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Effective Field Theories from GUT Scale to Nuclear Probes

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Based on CQS, J.H.Yu, arxiv:2504.XXXX

Contents

Effective Field Theory

Our dream is to understand the most fundamental (UV) theory. On the other hand, we can always do computation without knowing the exact theory.

For example

- $v \ll c \rightarrow$ no need of Special Relativity
- $x \gg \lambda \sim h/p \rightarrow$ no need of Quantum Mechanics
- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} \ll 1 \rightarrow$ no need of General Relativity

Consider a QFT with a fundamental high energy scale M but interested in observables as energy E << M.

Choose a cutoff $\Lambda <$ M and divide all quantum fields

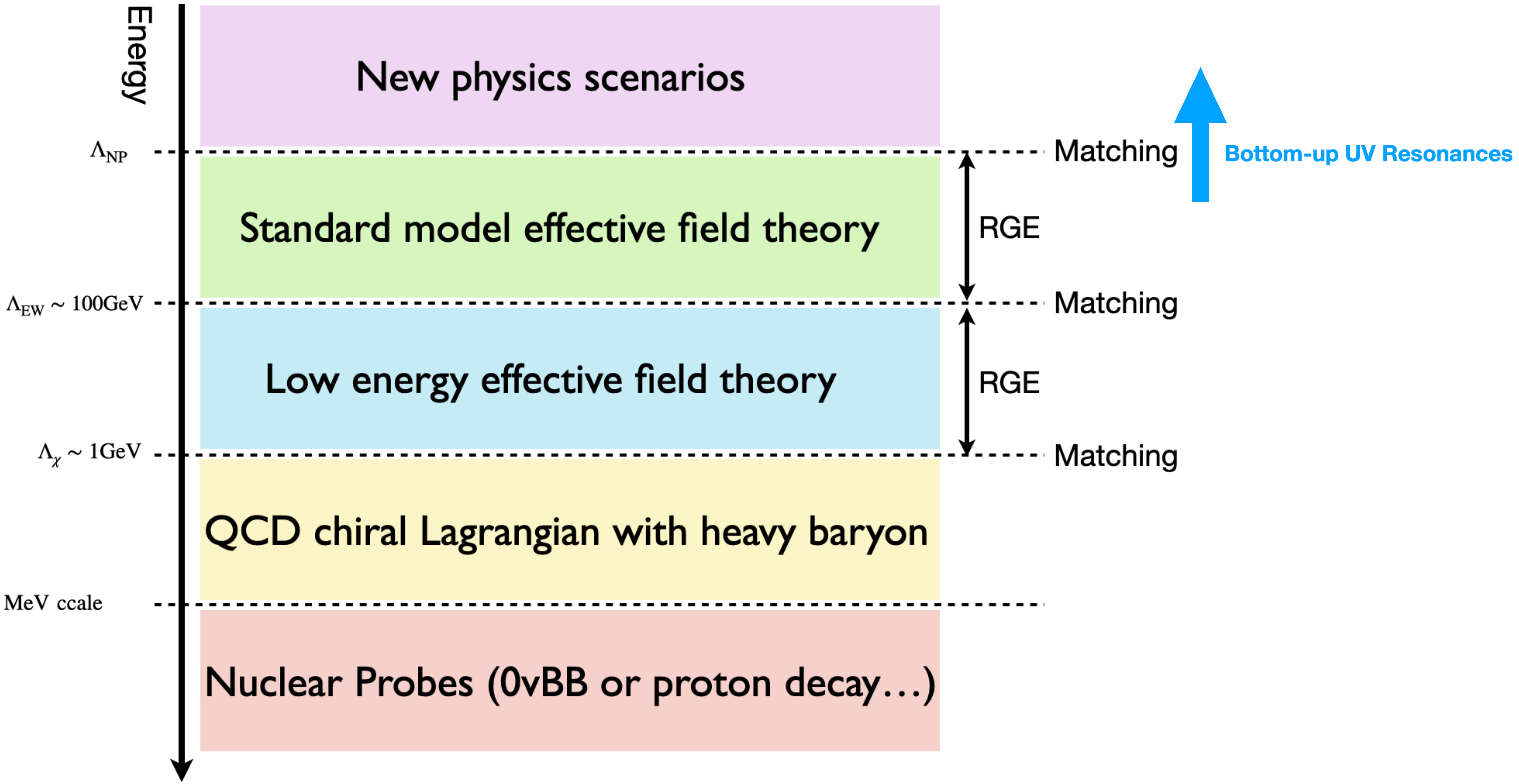
$$\varphi = \varphi_H + \varphi_L \ (\varphi_H : \omega > \Lambda \ \varphi_L : \omega < \Lambda)$$

Integrate-out

$$\int \mathcal{D}\,\varphi_H\, e^{i\int \mathcal{L}(\varphi_L, \varphi_H)} = e^{i\int \mathcal{L}_{eff}(\varphi_L)}$$

$$\mathcal{L}_{eff}(\varphi_L) = \mathcal{L}_{d\leq 4} + \sum_i \frac{O_i}{\Lambda^{dim(O_i)-4}}$$

Effective Field Theory



Grand Unified Theory

$\mathcal{G}_{\rm GUT}:\ SU(5)$ or $SO(10)$ ...

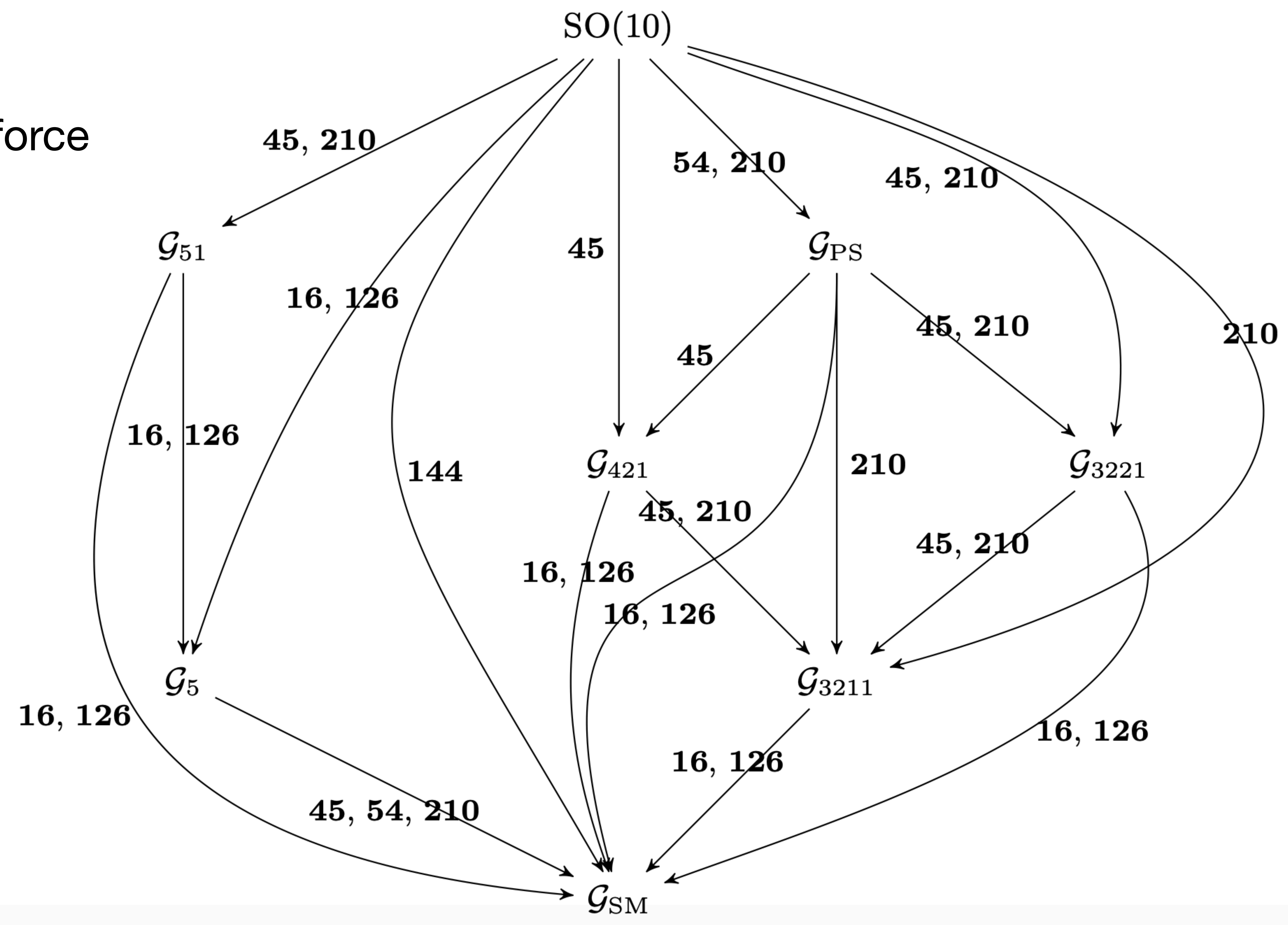
Symmetry breaking ↓ ↑ Unify into a single force

$$\mathcal{G}_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

Different breaking chains

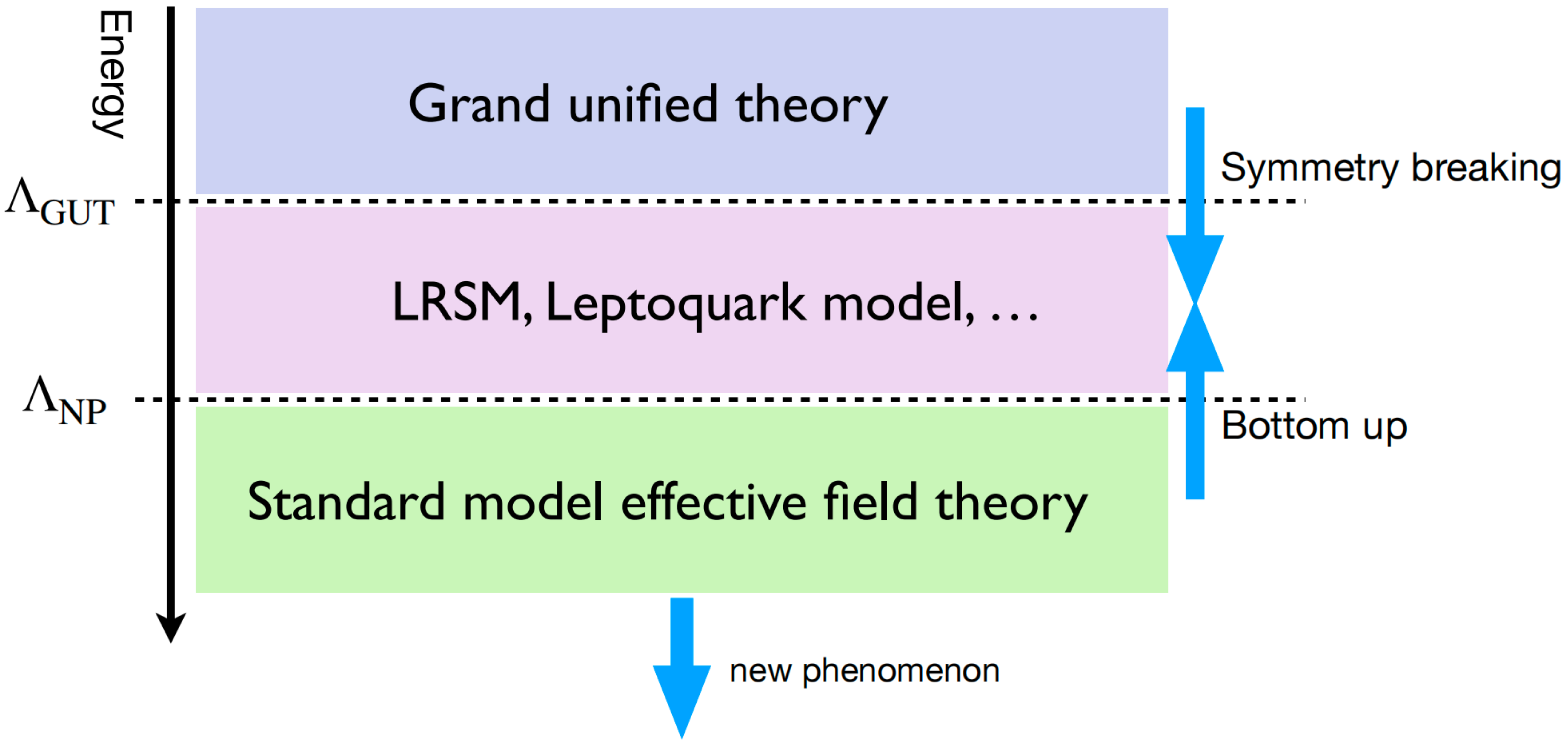
$$\begin{aligned}
\mathcal{G}_{\rm PS} &= SU(4)_C \times SU(2)_L \times SU(2)_R \\
\mathcal{G}_{421} &= SU(4)_C \times SU(2)_L \times U(1)_{B-L} \\
\mathcal{G}_{3221} &= SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\mathcal{G}_{3211} &= SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\
\mathcal{G}_{51} &= SU(5) \times U(1)_X
\end{aligned}$$

$$\begin{aligned}
\frac{1}{10}(-X+4Y) &= B-L, \\
Y &= B-L+T_R^3,
\end{aligned}$$



M.Pernow (2021)

SMEFT and GUT



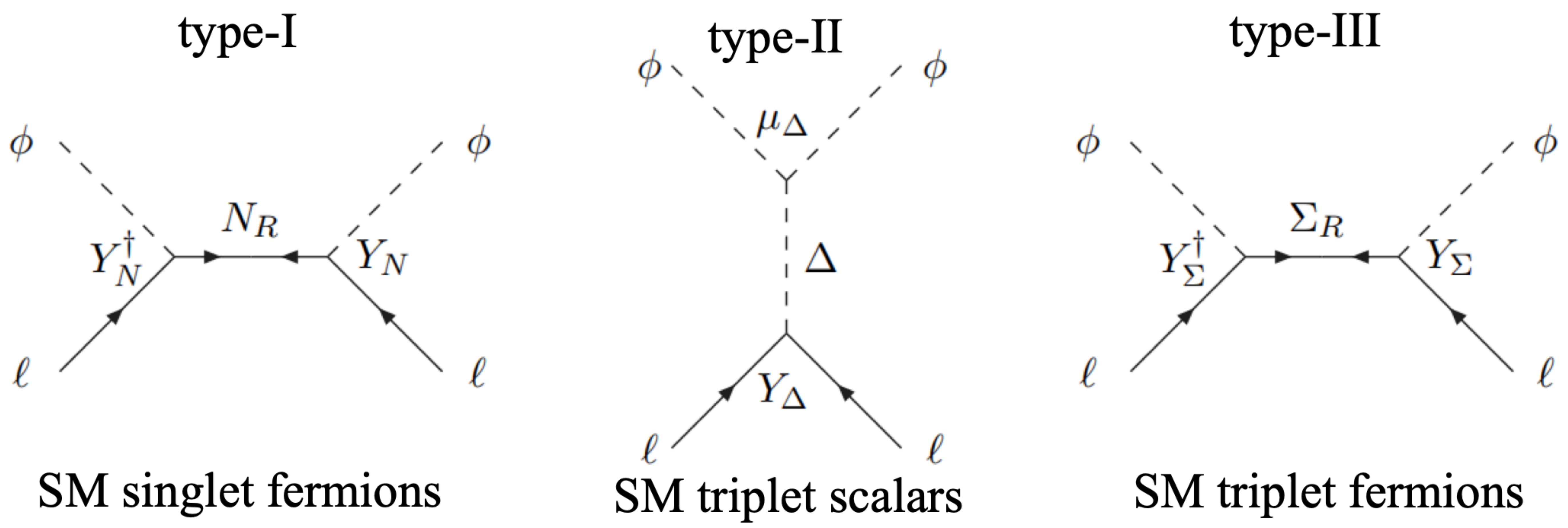
Lepton number violating **Baryon number violating**

Lepton Number Violating

Dimension-5

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n$$

Corresponding three types seesaw



The breaking chain in SO(10)

$SO(10)$	$\to$	$\mathcal{G}_{PS}$	$\to$	$\mathcal{G}_{\text{SM}}$
$\mathbf{45}_F$	$\to$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$\to$	$F_1(\mathbf{1},\mathbf{1})_0$
$\mathbf{45}_F$	$\to$	$(\mathbf{1},\mathbf{3},\mathbf{1})$	$\to$	$F_5(\mathbf{1},\mathbf{3})_0$
$\mathbf{54}_S$	$\to$	$(\mathbf{1},\mathbf{3},\mathbf{3})$	$\to$	$S_6(\mathbf{1},\mathbf{3})_1$

$SO(10)$	$\to$	$\mathcal{G}_{51}$	$\to$	$\mathcal{G}_{\text{SM}}$
$\mathbf{45}_F$	$\to$	$\mathbf{24}_0$	$\to$	$F_1(\mathbf{1},\mathbf{1})_0$
$\mathbf{45}_F$	$\to$	$\mathbf{24}_0$	$\to$	$F_5(\mathbf{1},\mathbf{3})_0$
$\mathbf{54}_S$	$\to$	$\mathbf{15}_4$	$\to$	$S_6(\mathbf{1},\mathbf{3})_1$

Lepton Number Violating

Dimension-7

Only L-violating			
Type: $\psi^2 H^4$		Type: $\psi^2 H^3 D$	
$\mathcal{O}_{LH}$	$\epsilon^{ik}\epsilon^{jl}(\ell_i^T C \ell_j) H_k H_l (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon^{ij}\epsilon^{kl}(\ell_i^T C \gamma^\mu e) H_j H_k (i D_\mu H_l)$
Type: $\psi^2 H^2 D^2$		Type: $\psi^2 H^2 X$	
$\mathcal{O}_{LDH1}$ $\mathcal{O}_{LDH2}$	$\epsilon^{ij}\epsilon^{kl}(\ell_i^T C D_\mu \ell_j)(H_k D^\mu H_l)$ $\epsilon^{ik}\epsilon^{jl}(\ell_i^T C D_\mu \ell_j)(H_k D^\mu H_l)$	$\mathcal{O}_{LHW}$ $\mathcal{O}_{LHB}$	$\epsilon^{ik}(\epsilon\tau^I)^{jl}(\ell_i^T C i\sigma^{\mu\nu}\ell_j) H_k H_l W^I_{\mu\nu}$ $\epsilon^{ik}\epsilon^{jl}(\ell_i^T C i\sigma^{\mu\nu}\ell_j) H_k H_l B_{\mu\nu}$
Type: $\psi^4 D$		Type: $\psi^4 H$	
$\mathcal{O}_{duLLD}$	$\epsilon^{ij}(\bar{d}^a \gamma^\mu u_a)(\ell_i^T C i D_\mu \ell_j)$	$\mathcal{O}_{eLLLH}$ $\mathcal{O}_{dLQLH1}$ $\mathcal{O}_{dLQLH2}$ $\mathcal{O}_{dLueH}$ $\mathcal{O}_{QuLLH}$	$\epsilon^{ij}\epsilon^{kl}(\bar{e}\ell_i)(\ell_j^T C \ell_k) H_l$ $\epsilon^{ij}\epsilon^{kl}(\bar{d}^a \ell_i)(q_{aj}^T C \ell_k) H_l$ $\epsilon^{ik}\epsilon^{jl}(\bar{d}^a \ell_i)(q_{aj}^T C \ell_k) H_l$ $\epsilon^{ij}(\bar{d}^a \ell_i)(u_a^T C e) H_j$ $\epsilon^{ij}(\bar{q}^{ak} u_a)(\ell_k^T C \ell_i) H_j$

$$\mathcal{L}_{eff} = \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \ldots$$

The dimension-7 operators are suppressed, the UV models which contribute to dimension-7 operators should exclude three types seesaw.

Lepton Number Violating

The UV models contribute to dimension-7 operators without seesaw.

(S_8, F_6)	$\rightarrow$	$\mathcal{O}_{LH}$
(F_3, V_2), (V_2, V_3)	$\rightarrow$	$\mathcal{O}_{LeHD}$
(S_2, S_4), (S_2, F_4)	$\rightarrow$	$\mathcal{O}_{eLLLH}$
(S_{12}, F_{14})	$\rightarrow$	$\mathcal{O}_{dLQLH1}$
(S_2, S_4), (S_2, F_9), (S_2, F_{10}), (S_{12}, F_9), (S_{12}, F_{14})	$\rightarrow$	$\mathcal{O}_{dLQLH2}$
(S_{12}, F_3), (S_{12}, F_{12}), (F_3, V_2), (F_{10}, V_3), (F_{12}, V_3), (F_{12}, V_5), (V_2, V_3)	$\rightarrow$	$\mathcal{O}_{dLueH}$
(S_2, S_4), (S_2, F_8), (S_2, F_{12}), (F_{12}, V_5), (F_{12}, V_9)	$\rightarrow$	$\mathcal{O}_{QuLLH}$

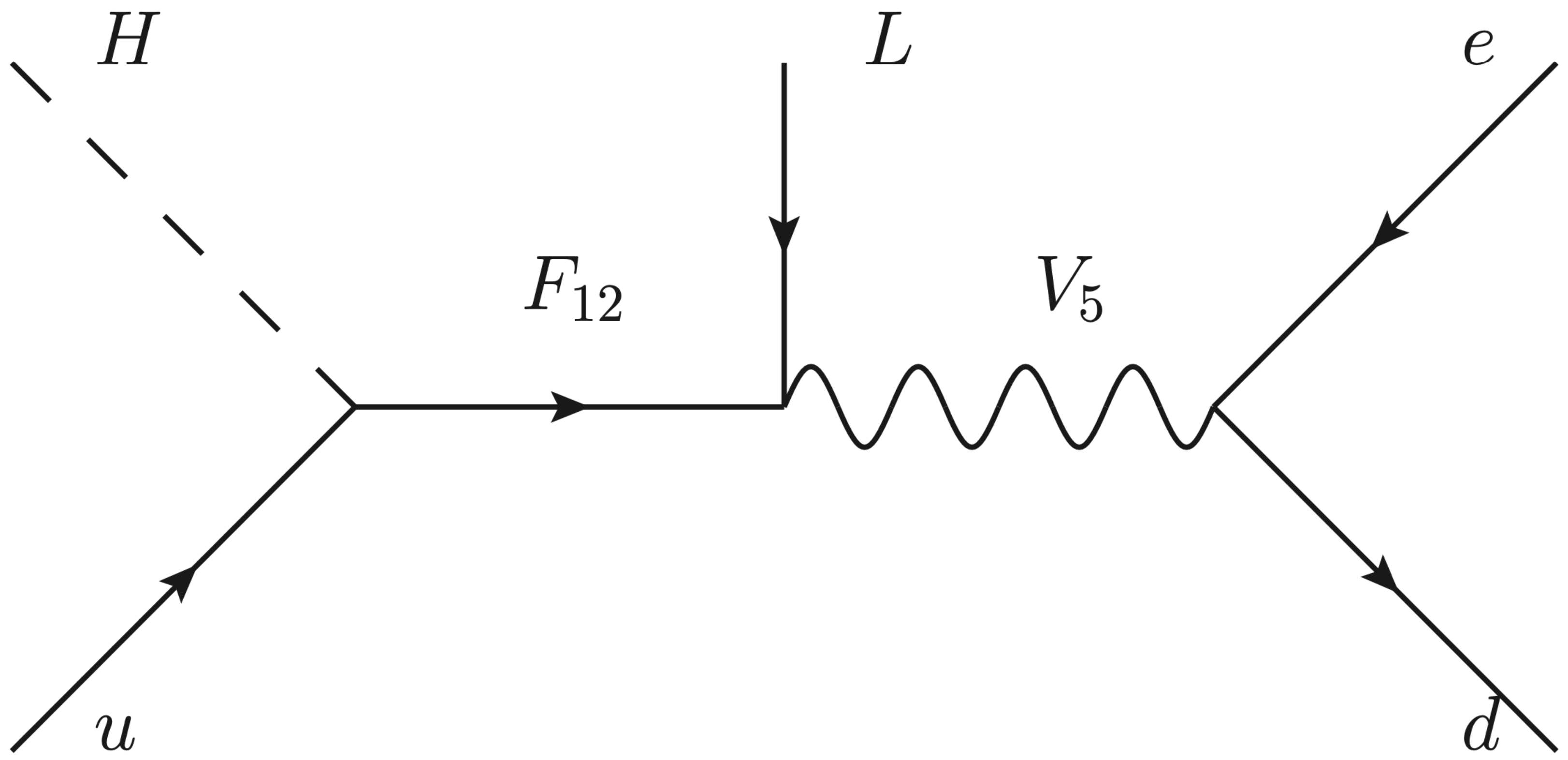
The breaking chain in SO(10).

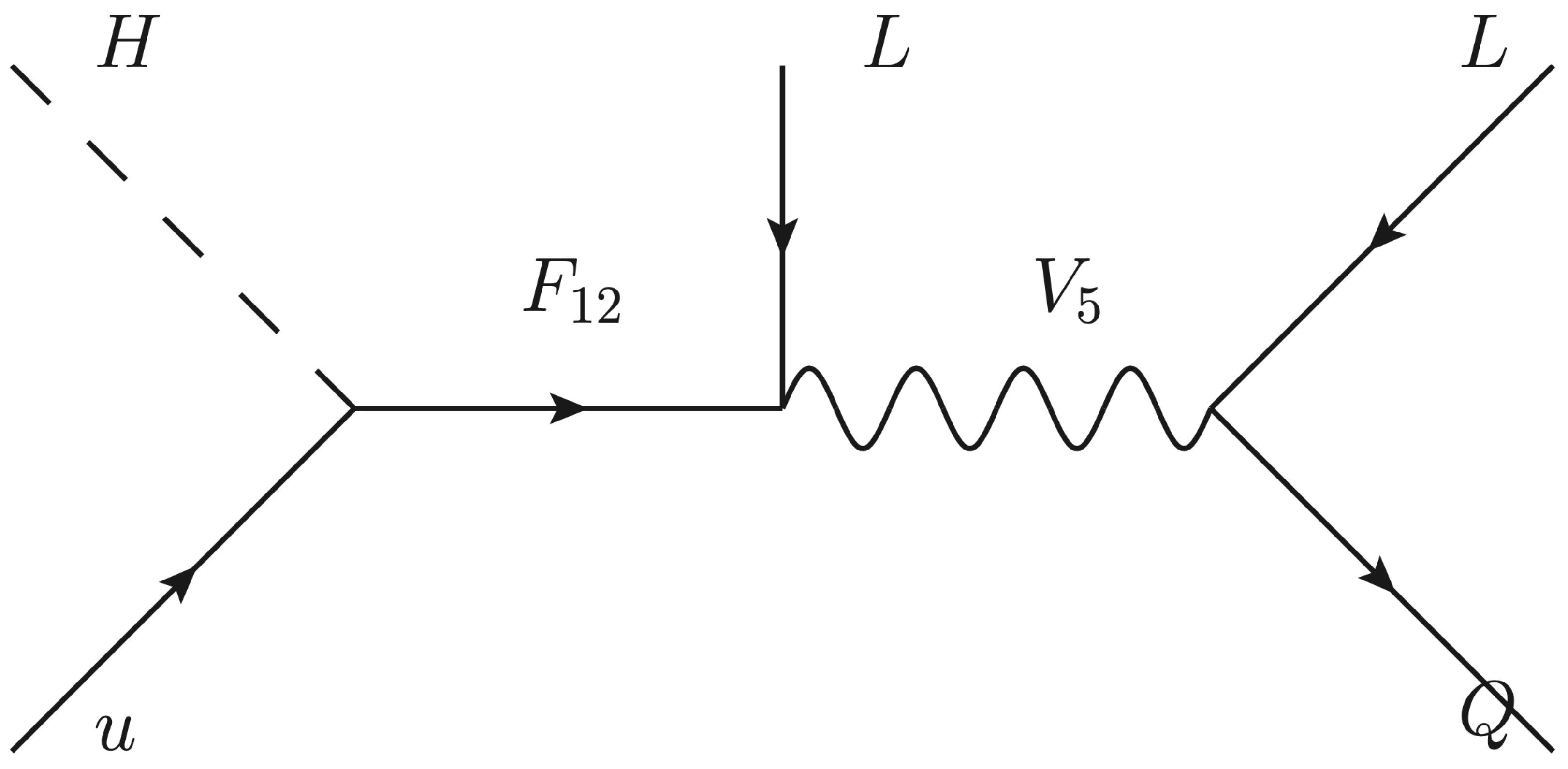
$SO(10)$	$\rightarrow$	$\mathcal{G}_{51}$	$\rightarrow$	$\mathcal{G}_{\rm SM}$
$\mathbf{10}_F$	$\rightarrow$	$\mathbf{5}_2$	$\rightarrow$	$F_3(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{210}_F$	$\rightarrow$	$\mathbf{40}_{-4}$	$\rightarrow$	$F_4(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$
$\mathbf{54}_F$	$\rightarrow$	$\mathbf{15}_4$	$\rightarrow$	$F_6(\mathbf{1},\mathbf{3})_1$
$\mathbf{10}_F$	$\rightarrow$	$\mathbf{5}_2$	$\rightarrow$	$F_8(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{45}_F$	$\rightarrow$	$\overline{\mathbf{10}}_{-4}$	$\rightarrow$	$F_9(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$
$\mathbf{45}_F$	$\rightarrow$	$\mathbf{24}_0$	$\rightarrow$	$F_{10}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\overline{\mathbf{126}}_F$	$\rightarrow$	$\overline{\mathbf{45}}_{-2}$	$\rightarrow$	$F_{12}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$
$\mathbf{120}_F$	$\rightarrow$	$\mathbf{45}_2$	$\rightarrow$	$F_{13}(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$
$\mathbf{210}_F$	$\rightarrow$	$\mathbf{40}_{-4}$	$\rightarrow$	$F_{14}(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$
$\mathbf{16}_S$	$\rightarrow$	$\mathbf{10}_{-1}$	$\rightarrow$	$S_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{10}_S$	$\rightarrow$	$\mathbf{5}_2$	$\rightarrow$	$S_4(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{16}_S$	$\rightarrow$	$\mathbf{10}_{-1}$	$\rightarrow$	$S_{12}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{16}_V$	$\rightarrow$	$\mathbf{10}_{-1}$	$\rightarrow$	$V_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{210}_V$	$\rightarrow$	$\mathbf{40}_{-4}$	$\rightarrow$	$V_3(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$
$\mathbf{45}_V$	$\rightarrow$	$\overline{\mathbf{10}}_{-4}$	$\rightarrow$	$V_5(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$
$\mathbf{210}_V$	$\rightarrow$	$\mathbf{40}_{-4}$	$\rightarrow$	$V_9(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$

$SO(10)$	$\rightarrow$	$\mathcal{G}_{PS}$	$\rightarrow$	$\mathcal{G}_{\rm SM}$
$\mathbf{10}_F$	$\rightarrow$	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$\rightarrow$	$F_3(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{210}_F$	$\rightarrow$	$(\overline{\mathbf{10}},\mathbf{2},\mathbf{2})$	$\rightarrow$	$F_4(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$
$\mathbf{54}_F$	$\rightarrow$	$(\mathbf{1},\mathbf{3},\mathbf{3})$	$\rightarrow$	$F_6(\mathbf{1},\mathbf{3})_1$
$\mathbf{10}_F$	$\rightarrow$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$\rightarrow$	$F_8(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{45}_F$	$\rightarrow$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\rightarrow$	$F_9(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$
$\mathbf{45}_F$	$\rightarrow$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\rightarrow$	$F_{10}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\overline{\mathbf{126}}_F$	$\rightarrow$	$(\mathbf{15},\mathbf{2},\mathbf{2})$	$\rightarrow$	$F_{12}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$
$\mathbf{120}_F$	$\rightarrow$	$(\mathbf{6},\mathbf{3},\mathbf{1})$	$\rightarrow$	$F_{13}(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$
$\mathbf{210}_F$	$\rightarrow$	$(\mathbf{15},\mathbf{3},\mathbf{1})$	$\rightarrow$	$F_{14}(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$
$\mathbf{16}_S$	$\rightarrow$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$	$\rightarrow$	$S_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{10}_S$	$\rightarrow$	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$\rightarrow$	$S_4(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{16}_S$	$\rightarrow$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\rightarrow$	$S_{12}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{16}_V$	$\rightarrow$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$	$\rightarrow$	$V_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{210}_V$	$\rightarrow$	$(\overline{\mathbf{10}},\mathbf{2},\mathbf{2})$	$\rightarrow$	$V_3(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$
$\mathbf{45}_V$	$\rightarrow$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\rightarrow$	$V_5(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$
$\mathbf{210}_V$	$\rightarrow$	$(\mathbf{15},\mathbf{3},\mathbf{1})$	$\rightarrow$	$V_9(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$

Lepton Number Violating

For example $F_{12}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$ $V_5(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$





Interaction

$$\begin{aligned}\mathcal{L}_{F_{12}V_5} =& -2\mathcal{D}^{pr}_{F_{12L}H^\dagger u^\dagger}[(\bar{u}_r)^a(F_{12p})_{ai}]H^{\dagger i} \\ & -2\mathcal{D}^{rsp}_{d^\dagger e V_5}[(\bar{d}_r)^a\gamma_\mu e_s](V_{5p})^\mu_a + 2\mathcal{D}^{srp}_{L^\dagger Q V_5^\dagger}[(\bar{l}_s)^i\gamma_\mu(q_r)_{ai}](V^\dagger_{5p})^{a\mu} \\ & + 2\mathcal{D}^{psr}_{F^\dagger_{12R}L^\dagger V_5}\epsilon_{ij}\delta^b_a[(\bar{l}_s)^j\gamma_\mu(F_{12p})^{ai}](V_{5r})^\mu_b\end{aligned}$$

Wilson coefficient

$$\mathcal{C}^{f_1f_2f_4f_5}_{dLueH} = \sum_{p_1p_2}\frac{16\mathcal{D}^{f_1f_4p_1}_{d^\dagger e V_5}\mathcal{D}^{p_2*f_5}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_2f_2p_1*}_{F^\dagger_{12R}L^\dagger V_5}}{M_{F_{12}}M^2_{V_5}}$$

$$\mathcal{C}^{f_1f_2f_4f_5}_{QuLLH} = \sum_{p_1p_2}\frac{16\mathcal{D}^{p_1*f_5}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_2p_2*}_{F^\dagger_{12R}L^\dagger V_5}\mathcal{D}^{f_1f_4p_2}_{L^\dagger Q V_5^\dagger}}{M_{F_{12}}M^2_{V_5}}$$

Effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{C}^{f_1f_2f_4f_5}_{dLueH}\times\epsilon^{ij}(\overline{d}^a_{f_1}\ell_{if_4})(u^T_{af_5}Ce_{f_2})H_j + \mathcal{C}^{f_1f_2f_4f_5}_{QuLLH}\times\epsilon^{ij}(\overline{q}^{ak}_{f_4}u_{af_5})(\ell^T_{kf_1}C\ell_{if_2})H_j$$

Baryon Number Violating

Dimension-6 $\Delta(B-L)=0$

Type: B-violating	
$\mathcal{O}_{duq}$	$\epsilon^{abc}\epsilon^{jk}(d_a^T C u_b)(q_{cj}^T C \ell_k)$
$\mathcal{O}_{qqu}$	$\epsilon^{abc}\epsilon_{jk}(q_{aj}^T C q_{bk})(u_c^T C e)$
$\mathcal{O}_{qqq}$	$\epsilon^{abc}\epsilon_{jn}\epsilon_{km}(q_{aj}^T C q_{bk})(q_{cm}^T C \ell_n)$
$\mathcal{O}_{duu}$	$\epsilon^{abc}(d_a^T C u_b)(u_c^T C e)$

The breaking chain in SO(10).

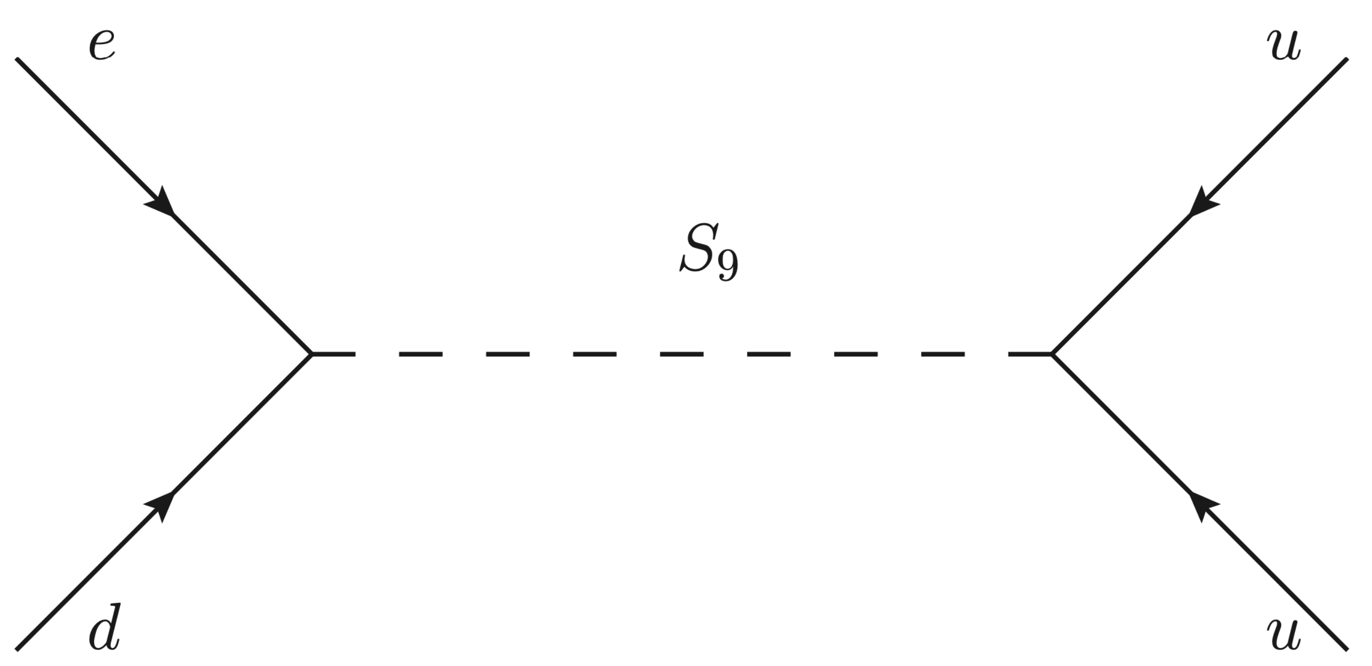
$SO(10)$	$\to$	$\mathcal{G}_{PS}$	$\to$	$\mathcal{G}_{\rm SM}$
$\mathbf{45}_V$	$\to$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\to$	$V_7(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\mathbf{16}_V$	$\to$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\to$	$V_8(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{120}_S$	$\to$	$(\mathbf{6},\mathbf{1},\mathbf{3})$	$\to$	$S_9(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$
$\mathbf{10}_S$	$\to$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$\to$	$S_{10}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{120}_S$	$\to$	$(\mathbf{6},\mathbf{3},\mathbf{1})$	$\to$	$S_{14}(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$

$SO(10)$	$\to$	$\mathcal{G}_{51}$	$\to$	$\mathcal{G}_{\rm SM}$
$\mathbf{45}_V$	$\to$	$\mathbf{24}_0$	$\to$	$V_7(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\mathbf{16}_V$	$\to$	$\mathbf{10}_{-1}$	$\to$	$V_8(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{120}_S$	$\to$	$\overline{\mathbf{45}}_{-2}$	$\to$	$S_9(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$
$\mathbf{10}_S$	$\to$	$\mathbf{5}_2$	$\to$	$S_{10}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{120}_S$	$\to$	$\mathbf{45}_2$	$\to$	$S_{14}(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$

The UV models contribute to dimension-6 operators.

$S_{10}\,,\ V_7\,,\ V_8$	$\to$	$\mathcal{O}_{duq}$
$S_{10}\,,\ V_7$	$\to$	$\mathcal{O}_{qqu}$
$S_{10}\,,\ S_{14}$	$\to$	$\mathcal{O}_{qqq}$
$S_9\,,\ S_{10}$	$\to$	$\mathcal{O}_{duu}$

For example



$$\mathcal{L}_{S_9} = -\,2\mathcal{D}^{rsp}_{d^\dagger e^\dagger S_9}[(\bar{d}_r)^a C \bar{e}_s](S_{9p})_a - 4\mathcal{D}^{prs}_{S_9^\dagger u^\dagger u^\dagger}\epsilon^{abc}[(\bar{u}_r)^b C(\bar{u}_s)^c](S^\dagger_{9p})^a\,,$$

Baryon Number Violating

Dimension-7 $\Delta(B-L)=2$

B-violating			
Type: $\psi^4 D$		Type: $\psi^4 H$	
$\mathcal{O}_{LQddD}$	$\epsilon^{abc}(\bar{\ell}^i\gamma^\mu q_{ai})(d_b^T C i D_\mu d_c)$	$\mathcal{O}_{LdudH}$	$\epsilon^{abc}\epsilon_{ij}(\bar{\ell}^i d_a)(u_b^T C d_c)H^{*j}$
$\mathcal{O}_{edddD}$	$\epsilon^{abc}(\bar{e}\gamma^\mu d_a)(d_b^T C i D_\mu d_c)$	$\mathcal{O}_{LdddH}$	$\epsilon^{abc}(\bar{\ell}^i d_a)(d_b^T C d_c)H_i$
		$\mathcal{O}_{eQddH}$	$-\epsilon^{abc}(\bar{e}Q_{ai})(d_b^T C d_c)H^{*i}$
		$\mathcal{O}_{LdQQH}$	$-\epsilon^{abc}(\bar{\ell}^k d_a)(q_{bk}^T C q_{ci})H^{*i}$

The UV models contribute to dimension-7 operators which do not contribute to dimension-6 operators.

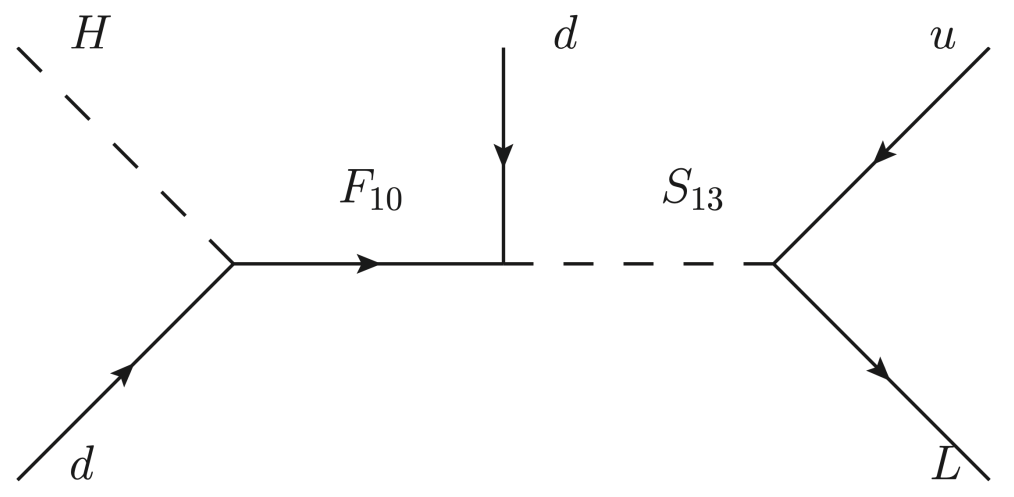
$(S_{11},S_{12})\,,\ (S_{11},F_2)\,,\ (S_{11},F_{11})$	$\to$	$\mathcal{O}_{LdddH}$
$(S_{11},S_{13})\,,\ (S_{11},F_1)\,,\ (S_{11},F_{11})\,,\ (S_{13},F_{10})$	$\to$	$\mathcal{O}_{LdudH}$
$(S_{11},S_{13})\,,\ (S_{11},F_3)\,,\ (S_{11},F_8)\,,\ (S_{13},F_{10})$	$\to$	$\mathcal{O}_{eQddH}$

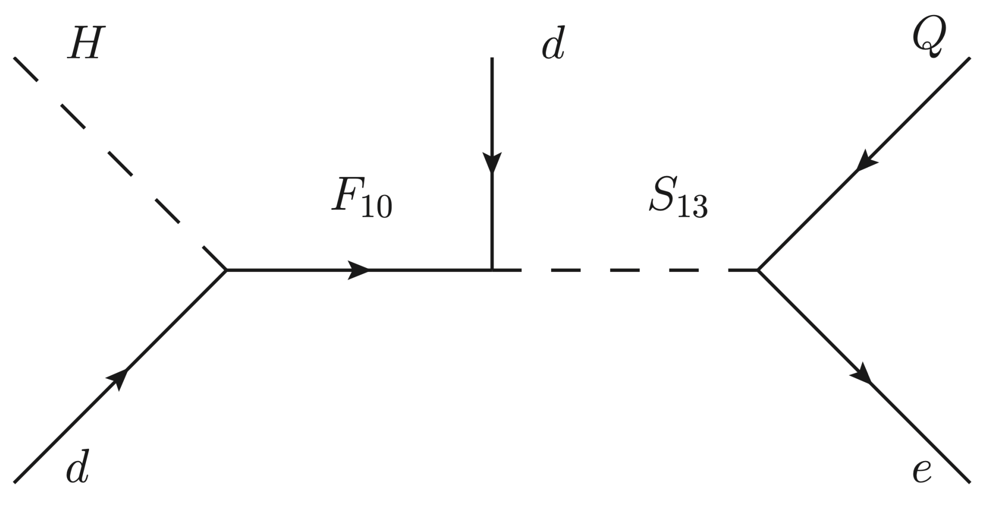
The breaking chain in SO(10).

$SO(10)$	$\to$	$\mathcal{G}_{51}$	$\to$	$\mathcal{G}_{\rm SM}$
$\mathbf{45}_F$	$\to$	$\mathbf{24}_0$	$\to$	$F_1(\mathbf{1},\mathbf{1})_0$
$\mathbf{16}_F$	$\to$	$\overline{\mathbf{10}}_{-1}$	$\to$	$F_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{10}_F$	$\to$	$\mathbf{5}_2$	$\to$	$F_3(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{10}_F$	$\to$	$\mathbf{5}_2$	$\to$	$F_8(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{45}_F$	$\to$	$\mathbf{24}_0$	$\to$	$F_{10}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\mathbf{16}_F$	$\to$	$\mathbf{10}_{-1}$	$\to$	$F_{11}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{16}_S$	$\to$	$\mathbf{5}_2$	$\to$	$S_{11}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{16}_S$	$\to$	$\mathbf{10}_{-1}$	$\to$	$S_{12}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\overline{\mathbf{126}}_S$	$\to$	$\overline{\mathbf{45}}_{-2}$	$\to$	$S_{13}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$

$SO(10)$	$\to$	$\mathcal{G}_{PS}$	$\to$	$\mathcal{G}_{\rm SM}$
$\mathbf{45}_F$	$\to$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$\to$	$F_1(\mathbf{1},\mathbf{1})_0$
$\mathbf{16}_F$	$\to$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$\to$	$F_2(\mathbf{1},\mathbf{1})_1$
$\mathbf{10}_F$	$\to$	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$\to$	$F_3(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\mathbf{10}_F$	$\to$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$\to$	$F_8(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{45}_F$	$\to$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\to$	$F_{10}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
$\mathbf{16}_F$	$\to$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\to$	$F_{11}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\mathbf{16}_S$	$\to$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$\to$	$S_{11}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathbf{16}_S$	$\to$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\to$	$S_{12}(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\overline{\mathbf{126}}_S$	$\to$	$(\mathbf{15},\mathbf{2},\mathbf{2})$	$\to$	$S_{13}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$

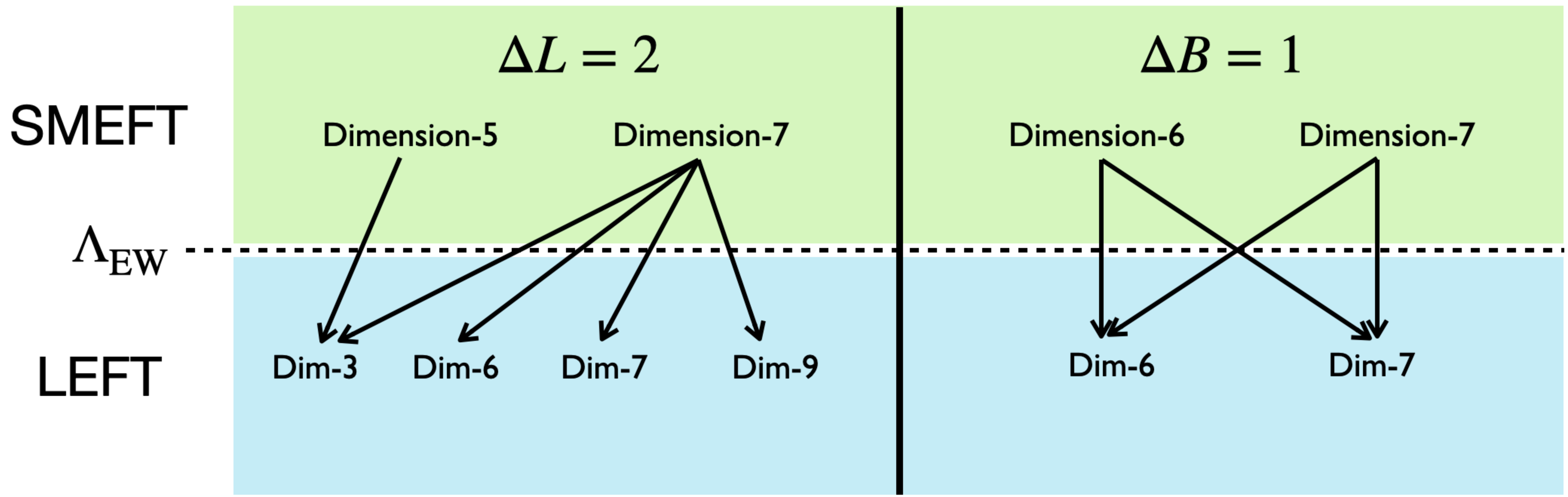
For example





$$\mathcal{L}_{S_{13}F_{10}} = -2\mathcal{D}^{rsp}_{e^\dagger Q S^\dagger_{13}}[\bar{e}_r(q_s)_{ai}](S^\dagger_{13p})^{ai} - 2\mathcal{D}^{rps}_{LS_{13}u^\dagger}\epsilon^{ji}[(\bar{u}_s)^a(l_r)_j](S_{13p})_{ai}$$
$$- 2\mathcal{D}^{rp}_{d^\dagger F_{10L}H}\epsilon^{ij}[(\bar{d}_r)^a(F_{10p})_{ai}]H_j - 4\mathcal{D}^{spr}_{d^\dagger F^\dagger_{10R}S^\dagger_{13}}\epsilon_{ij}\epsilon^{cab}[(\bar{d}_s)^c(F_{10p})^{ai}](S^\dagger_{13r})^{bj}$$

Matching between SMEFT and LEFT



The particles in LEFT : $(u_R\,,\, u_L\,,\, d_L\,, d_R\,,\, e_R\,, e_L\,,\, \nu_L)\,,\quad n_u = 2\,, n_d = 3\,, n_e = 3\,, n_\nu = 3\,.$

The involving LEFT operator form : $(\bar{q}q\bar{l}l)\,,\,(\bar{q}q\bar{l}lD)\,,\,(\bar{q}q\bar{q}qll)\,,\,(qqql)\,,\,(qqq\bar{l})\,,\,(qqqlD)\,,\,(qqq\bar{l}D)\,,\quad q=(u,d,s)^T$

Matching between LEFT and ChPT

The QCD Lagrangian with external source

$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \mathcal{L}_{\rm ext} \ ,$$

$$\mathcal{L}_{\rm ext} = \bar{q}\gamma^\mu \left(v_\mu + \gamma^5 a_\mu\right) q - \bar{q}\left(s - i\gamma^5 p\right) q + \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu} q \ ,$$

$SU(3)_L \otimes SU(3)_R$ chiral symmetry

symmetry breaking →

The chiral Lagrangian

$$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] + \frac{F_0^2}{4}\,\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

$$\mathcal{L}^{(1)}_{MB} = \langle \bar{B}(i\gamma^\mu D_\mu - M_0)B\rangle + \frac{D}{2}\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle \ .$$

$SU(3)_V$ symmetry

Dimension-6 and dimension-7 LEFT LNV operators

$$(\bar{q}\Gamma q)(l\Gamma l^c)$$

Dimension-9 LEFT LNV operators

$$(\bar{q}\Gamma q)(\bar{q}\Gamma q)(l\Gamma l^c)$$

$SU(3)_L \otimes SU(3)_R$ chiral symmetry

symmetry breaking →

The chiral Lagrangian

$$(B + U + ...)(l\Gamma l^c)$$

$SU(3)_V$ symmetry

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right), \ \phi(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} \quad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Matching between LEFT and ChPT

Quark bilinear with spurion

$$(\bar{q}_L \Gamma \Sigma_L q_L), \quad (\bar{q}_R \Gamma \Sigma_R q_R), \quad (\bar{q}_R \Gamma \Sigma q_L), \quad (\bar{q}_L \Gamma \Sigma^\dagger q_R)\ ,$$

	$\bar{q}q$	$\bar{q}\gamma^5 q$	$\bar{q}\gamma^\mu q$	$\bar{q}\gamma^5\gamma^\mu q$	$\bar{q}\sigma^{\mu\nu} q$	$\bar{q}\overleftrightarrow{\partial}^\mu q$	$\bar{q}\gamma^5\overleftrightarrow{\partial}^\mu q$	$\bar{q}\gamma^\mu\overleftrightarrow{\partial}^\nu q$	$\bar{q}\gamma^5\gamma^\mu\overleftrightarrow{\partial}^\nu q$
C	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$+$	$-$
P	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$-$

	$\bar{B}B$	$\bar{B}\gamma^5 B$	$\bar{B}\gamma^\mu B$	$\bar{B}\gamma^5\gamma^\mu B$	$\bar{B}\sigma^{\mu\nu} B$	$\bar{B}\overleftrightarrow{D}^\mu B$
C	$+$	$+$	$-$	$+$	$-$	$-$
P	$+$	$-$	$+$	$-$	$+$	$+$
	$\bar{B}\gamma^5\overleftrightarrow{D}^\mu B$	$\bar{B}\gamma^\mu\overleftrightarrow{D}^\nu B$	$\bar{B}\gamma^5\gamma^\mu\overleftrightarrow{D}^\nu B$	u_μ	$\Sigma_\pm$	$Q_\pm$
C	$-$	$+$	$-$	$+$	$+$	$\mp$
P	$-$	$+$	$-$	$-$	$\pm$	$\pm$

$$\Sigma_\pm = u\left(\Sigma^\dagger \pm \Sigma\right)u \pm u^\dagger\left(\Sigma^\dagger \pm \Sigma\right)u^\dagger\ ,$$

$$Q_\mp = u^\dagger\left(\Sigma_R \pm \Sigma_L\right)u \mp u\left(\Sigma_R \pm \Sigma_L\right)u^\dagger\ ,$$

$$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)\ ,$$

$$\nabla_\mu X = \partial_\mu + [\Gamma_\mu, X]\ , \quad \Gamma_\mu = \frac{1}{2}(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger)\ .$$

$SU(3)_L \otimes SU(3)_R$

$$\begin{pmatrix} q_L \\ q_R \\ \bar{q}_L \\ \bar{q}_R \\ \Sigma \\ \Sigma^\dagger \\ \Sigma_L \\ \Sigma_R \end{pmatrix} \to \begin{pmatrix} Lq_L \\ Rq_R \\ \bar{q}_L L^\dagger \\ \bar{q}_R R^\dagger \\ R\Sigma L^\dagger \\ L\Sigma^\dagger R^\dagger \\ L\Sigma_L L^\dagger \\ R\Sigma_R R^\dagger \end{pmatrix}, \quad L \in SU(3)_L\,, R \in SU(3)_R\ .$$

$$u \to RuV^\dagger = VuL^\dagger \qquad u = \sqrt{U}$$

$$\begin{pmatrix} u_\mu \\ \Sigma_- \\ \Sigma_+ \\ Q_- \\ Q_+ \\ B \end{pmatrix} \to \begin{pmatrix} Vu_\mu V^\dagger \\ V\Sigma_- V^\dagger \\ V\Sigma_+ V^\dagger \\ VQ_- V^\dagger \\ VQ_+ V^\dagger \\ VBV^\dagger \end{pmatrix}, \quad V \in SU(3)_V\ .$$

Matching between LEFT and ChPT

Quark bilinear with two spurion

$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_L\Gamma\,\Sigma_L q_L)\,,\quad (\bar{q}_R\Gamma\,\Sigma_R q_R)(\bar{q}_R\Gamma\,\Sigma_R q_R)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_R\Gamma\,\Sigma_R q_R)\,,\quad (\bar{q}_L\Gamma\,\Sigma^\dagger q_R)(\bar{q}_L\Gamma\,\Sigma q_R)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)(\bar{q}_R\Gamma\,\Sigma^\dagger q_L)\,,\quad (\bar{q}_R\Gamma\,\Sigma q_L)(\bar{q}_R\Gamma\,\Sigma q_L)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_R\Gamma\,\Sigma q_L)\,,\quad (\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)\,,$$
$$(\bar{q}_R\Gamma\Sigma_R q_R)(\bar{q}_R\Gamma\,\Sigma q_L)\,,\quad (\bar{q}_R\Gamma\Sigma_R q_R)(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)\,.$$

→

$$Q_{1+} = u^\dagger(\Sigma_{1R}-\Sigma_{1L})u + u(\Sigma_{1R}-\Sigma_{1L})u^\dagger\,,$$
$$Q_{1-} = u^\dagger(\Sigma_{1R}+\Sigma_{1L})u - u(\Sigma_{1R}+\Sigma_{1L})u^\dagger\,,$$
$$Q_{2+} = u^\dagger(\Sigma_{2R}-\Sigma_{2L})u + u(\Sigma_{2R}-\Sigma_{2L})u^\dagger\,,$$
$$Q_{2-} = u^\dagger(\Sigma_{2R}+\Sigma_{2L})u - u(\Sigma_{2R}+\Sigma_{2L})u^\dagger\,.$$

$$\Sigma_{1+} = u^\dagger(\Sigma_1+\Sigma_1^\dagger)u^\dagger + u(\Sigma_1+\Sigma_1^\dagger)u\,,$$
$$\Sigma_{1-} = u^\dagger(\Sigma_1-\Sigma_1^\dagger)u^\dagger - u(\Sigma_1-\Sigma_1^\dagger)u\,,$$
$$\Sigma_{2+} = u^\dagger(\Sigma_2+\Sigma_2^\dagger)u^\dagger + u(\Sigma_2+\Sigma_2^\dagger)u\,,$$
$$\Sigma_{2-} = u^\dagger(\Sigma_2-\Sigma_2^\dagger)u^\dagger - u(\Sigma_2-\Sigma_2^\dagger)u\,.$$

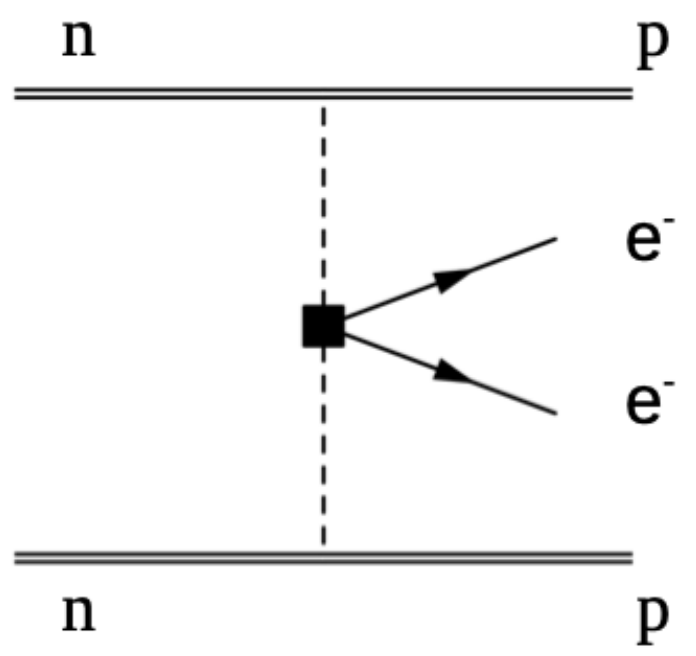
$0\nu\beta\beta$ operators

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{1}{v^5}\sum_i \left[\left(C^{(9)}_{i\mathrm{R}}\,\bar{e}_R C\bar{e}_R^T + C^{(9)}_{i\mathrm{L}}\,\bar{e}_L C\bar{e}_L^T\right) O_i^{(9)} + C_i^{(9)}\bar{e}\gamma^\mu\gamma^5 C\bar{e}^T\,O_i^{\mu(9)}\right]$$

$$O^{(9)}_{1+} = \bar{q}^\alpha_L\gamma_\mu\tau^a q^\alpha_L\;\bar{q}^\beta_L\gamma^\mu\tau^a q^\beta_L + \bar{q}^\alpha_R\gamma_\mu\tau^a q^\alpha_R\;\bar{q}^\beta_R\gamma^\mu\tau^a q^\beta_R$$
$$O^{(9)}_{2-} = \bar{q}^\alpha_R\gamma_\mu\tau^a q^\alpha_R\;\bar{q}^\beta_R\gamma^\mu\tau^a q^\beta_R - \bar{q}^\alpha_L\gamma_\mu\tau^a q^\alpha_L\;\bar{q}^\beta_L\gamma^\mu\tau^a q^\beta_L$$
$$O^{(9)}_{7+} = \bar{q}^\alpha_L\gamma_\mu\tau^a q^\alpha_L\;\bar{q}^\beta_R\gamma^\mu\tau^a q^\beta_R$$
$$O^{(9)}_{8+} = \bar{q}^\alpha_L\gamma_\mu\tau^a q^\beta_L\;\bar{q}^\beta_R\gamma^\mu\tau^a q^\alpha_R$$

→

$$O^{(9)}_{1+} \to 2\langle Q_{a-}Q_{a-}\rangle + 2\langle Q_{a+}Q_{a+}\rangle = 0\,,$$
$$O^{(9)}_{2-} \to -2\langle Q_{a-}Q_{a+}\rangle - 2\langle Q_{a+}Q_{a-}\rangle = 0\,,$$
$$O^{(9)}_{7+}\,,O^{(9)}_{8+} \to \langle Q_{a-}Q_{a-}\rangle - \langle Q_{a+}Q_{a+}\rangle - \langle Q_{a-}Q_{a+}\rangle + \langle Q_{a+}Q_{a-}\rangle = 4\langle U^\dagger\tau^\pm U\tau^\pm\rangle\,,$$



The vertex $\pi\pi ee$ can be obtained for $0\nu\beta\beta$.

Matching between LEFT and ChPT

Dimension-6 and dimension-7 LEFT BNV operators

$$(q_L q_L q_L)\ell\,,\ (q_R q_R q_R)\ell\,,\ (q_L q_L q_R)\ell\,,\ (q_R q_R q_L)\ell$$

Three spurions can be added

$$(T_L q_L T_L q_L T_L q_L)\ell\,,\quad (T_R q_R T_R q_R T_R q_R)\ell\,,\quad (T_L q_L T_R q_R T_R q_R)\ell\,,\quad (T_L q_L T_L q_L T_R q_R)\ell\,,$$

$$\begin{pmatrix} q_L \\ q_R \\ T_L \\ T_R \end{pmatrix} \to \begin{pmatrix} L q_L \\ R q_R \\ T_L L^\dagger \\ T_R R^\dagger \end{pmatrix}\,, \quad L \in SU(3)_L\,, R \in SU(3)_R\,.$$

$$\begin{aligned} (T_L u) &\to (T_L u) V^\dagger\,, \\ (T_R u^\dagger) &\to (T_R u^\dagger) V^\dagger\,, \\ B &\to V u V^\dagger\,, \\ u^\mu &\to V u^\mu V^\dagger\,, \end{aligned}$$

Type $BT_L T_L T_L$ LO

$$\epsilon^{acd}(\lambda^A)^b_d (T_L u)_a (T_L u)_b (T_L u)_c (B^{A^T} CL)\,,$$
$$\epsilon^{bcd}(\lambda^A)^a_d (T_L u)_a (T_L u)_b (T_L u)_c (B^{A^T} CL)\,,$$

NLO

$$\epsilon^{abc}(T_L u)_a (T_L u)_b (T_L u)_c (B^{A^T} C\gamma_\mu L) u^A_\mu\,,$$
$$f_{ABC}\epsilon^{bcd}(\lambda^A)^a_d (T_L u)_a (T_L u)_b (T_L u)_c (B^{B^T} C\gamma_\mu L) u^C_\mu\,,$$
$$d_{ABC}\epsilon^{bcd}(\lambda^A)^a_d (T_L u)_a (T_L u)_b (T_L u)_c (B^{B^T} C\gamma_\mu L) u^C_\mu\,,$$

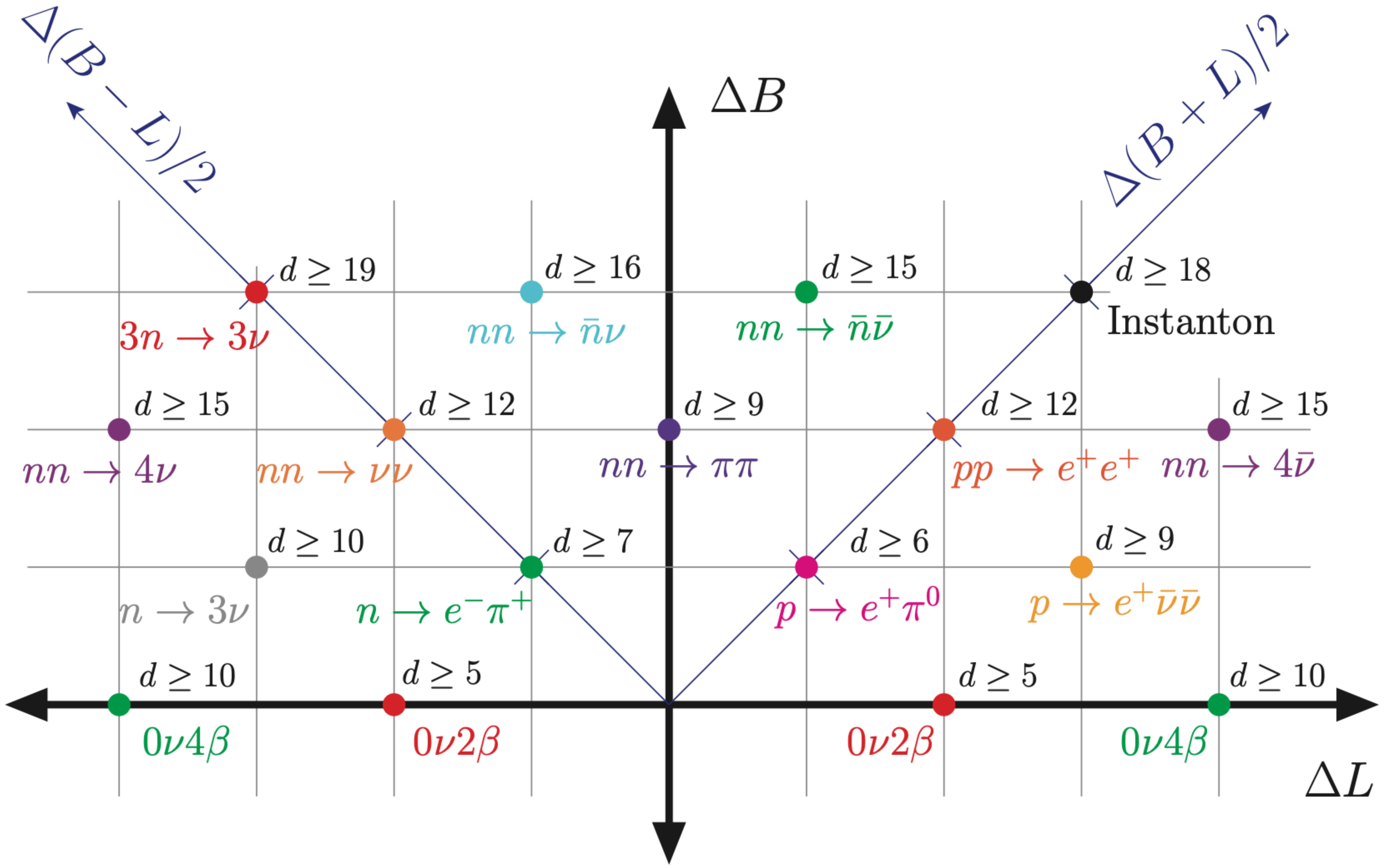
The corresponding chiral Lagrangian for $(u_L u_L d_L L)$ can be expand as

$$\begin{aligned}
&\epsilon^{acd}(\lambda^A)^b_d (T_L u)_a (T_L u)_b (T_L u)_c (B^{A^T} CL) \\
=&\epsilon^{123}(\lambda^A)^b_3 (T_L u)_1 (T_L u)_b (T_L u)_2 (B^{A^T} CL) + \epsilon^{231}(\lambda^A)^b_1 (T_L u)_2 (T_L u)_b (T_L u)_3 (B^{A^T} CL) \\
&+ \epsilon^{312}(\lambda^A)^b_2 (T_L u)_3 (T_L u)_b (T_L u)_1 (B^{A^T} CL) + \epsilon^{132}(\lambda^A)^b_2 (T_L u)_1 (T_L u)_b (T_L u)_3 (B^{A^T} CL) \\
&+ \epsilon^{213}(\lambda^A)^b_3 (T_L u)_2 (T_L u)_b (T_L u)_1 (B^{A^T} CL) + \epsilon^{321}(\lambda^A)^b_1 (T_L u)_3 (T_L u)_b (T_L u)_2 (B^{A^T} CL) \\
=&p^T CL + (\pi^0 + \frac{1}{\sqrt{3}}\eta\,, \sqrt{2}\pi^+\,, \sqrt{2}K^+)\cdot(p, n, -\frac{2}{\sqrt{6}}\Lambda)^T CL + (\pi^0 + \frac{1}{\sqrt{3}}\eta) p^T CL + (-\pi^0 + \frac{1}{\sqrt{3}}\eta) p^T CL \\
&- \sqrt{2}K^0 \Sigma^{+T} CL - \sqrt{2}K^- (\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda)^T CL + \ldots \\
&+ (\pi^0 + \frac{1}{\sqrt{3}}\eta)(-\pi^0 + \frac{1}{\sqrt{3}})(\pi^0 + \frac{1}{\sqrt{3}}\eta\,, \sqrt{2}\pi^+\,, \sqrt{2}K^+)\cdot(p, n, -\frac{2}{\sqrt{6}})^T \\
&+ 2\pi^+ K^0 (\pi^0 + \frac{1}{\sqrt{3}}\eta\,, \sqrt{2}\pi^+\,, \sqrt{2}K^+)(\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda, \Sigma^-, \Xi^-)^T + \ldots\,.
\end{aligned}$$

Contribute to the processes

$$p \to \pi^0 + e^+\,,\qquad p \to \eta + e^+\,,\qquad n \to \pi^- + e^+\,,$$

$$\begin{aligned} p &\to \pi^0 + \pi^0 + e^+\,, \\ p &\to \pi^+ + \pi^- + e^+\,, \\ p &\to \eta + \pi^0 + e^+\,, \\ n &\to \pi^0 + \pi^- + e^+\,, \\ n &\to \eta + \pi^- + e^+\,, \end{aligned}$$



Heeck, Takhistov 1910.07647

Summary

$\Delta L = 2$		$\Delta B = 1$
$(\overline{\mathbf{126}}_F\,, \mathbf{45}_V)$	Grand unified theory	$(\mathbf{45}_F\,, \overline{\mathbf{126}}_S)$
$[F_{12}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}, V_5(\mathbf{3},\mathbf{1})_{\frac{2}{3}}]$	LRSM, Leptoquark model, …	$[F_{10}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}, S_{13}(\mathbf{3},\mathbf{2})_{\frac{7}{6}}]$
$\epsilon^{ij}(\overline{d}^a_{f_1}\ell_{if_4})(u^T_{af_5}Ce_{f_2})H_j$ $\epsilon^{ij}(\overline{q}^{ak}_{f_4}u_{af_5})(\ell^T_{kf_1}C\ell_{if_2})H_j$	Standard model effective field theory	$-\epsilon^{abc}(\overline{e}Q_{ai})(d^T_bCd_c)H^{*i}$ $-\epsilon^{abc}(\overline{\ell}^k d_a)(q^T_{bk}Cq_{ci})H^{*i}$
$\bar{u}_R\gamma^\mu d_R\,\bar{e}_{R,i}\,\gamma_\mu\,C\bar{\nu}^T_{L,j}$ $\bar{u}_R d_L\,\bar{e}_{L,i}\,C\bar{\nu}^T_{L,j}$	Low energy effective field theory	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Rp}Cd^\beta_{Rr})(\bar{e}_{Rs}d^\gamma_{Lt})$ $\epsilon_{\alpha\beta\gamma}(u^{\alpha T}_{Rp}Cd^\beta_{Rr})(\bar{\nu}_{Ls}d^\gamma_{Rt})$
$\langle\bar{B}Q_+\gamma^\mu B\rangle(\bar{e}_R\gamma_\mu C\bar{\nu}^T_L)$ $\langle\bar{B}\Sigma_+ B\rangle(\bar{e}_L C\bar{\nu}^T_L)$	QCD chiral Lagrangian with heavy baryon	$\epsilon^{acd}(\lambda^A)^b_d(T_Lu)_a(T_Lu)_b(T_Lu)_c(\bar{e}_RB^A_L)\,,$ $\epsilon^{bcd}(\lambda^A)^a_d(T_Lu)_a(T_Lu)_b(T_Lu)_c(\bar{\nu}_LB_R)\,,$
$n \to p + e + \nu^c$	Nuclear Probes (0vBB or proton decay…)	$p \to \pi^+ + \nu$

Thanks !