the hadron spectrum, which shows the pattern: $m_\pi \ll m_p$



What is hidden behind them?

It inspires Nambu to introduce spontaneously breaking to QFT and discover the chiral symmetry breaking

Pre-QCD: the NJL (Nambu–Jona-Lasinio) model (1961)

To understand the origin of hadron mass, NJL model was proposed to mimic the low energy QCD, including the **four-fermion terms**

$$\mathcal{L}_{\text{chiral}} = \bar{q} i \partial\!\!\!/ q + G_S \sum_{a=1}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q} i \gamma^5 \lambda^a q)^2 \right]$$

$$+ G_D \left[\det(\bar{q}(1-\gamma^5)q + \bar{q}(1+\gamma^5)q) \right]$$

No hints of gluon (color was not discovered before 1970s)

The KMT (Kobayashi-Maskawa-'t Hooft, 1976~1978) determinant term from instanton effect to break $\mathrm{U}(1)_A$ for η'

Later, we will show that, the NJL model determines the QCD vacuum with $SU(2)_A$ chiral symmetry spontaneously breaking due to quark condensation $\langle \bar{q} q \rangle \neq 0$, and π are the 3 pGSB!

The remaining $\mathrm{SU}(2)_V$ is seen in the hadron spectrum

Bosonization via Hubbard-Stratonovich transformation

J. Hubbard, PRL, 1959

To describe the four-fermion terms effect, one induces the collective fields $\boldsymbol{\sigma} = 4G_S\langle\bar{q}q\rangle$ and $\boldsymbol{\pi}$ in the partial function

$$Const = \int \mathcal{D}[\sigma]\mathcal{D}[\pi_i] \exp\left(i\int d^4x \frac{-1}{4G}\left[(\sigma + 2G\bar{q}q)^2 + (\pi_i + 2Gi\bar{q}\gamma_5\tau_i q)^2\right]\right)$$

$$\mathcal{L}^{eff}_{NJL} = \bar{q}_i[i\not{\partial} - M_i(\sigma)]q_i - V^{tree}_{NJL}(\sigma).$$

The quarks gain constitute mass **(the main mass origin)** from the QCD vacuum with quark condensate

$$M_i(\sigma) = -\sigma - \frac{G_D\sigma^2}{64G_S^2},$$

The tree-level effective potential, plus a zero point energy (next page), used to check if quark condensate could develop

$$\mathcal{V}^{tree}_{NJL}(\sigma) = \frac{3\sigma^2}{8G_S} + \frac{G_D\sigma^3}{128G_S^3}.$$

Chiral symmetry breaking

Zero-point energy for each flavor $\frac{1}{2}\omega_i(p) = \frac{1}{2}\left(p^2 + M_i^2(\sigma)\right)^{1/2}$ contributes an effective potential at quantum level

$$\mathcal{V}^{vac}_{NJL} = -4NN_f\int^{\Lambda}\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2}\sqrt{\mathbf{p}^2+M_i^2(\sigma)}$$
$$= -2N_fN\frac{\Lambda^4}{16\pi^2}\left[(2+\zeta^2)\sqrt{1+\zeta^2}+\frac{\zeta^4}{2}\log\left(\frac{\sqrt{1+\zeta^2}-1}{\sqrt{1+\zeta^2}+1}\right)\right]$$

condense occurs & $M(\sigma) < \Lambda \Rightarrow$ $\frac{\pi^2}{6} \le \boldsymbol{G_S\Lambda^2} \le \boldsymbol{3}$, and this condense $\langle\sigma\rangle$ will be the order parameter for chiral PT

1.the cut-off $\Lambda \sim \Lambda_{\rm QCD}$ is built-in in the effective model NJL
2. It is fixed by meson spectrum in real QCD, along with G_S and G_D

02

$T \neq 0$: interplay with gluons in the PNJL

Kenji Fukushima，PLB 2004

Confinement transition in hot pure gauge system

At $T_c \sim \Lambda$, the $SU(N \geq 3)$ experiences a first order confinement phase transition from the quasi-gluon phase to the glueball phase

The order parameter is the traced Polyakov loop $\boldsymbol{\ell = \mathrm{Tr_c L} = \mathrm{Tr}_c \exp(ig\beta A_4^a T^a)}$, gauge invariant and charged under Z_N

The Z_N-invariant Landau free energy $L[\ell]$ to study PT may be constructed, following the phenomenological approaches, e.g.,

Unlike EWPT, the free energy cannot be calculated simply via the perturbative approach; try this way

$$V_{PLM}(l, l^*, T) = T^4 \left[\frac{-b_2(T)}{2} l l^* + \frac{b_3}{6}(l^3 + l^{*3}) + \frac{b_4}{4}(l l^*)^2 \right]$$

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3$$

Color number	a_0	a_1	a_2	a_3	b_3	b_4
$N = 3$	6.75	-1.95	2.625	-7.44	0.75	7.5

Fix b_i by **lattice data: order+ thermodynamics**

Zhaofeng Kang, Jiang Zhu, and Jun Guo， PRD 23

Convey the confinement effect to quarks

The information of gluon, in particular, confinement is absent in the NJL model, thus considering the A_0 background $\boldsymbol{D_\mu = \partial_\mu + iA_0\delta_{\mu 0}}$

$$\mathcal{V}^T_{NJL} = -\frac{3T^4}{\pi^2}\int dx x^2 G(x,\sigma,T,\mu)$$

Calculable free energy

$\mathrm{V}\sim\mathrm{Tr}\log[\boldsymbol{D^2(A_0)} + M^2(\sigma)]$

When the system has quark asymmetry, at finite T one should replace $H \rightarrow H - \mu N$ with N **the conserved charge density, i.e., $\boldsymbol{i\mu\bar{q}\gamma^0 q}$**

When there is a constant A_4 background, simply $\mu \rightarrow \mu - iA_4$, and thus **$\boldsymbol{A_4}$ is dubbed as the imaginary chemical** potential

From canonical to grand canonical ensemble

Convey the confinement effect to quarks

Where is the color confinement effect?

$$G(x,\sigma,T,\mu) = \log\left[1 + e^{-3\left(\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}+\frac{\mu}{T}\right)} + 3l e^{-\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}+\frac{\mu}{T}} + 3l^* e^{-2\left(\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}+\frac{\mu}{T}\right)}\right]$$

$$+ \log\left[1 + e^{-3\left(\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}-\frac{\mu}{T}\right)} + 3l^* e^{-\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}+\frac{\mu}{T}} + 3l e^{-2\left(\sqrt{x^2+\frac{M(\sigma)^2}{T^2}}-\frac{\mu}{T}\right)}\right]$$

1. At the deconfinement vacuum, $l \to 1$ at very high T and it reduces to a gas of free & massless Dirac fermions

2. At the confinement vacuum, the Z_3 recovered phase, the order parameter $l = 0$, and the **baryonic** term dominates, signing the **statistical confinement effect**

For vanishing chemical potential, the two lines are complex conjugate to each other, thus a real $\boldsymbol{G}$, otherwise we encounter the sign problem

M. Reichert, F Sannino, Zhi-Wei Wang, and Chen Zhang, JHEP,2022

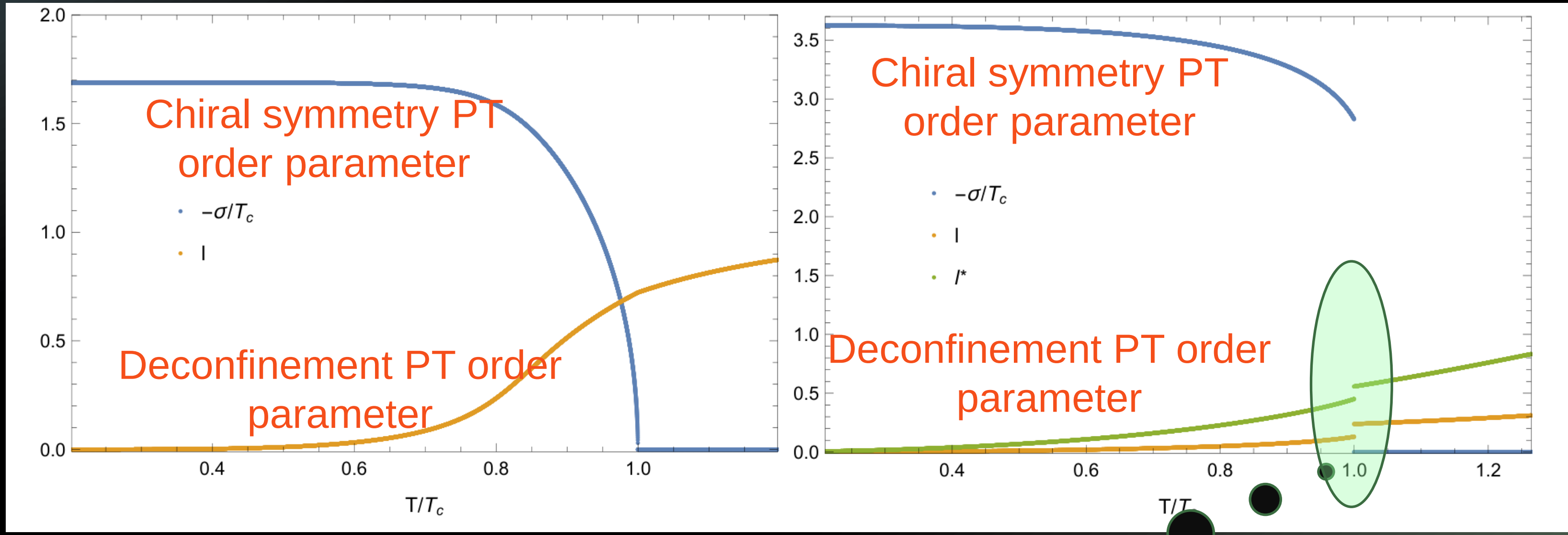
If the KMT coupling G_D is quite large, then it can drive chiral PT first order, for instance in

A new story: μ-driven cosmic chiral PT

03

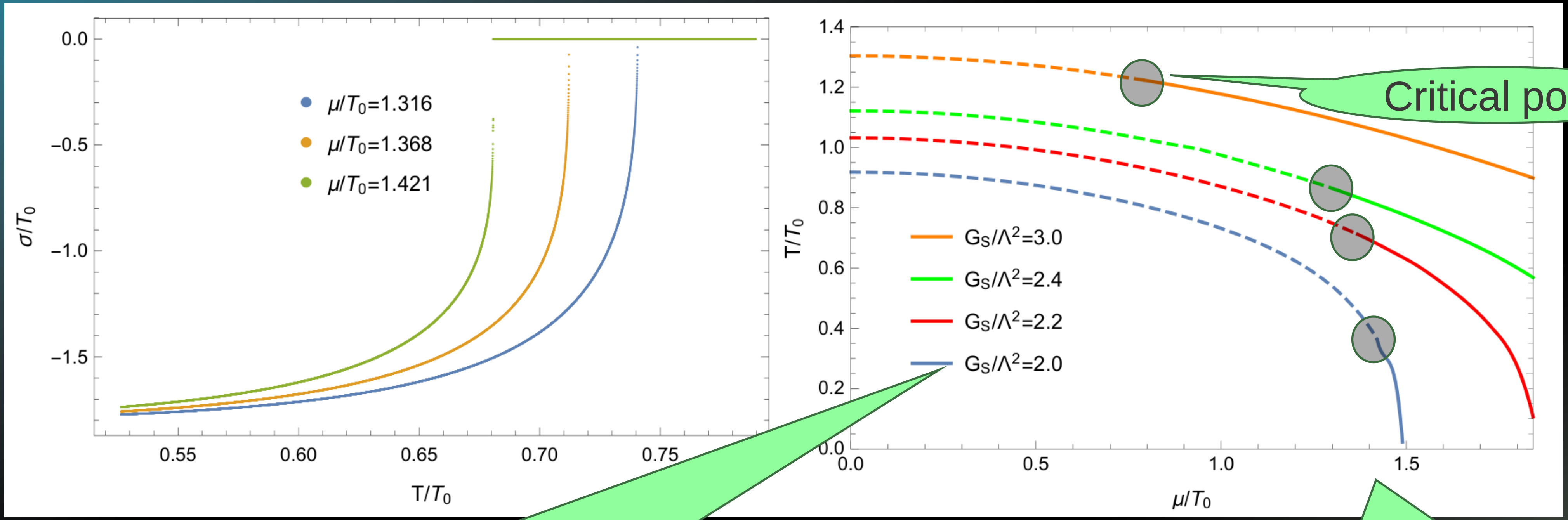
To plot the phase diagram

To determine the order of PT, we need to determine the three order fields $(l(T), l^*, \sigma)$ by solving $\frac{\partial V_{eff}(T)}{\partial l} = 0, \frac{\partial V_{eff}}{\partial l^*} = 0$ & $\frac{\partial V_{eff}}{\partial \sigma} = 0$



Chiral PT is first order with σ jumping around T_c, but **l is almost continuous during chiral PT**, which helps us to reduce the problem into one-dimension

The μ-driven dark-QCD phase diagram



1. Require $G_S\Lambda^2 > 1.8$ to realize FOPT

2. For a larger G_S , the critical point shifts towards the obviously smaller chemical potential. This results in a more pronounced FOPT.

3. Along each line, as μ increases, the critical temperature T_c rapidly decreases and the latent heat release (not shown) also decreases

Cosmic chiral PT: modified bounce equation for σ

The composite σ field has **no tree level kinematic term**, which arises at one-loop level via the wave function renormalization Z_σ (see later)

$$S_E = 4\pi \int dr r^2 \left(\frac{1}{2Z_\sigma} \left(\frac{d\sigma}{dr} \right)^2 + V_{PNJL}[\sigma, l(\sigma), l^*(\sigma), T, \mu] \right)$$

$$\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - \frac{1}{2}\frac{Z'_\sigma}{Z_\sigma}\left(\frac{d\sigma}{dr}\right)^2 = Z_\sigma V'_{PNJL},$$

$$\frac{d\sigma}{dr}\Big|_{r=0} = 0, \quad \sigma|_{r\to\infty} = 0,$$

$Z_\sigma(\sigma)$ depends on the order parameter field, inducing a “kinematic term”

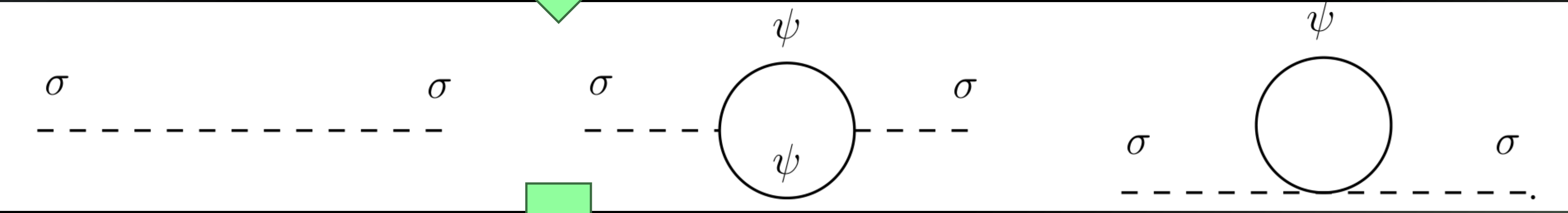
How to solve this kind of modified bounce equation? Please use the public program by Jiang Zhu & Banghui Hua, 2501.15236

Cosmic chiral PT: renormalization factor for σ

$Z_\sigma^{-1} = -\frac{d\Gamma_{\sigma\sigma}(q_0,\mathbf{q},\sigma)}{d\mathbf{q}^2}\Big|_{q_0=0,\mathbf{q}^2=0}$ with $\Gamma_{\sigma\sigma}$ the 1PI 2-point function

$$\mathcal{L}^{eff}_{NJL} = \bar{q}[i\partial\!\!\!/ - M(\sigma) + \gamma^0(\mu - iA_4)]q - V^{tree}_{NJL},$$

momentum of the quasi quark on the PL background: $k = (k_0 + \mu - iA_4, \mathbf{k})$



$$Z_\sigma^{-1} = 18\left(1 - \frac{G_D\sigma}{4G_S^2}\right)^2\left[I(0,\sigma) + 4M^2(\sigma)\frac{dI(0,\sigma)}{d\mathbf{q}^2}\right]$$

Order field σ dependent!

Cosmic chiral PT: renormalization factor for σ

Calculation of $I(0,\sigma)$ requires certain technique and we refer to our work, arxiv 2501.15242

$$I(0,\sigma)=\int\frac{d\mathbf{k}^3}{(2\pi)^3}\frac{1}{4E_k^3}\left[1-f_+(\omega,l)-f_-(\omega,l)+E_k\left(\frac{\partial f_+(\omega,l)}{\partial\omega}+\frac{\partial f_-(\omega,l)}{\partial\omega}\right)\right]_{\omega=E_k}$$

$$\frac{dI(0,\sigma)}{d\mathbf{q}^2}=\int\frac{d\mathbf{k}^3}{(2\pi)^3}\frac{1}{16E_k^5}\left[3\Big(f_+(\omega,l)+f_-(\omega,l)\Big)-E_k\left(\frac{\partial f_+(\omega,l)}{\partial\omega}+\frac{\partial f_-(\omega,l)}{\partial\omega}\right)-1\right]_{\omega=E_k}$$

$$\frac{(l^*+2le^{-\Omega_+})e^{-\Omega_+}+e^{-3\Omega_+}}{1+3(l+l^*e^{-\Omega_+})e^{-\Omega_+}+e^{-3\Omega_+}}$$

$f_-(\omega,l)$ is obtained with $l\to l^*$ and $\Omega_+\to\Omega_-$ with $\Omega_\pm\equiv(\omega\pm\mu)/T$

$Z(\sigma)<0$ in some region of σ, indicating that the condensation is unstable and thus unphysical.

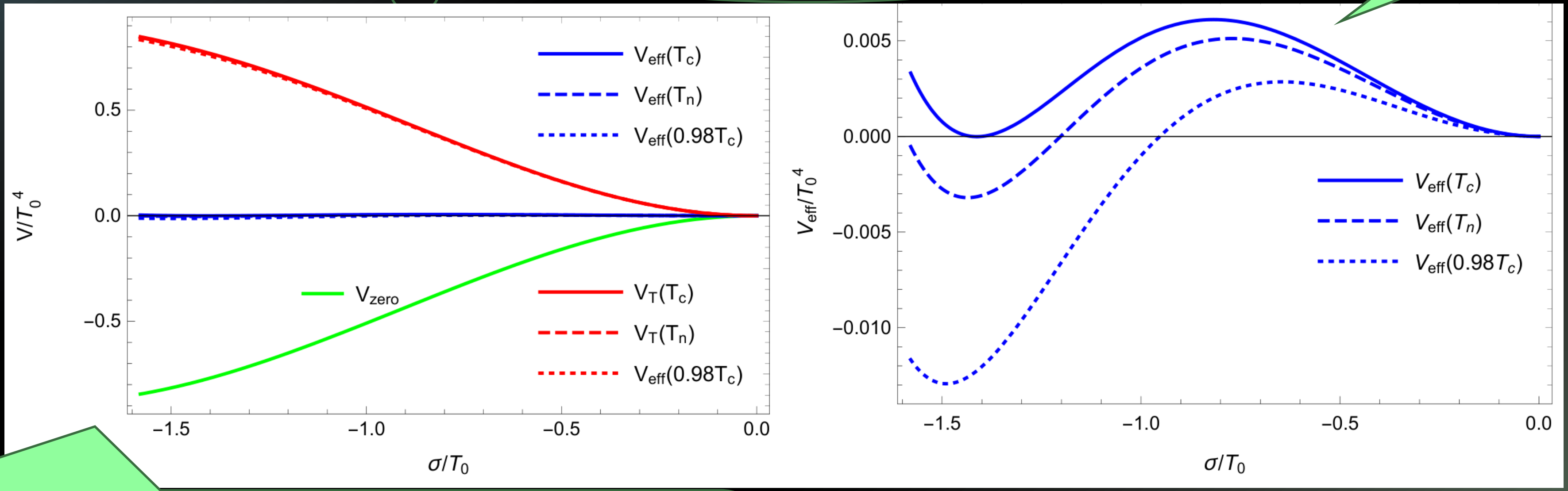
μ-enhanced GW prospect

04

Cosmic μ-driven chiral PT: dynamical tunning

GW prospect of QCD-like PT is hampered by short duration, $\tilde{\beta} = \frac{Td\left(\frac{S_E}{T}\right)}{dT}\Big|_{T=T_n} \gg 10^3$, why??

Also, in the confinement PT



1. The critical temperature T_c is dynamically tuned to **cancel two large parts, V_{zero} and $V_{zero}(T)$ in one vacuum to approach zero**, so as T drops a little below T_c, $\Delta V(T) = V(\sigma_t, T) - V(\sigma_f, T)$ changes dramatically with T

2. in the thin wall limit, where $S_3(T) \propto 1/\Delta V(T)$ and thus $\tilde{\beta} \sim -\frac{1}{\Delta V(T)^2}\frac{d\Delta V(T)}{dT}$ tends to be huge, $\sim \mathcal{O}(10^5)$

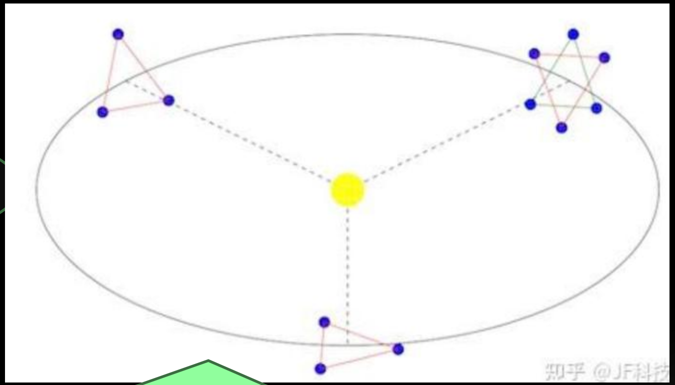
Cosmic μ-driven chiral PT: μ-flatten

Increasing $\mu \sim T_c$ helps to flatten the potential,
thus decreasing T_n and $\tilde{\beta}$ at the same time

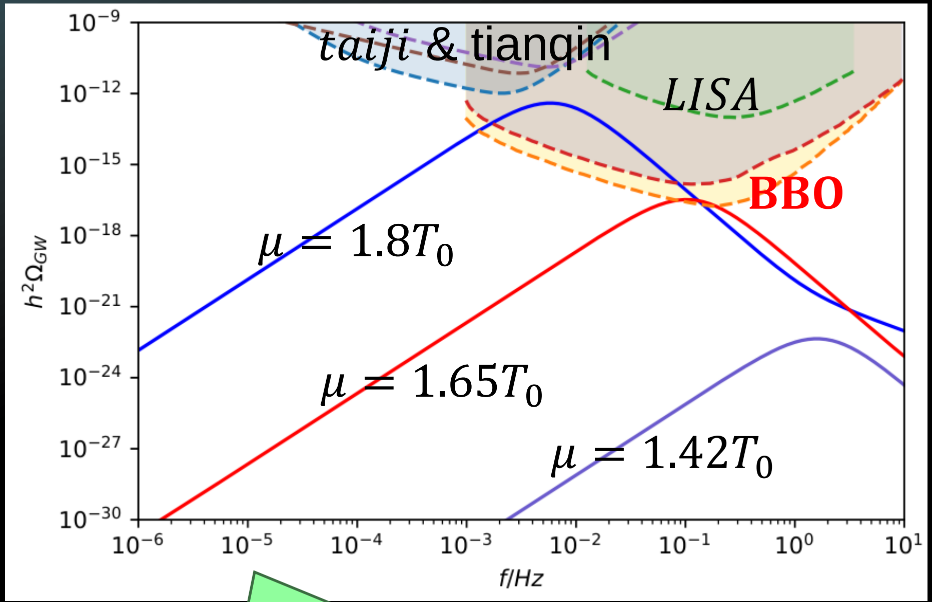
Phase Transition Parameters of Benchmark B							
Benchmark	μ/T_0	T_0	T_c	T_n	g_*	α	$\tilde{\beta}$
B	1.421	$1.4695T_c$	10GeV	$0.9992T_c$	138	0.0234	791545
B	1.65	$1.9309T_c$	10MeV	$0.99348T_c$	53	0.5628	51083
B	1.65	$1.9309T_c$	100MeV	$0.9948T_c$	101	0.2953	51083.0
B	1.65	$1.9309T_c$	1GeV	$0.9948T_c$	129	0.2312	51083.0
B	1.65	$1.9309T_c$	10GeV	$0.9948T_c$	138	0.2167	51083.0
B	1.65	$1.9309T_c$	100GeV	$0.9948T_c$	151	0.1976	51083.0
B	1.8	$3.6122T_c$	10MeV	$0.9261T_c$	53	2.2794	2901.27
B	1.8	$3.6122T_c$	100MeV	$0.9261T_c$	101	1.1961	2901.27
B	1.8	$3.6122T_c$	1GeV	$0.9261T_c$	129	0.9365	2901.27
B	1.8	$3.6122T_c$	10GeV	$0.9261T_c$	138	0.8754	2901.27
B	1.8	$3.6122T_c$	100GeV	$0.9261T_c$	151	0.8000	2901.27

Cosmic μ-driven chiral PT: GW prospect

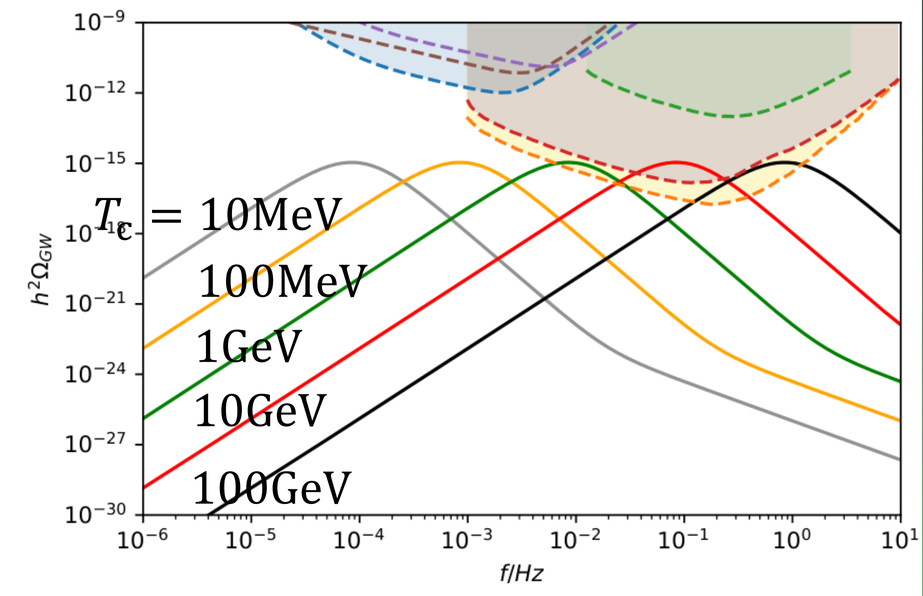
Enhanced gravitational wave prospect due to a large chemical potential $\mu \sim T_c$!



dark-QCD with $1\mathrm{GeV} < \mathrm{T_c} < 100\ \mathrm{GeV}$ has a good prospect at the Big Bang Observer (BBO)



Both amplitude and peak are sensitive to μ; $G_S = 2.2/\Lambda^2$



A unification way for asymmetry genesis

Why chemical potential, or asymmetric dark quarks, is of interest?

The SM lacks the ingredients to produce the baryon asymmetric universe

So, why not let the dark-QCD to seed the visible asymmetry?

To obtain a large chemical potential, we take the Affleck-Dine mechanism, generating charge via an evolving classical field: $n_B = ib(\dot{\phi}^*\phi - \phi^*\dot{\phi})$

$$\mathcal{L}_{AD} = |D_\mu\phi_A|^2 - m^2|\phi_A|^2 + \lambda|\phi_A|^4 + \left(\epsilon|\phi_A|^2\mathrm{Tr}(\phi_A^2) + \delta\mathrm{Tr}(\phi_A^4) + c.c.\right)$$
$$+ \left(y^L_{nm}\phi_A^{ij}\psi^T_{nLi}C\psi_{mLj} + y^R_{nm}\phi_A^{ij}\psi^T_{nRi}C\psi_{mRj} + c.c.\right) ,$$

ϕ_A is in **the antisymmetric rank two** tensor representation of $SU(N)$, and either term can break the dark baryon number

Integrate over time to accumulate the asymmetry

$$\frac{\mu}{T} \sim \left(\epsilon\frac{18.3\pi^2 g_*^{3/4}}{g_\psi}\right)^{1/3}\left(\frac{\phi_0}{m_\phi}\right)^{5/6}\left(\frac{\phi_0}{M_{\rm PL}}\right)^{1/2}$$

Conclusion and discussion

A dark-QCD sector is well motivated in new physics models, and it may experience an intrinsic first order PT, thus a source of stochastic GW signal

However, the resulting GW amplitude usually is very small because of the very short PT duration (much shorter than the Hubble time scale), leading to the suppressed GW release

We explore the chiral PT driven by a large chemical potential, and find that it can change the situation and makes chiral PT with $T_c \sim$(1 GeV, 100GeV) observable at BBO

Thanks for your attention!