Non-Abelian Domain walls

Bowen Fu (付博文) 21 April 2025

The 2nd Workshop on Grand Unified Theories: Phenomenology and Cosmology (GUTPC 2025)

Content

- Brief introduction of Z_2 domain walls
- S_4 domain walls
- A_4 domain walls

Content

- Brief introduction of Z_2 domain walls
- S_4 domain walls
- A_4 domain walls

Based on **BF**, S. F. King, L. Marsili, S. Pascoli, J. Turner, Y-L Zhou, 2409.16359 and follow up works

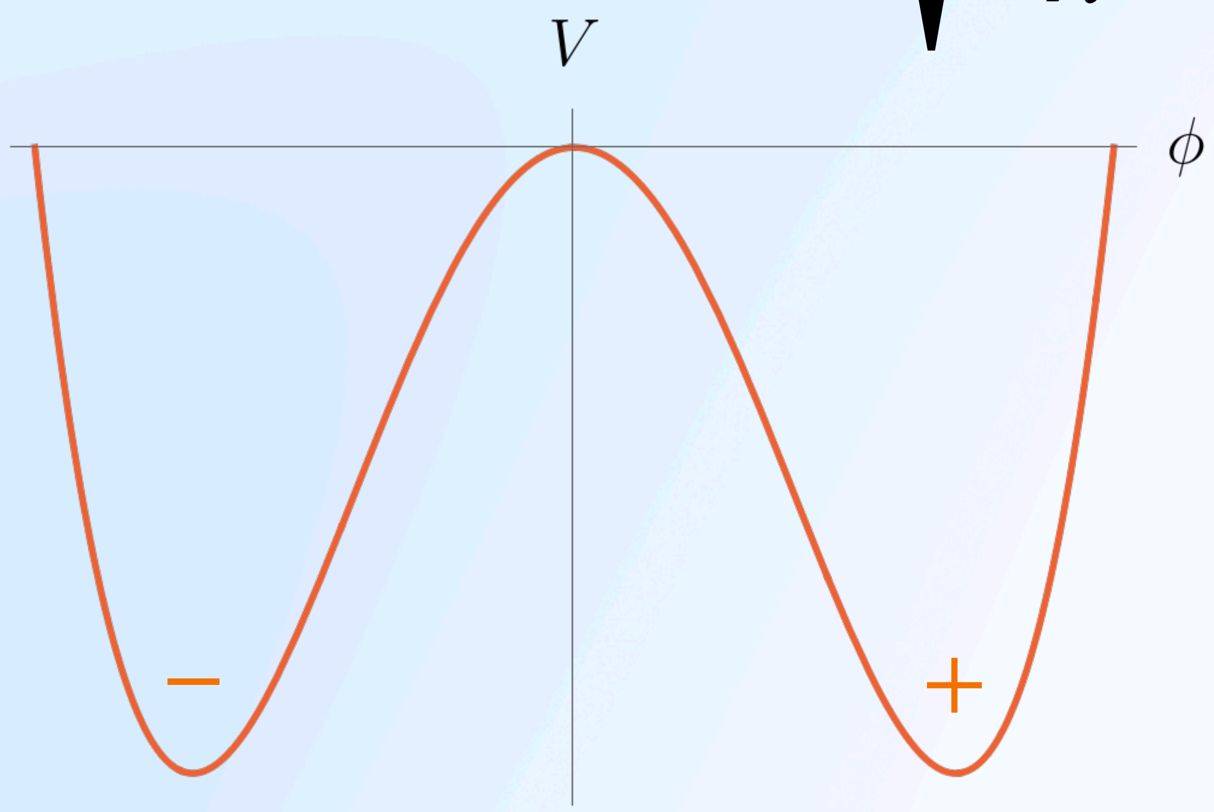
Domain wall formation

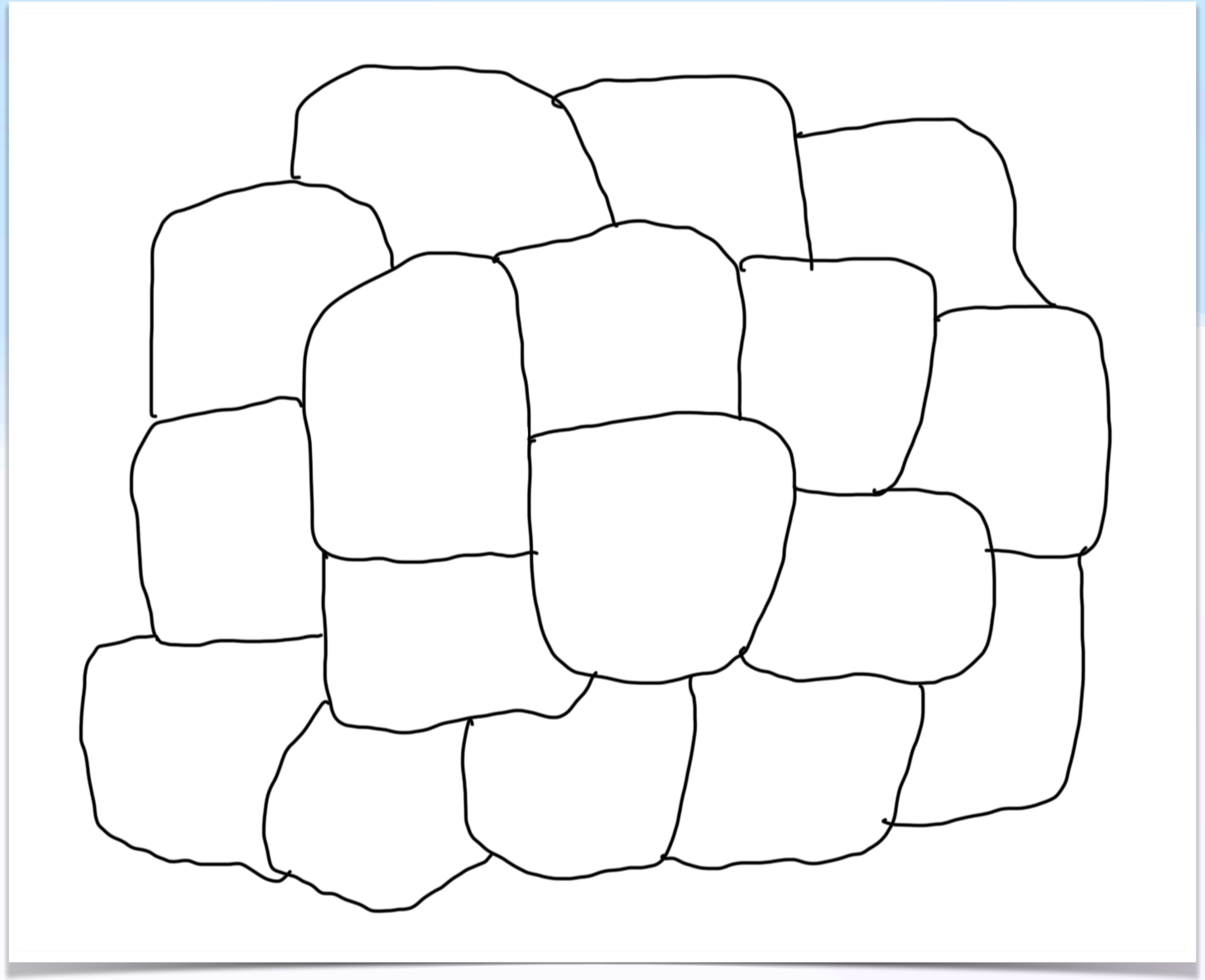
Kibble mechanism:

Z_2 - a simplest case

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\langle\phi\rangle = \pm v \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$





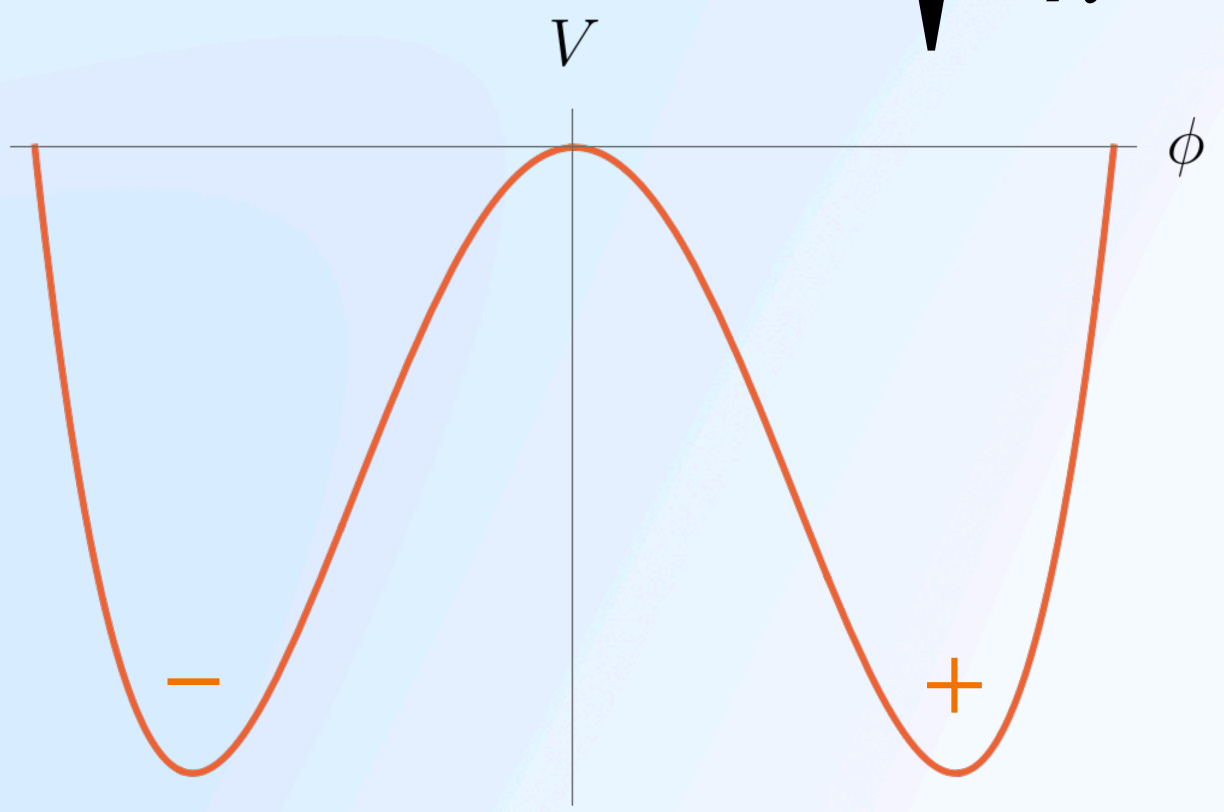
Domain wall formation

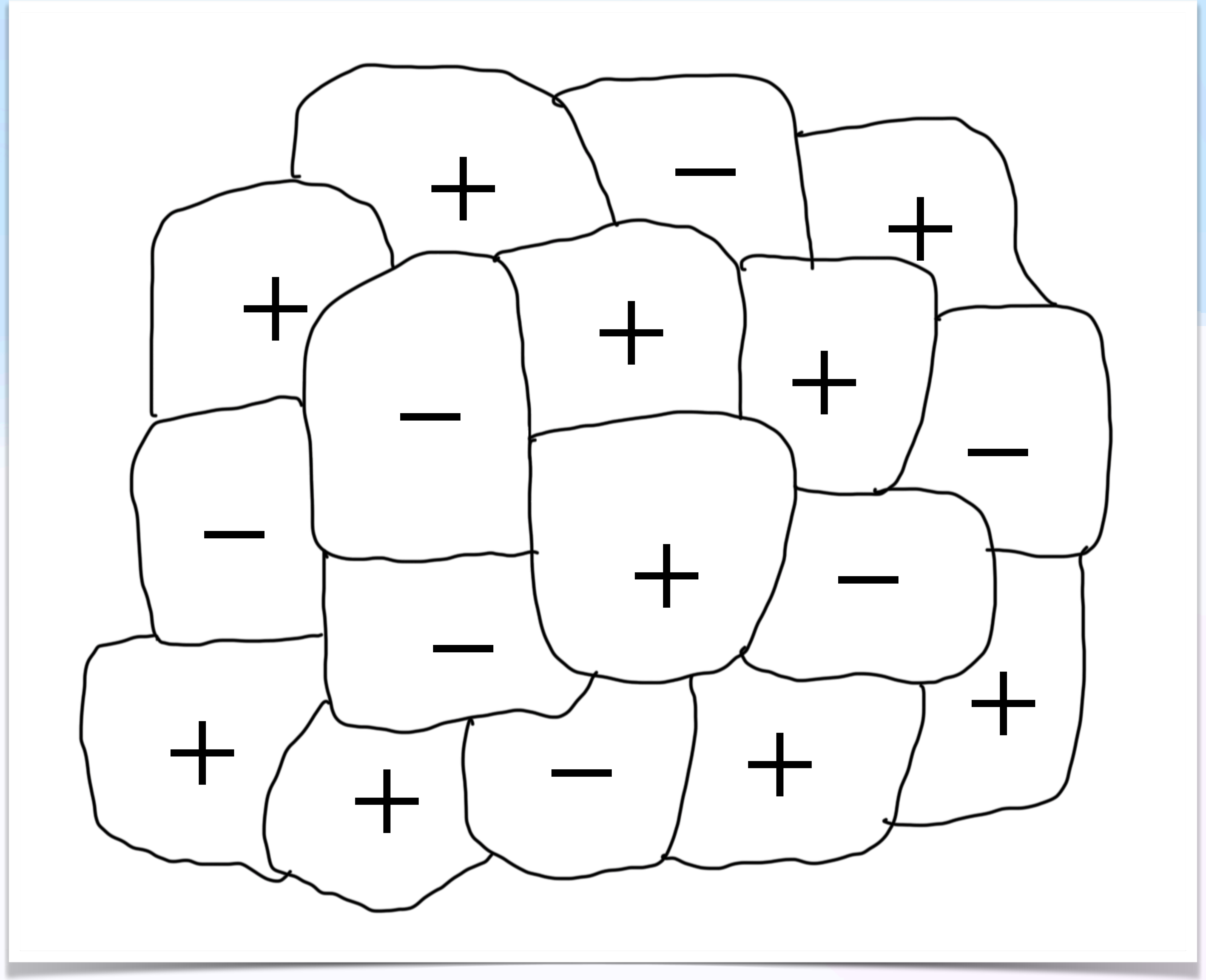
Kibble mechanism:

Z_2 - a simplest case

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\langle\phi\rangle = \pm v \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$





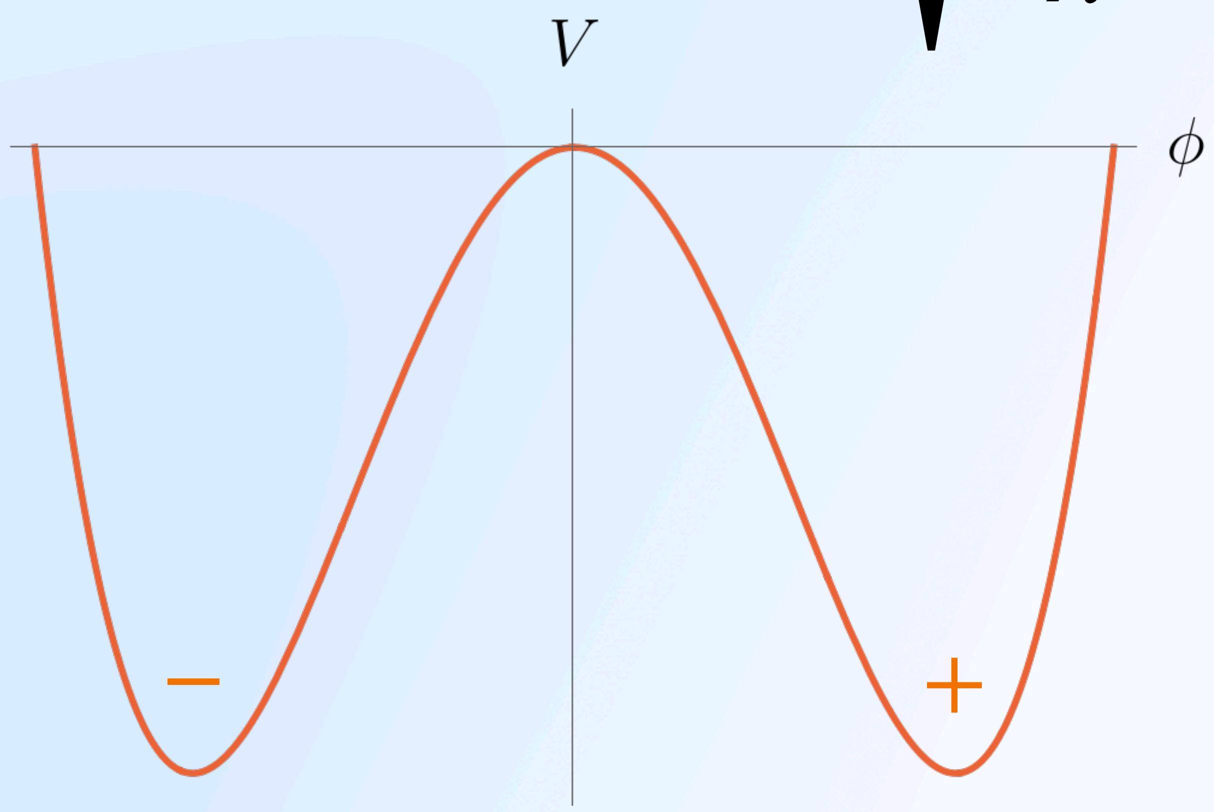
Domain wall formation

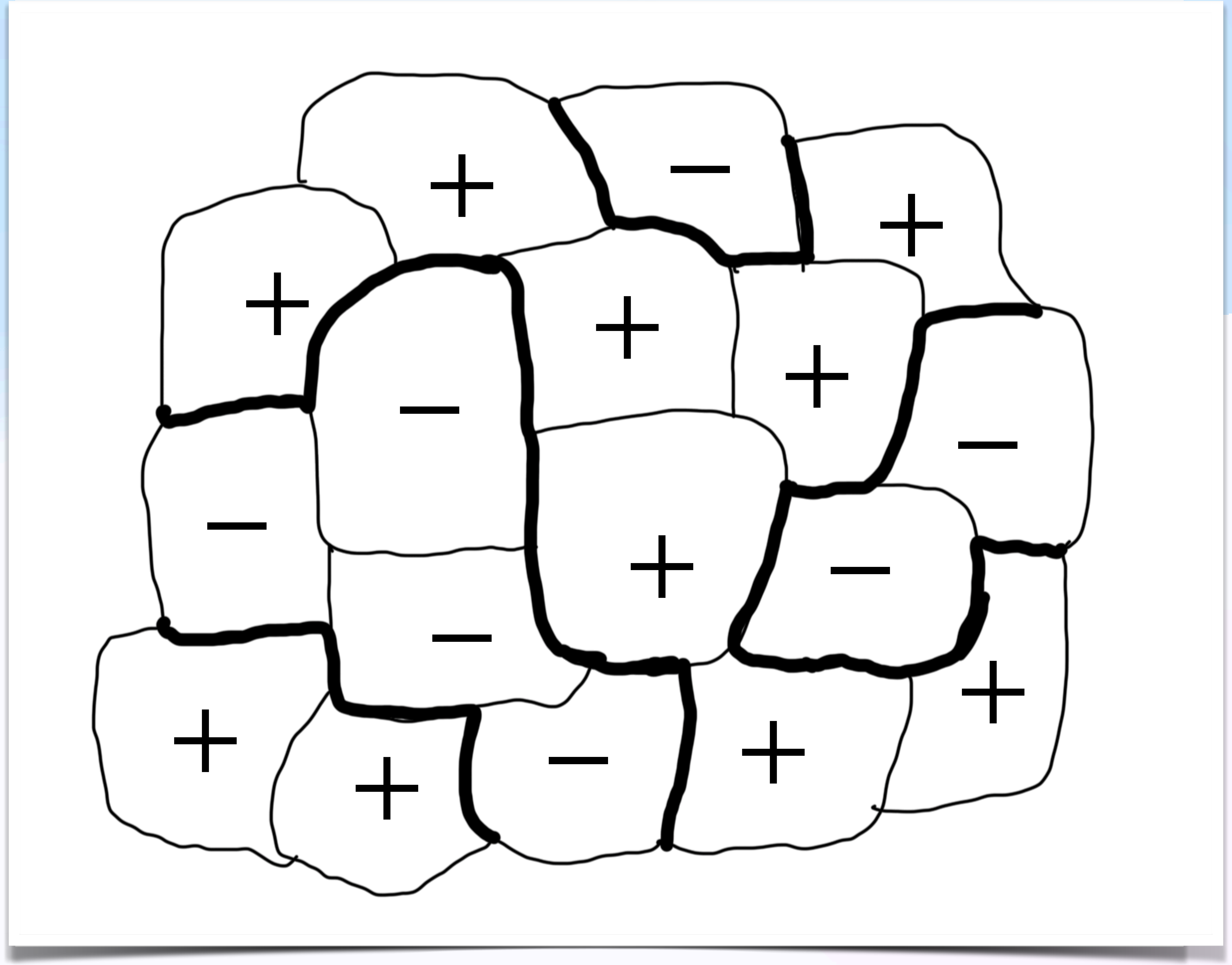
Kibble mechanism:

Z_2 - a simplest case

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\langle\phi\rangle = \pm v \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$



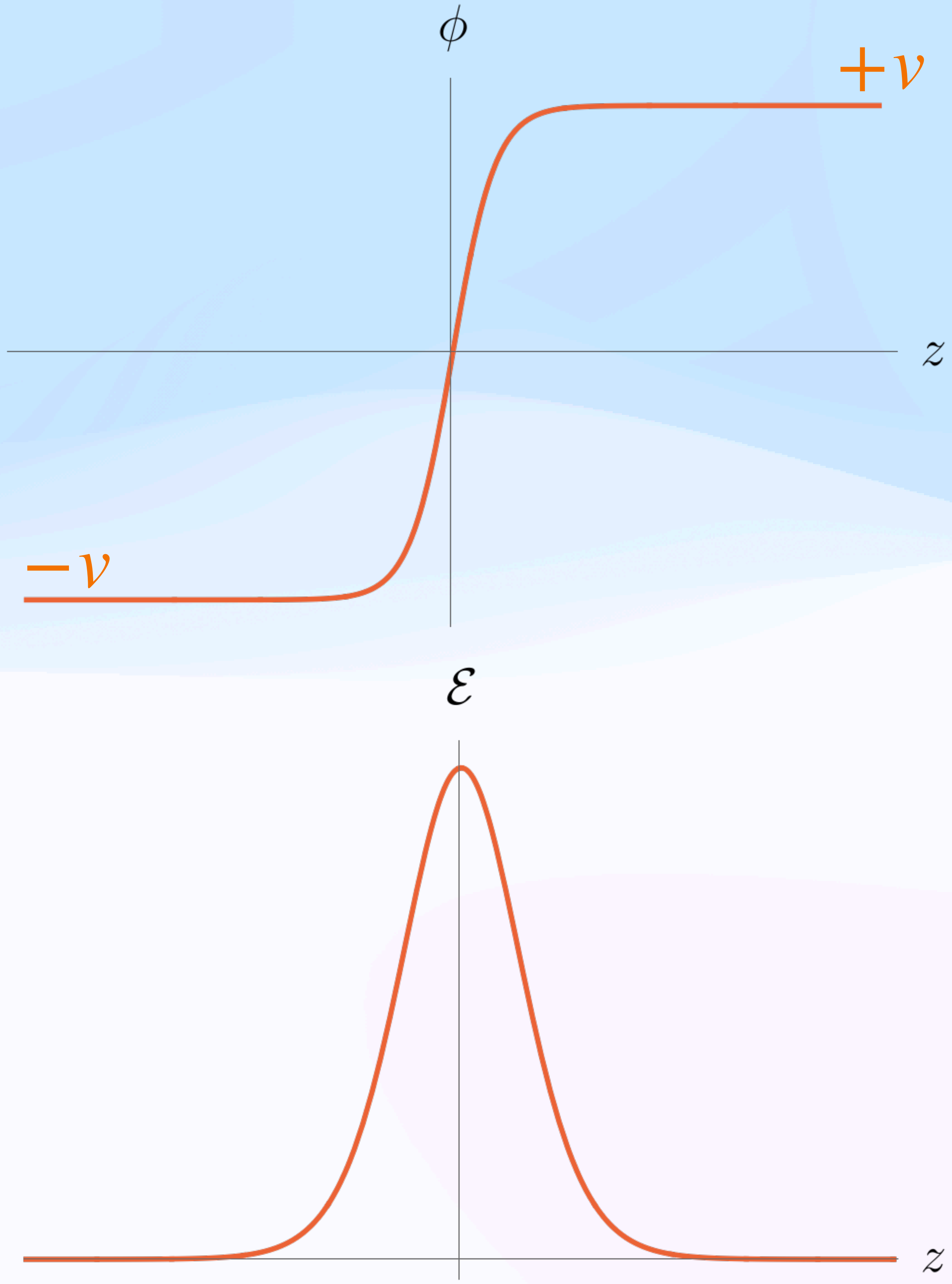


Z_2 domain wall

$$\phi(z) = v \tanh \frac{z}{\Delta} \qquad \Delta = \sqrt{\frac{2}{\lambda v^2}}$$

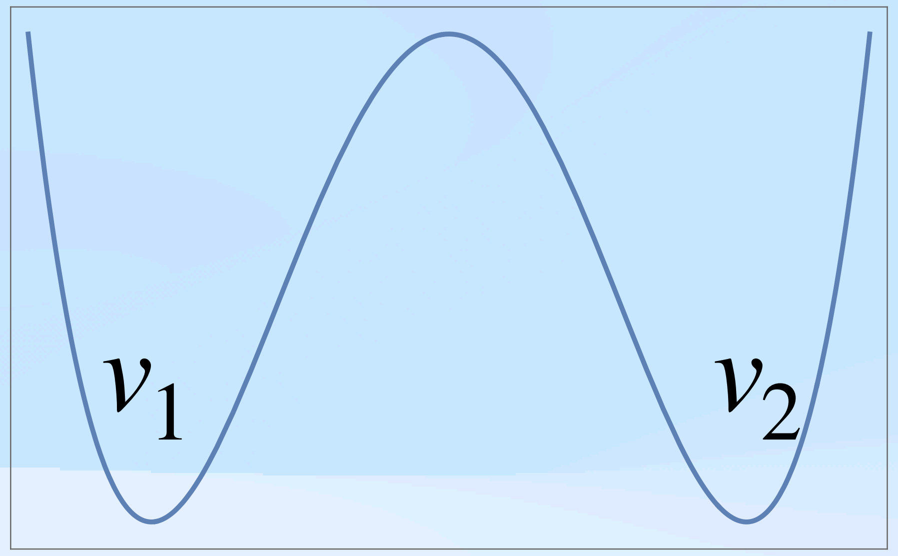
$$\mathcal{E} = \frac{1}{2}\phi'^2 + \Delta V \qquad \sigma = \int \mathcal{E}\, dz = \frac{2\sqrt{2}}{3}\sqrt{\lambda} v^3$$

Tension (surface energy density)

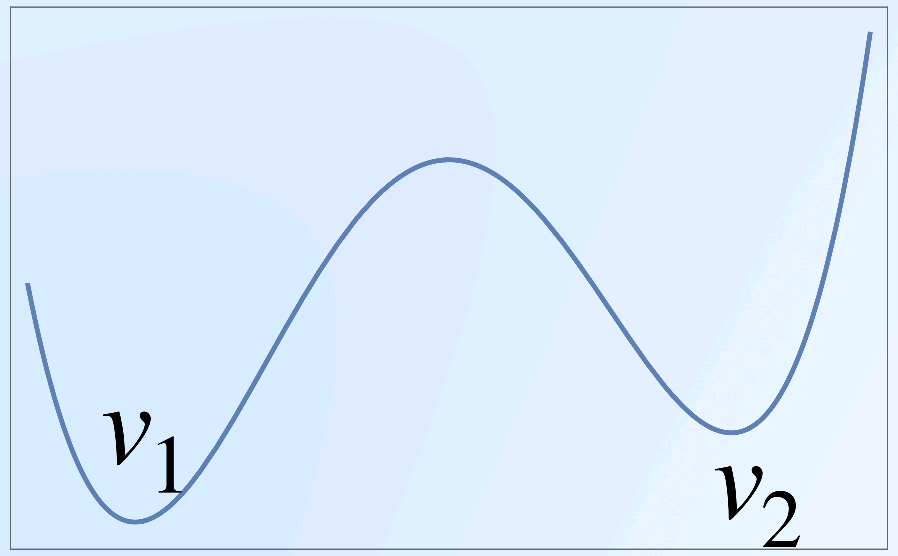


Gravitational wave from DWs

- Exact discrete symmetry $\Longrightarrow$ stable DWs

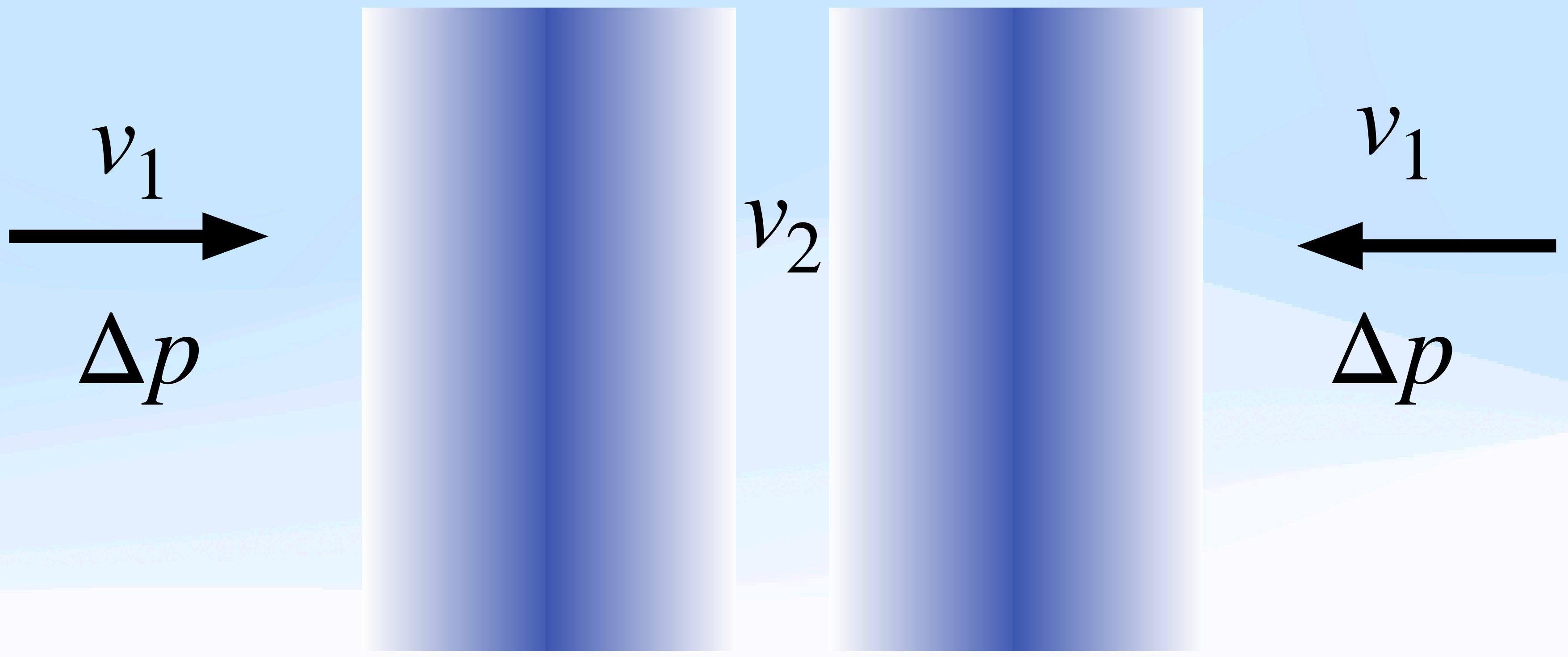


- Bias $\Longrightarrow$ unstable DWs



$$V_{\rm bias} \equiv V(v_2) - V(v_1)$$

Pressure difference $\Delta p \propto V_{\rm bias}$



$$\Omega_{\rm GW}^{\rm peak}\left(\sigma, V_{\rm bias}\right)$$

$$\begin{cases} f > f_{\rm peak}, \Omega_{\rm GW} \propto f^{-1} \\ f < f_{\rm peak}, \Omega_{\rm GW} \propto f^{3} \end{cases}$$

Saikawa [2017]

Discrete Symmetries

- Abelian: Z_n
- Non-Abelian： $A_n, S_n, \Delta(27)$……

Roles: flavour symmetries, dark matter, …

S_4 scalar theory

BF, S.F. King, L. Marsili, S. Pascoli,
J. Turner, Y-L. Zhou, 2409.16359

- The octahedral/cube group S_4:

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- The most general renormalisable flavon potential:

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

Also for $A_4 \times Z_2$

S_4 vacuum structure

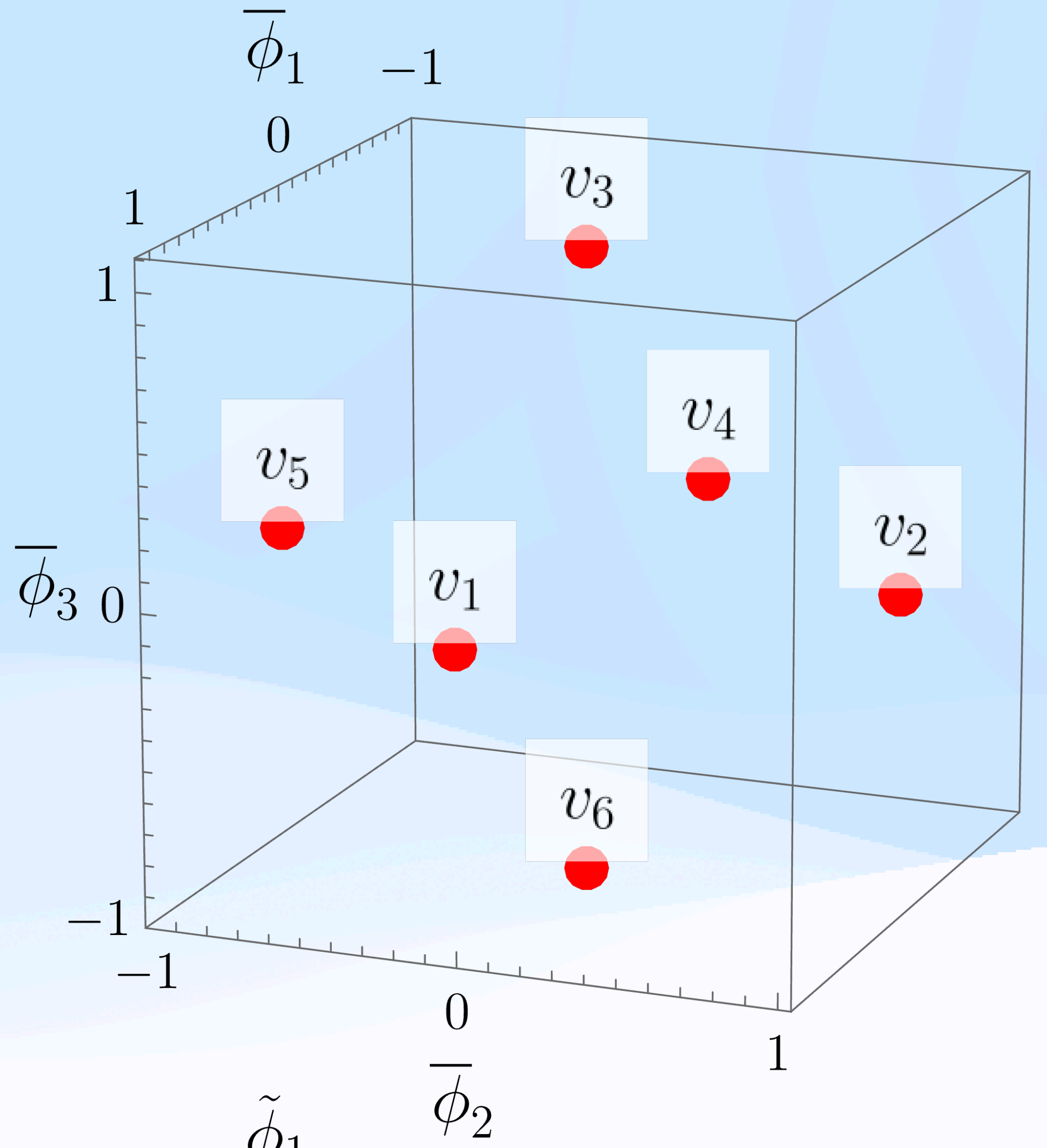
BF, S.F. King, L. Marsili, S. Pascoli,
J. Turner, Y-L. Zhou, 2409.16359

$g_2 > 0$

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}-1\\0\\0\end{pmatrix},\begin{pmatrix}0\\-1\\0\end{pmatrix},\begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}v$$

$$v = \frac{\mu}{\sqrt{g_1}}$$

$$\bar{\phi}_i = \frac{\phi_i}{v}$$

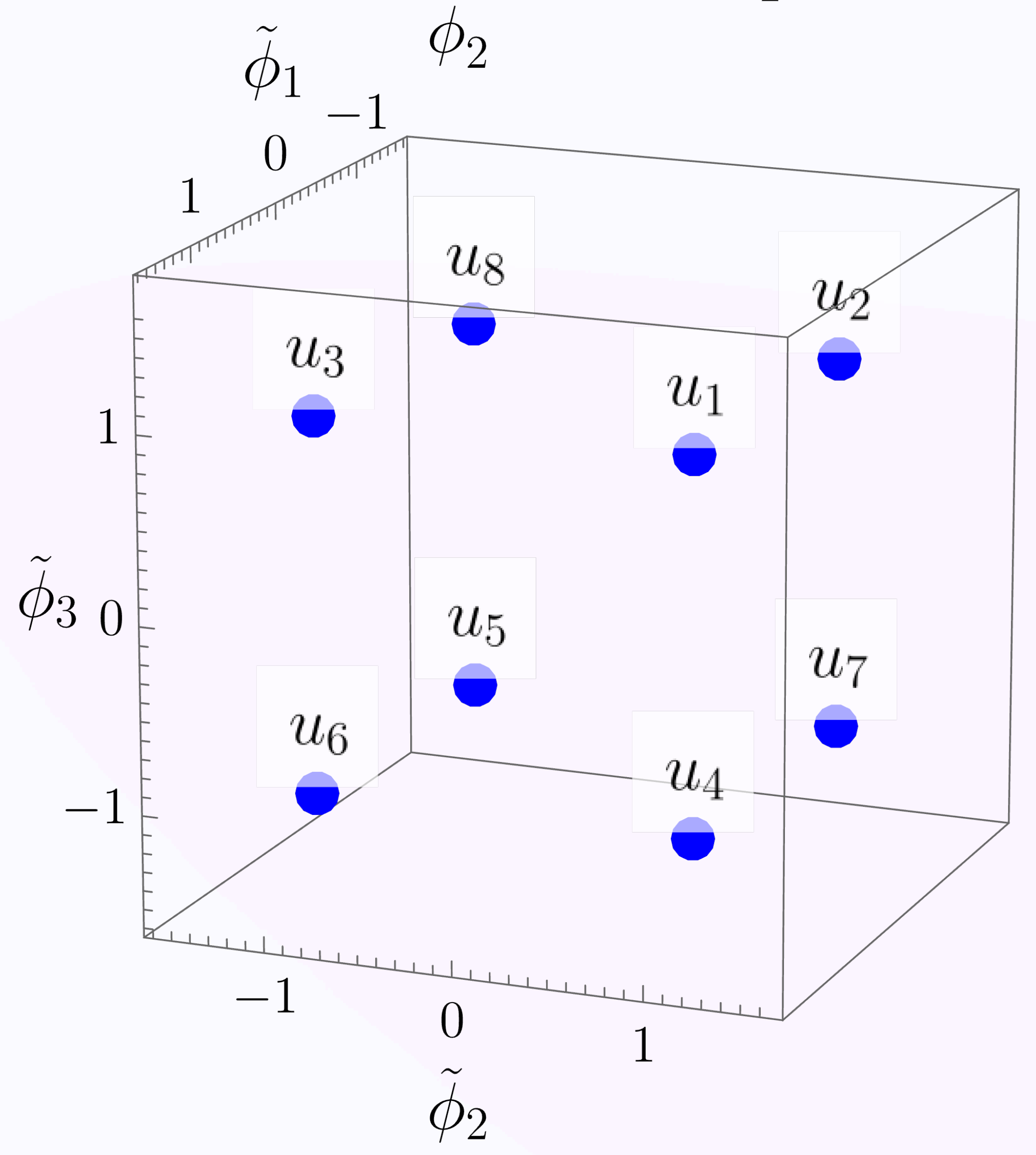


$$-\frac{3}{2}g_1 < g_2 < 0$$

$$\left\{\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\end{pmatrix},\begin{pmatrix}-1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}u$$

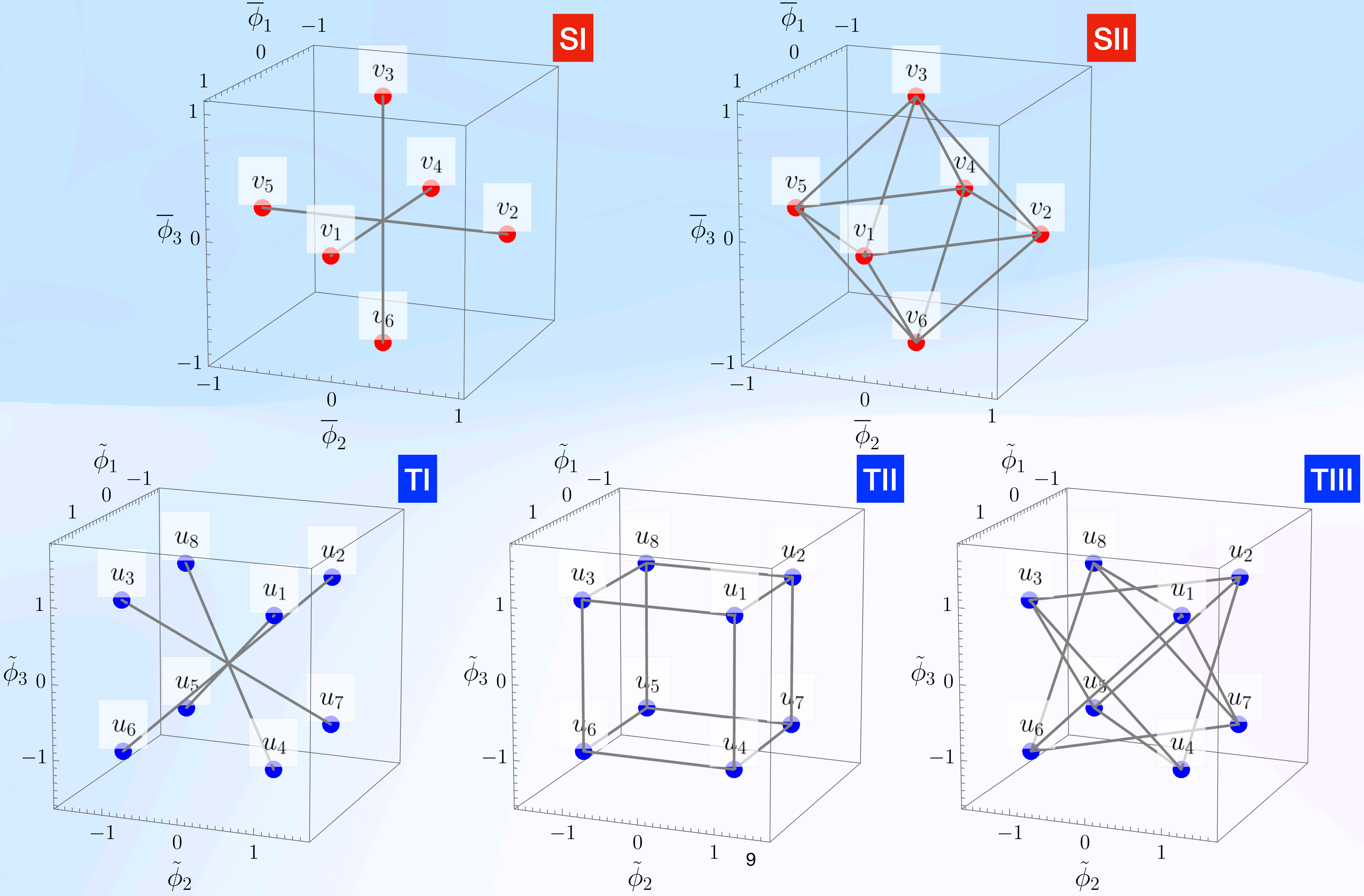
$$u = \frac{\mu}{\sqrt{3g_1 + 2g_2}}$$

$$\tilde{\phi}_i = \frac{\phi_i}{u}$$



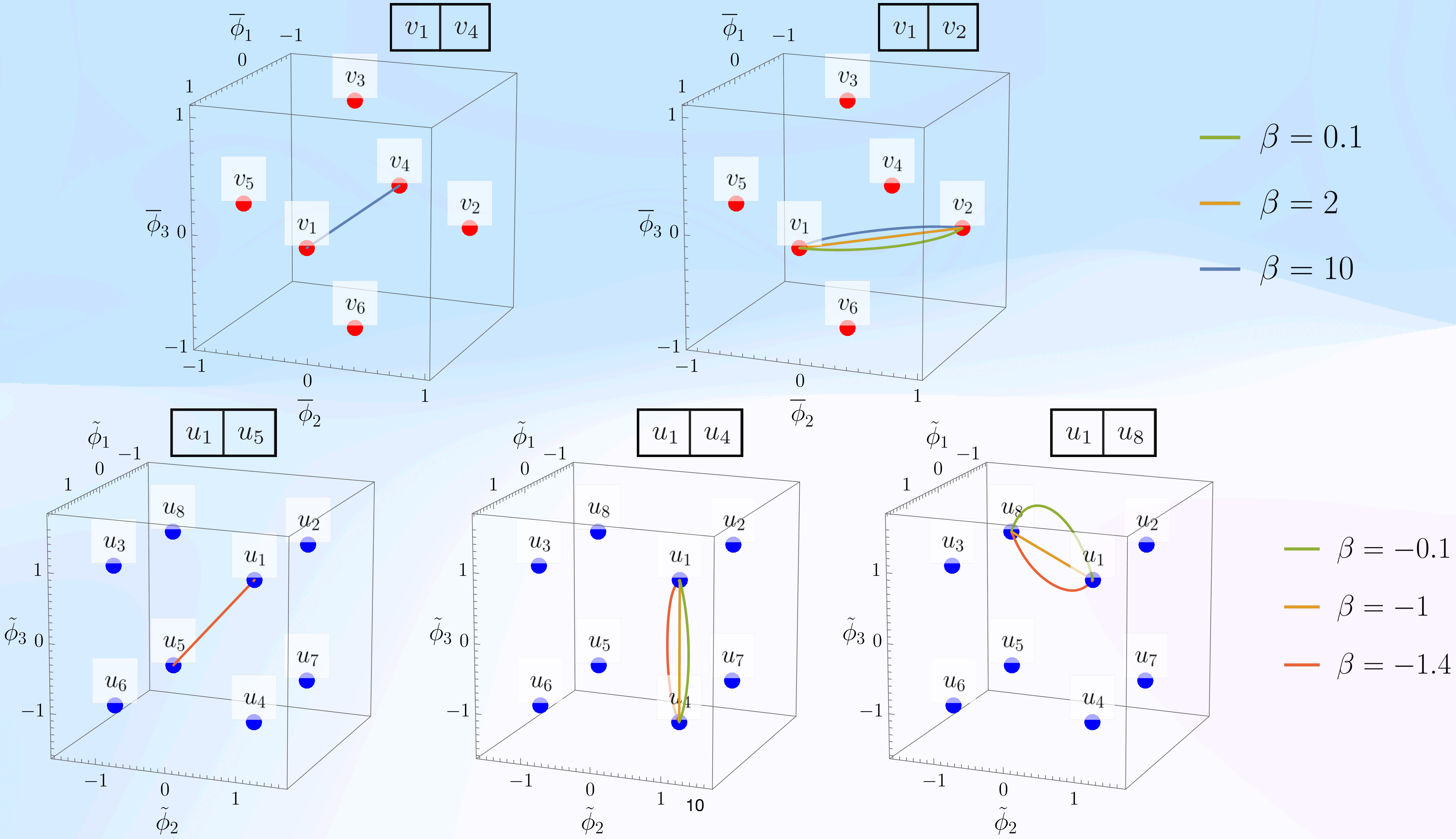
S_4 domain walls

BF, S.F. King, L. Marsili, S. Pascoli,
J. Turner, Y-L. Zhou, 2409.16359



S_4 domain walls

BF, S.F. King, L. Marsili, S. Pascoli, J. Turner, Y-L. Zhou, 2409.16359

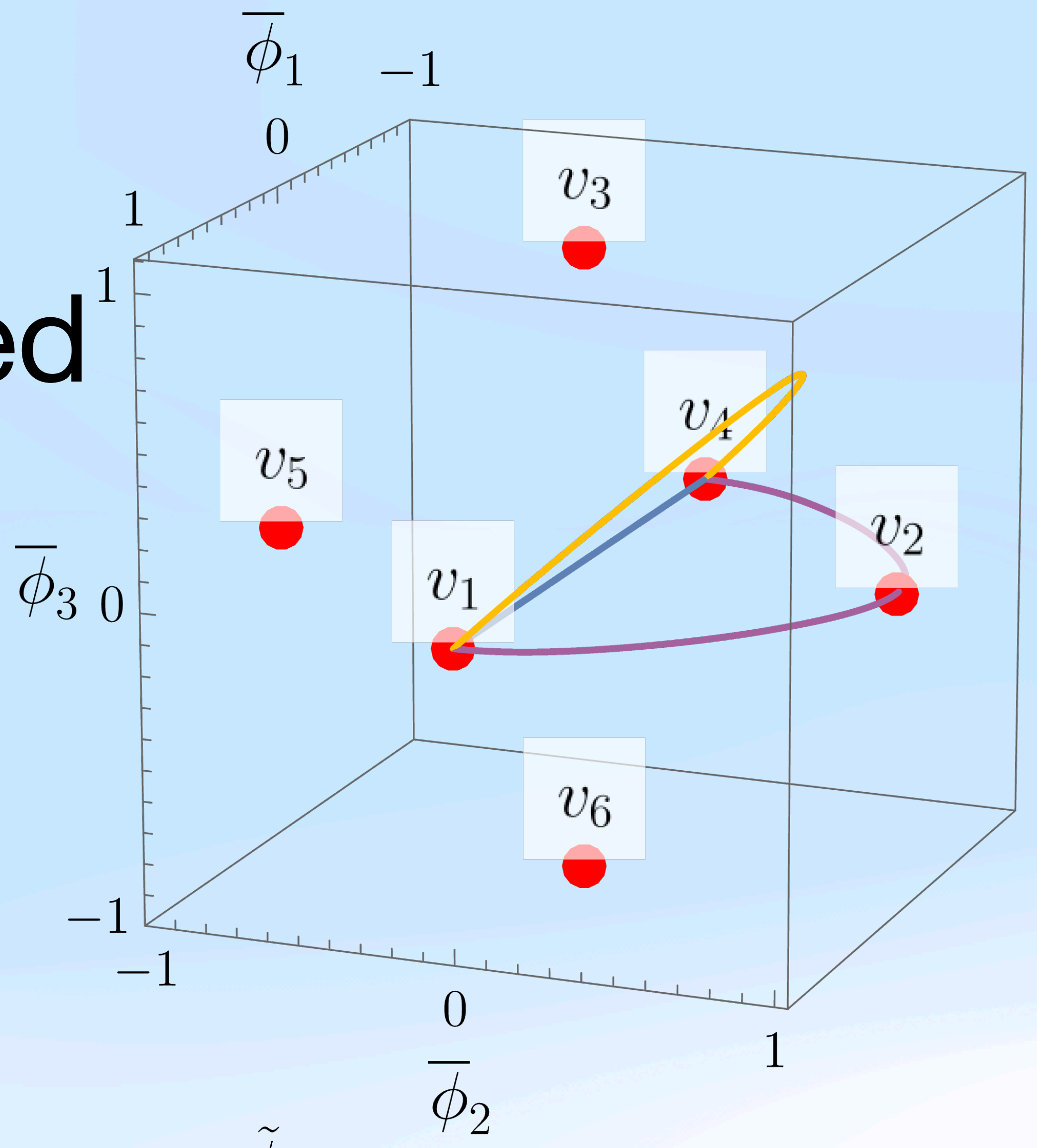


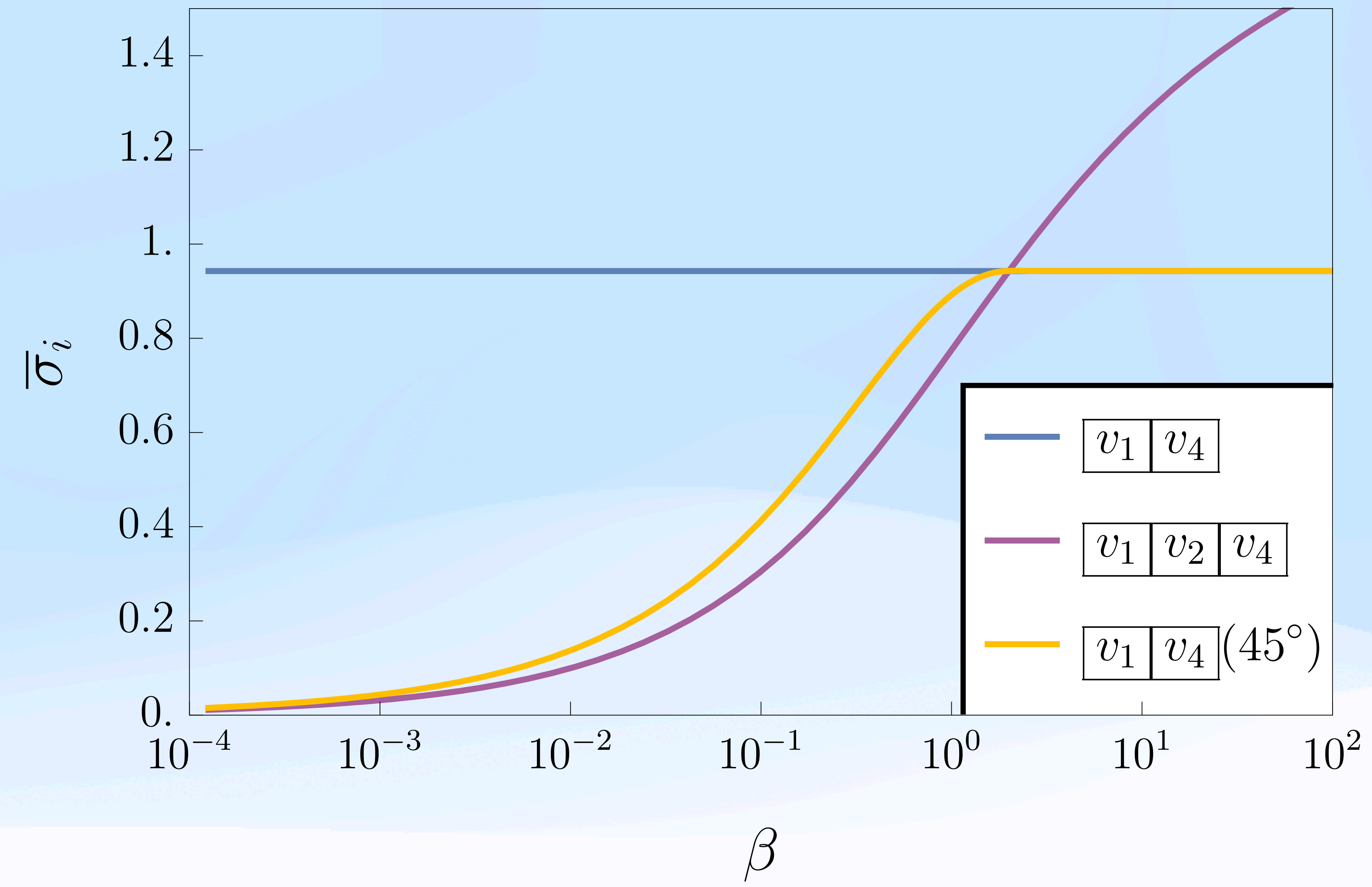
Stability of DWs

BF, S.F. King, L. Marsili, S. Pascoli,
J. Turner, Y-L. Zhou, 2409.16359

$\boxed{v_1 \mid v_4}$: straight or curved

$\boxed{v_1 \mid v_2 \mid v_4}$: 2 SII DWs

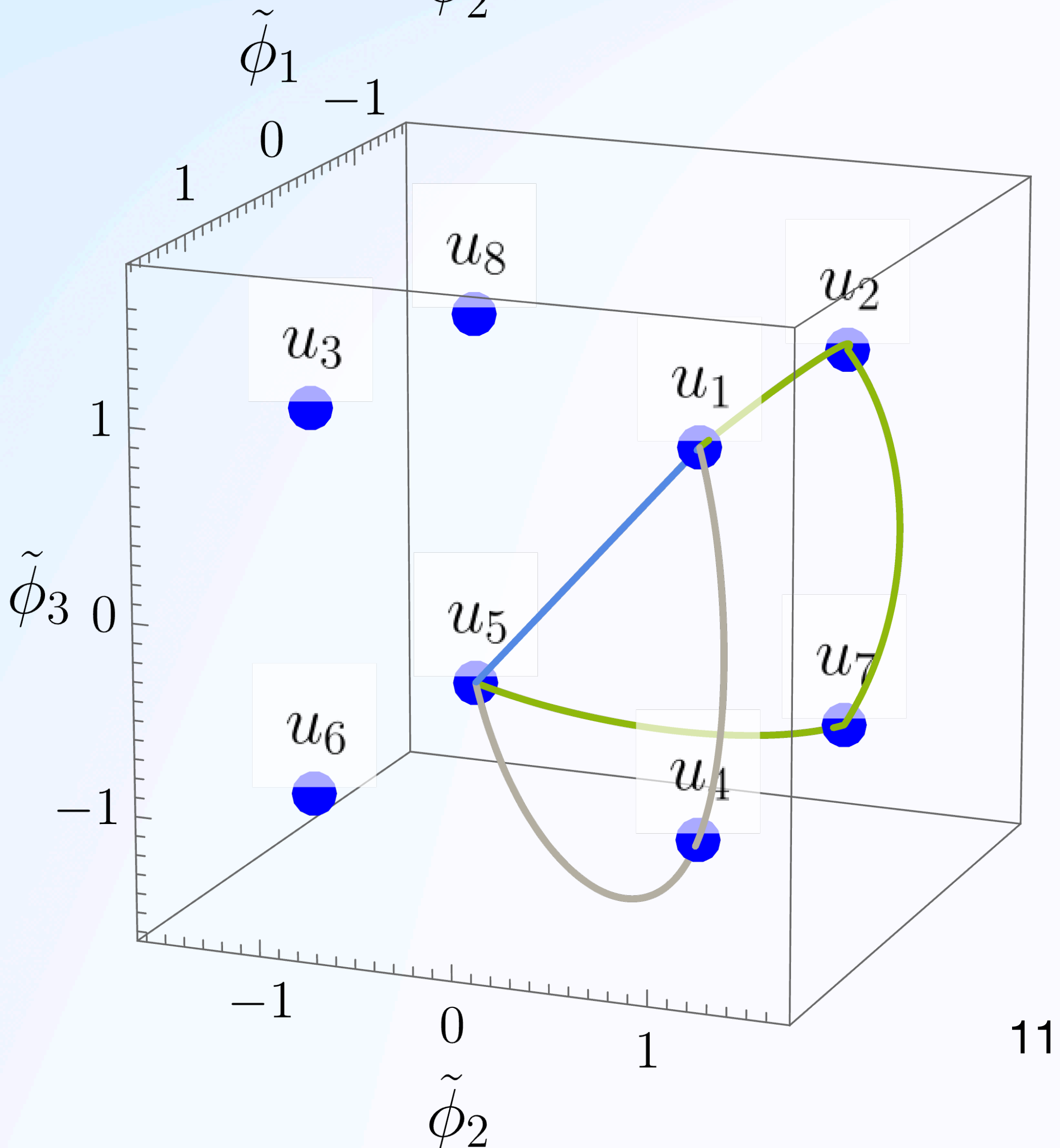


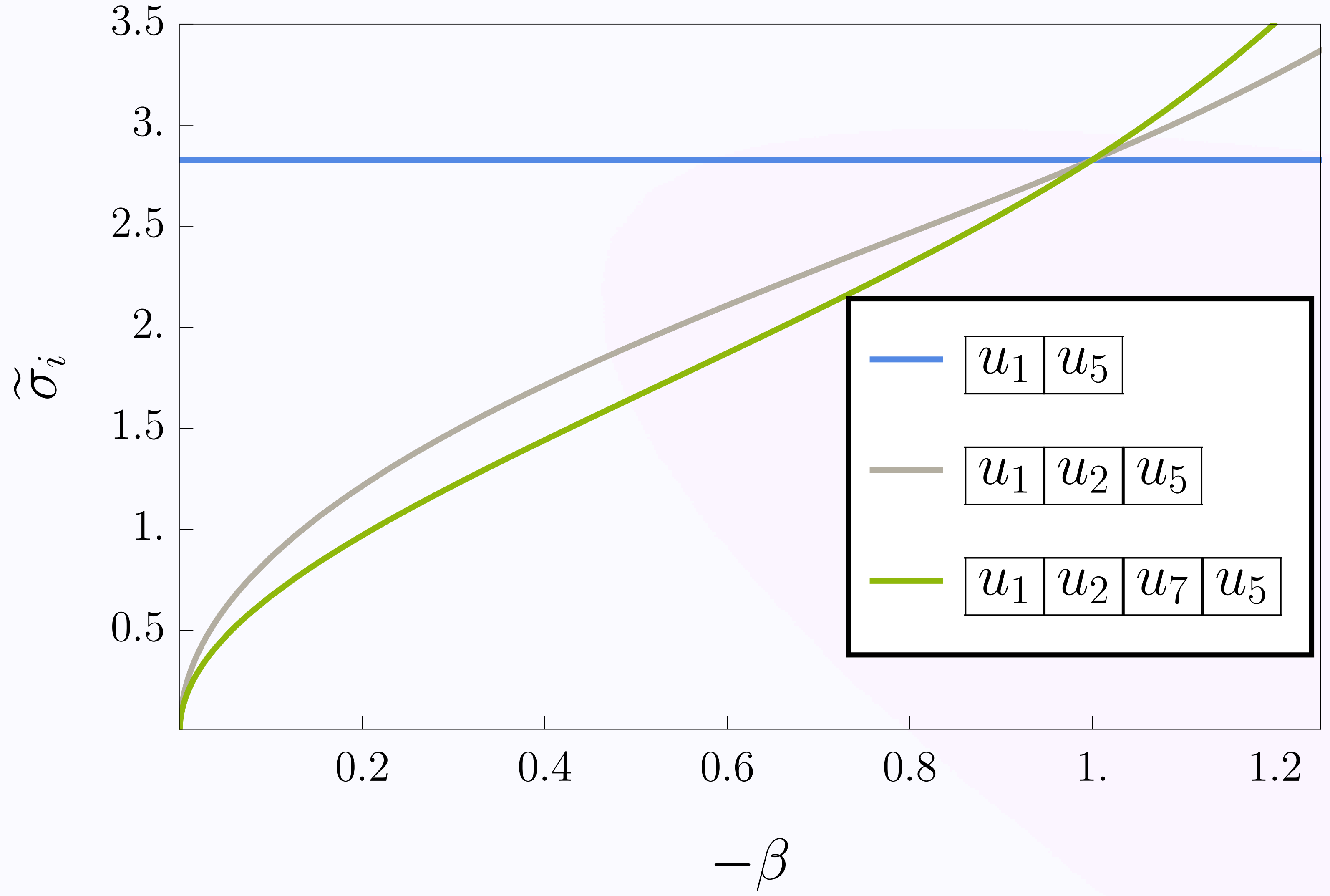


$\boxed{u_1 \mid u_5}$: straight

$\boxed{u_1 \mid u_4 \mid u_5}$: TII + TIII

$\boxed{u_1 \mid u_2 \mid u_7 \mid u_5}$: 3 TII DWs



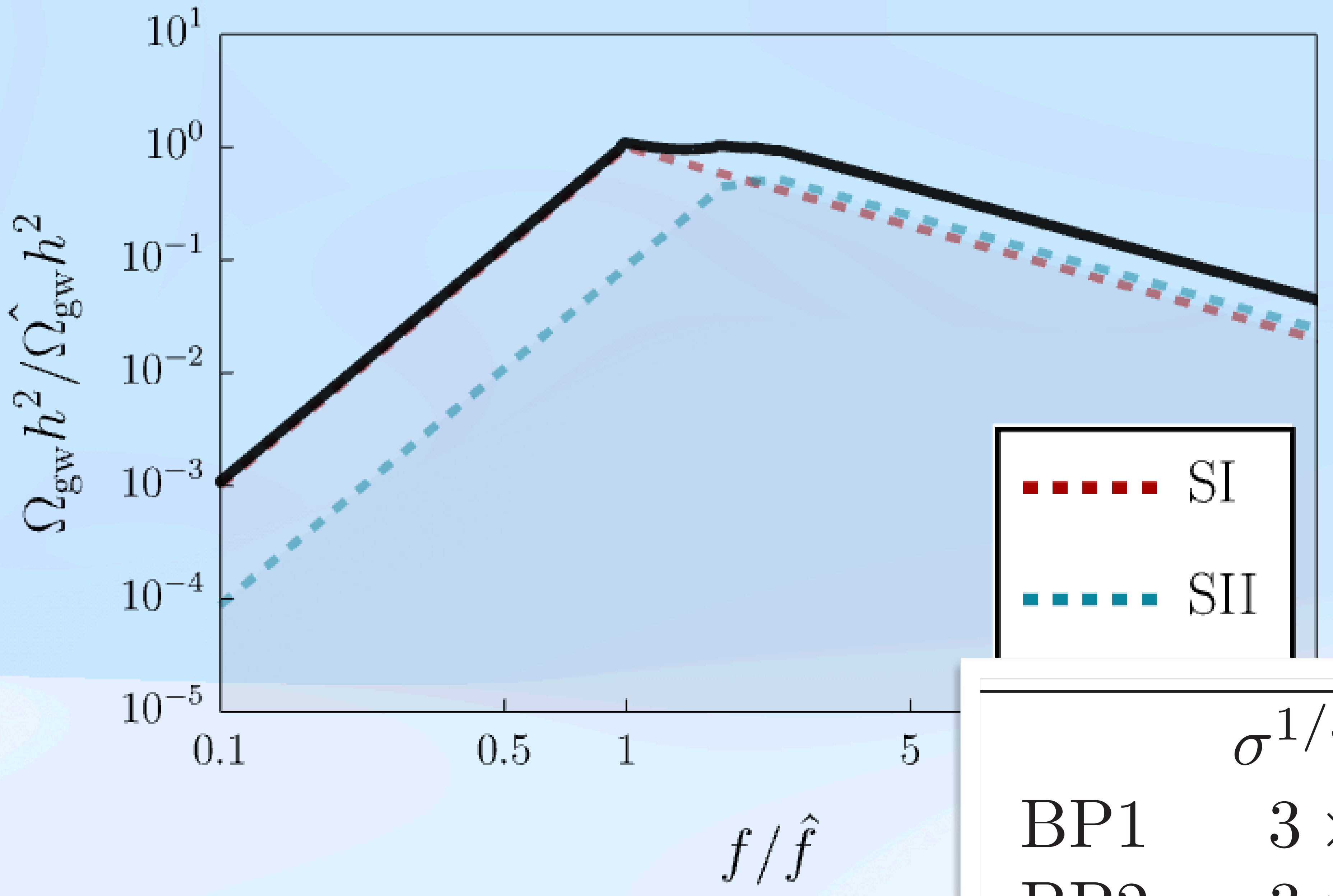


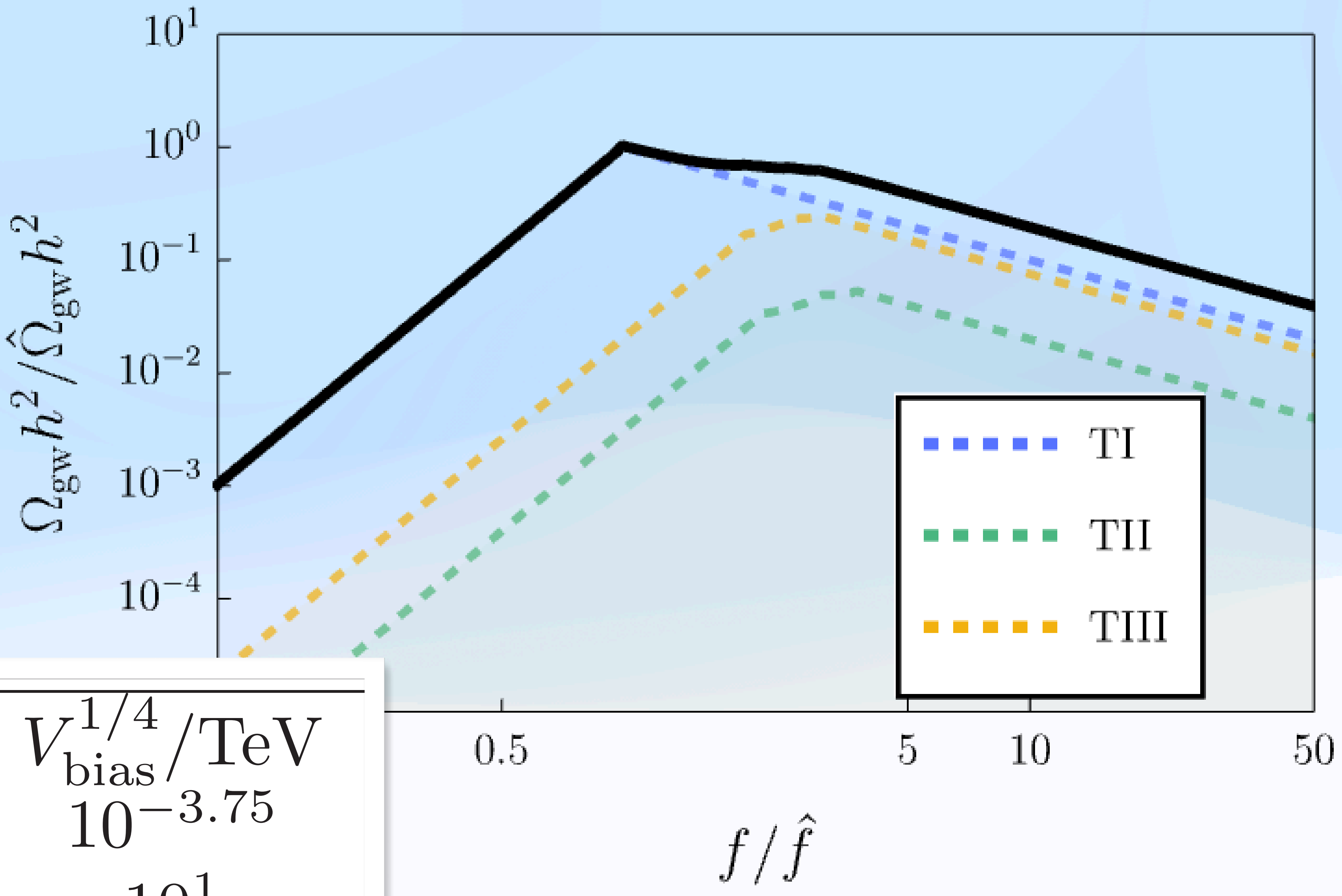
Gravitational wave

ϵ^v_{12}	ϵ^v_{13}	ϵ^v_{14}	ϵ^v_{15}	ϵ^v_{16}
$2\hat{\epsilon}$	$3\hat{\epsilon}$	$\hat{\epsilon}$	$4\hat{\epsilon}$	$5\hat{\epsilon}$

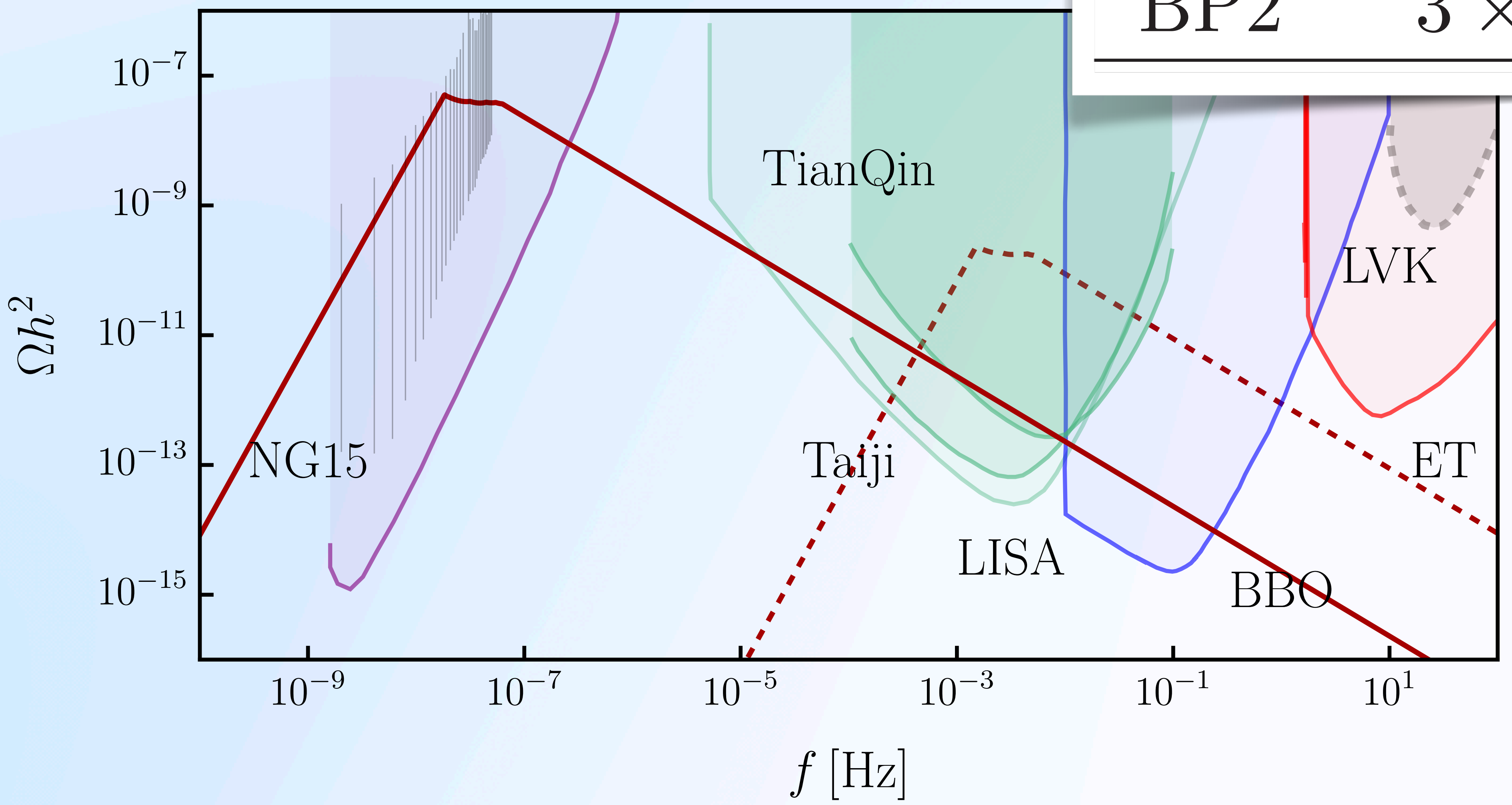
ϵ^u_{12}	ϵ^u_{13}	ϵ^u_{14}	ϵ^u_{15}	ϵ^u_{16}	ϵ^u_{17}	ϵ^u_{18}
$2\hat{\epsilon}$	$4\hat{\epsilon}$	$6\hat{\epsilon}$	$\hat{\epsilon}$	$3\hat{\epsilon}$	$5\hat{\epsilon}$	$7\hat{\epsilon}$

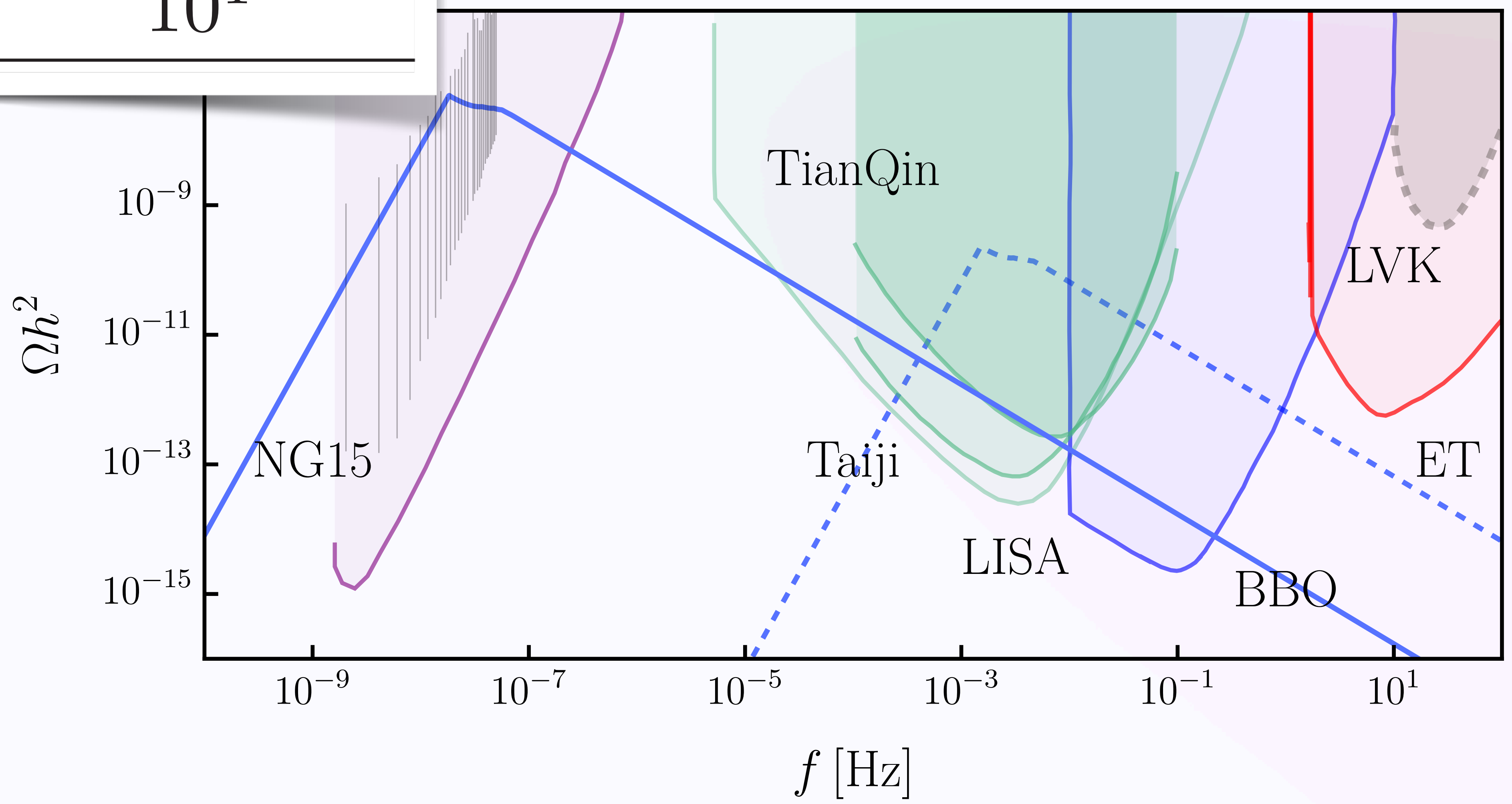
Naive estimation





	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1





A_4 scalar theory

S_4

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 scalar theory

S_4

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\Rightarrow$

A_4

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

A_4 scalar theory

S_4

$$V(\phi) = -\frac{\mu^2}{2}I_1 + \frac{g_1}{4}I_1^2 + \frac{g_2}{2}I_2$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

$$I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

$\Rightarrow$

A_4

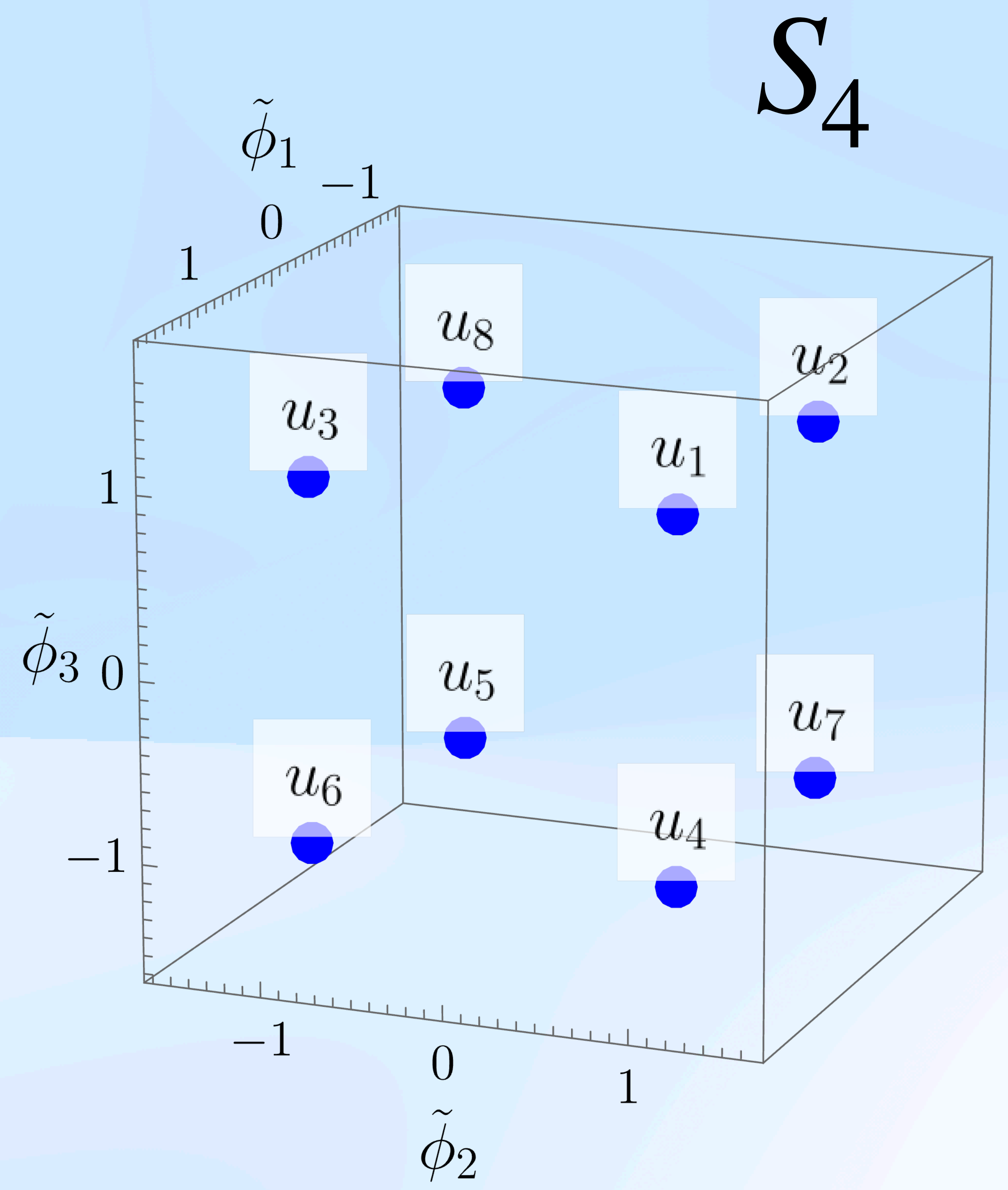
$$V(\phi) = -\frac{\mu^2}{2}I_1 + \frac{g_1}{4}I_1^2 + \frac{g_2}{2}I_2 + AI_3$$

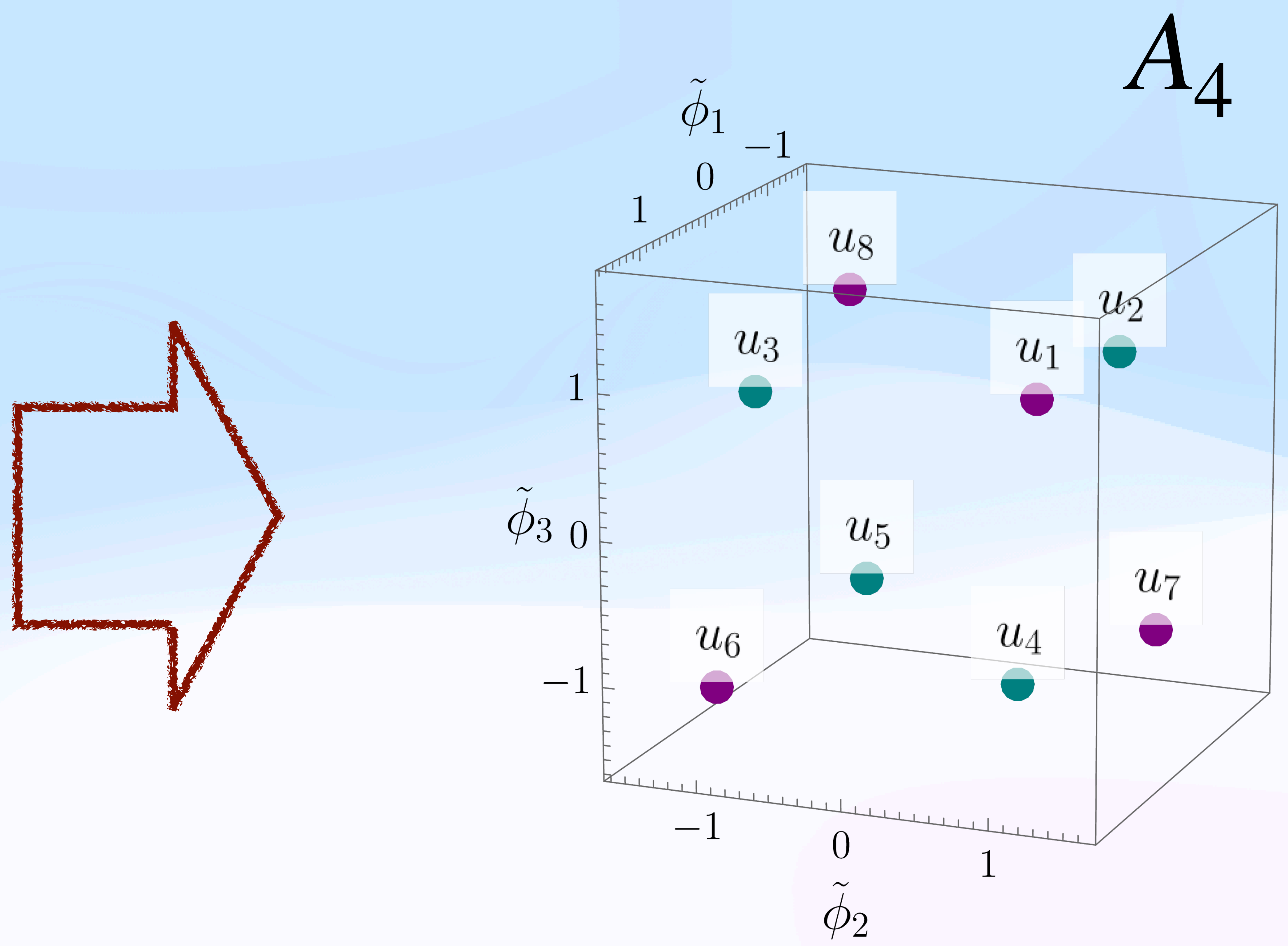
$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

$$I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

$$I_3 = \phi_1\phi_2\phi_3$$

A_4 domain wall

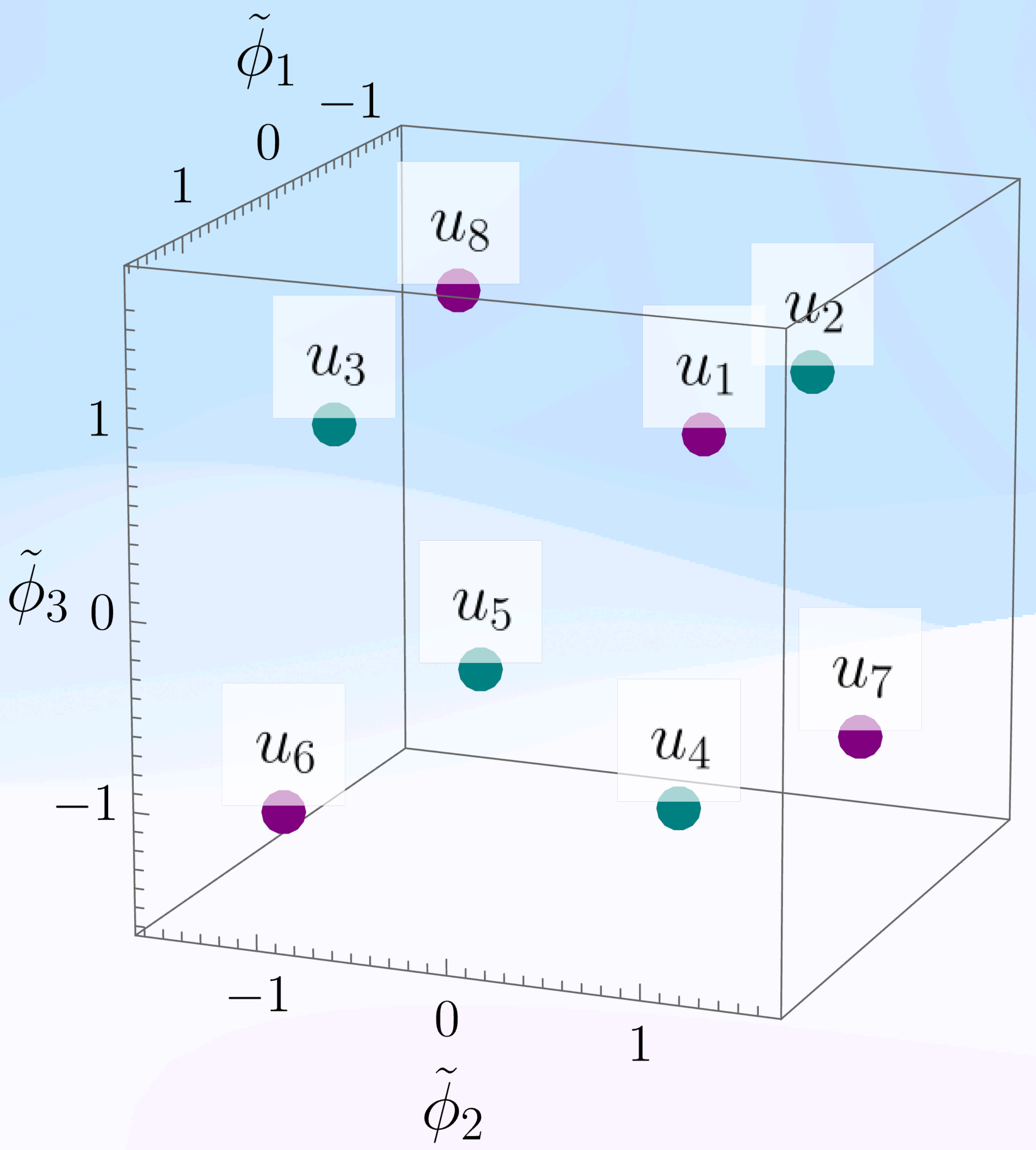




A_4 vacuum structure

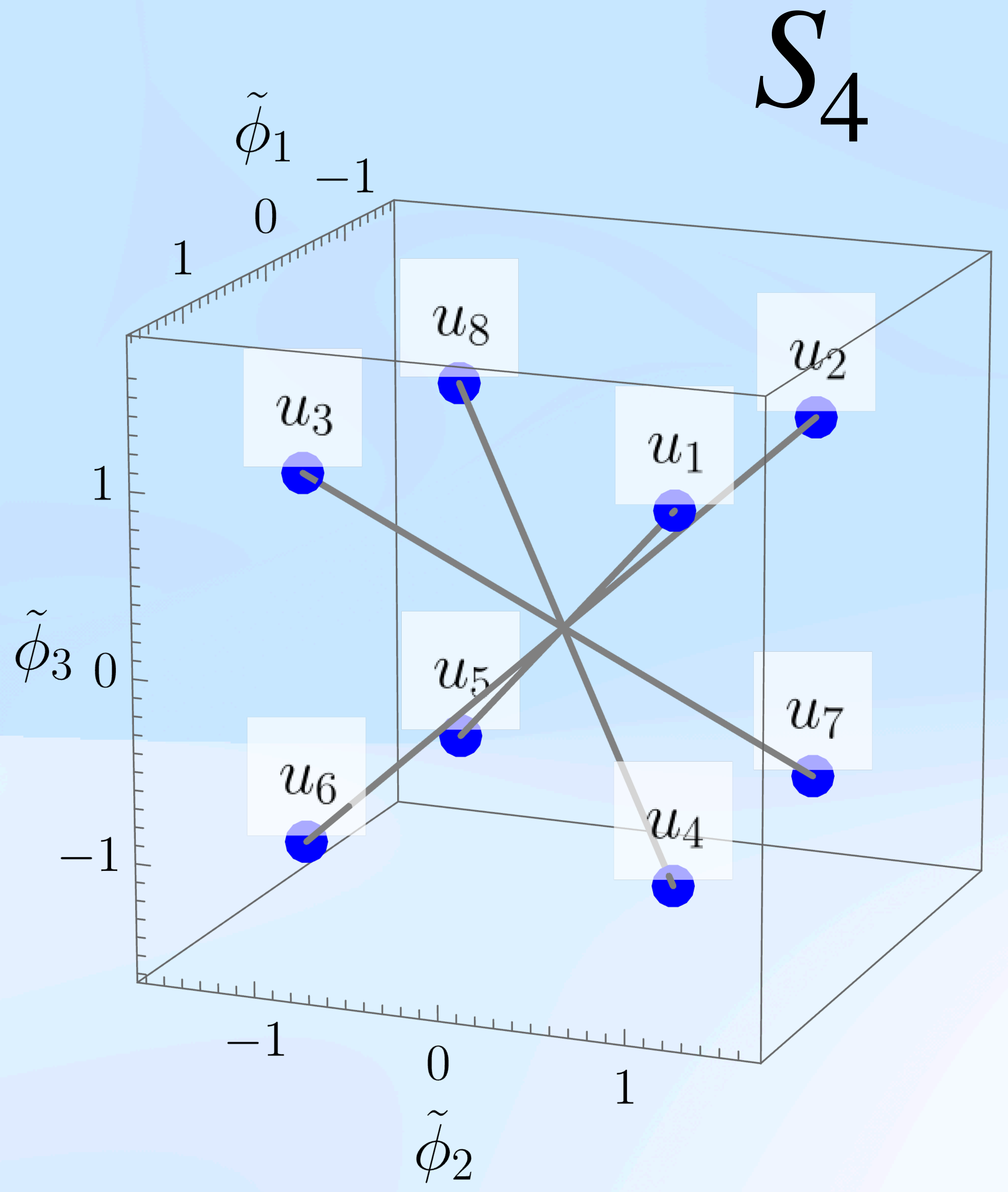
$$T_- : \quad u_{1,6,7,8} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\} u_-$$

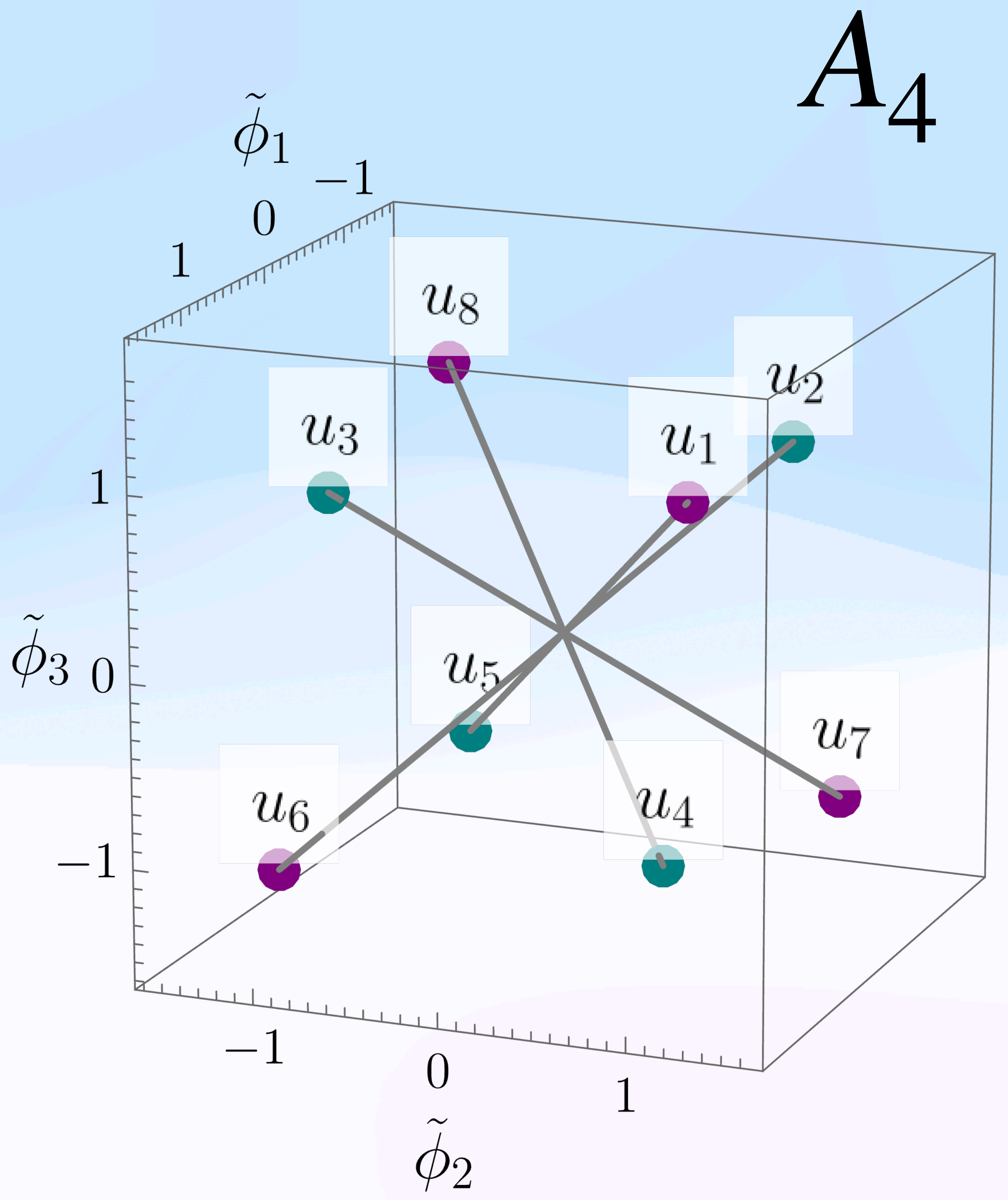
$$T_+ : \quad u_{2,3,4,5} = \left\{ \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \right\} u_+$$



$$u_\mp = \frac{\mu}{\sqrt{3g_1 + 2g_2}} \left(\sqrt{a^2 + 1} \mp a \right), \quad \text{with} \quad a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}}$$

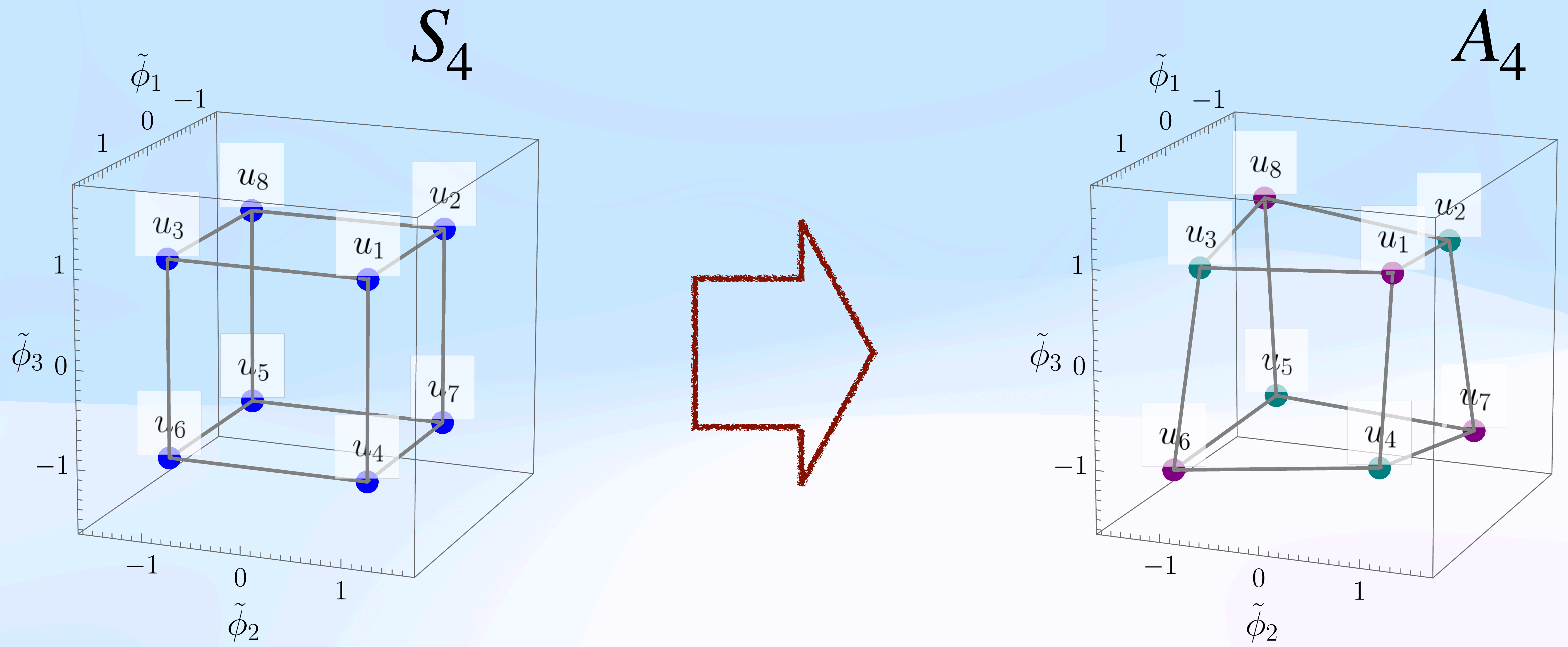
A_4 domain wall





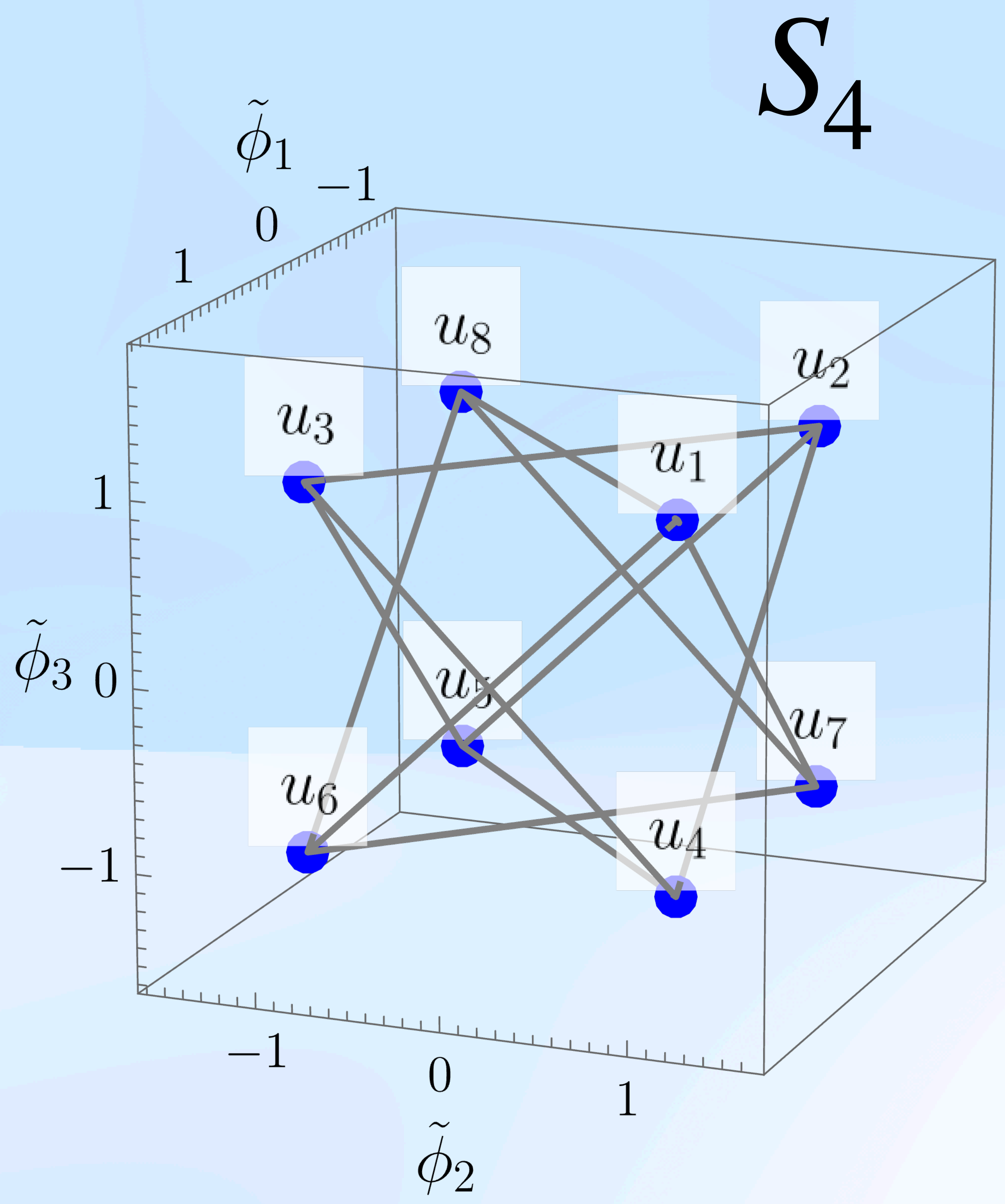
TI

A_4 domain wall

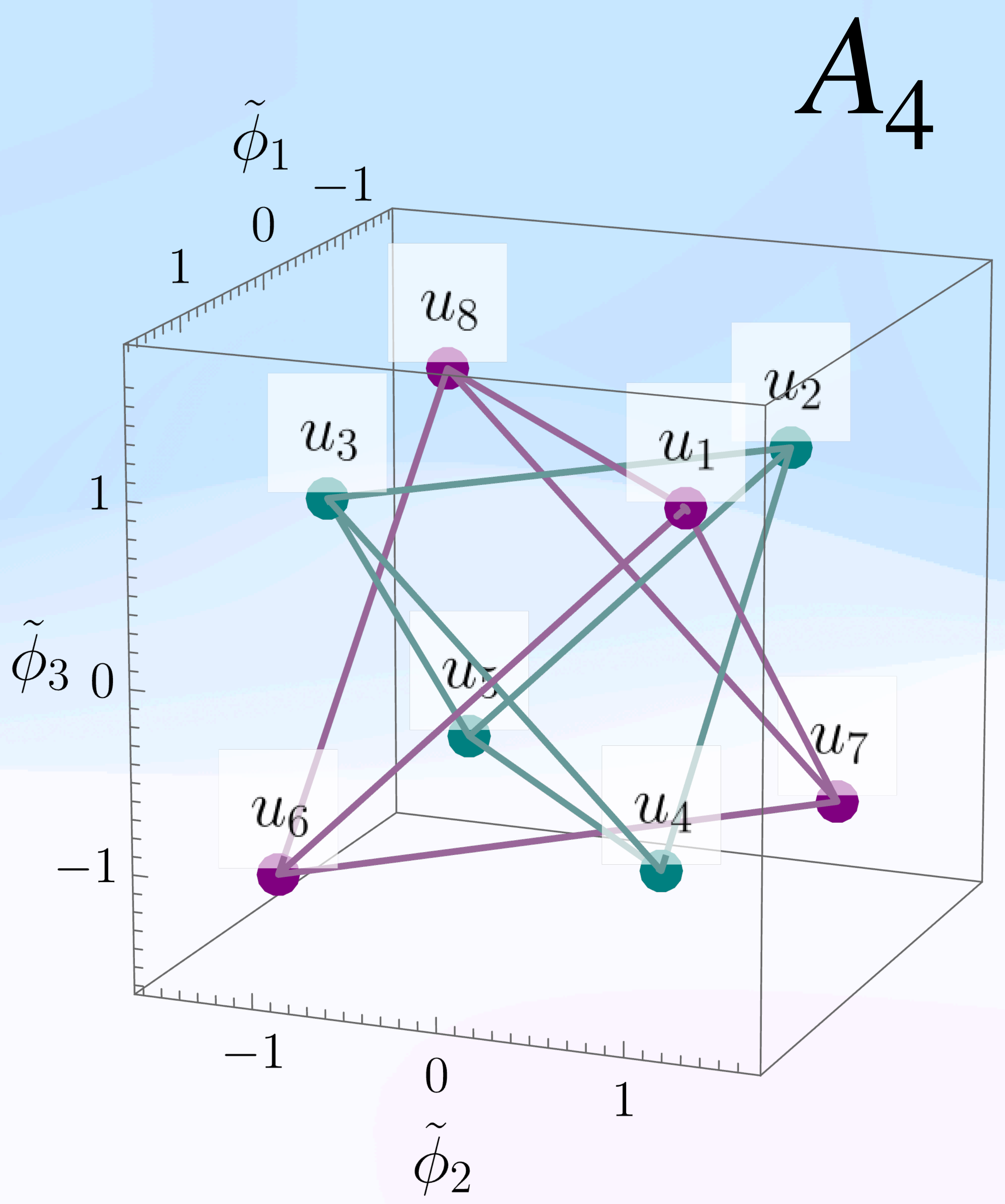


TII

A_4 domain wall



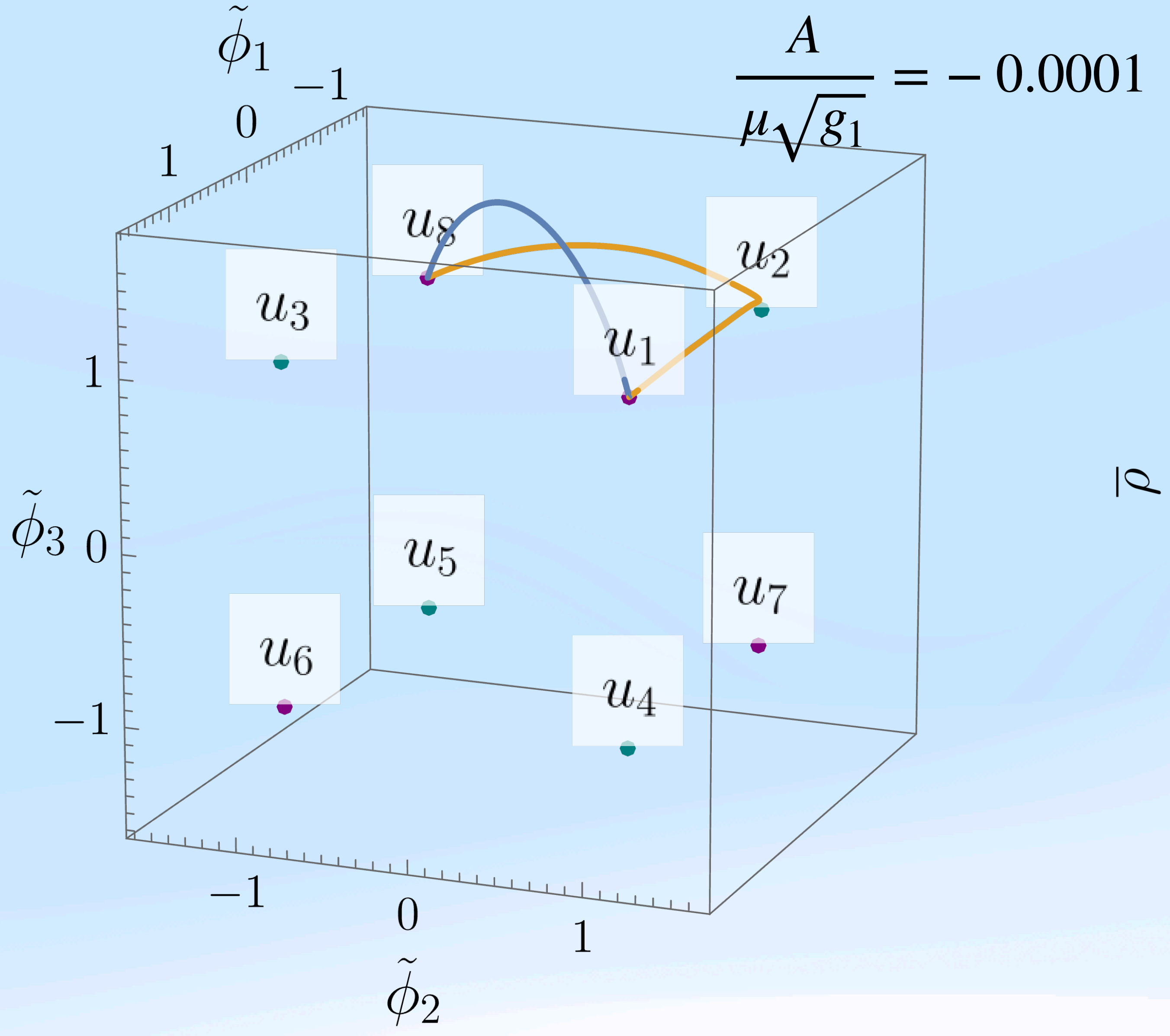
TIII

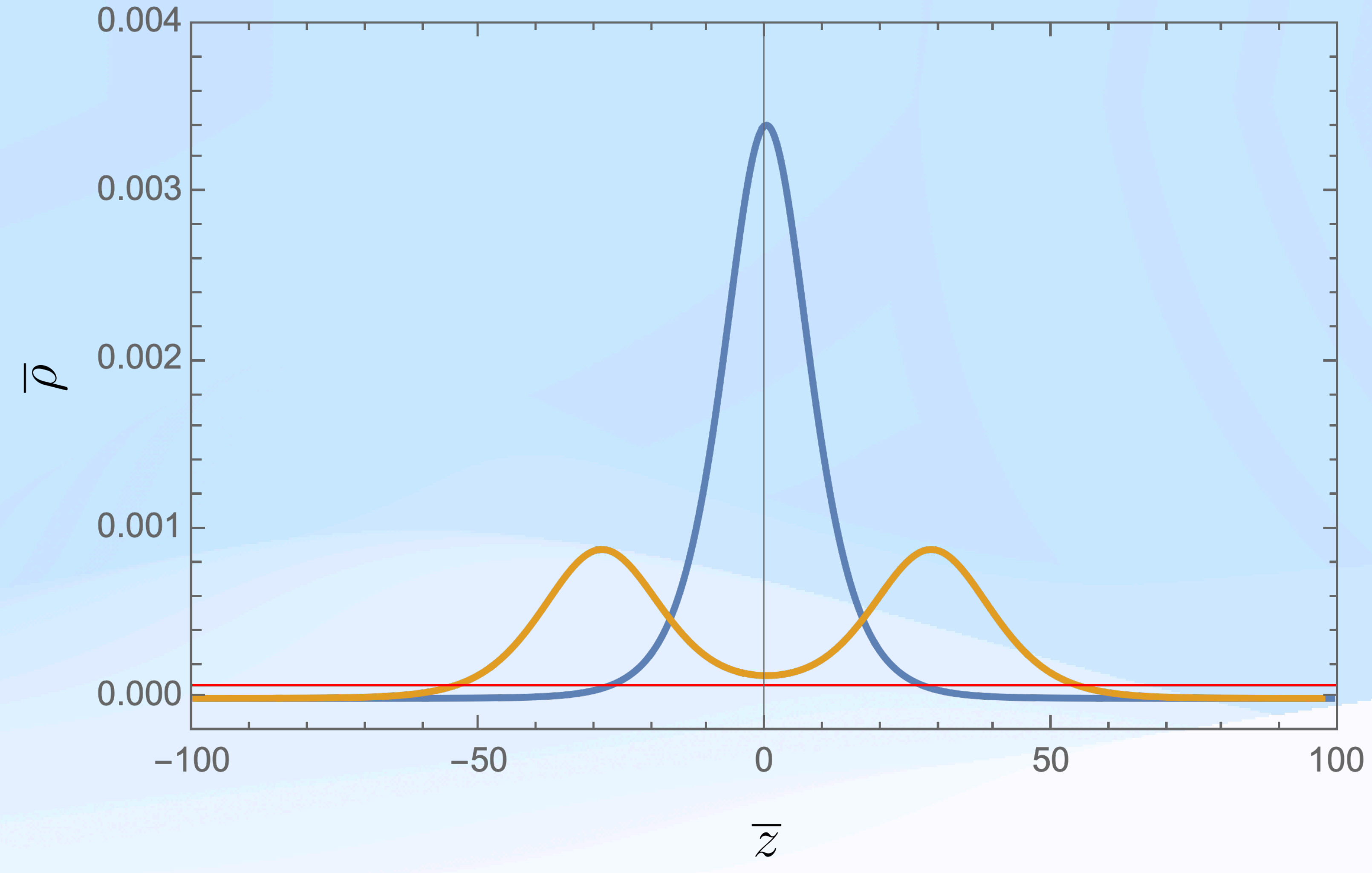


TIII+, TIII-

Mestastable A_4 domain wall

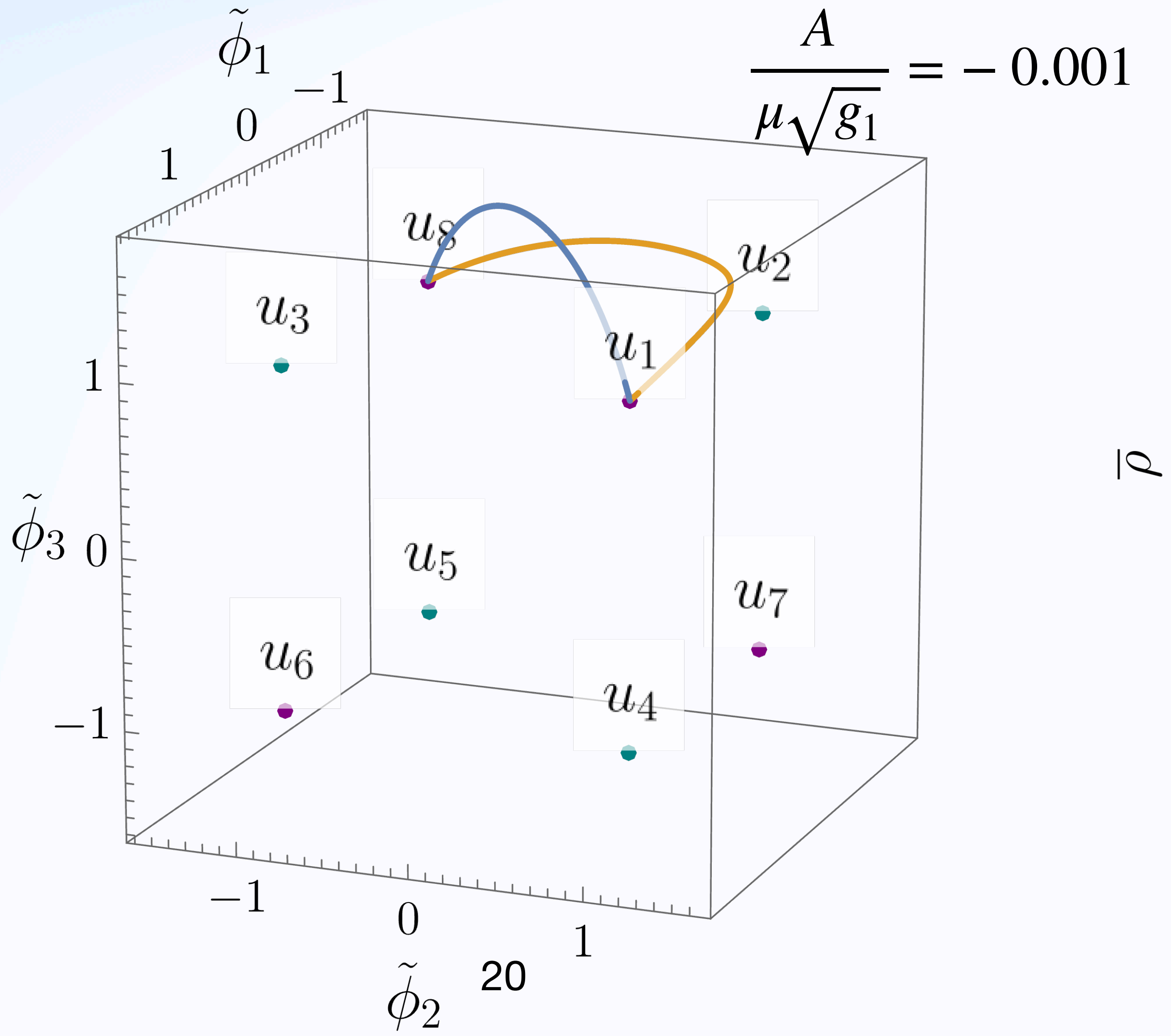
$$V_{\mp} = -\frac{3\mu^4}{4(3g_1 + 2g_2)}\left(1 + \frac{2}{3}a^2 \mp \frac{2}{3}a\sqrt{a^2+1}\right)\left(\sqrt{a^2+1} \mp a\right)^2$$

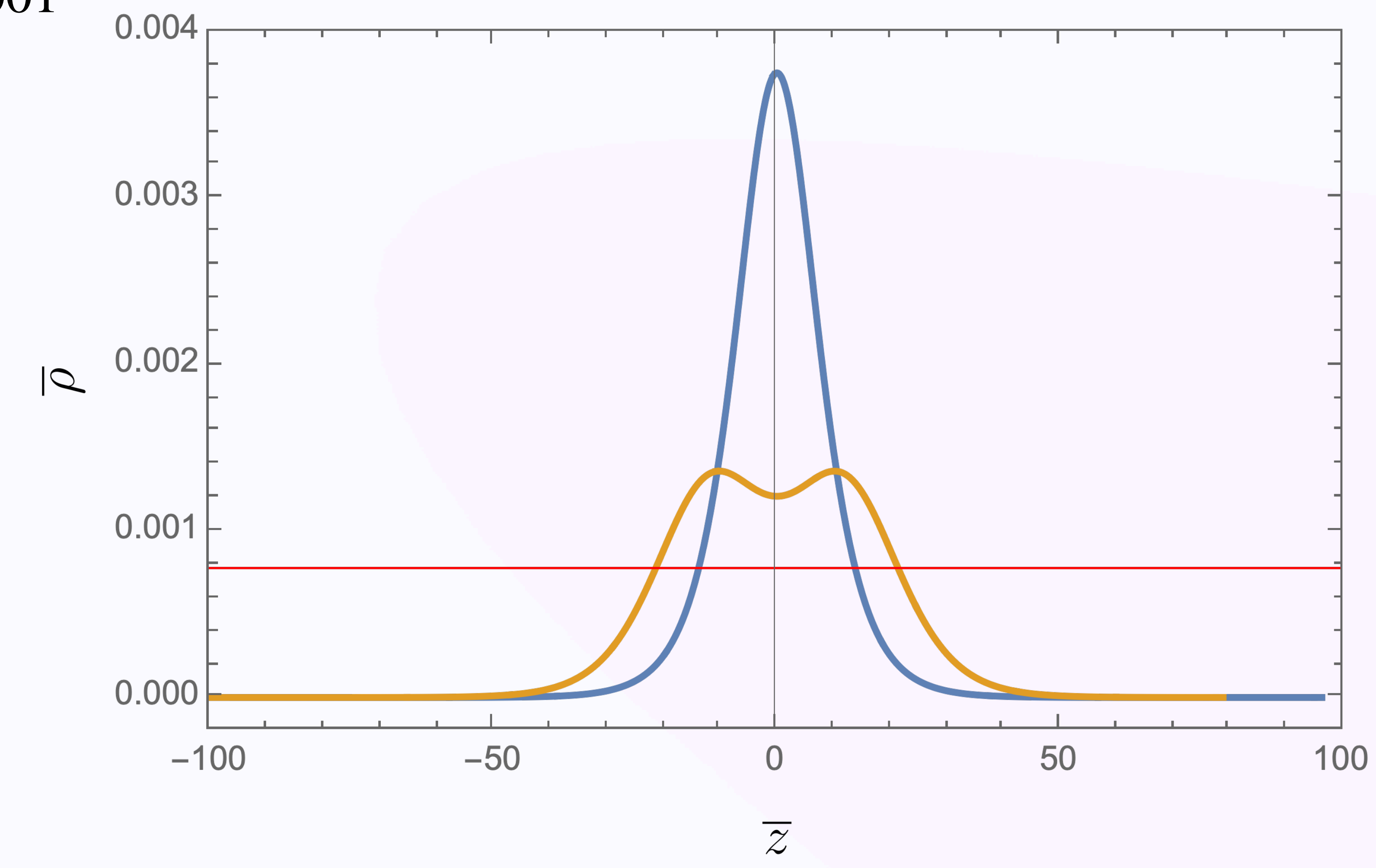




$$a < 0 \implies V_+ > V_-$$

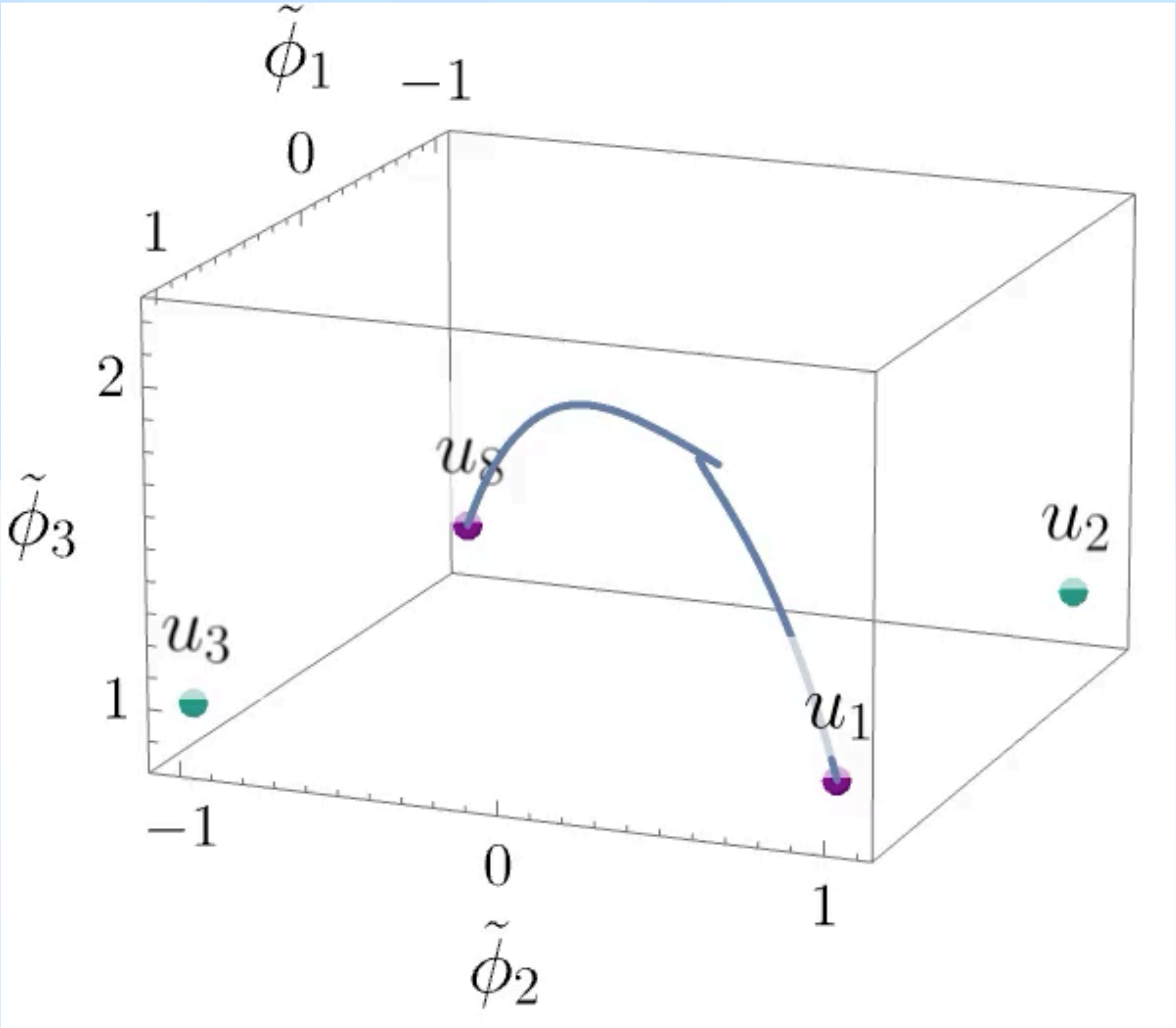
$$V(u_2) > V(u_1), V(u_8)$$

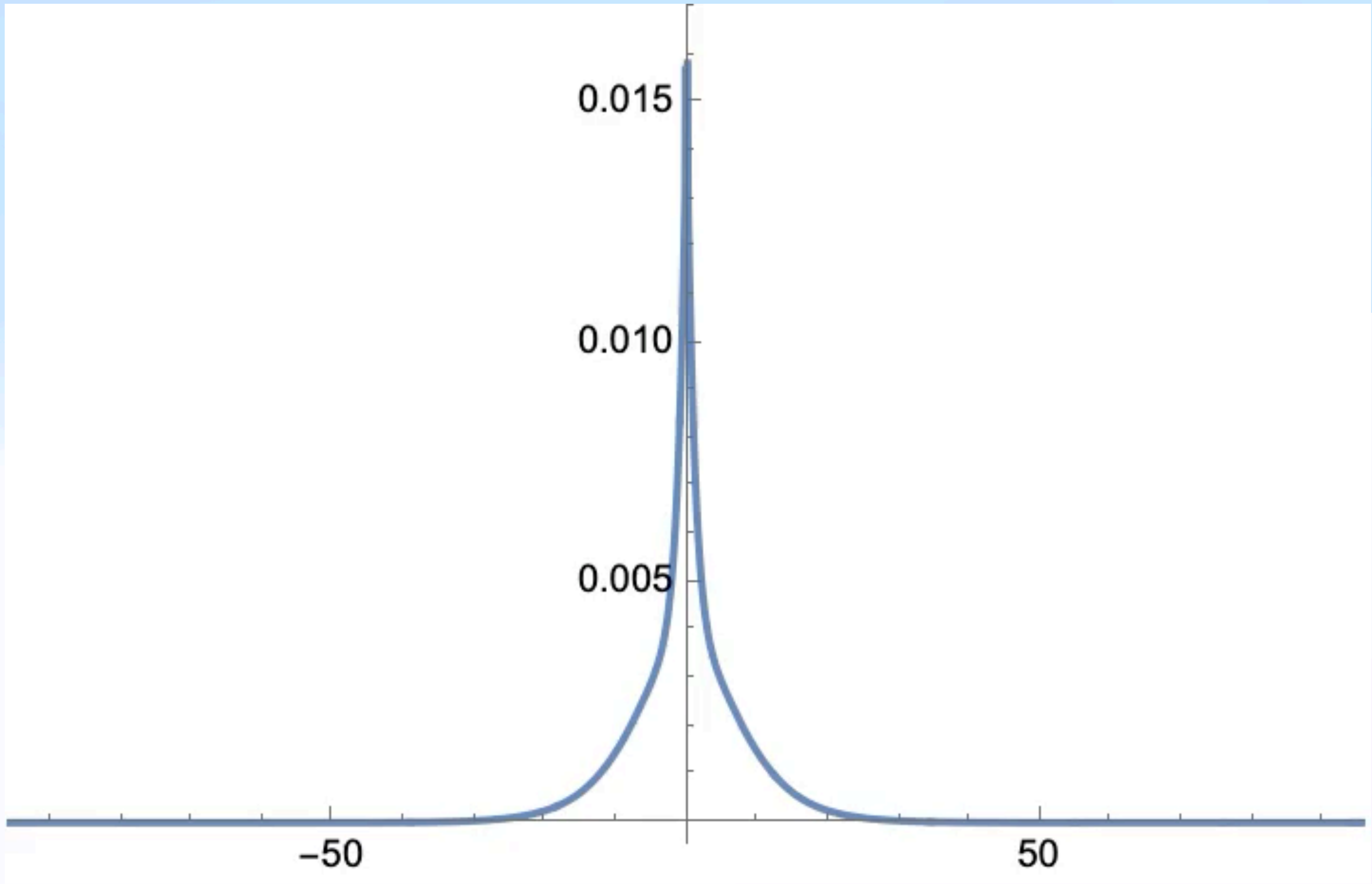




Unstable or Metastable?

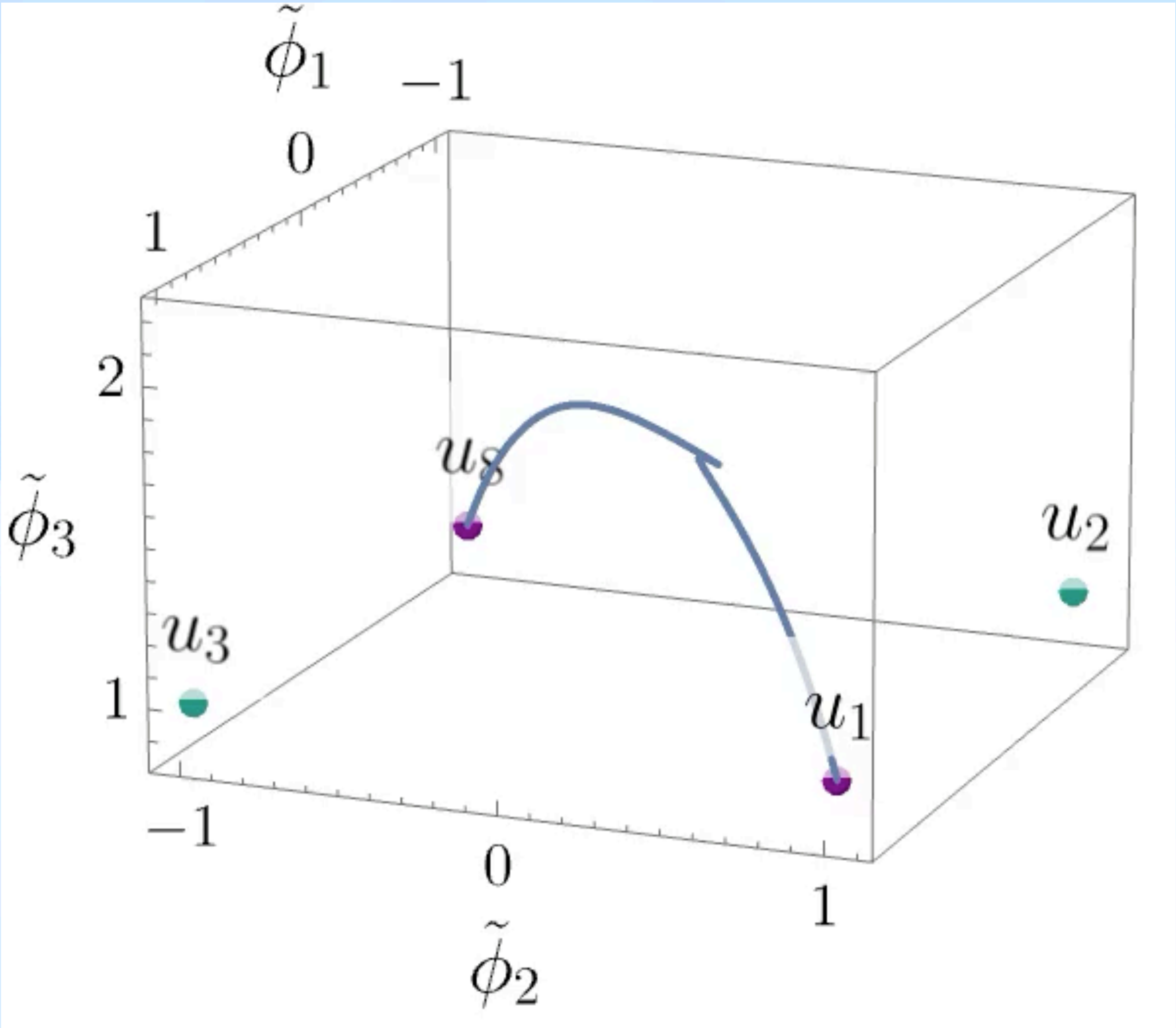
$$\frac{\partial^2 \phi_i}{\partial t^2} - \frac{\partial^2 \phi_i}{\partial z^2} + \frac{\partial V}{\partial \phi_i} = 0$$

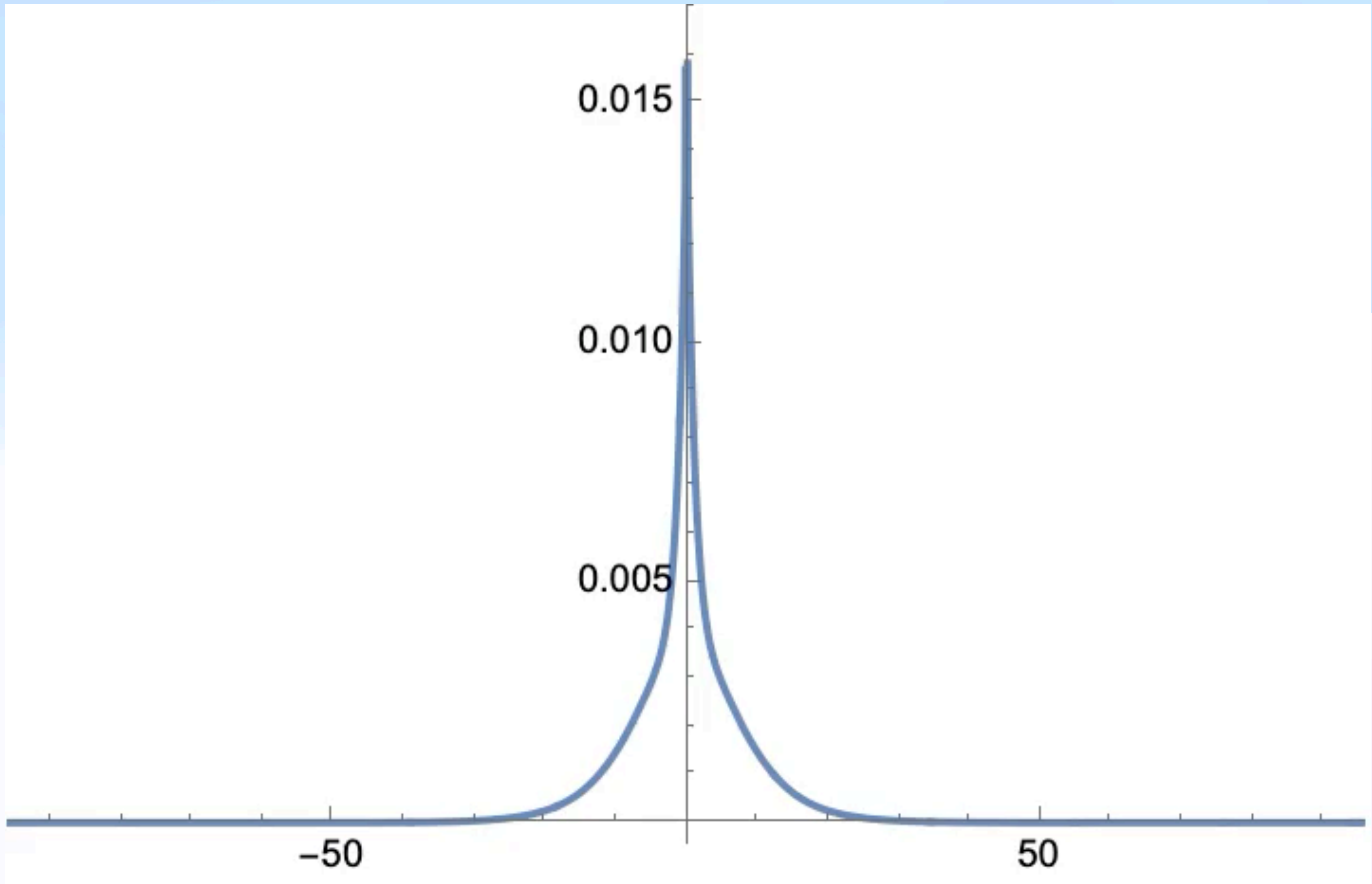




Unstable or Metastable?

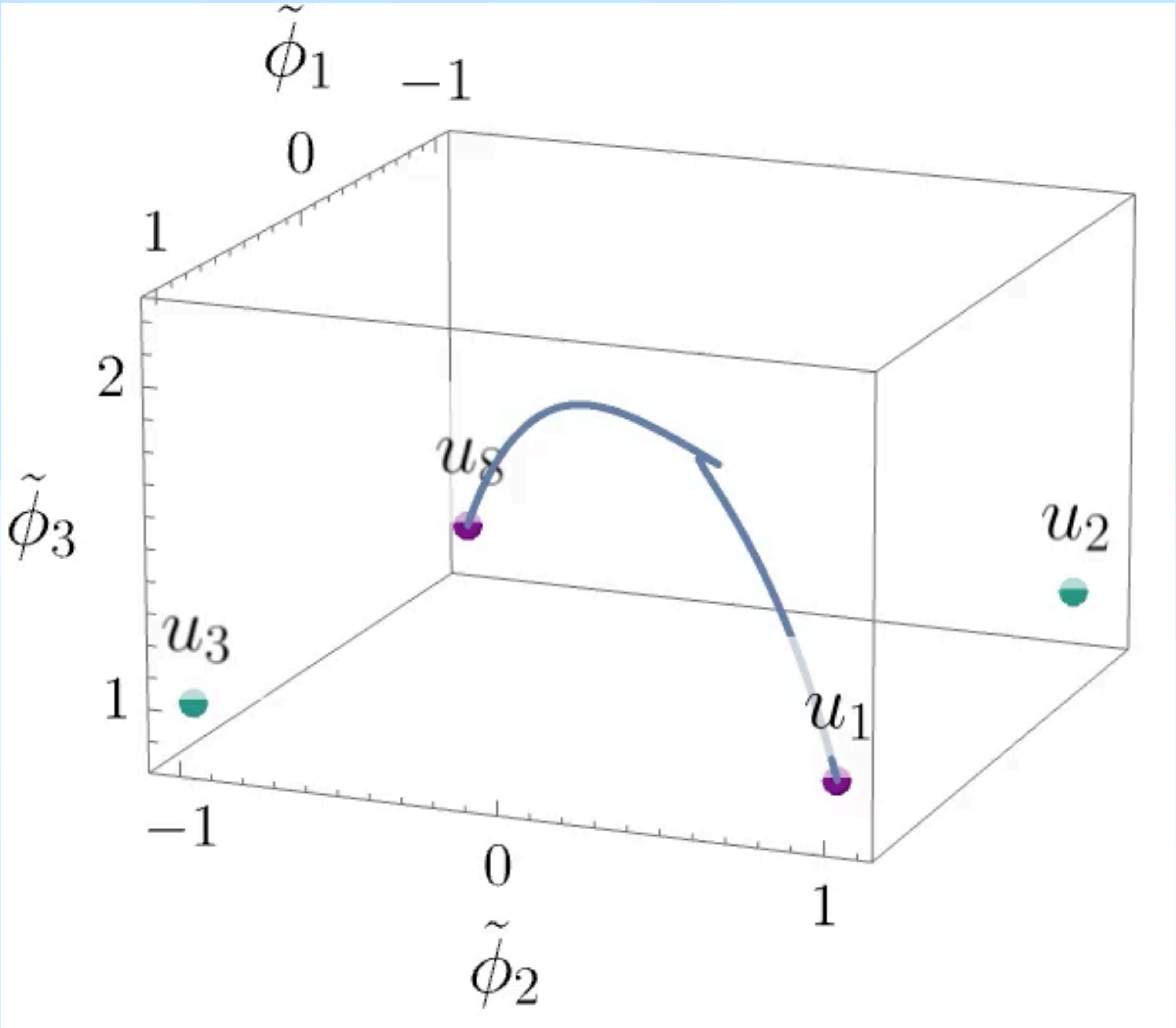
$$\frac{\partial^2 \phi_i}{\partial t^2} - \frac{\partial^2 \phi_i}{\partial z^2} + \frac{\partial V}{\partial \phi_i} = 0$$

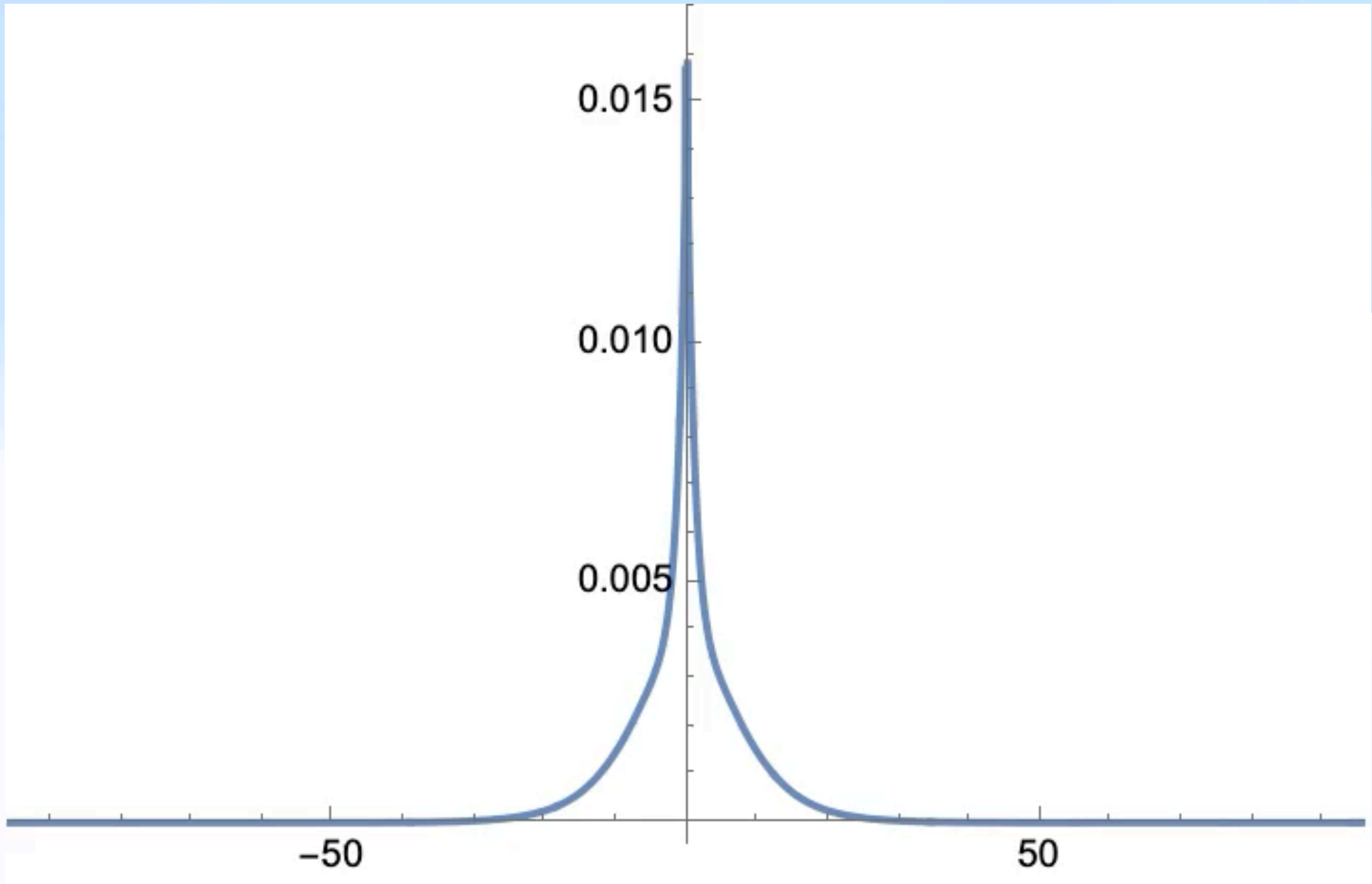




Unstable or Metastable?

$$\frac{\partial^2 \phi_i}{\partial t^2} - \frac{\partial^2 \phi_i}{\partial z^2} + \frac{\partial V}{\partial \phi_i} = 0$$





Summary and Outlook

- Non-abelian DWs have more interesting and non-trivial structure and phenomena
- $S_4, A_4, \ldots$
- Real scalar to complex scalar
- Stability of the DWs need to be explored in detail
- Collision of two DWs into another DW