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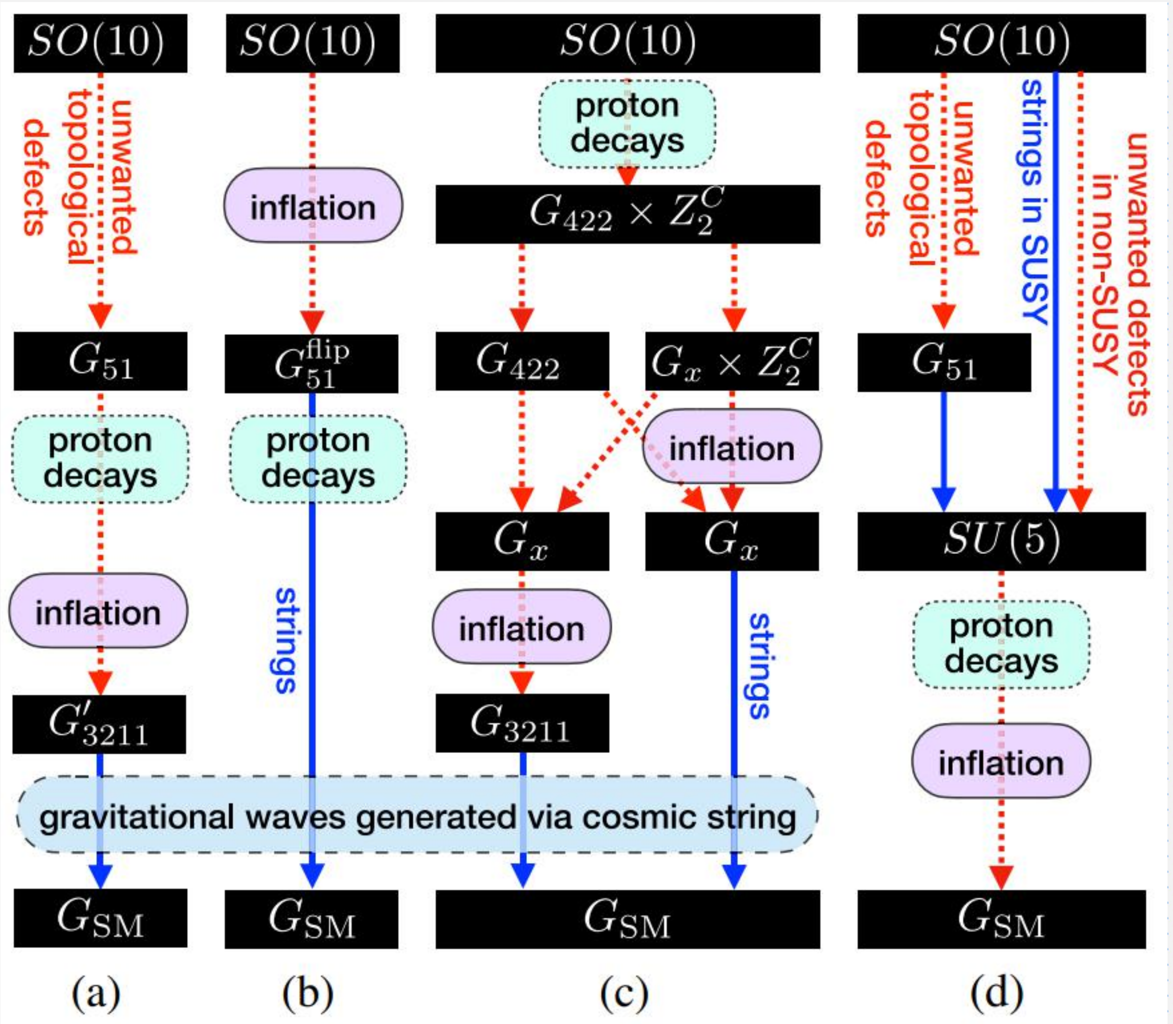
# Detecting Gravitational Waves from Cosmic Phase Transitions

Huaike Guo

April 18, 2025

Alba Romero, ..., HG, ..., PRL [2102.01714]
Shuo Guan, HG, Dian Jiao, Qingyuan Liang, Lei Wu, Yang Zhang (to appear)

# GUT Comes with Symmetry Breakings

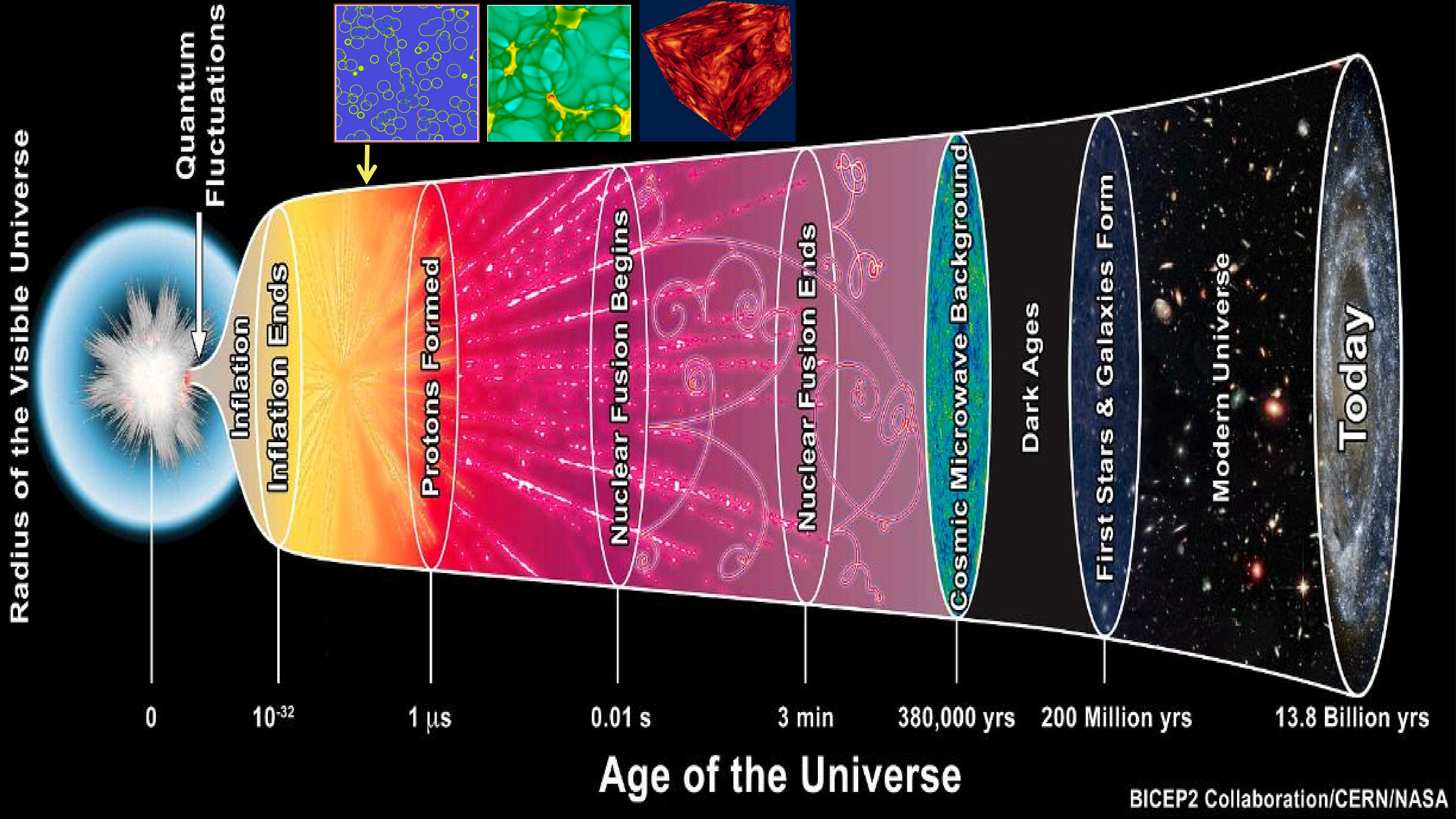



King, Pascoli, Turner, Zhou, PRL [2005.13549], JHEP [2106.15634]

Consequences of symmetry breakings:

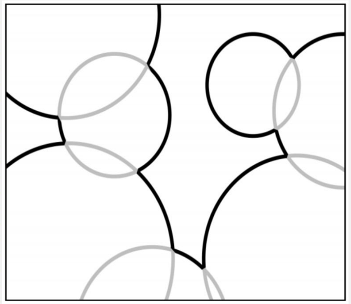
- Phase transitions
- Topological solitons (topological defects)
  monopoles, cosmic strings, domain walls, ...
- Non-Topological solitons
  Fermiballs (or PBH), ...

Multiple Sources for gravitational waves!


Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark Ages
First Stars & Galaxies Form
Modern Universe
Today
0
10^-32
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA

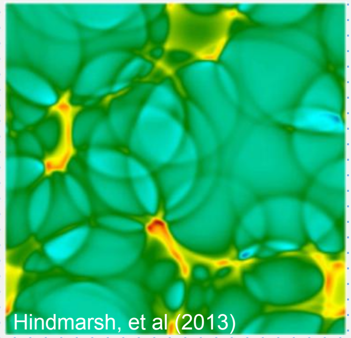
# The Spectra

bubble collision



$$\Omega_{\rm coll}(f)h^2 = 1.67\times10^{-5}\Delta\left(\frac{H_{\rm pt}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 \times\left(\frac{100}{g_*}\right)^{1/3}S_{\rm env}(f),$$

sound waves




$$\Omega_{\rm sw}(f)h^2 = 2.65\times10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2} \quad \text{(RD)}$$

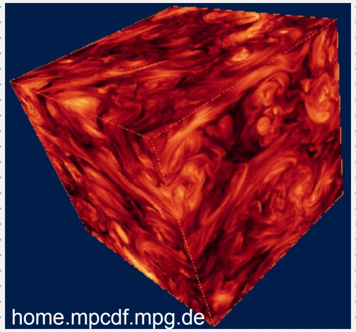
HG, Sinha, Vagie, White, JCAP [2007.08537]

$$\Upsilon = \frac{2[1-y^{3(w-1)/2}]}{3(1-w)}$$

HG, Yang Xiao, ... [2410.23666]

Reduces to Ellis, et al, JCAP [2003.07360]

MHD




$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\,S_{\rm turb}(f)$$

Chiara Caprini et al JCAP [1512.06239]

# Stochastic Gravitational Waves

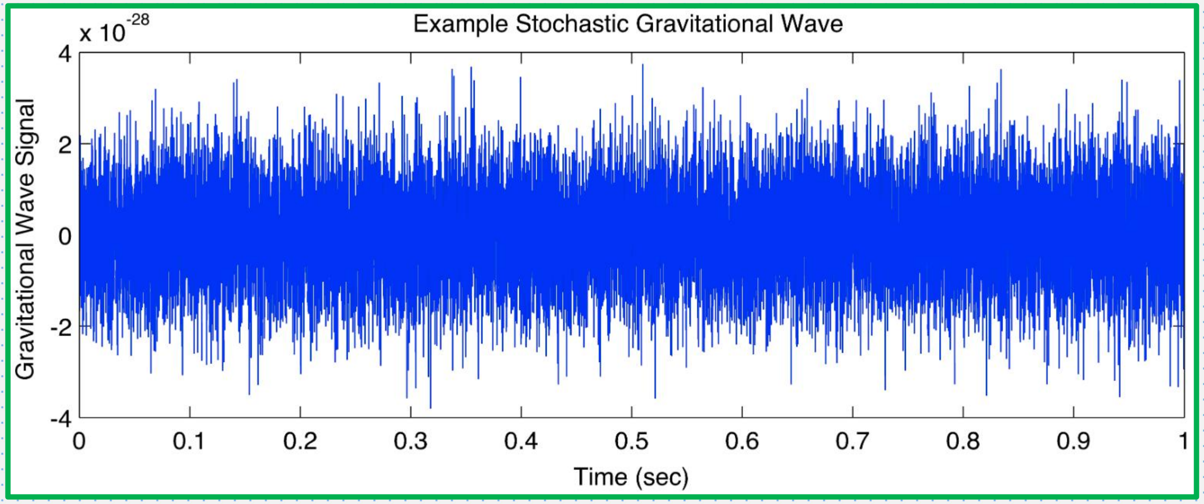
$$h_{ij}(t,\mathbf{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\mathbf{n}}\, \tilde{h}_A(f,\hat{\mathbf{n}})\, e_{ij}^A(\hat{\mathbf{n}})\, e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)}$$

- ✓ Gaussian
- ✓ Stationary
- ✓ Isotropic
- ✓ Unpolarized

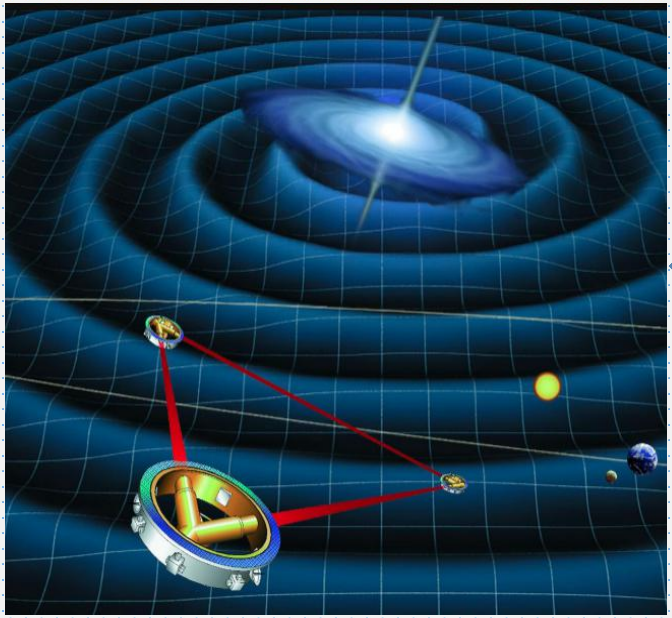
$$\langle \tilde{h}_A^*(f,\hat{\mathbf{n}})\tilde{h}_{A'}(f',\hat{\mathbf{n}}')\rangle = \frac{3H_0^2}{32\pi^3}\delta^2(\hat{\mathbf{n}},\hat{\mathbf{n}}')\delta_{AA'}\delta(f-f')f^{-3}\Omega_{\rm GW}(f)$$

**Energy density Spectrum**

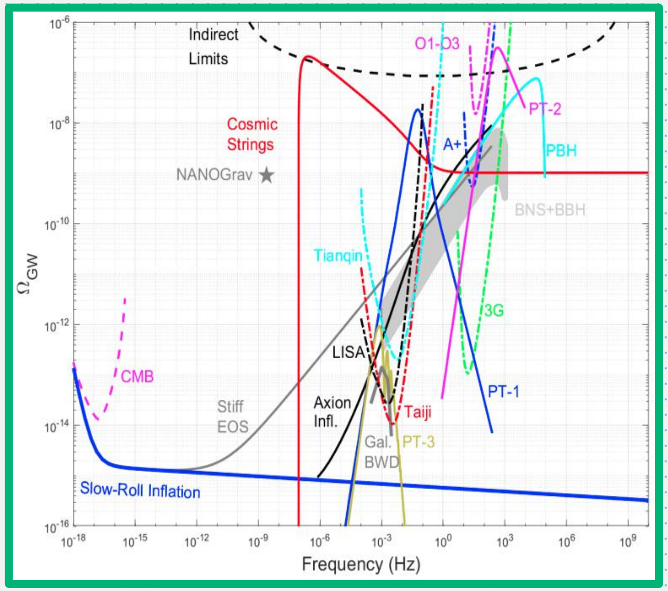
$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$



# From Theory to Experiment



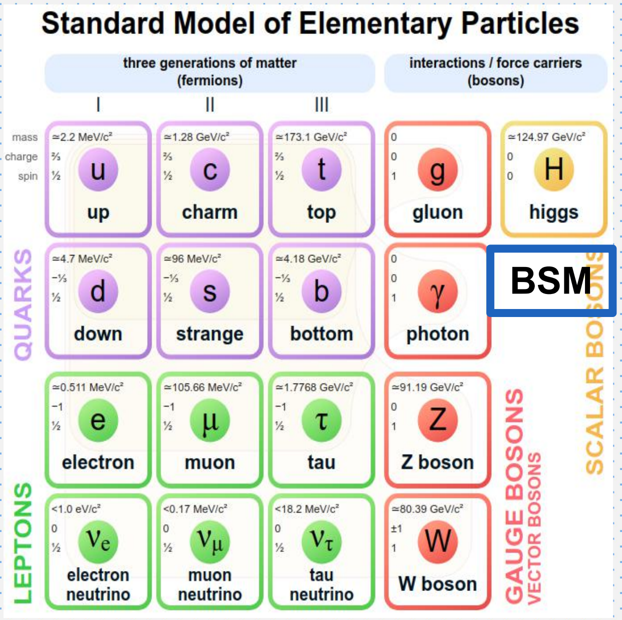
LIGO, LISA/Taiji/Tianqin, PTA, ...




Gravitational Wave Spectrum

$\alpha$
$\beta$
$v_w$
$T_*$
$g_s$
…

Phase Transition Parameters



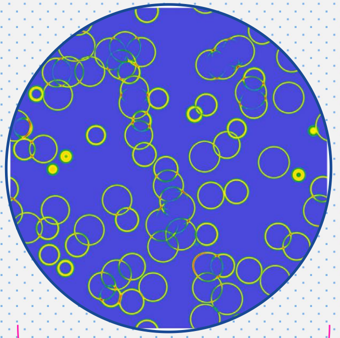

Particle Physics Model

this way

# Questions to Answer

- Set limits when signal is absent
- Parameter estimation when signal is discovered
  - What is precise shape of the signal spectrum?
  - What are the values of alpha, beta, vw, T*, etc?
  - What is the underlying particle physics model?
    - What are the values of the model parameters?
    - What this implies for collider experiments?
  - ...

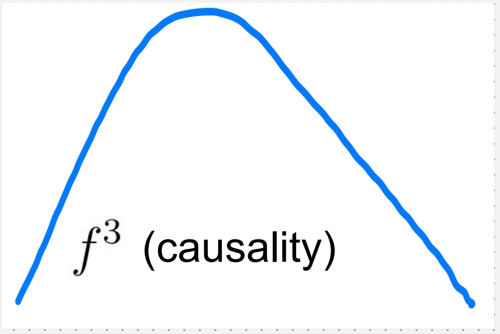
# Basic Properties



Hubble size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000




Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale

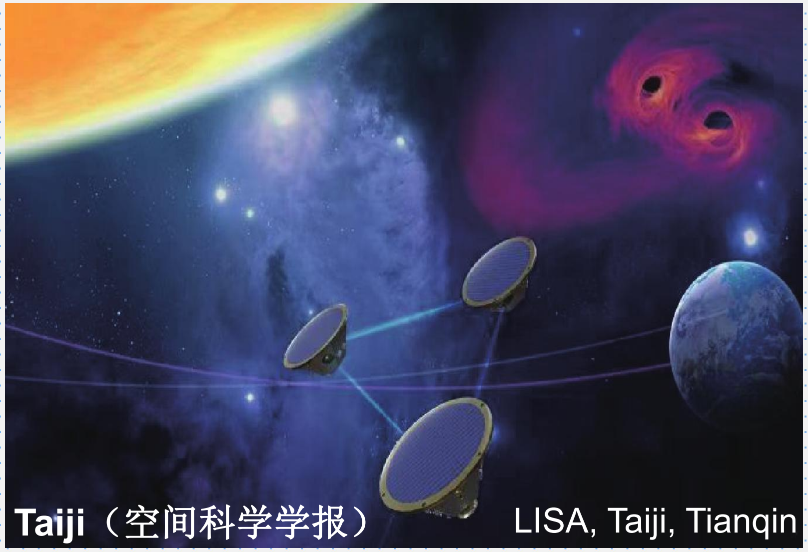
EPTA

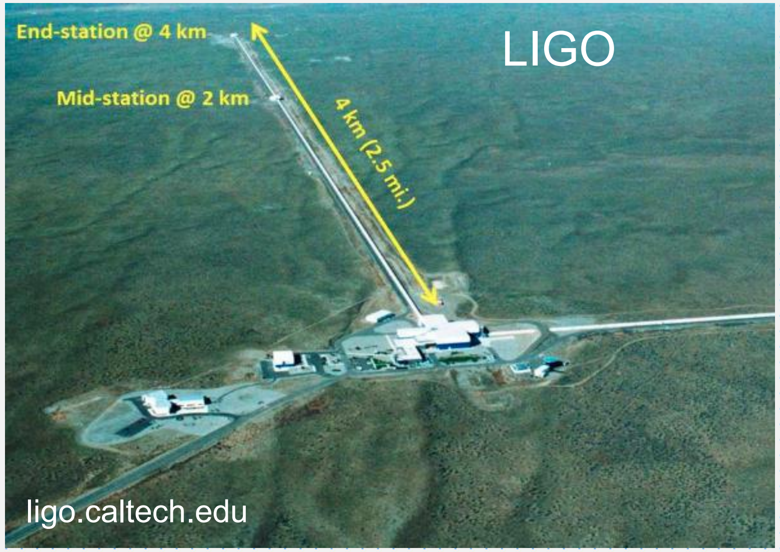
NANOGrav Physics Frontiers Center

PPTA

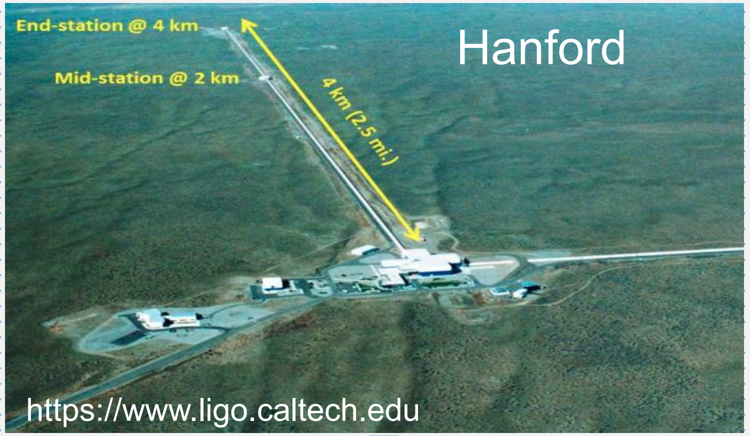
InPTA Indian Pulsar Timing Array

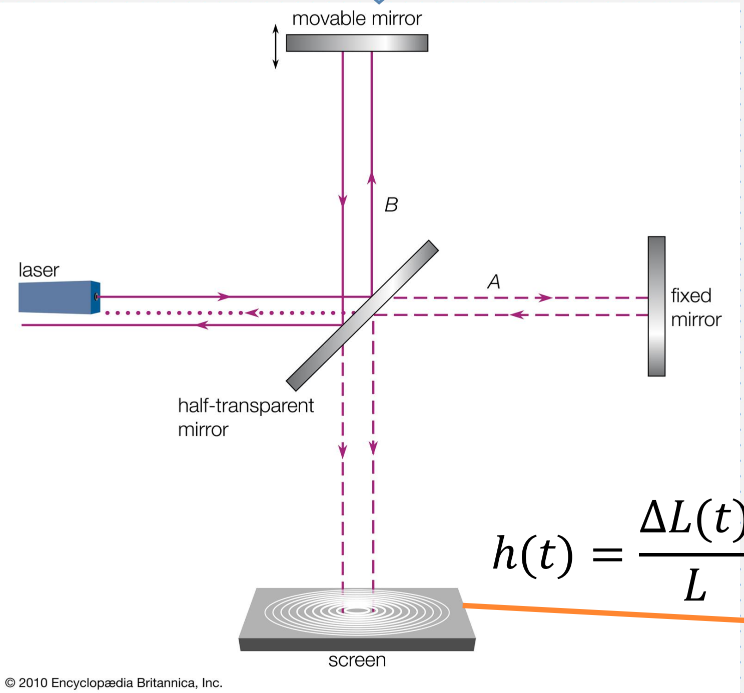
中国脉冲星测时阵列（CPTA）



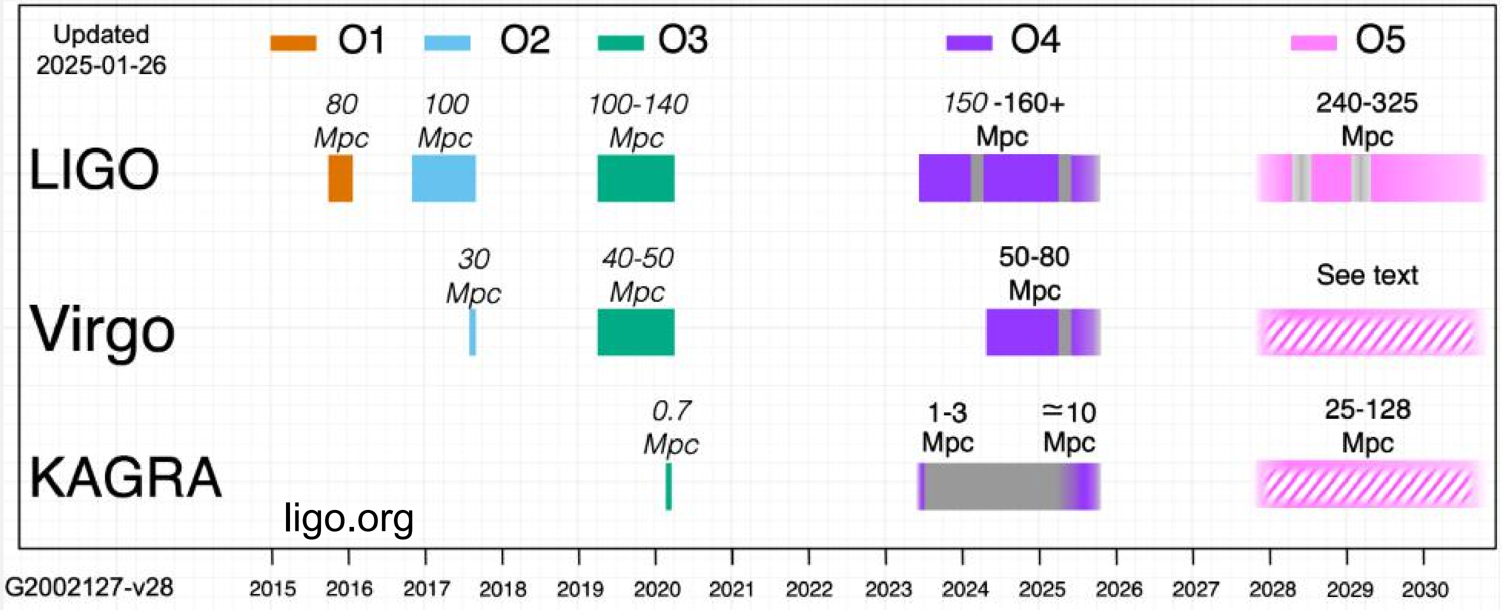



# LIGO-Virgo-Kagra




$$h(t) = \frac{\Delta L(t)}{L}$$

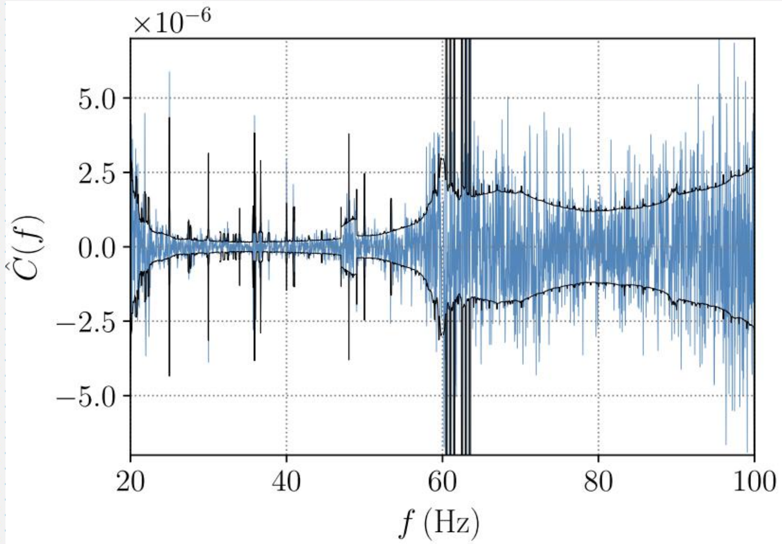



signal noise

$$h(t) = D_{ij} h_{ij}^{TT}(t) + n(t)$$

# Cross-Correlation Method


$$\hat{C}^{IJ}(f) = \frac{2}{T}\frac{\mathrm{Re}[\tilde{s}_I^\star(f)\tilde{s}_J(f)]}{\gamma_{IJ}(f)S_0(f)}$$




- Glitches and gating
- Narrow spectral artifacts
- Non-Gaussian noise
- Magnetic noise
- Calibration uncertainty

$$\langle \hat{C}^{IJ}(f)\rangle = \Omega_{\rm GW}(f)$$

LVK collaborations, PRD [2101.12130]

# The Bayesian Analysis Framework

calibration uncertainty

Likelihood

$$\log p(\hat{C}_{IJ}(f)|\boldsymbol{\theta}_{\rm gw}, \lambda) \propto -\frac{1}{2}\sum_f \frac{\left[\hat{C}_{IJ}(f) - \lambda\Omega_{\rm gw}(f, \boldsymbol{\theta}_{\rm gw})\right]^2}{\sigma^2_{IJ}(f)}$$

Gaussian noise

$$\sigma^2_{IJ}(f) \approx \frac{1}{2T\Delta f}\frac{P_I(f)P_J(f)}{\gamma^2_{IJ}(f)S_0^2(f)}$$

Priors for two analysis strategies:

## broken power law

$$\Omega_{\rm bpl}(f) = \Omega_* \left(\frac{f}{f_*}\right)^{n_1}\left[1 + \left(\frac{f}{f_*}\right)^{\Delta}\right]^{(n_2-n_1)/\Delta}$$

| **Broken power law model** | |
|---|---|
| Parameter | Prior |
| $\Omega_{\rm ref}$ | LogUniform($10^{-10}$, $10^{-7}$) |
| $\Omega_*$ | LogUniform($10^{-9}$, $10^{-4}$) |
| $f_*$ | Uniform(20, 256 Hz) |
| $n_1$ | 3 |
| $n_2$ | Uniform(-8,0) |
| $\Delta$ | 2 |

## sound waves, or bubble collision

| **Phenomenological model** | |
|---|---|
| Parameter | Prior |
| $\Omega_{\rm ref}$ | LogUniform($10^{-10}$, $10^{-7}$) |
| $\alpha$ | LogUniform ($10^{-3}$, 10) |
| $\beta/H_{\rm pt}$ | LogUniform ($10^{-1}$, $10^{3}$) |
| $T_{\rm pt}$ | LogUniform ($10^{5}$, $10^{9}$ GeV) |
| $v_{\rm w}$ | 1 |
| $\kappa_\phi$ | 1 |
| $\kappa_{\rm sw}$ | $f(\alpha, v_{\rm w}) \in [0.1 - 0.9]$ |

# Results

O1+O2+O3@LIGO (H1, L1), Virgo

- No Evidence for Broken Power Law Signal
- No Evidence for Bubble Collision Domination Signal
- No Evidence for Sound Waves Domination Signal

Bubble Collision

95% CL UL with fixed Tpt and beta/Hpt

| Phenomenological model (bubble collisions) | | | | |
|---|---|---|---|---|
| $\Omega^{95\%}_{\rm coll}(25\ {\rm Hz})$ | | | | |
| $\beta/H_{\rm pt}\backslash T_{\rm pt}$ | $10^7$ GeV | $10^8$ GeV | $10^9$ GeV | $10^{10}$ GeV |
| 0.1 | $9.2\times10^{-9}$ | $8.8\times10^{-9}$ | $1.0\times10^{-8}$ | $7.2\times10^{-9}$ |
| 1 | $1.0\times10^{-8}$ | $8.4\times10^{-9}$ | $5.0\times10^{-9}$ | … |
| 10 | $4.0\times10^{-9}$ | $6.3\times10^{-9}$ | … | … |

no sensitivity

Broken Power Law

95% CL UL (CBC+BPL)

$$\Omega_{\rm ref} = 6.1\times10^{-9}$$
$$\Omega_* = 5.6\times10^{-7}$$
$$\Omega_{\rm BPL}(25\ {\rm Hz}) = 4.4\times10^{-9}$$

Sound Waves

95% CL UL

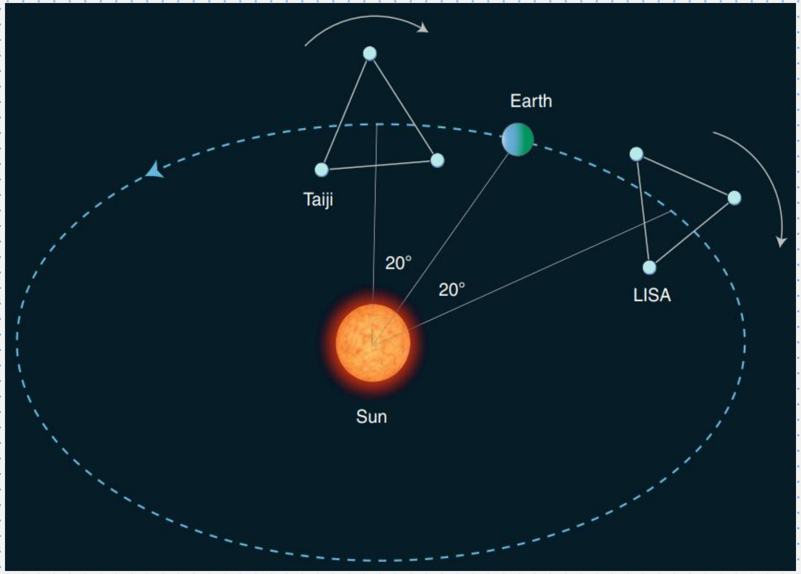
$$\Omega_{\rm sw}(25\ {\rm Hz}) \quad 5.9\times10^{-9}$$
$$\beta/H_{\rm pt} < 1 \text{ and } T_{\rm pt} > 10^8\ {\rm GeV}$$

See also Jiang, Huang, JCAP[2203.11781], Yu, Wang, PRD[2211.13111]

# Detection in Space

Stochastic GW detection in space:

- Complicated, and correlated noise
- Complications from time-delay interferometry
- Solution: null channel method, or with a detector network




Ruan, Liu, Guo, Wu, Cai, Nature Astron [2002.03603]

Studies on PT detection in space:

Gowling, Hindmarsh, Hooper, Torrado, JCAP [2209.13551]
Gowling, Hindmarsh, JCAP [2106.05984]
Boileau, et al, JCAP [2209.13277]
Lewicki, et al, PRD [2403.03769]
Caprini, et al, JCAP [2403.03723]

Cosmo SGB detectable down to $\Omega_{GW} \sim O(10^{-13})$

Boileau et al, MNRAS [2105.04283]

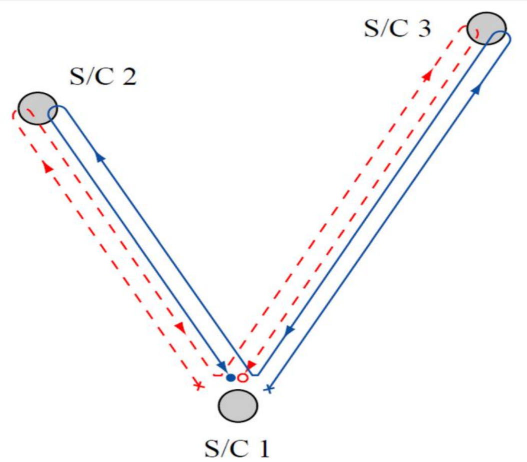
# Detector Response

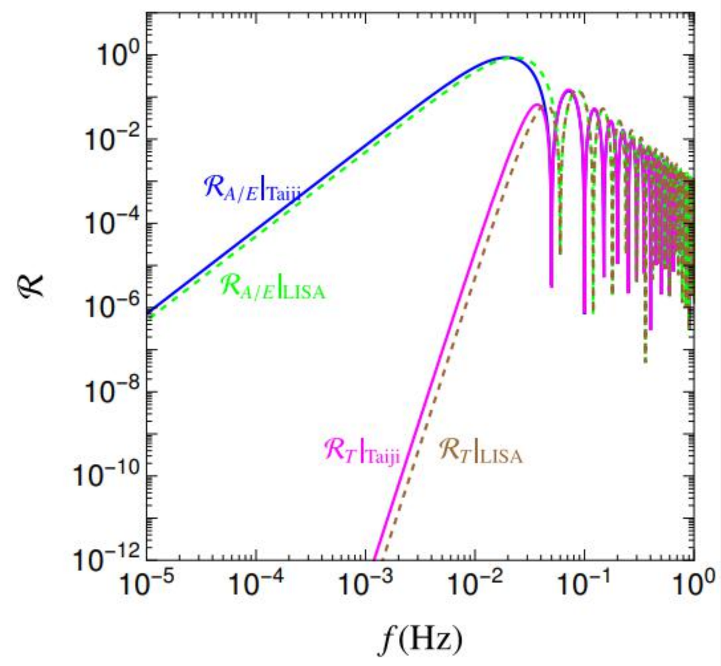
3 orthogonal TDI channels

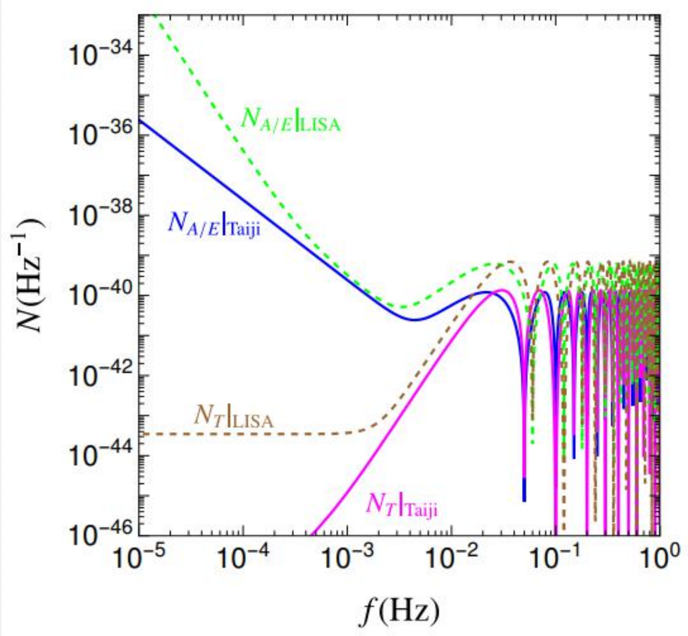
TDI: time-delay interferometry
Tinto, Dhurandhar, LRR, 2021

$$\langle \tilde{d}_a(f)\tilde{d}_b^\star(f')\rangle = \frac{1}{2}P_a(f)\delta_{ab}\delta(f-f')$$

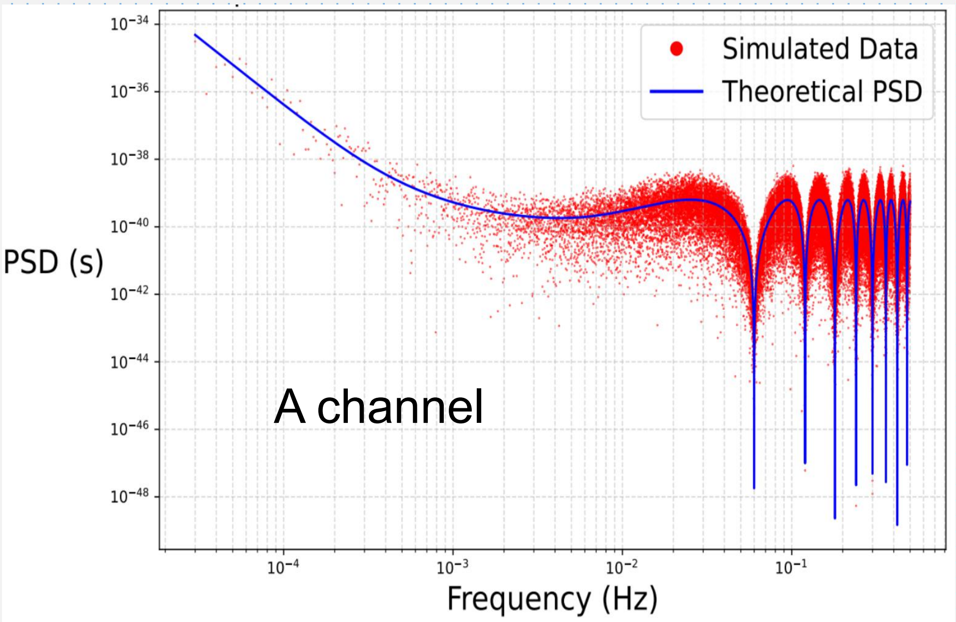
$$P_a(f) = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{\rm GW}}{f^3}R_a(f) + N_a(f)$$



# Likelihood




The core of the statistical analysis: likelihood

both signal and noise behave as random variables

$$\mathcal{L} = \prod_{\kappa=1}^{N_0} \prod_{k=1}^{N/2} \frac{1}{8\pi^3 \sigma_A^2 \sigma_E^2 \sigma_T^2} \exp\left[ - \sum_{a=A,E,T} \frac{\left| \tilde{d}_a^\kappa(f_k) \right|^2}{2\sigma_a^2} \right]$$

$N_0 = 126$ : number of segments

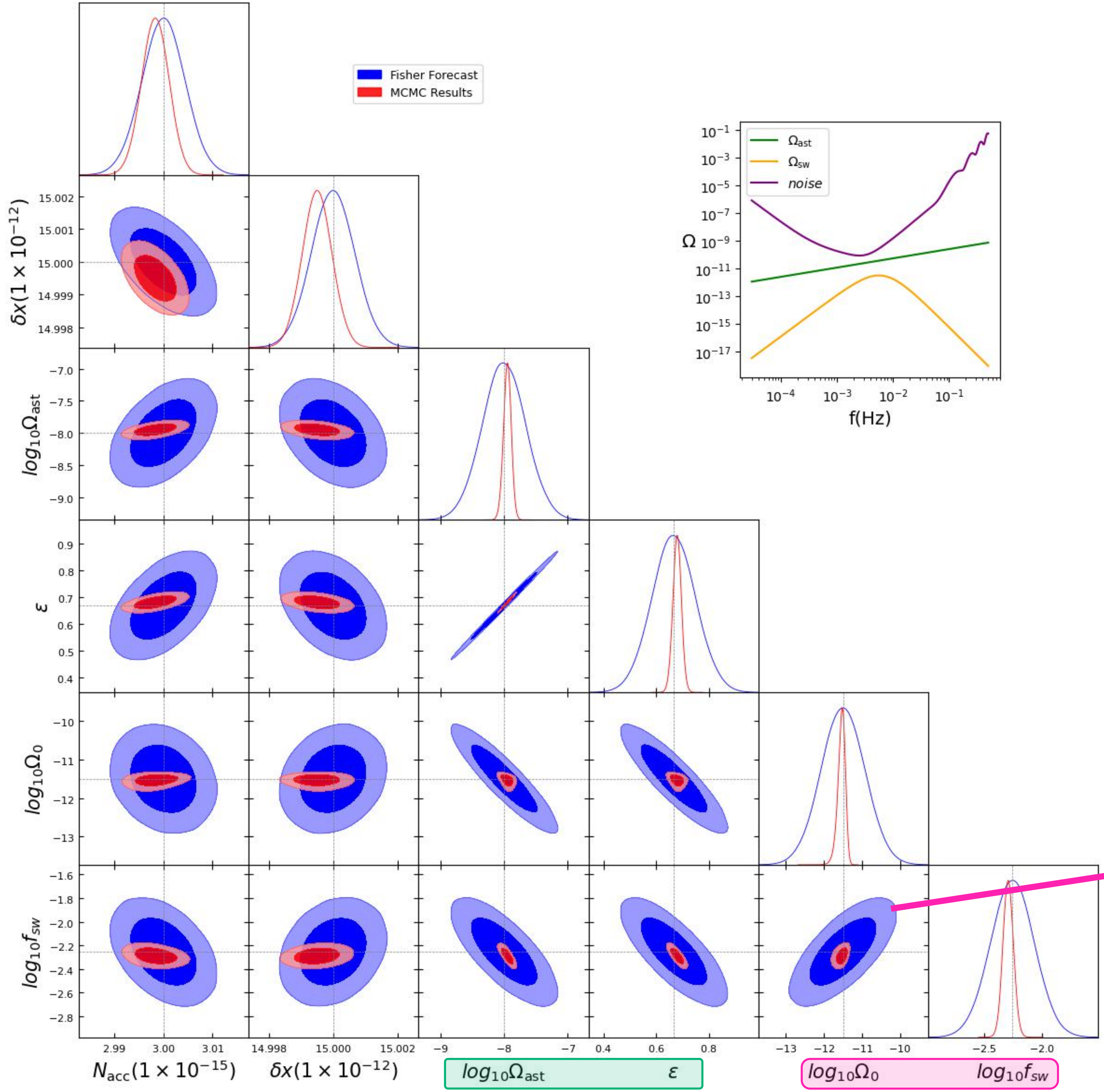
$N = 10^6$ : frequency bins in one segment

$T = 1.26 \times 10^8\,\mathrm{s}$ : observation time

$f_s = 1\,\mathrm{Hz}$ : sampling frequency

$$\sigma_a^2 = \frac{T f_s^2}{4} \left[ \frac{3H_0^2}{4\pi^2} \frac{\Omega_{\mathrm{GW}}}{f^3} R_a(f) + N_a(f) \right]$$

astrophysical background
cosmological background (PT, etc)

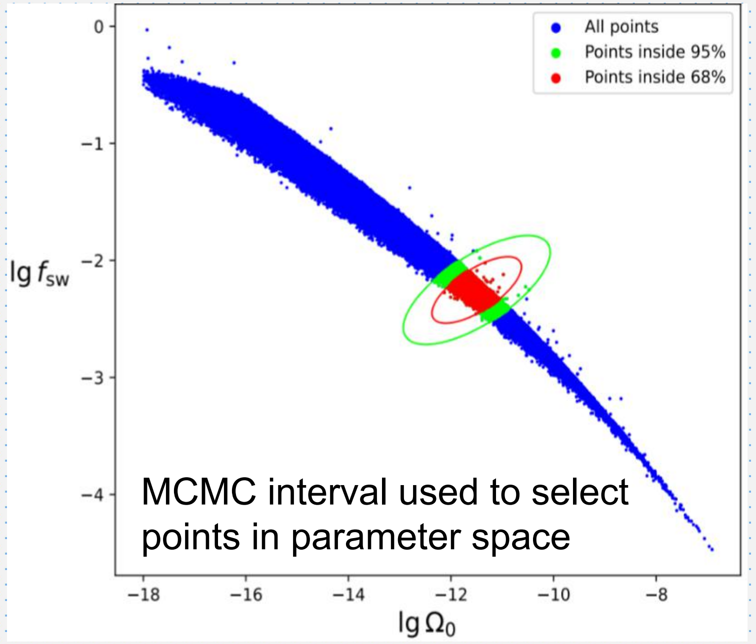


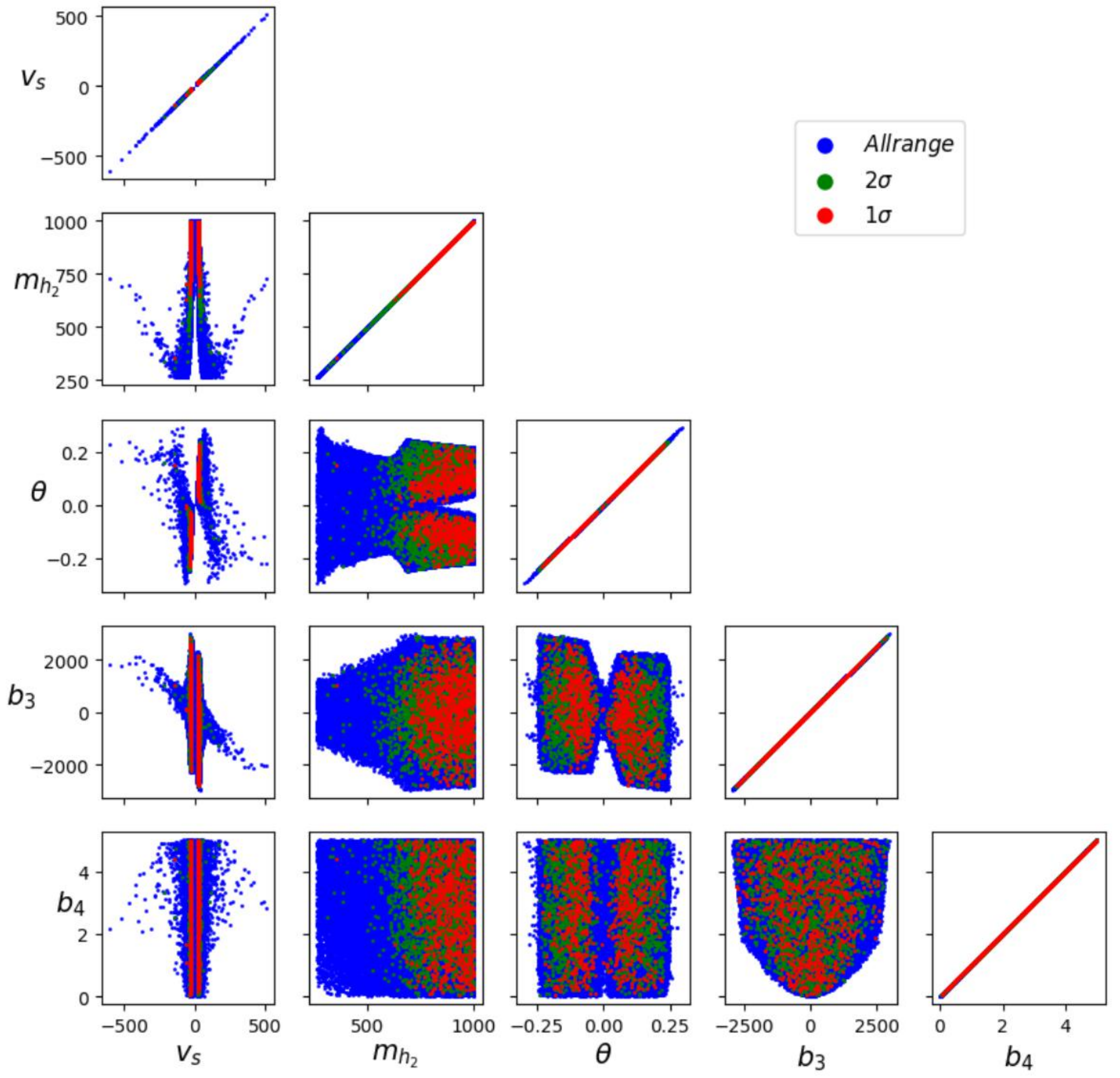

Simplified SW spectrum with no parameter degeneracy

$$\Omega_{\rm sw}(f) = \Omega_0 \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

$$\Omega_{\rm astro}(f) = \Omega_{\rm ast} \left(\frac{f}{f_{\rm ref}}\right)^{\varepsilon}$$

points from scan of xSM, evading physical constraints






# xSM Model

Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy, PRD [0706.4311]

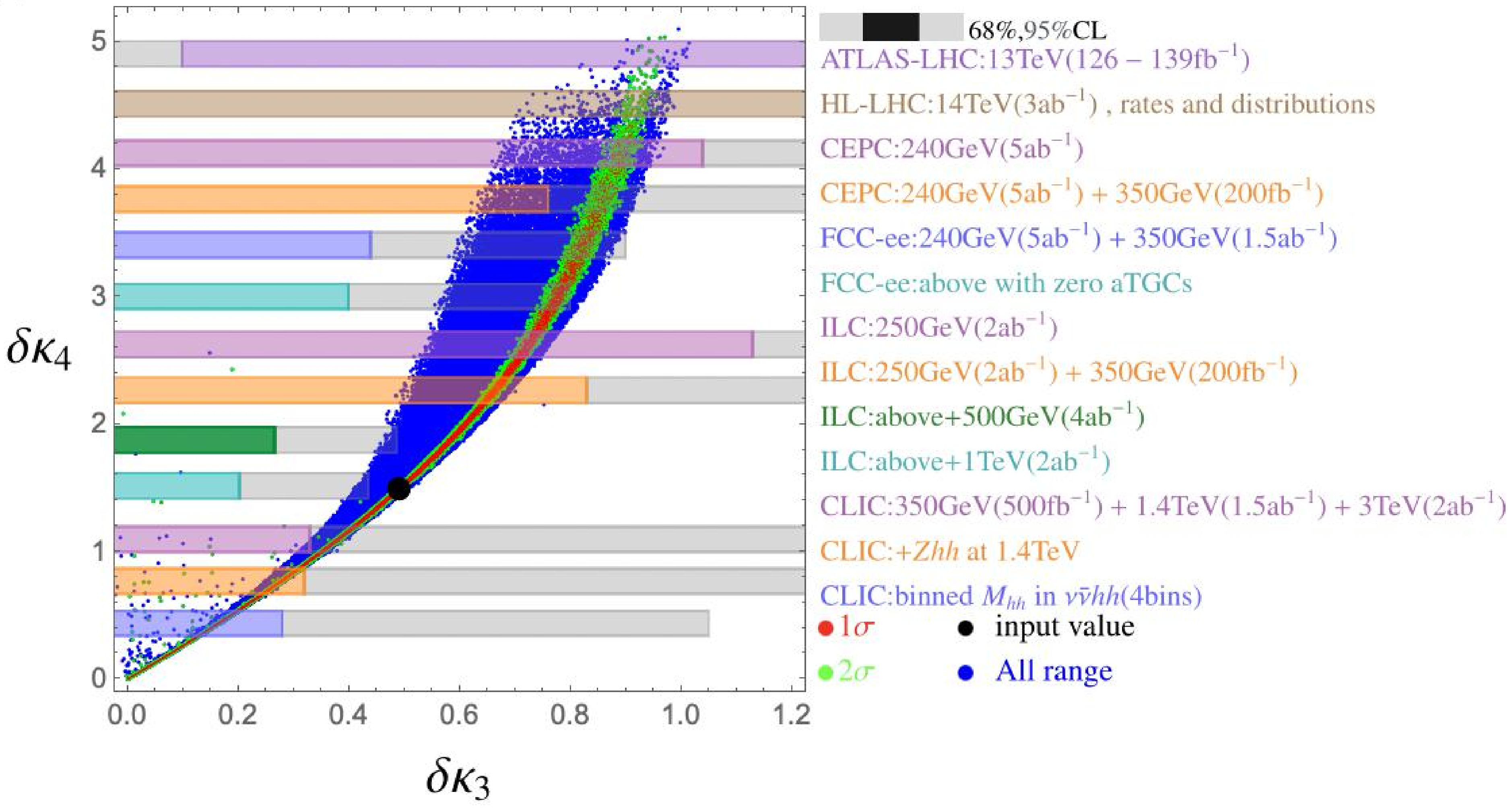
$$V(\phi, S) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4$$

$$v_s, \quad m_{h_2}, \quad \theta, \quad b_3, \quad b_4$$

Points survive pheno constraints.

Alves, Ghosh, HG, Sinha, Vagie, JHEP [1812.09333]

# Higgs Self-Couplings



# Thanks!