Gravitational Waves from Dark Sector Phase Transitions

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with Haipeng An , arXiv: 250x.xxxx

Introduction of dark sector:

Dark matter has a lot of candidates:

WIMPs, axions, …

Dark sector:

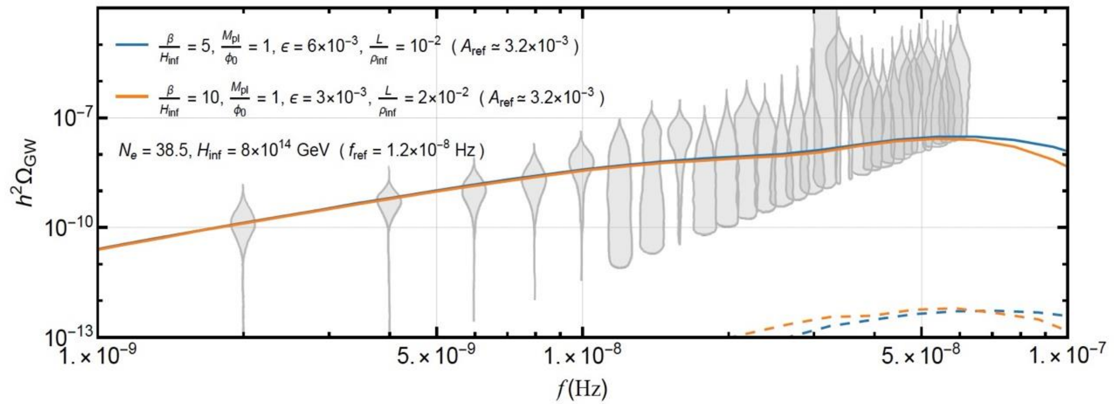
- dark big bang
- dark landscape
- may only couple to SM particles through gravity

DS may stay in a meta-stable vacuum.

⇒ phase transition / GW

What GWs can tell us?

Frequency: typical scale of the system
when the phase transition happens



PTA in 2023

the best fit of IR is flatter than k^3

Peak: information of the energy density

FOPT: $\Delta\rho_L/\rho_{tot}$

cosmic string: $G\mu$

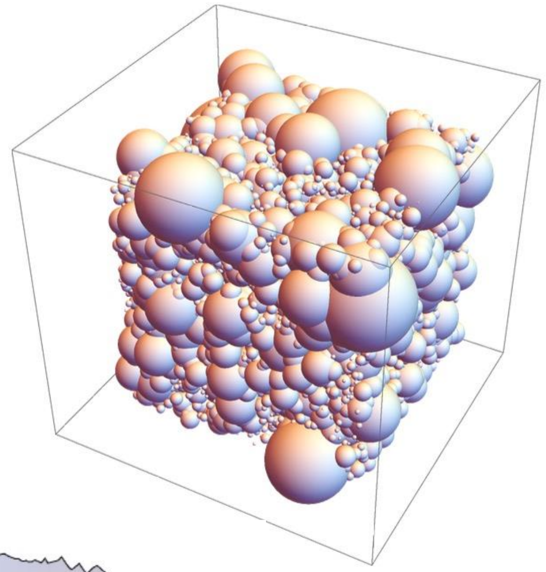
fluid: $\bar{v}$

Shape: dynamics of the system, expansion history of Universe

IR: k^3 causality, envelope

UV part: model parameter dependence

First order phase transition of dark sector



In the dark sector, the nucleation rate per unit volume:

$$\frac{\Gamma}{V} \sim C m_0^4 \mathrm{e}^{-S_\mathrm{b}}$$

The condition for first order phase transition:

$$\Gamma / H^4 \sim 1$$

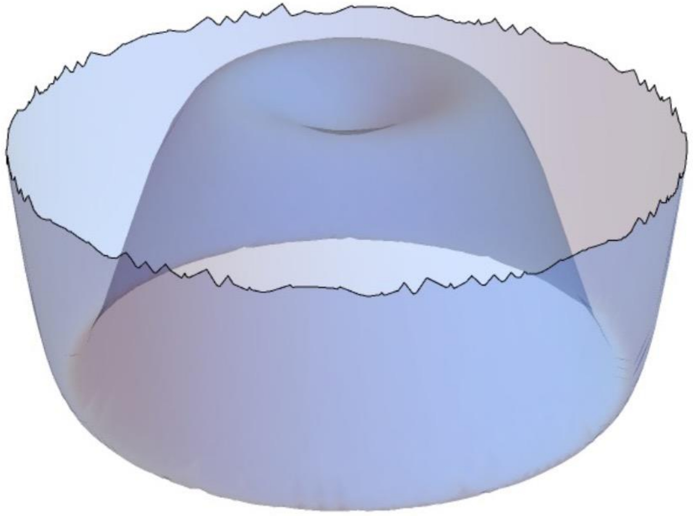
Assume that the dark sector only takes a subdominant fraction of the whole Universe, and PT happens in RD

We consider a complex field model

$$\mathcal{L} = \frac{1}{2}|\partial_\mu \Phi|^2 - V(|\Phi|)$$

$$V(|\Phi|) = \frac{1}{2}m^2|\Phi|^2 - \frac{\lambda}{4}|\Phi|^4 + \frac{\kappa}{6}|\Phi|^6$$

$$\Phi = \rho e^{i\theta}$$



$V(\Phi)$

Dynamics of bubble growth

In a dark sector phase transition, suppose the system is scalar-dominated

$$E_{\rm bub} = 4\pi R^2 \sigma^{\rm tw}\gamma - \frac{4}{3}\pi R^3 \rho_{\rm vac}$$

The boost factor of the bubble walls

$$\gamma(R) = \frac{R\rho_{\rm vac}}{3\sigma^{\rm tw}} = \frac{R}{R_0}$$

In our case

$$R \sim \Gamma^{-\frac{1}{4}} \sim H^{-1} \qquad R_0 \sim m^{-1}$$

When the bubble collisions occur, the boost factor of bubble walls can be extremely large

$$\gamma_{coll} \sim \frac{m}{H} \sim e^{S_b/4}$$

GW calculation

In RD $$\mathrm{h}_{ij}^{TT}(\eta,\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_i}^{\eta} \mathrm{d}\eta' \sin\left[k\left(\eta-\eta'\right)\right] a\left(\eta'\right) T_{ij}^{TT}\left(\eta',\mathbf{k}\right)$$

For a complex field $$T_{ij}(\mathbf{x},t) = T_{ij}^{\rho}(\mathbf{x},t) + T_{ij}^{\theta}(\mathbf{x},t) = \partial_i\rho\partial_j\rho + \rho^2\partial_i\theta\partial_j\theta$$

The GW spectrum function $$\Omega_{GW} = \frac{1}{16Ga^4(\eta)V}\frac{k^3}{(2\pi)^4}\left\langle\left|\partial_\eta \mathrm{h}_{ij}^{TT}(\eta,\mathbf{k})\right|^2\right\rangle$$

GWs from bubble collisions

Envelope approximation:

T_{ij} of the bubble walls vanishes after collision

the IR part scales as k^3

Beyond envelope approximation:

the bubble wall surface energy density drops as R^{-2} with a damping term after collision

the IR part transit from k^1 to k^3

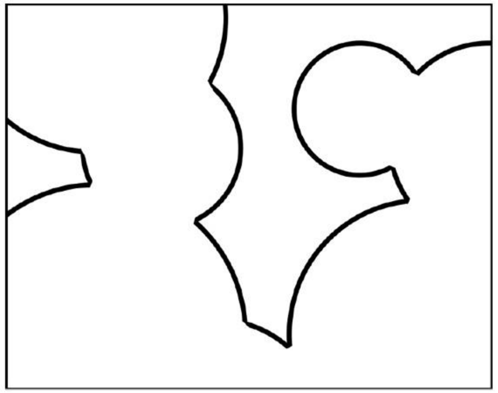
In our case:

scalar dominant

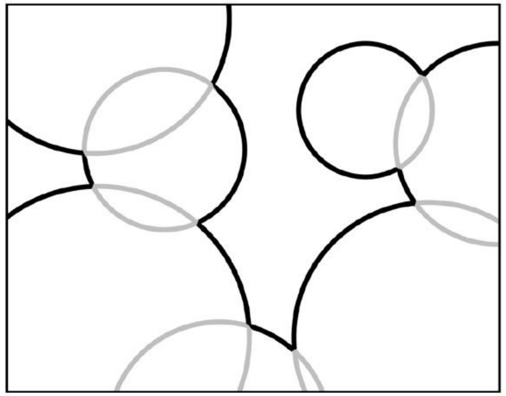
influence of expansion of Universe

detailed shape of IR

we need full numerical simulation



envelope case, arXiv: 1605.01403v2



beyond envelope, arXiv: 1707.03111

Difficulties of simulating large boost dynamics

3+1 D simulation:

If we fix the model and the number of bubbles in simulation

The physical size of the simulation box: $L_{box} \sim N_b^{1/3} R_c \sim N_b^{1/3} \gamma R_0$

Lattice spacing: $\delta x \sim R_0/\gamma$

The grids per spatial dimension $N = L_{box}/\delta x \propto \gamma^2$

The total resource: $N^3 T \propto \gamma^8$

1+1 D simulation:

Bubbles nucleate on z axis in flat space time

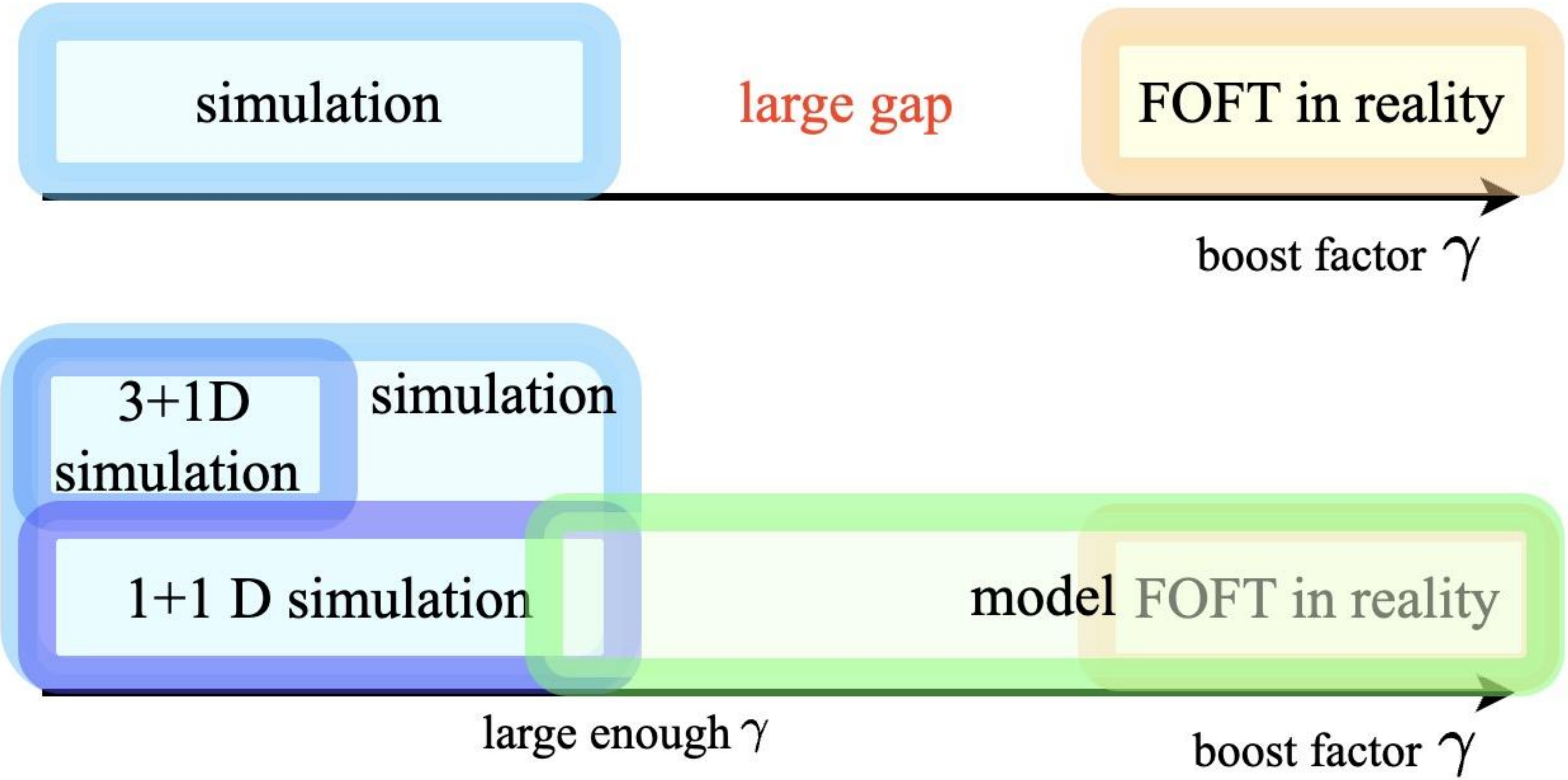
The initial condition has SO(2,1) symmetry

The total resource $NT \propto \gamma^4$

$$\Phi_i = \sum_{j=0}^{n} \Phi_{bounce}\Big(\sqrt{\underbrace{t^2 - x^2 - y^2}_{\text{SO(2,1)}} - (z - z_j)^2}\Big)$$

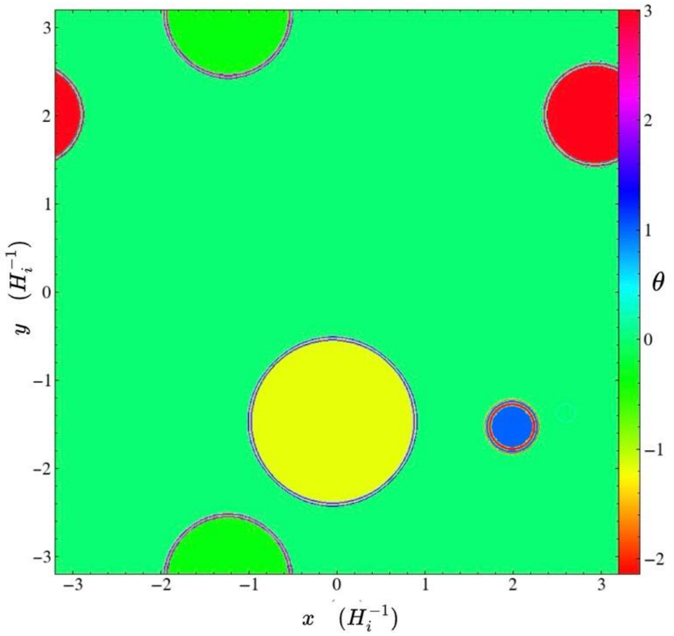
For large boost factor, the resource grows quickly

Fill the gap

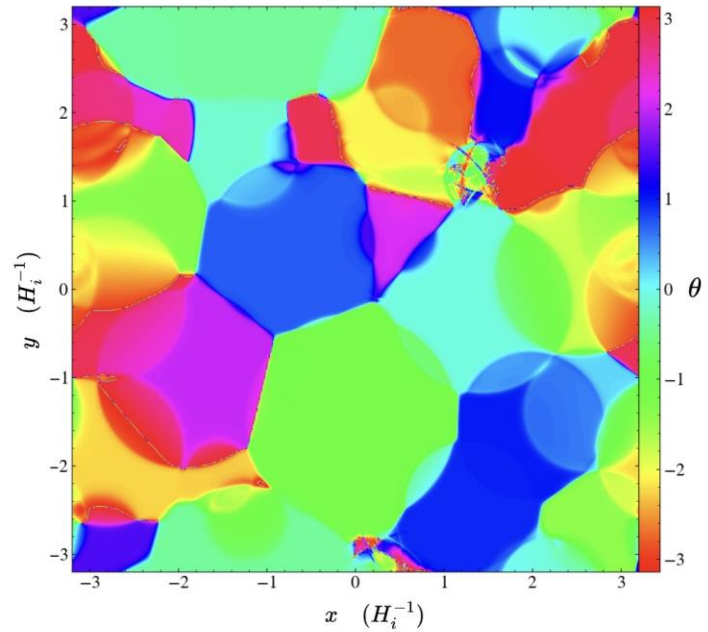

simulation
large gap
FOFT in reality
boost factor γ
3+1D simulation
simulation
1+1 D simulation
model
FOFT in reality
large enough γ
boost factor γ

Bubble configuration

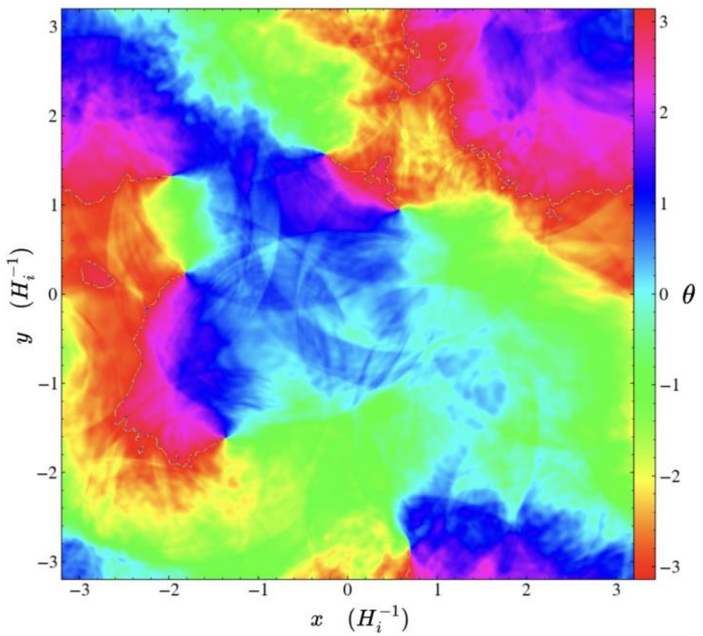
Phase structure:



$a/a_i = 1.1$



$a/a_i = 2.1$



$a/a_i = 3.1$

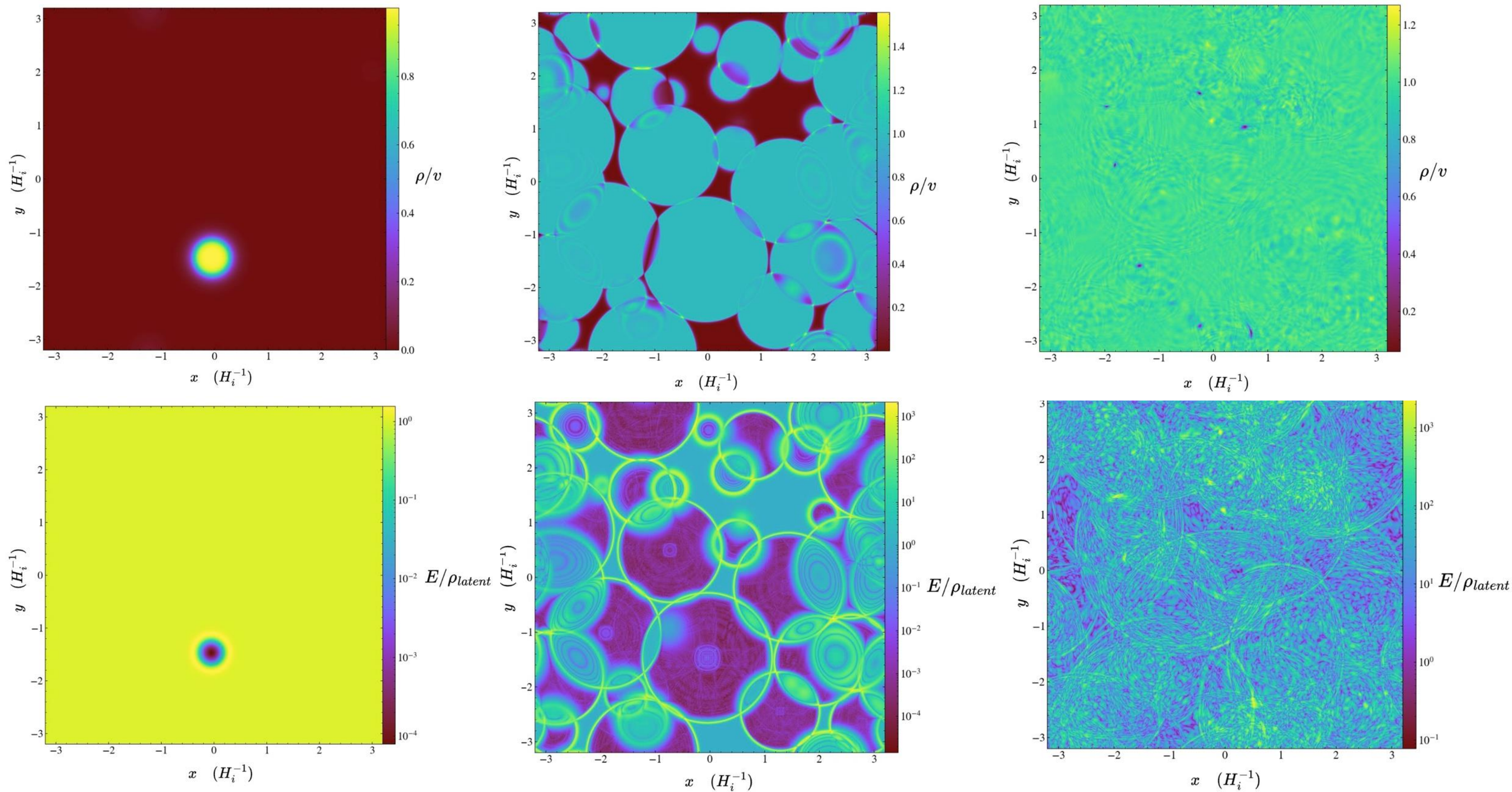
The cosmic string can form in phase transition, but for $m \gg H$ case, the energy density of string is much small than bubble walls

Bubble configuration

$a/a_i = 1.1$ | $a/a_i = 2.1$ | $a/a_i = 3.1$

Radial mode:

Energy density:



Structure of T_{ij} for bubble collisions

After bubble collision, the bubble wall is still very thin
Introduce a parameter δ to describe the width of the wall

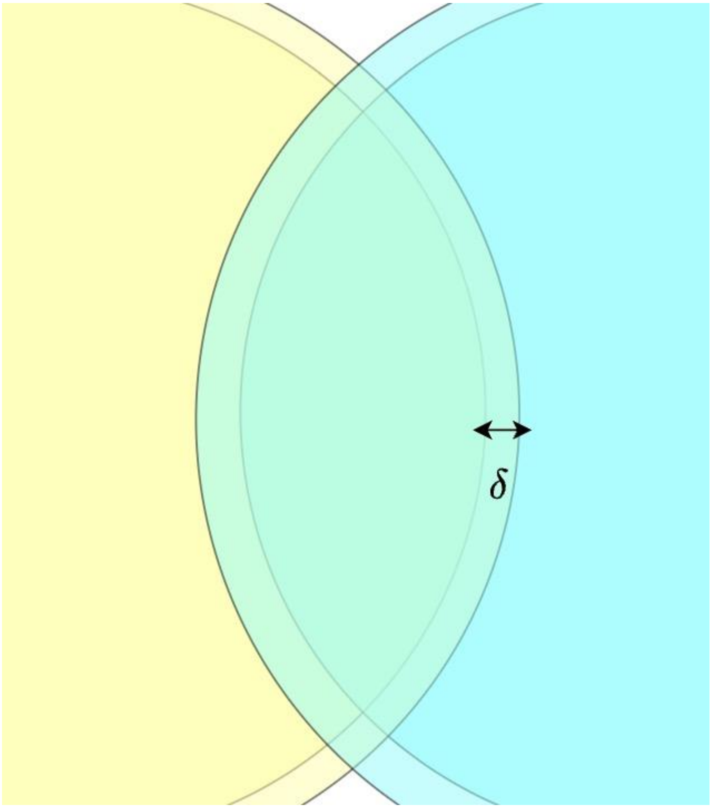
$$T_{ij}^{TT}(\eta, \mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) \int_{\text{sur}} T_{kl}(\eta, \mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \delta \, \mathrm{d}S \qquad \text{for} \quad k \ll \delta^{-1}$$

$$\partial\phi \sim \Delta\phi/\delta \qquad T_{kl}(\eta, \mathbf{x}) = \partial_k \phi \partial_l \phi \propto \delta^{-2}$$

Then $\quad T_{ij,\delta}^{TT}(\eta, \mathbf{k}) \propto \delta^{-1}$
From another point of view $\quad \phi(\eta, \mathbf{k}, \delta) = \sum_{i=0}^{\infty} C_i(\eta, \mathbf{k}) \delta^i$

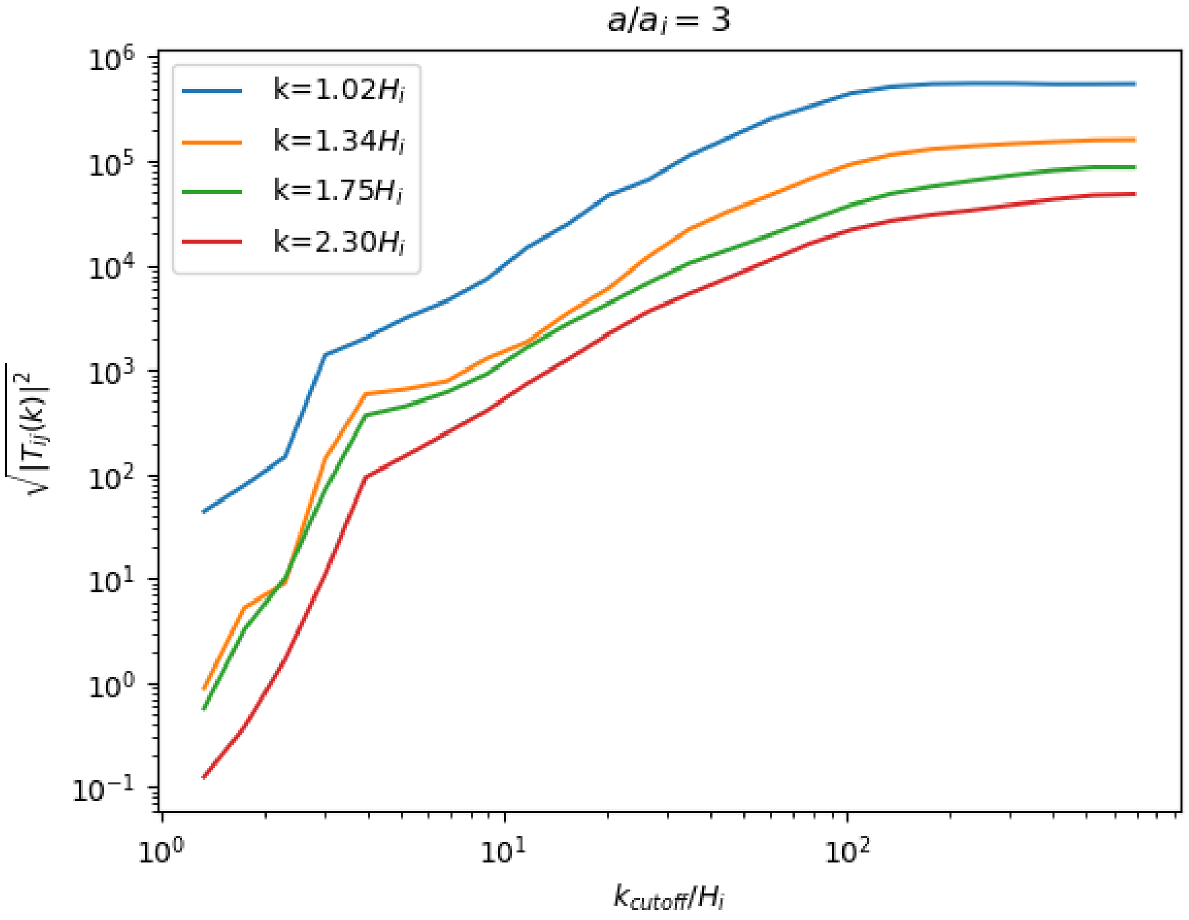
$$T_{ij}^{TT}(\eta, \mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) \int_0^{\delta^{-1}} \frac{p^4 \, \mathrm{d}p}{(2\pi)^3} \int \mathrm{d}\Omega \hat{p}_k \hat{p}_l \phi(\eta, \mathbf{p}) \phi(\eta, \mathbf{k} - \mathbf{p})$$

The integration of p is UV dominant for $\quad k \ll \delta^{-1}$

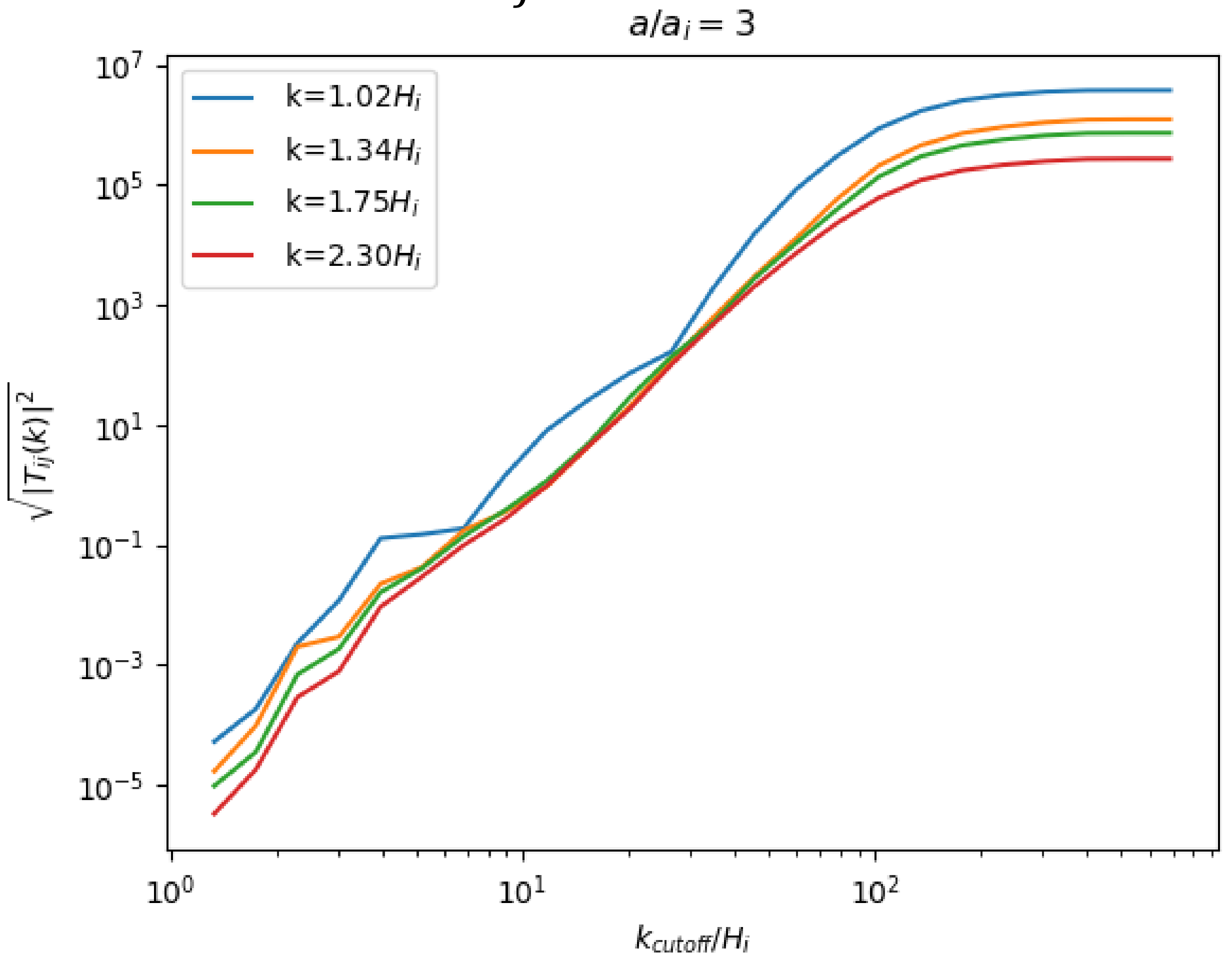


Structure of T_{ij} for bubble collisions

Goldstone mode T^{θ}_{ij}:



Radial mode T^{ρ}_{ij}:



Free field approximation

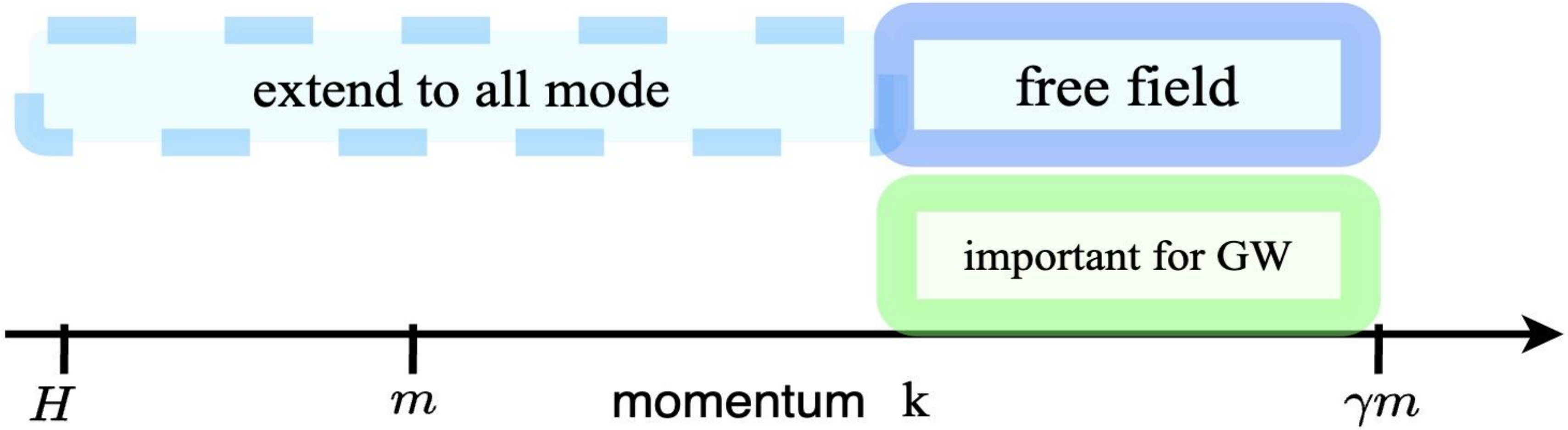
The bubble collision is a nonlinear process

But, for the largest k mode $k_b \sim \gamma m \gg m$

The kinetic energy is much larger than the potential term

$$\partial_\mu \partial^\mu \Phi + V'(\Phi) = 0 \quad \Longrightarrow \quad \partial_\mu \partial^\mu \Phi \approx 0$$

for large k mode



1+1 D case

If we treat both the radial mode and the goldstone as a free scalar

$$\partial_s^2\rho + \frac{2}{s}\partial_s\rho - \partial_z^2\rho = 0$$

$$\partial_s^2\theta + \frac{2}{s}\partial_s\theta - \partial_z^2\theta = 0$$

$$s = \sqrt{t^2 - x^2 - y^2}$$

For both kind of modes

$$\rho(s,k) = \frac{1}{s}\left(\rho_+(k)e^{-ik(s-s_0)} + \rho_-(k)e^{ik(s-s_0)}\right)$$

then the zz component of energy momentum tensor in 1+1 D

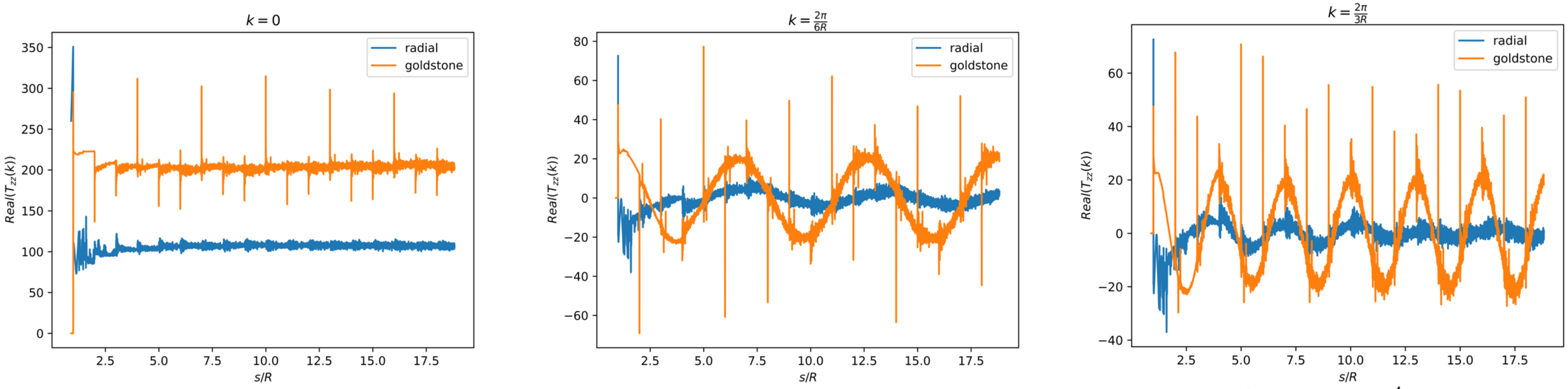
$$\begin{aligned} T^{\rho}_{zz}(s,k) &= \int \frac{\mathrm{d}p}{2\pi} p(k-p)\rho(s,p)\rho(s,k-p) \\ &= s^{-2}\int \frac{\mathrm{d}p}{2\pi} p(k-p)\sum_{c_n=+,-}\rho_{c_1}(p)\rho_{c_2}(k-p)e^{-i(c_1 p + c_2(k-p))(s-s_0)} \end{aligned}$$

1+1 D case

$$T^{\rho}_{zz}(s,k) = s^{-2}\int \frac{\mathrm{d}p}{2\pi} p(k-p) \sum_{c=+,-} \rho_c(p)\rho_c(k-p)e^{-ick(s-s_0)}$$
$$= s^{-2}\sum_{c=+,-} T_c(k)e^{-icks}$$

$T^{\rho}_{zz}(s,k)$ oscillates in s, and the frequency is k

We impose several 1+1 D simulations, the numerical results: yellow: T^{θ}_{zz} blue: T^{ρ}_{zz}



$k=0$ $k=2\pi/6R$ $k=2\pi/3R$

In expanding universe

A free massless field in RD:

$$\partial_\eta^2 \phi(\eta, \mathbf{k}) + \frac{2a'}{a} \partial_\eta \phi(\eta, \mathbf{k}) + k^2 \phi(\eta, \mathbf{k}) = 0$$

Mode function:

$$\phi(\eta, \mathbf{k}) = \frac{1}{a(\eta)} \left(\mathcal{A}_+(\mathbf{k}) e^{-ik(\eta-\eta_i)} + \mathcal{A}_-(\mathbf{k}) e^{ik(\eta-\eta_i)} \right)$$

Energy momentum tensor

$$T_{ij}^{TT}(\eta, \mathbf{k}) = \frac{\Lambda_{ij,kl}(\hat{\mathbf{k}})}{a^2(\eta)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} p_k p_l \sum_{c_n=+,-} \mathcal{A}_{c_1}(\mathbf{p}) \mathcal{A}_{c_2}(\mathbf{k}-\mathbf{p}) e^{-ic_1 p(\eta-\eta_i) - ic_2 |\mathbf{k}-\mathbf{p}|(\eta-\eta_i)}$$

and h_{ij}

$$h_{ij}^c(\mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) \int \frac{\mathrm{d}^3 p}{(2\pi)^3} p_k p_l \sum_{c_1=+,-} |\mathcal{A}_{c_1}(\mathbf{p})|^2 \left(\int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{a(\eta)} \cos k\eta \, e^{-ic_1 \hat{\mathbf{p}} \cdot \mathbf{k}(\eta-\eta_i)} \right)$$

In expanding universe

Integrate the time first:

$$\int_{\eta_i}^{\infty} \frac{\mathrm{d}\eta}{a(\eta)} \cos k\eta e^{-ic_1 \hat{\mathbf{p}}\cdot\mathbf{k}(\eta-\eta_i)} = -\frac{e^{ic_1\hat{\mathbf{p}}\cdot\mathbf{k}\eta_i}}{2H_0}\Big(Ei\Big(-i(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}-1)k\eta_i\Big)+Ei\Big(-i(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}+1)k\eta_i\Big)\Big)(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}\neq 1)$$

In IR limit

$$h_{ij}^c(\mathbf{k}) = -\frac{\log(k\eta_i)}{2H_0}\Lambda_{ij,kl}(\hat{\mathbf{k}})\int \frac{\mathrm{d}^3 p}{(2\pi)^3} p_k p_l \sum_{c_1=+,-} |\mathcal{A}_{c_1}(\mathbf{p})|^2 = -\frac{\log(k\eta_i)}{2H_0} T_{ij}^{TT}(\eta_i, \mathbf{0})$$

The GW spectrum function $\Omega_{GW} \propto k^3 \log^2(k\eta_i), \quad k\eta_i \ll 1$

But for radial mode $\partial_\eta^2 \rho(\eta,\mathbf{k}) + \frac{2a'}{a}\partial_\eta \rho(\eta,\mathbf{k}) + k^2\rho(\eta,\mathbf{k}) + a^2 m^2 \rho(\eta,\mathbf{k}) = 0$

The contribution of radial mode to the integration stops at $\eta \sim \gamma/H_0$

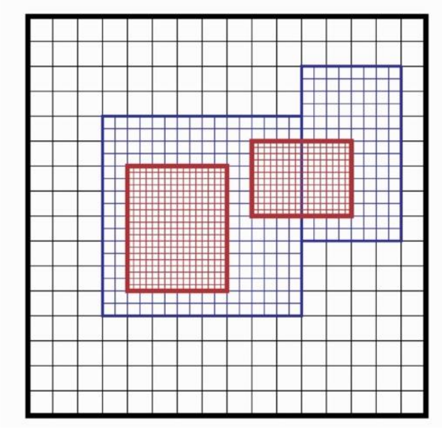
Therefore, the IR part of GW spectrum is mainly produced by the goldstone mode

Results

We do a 1536^3 size numerical simulation

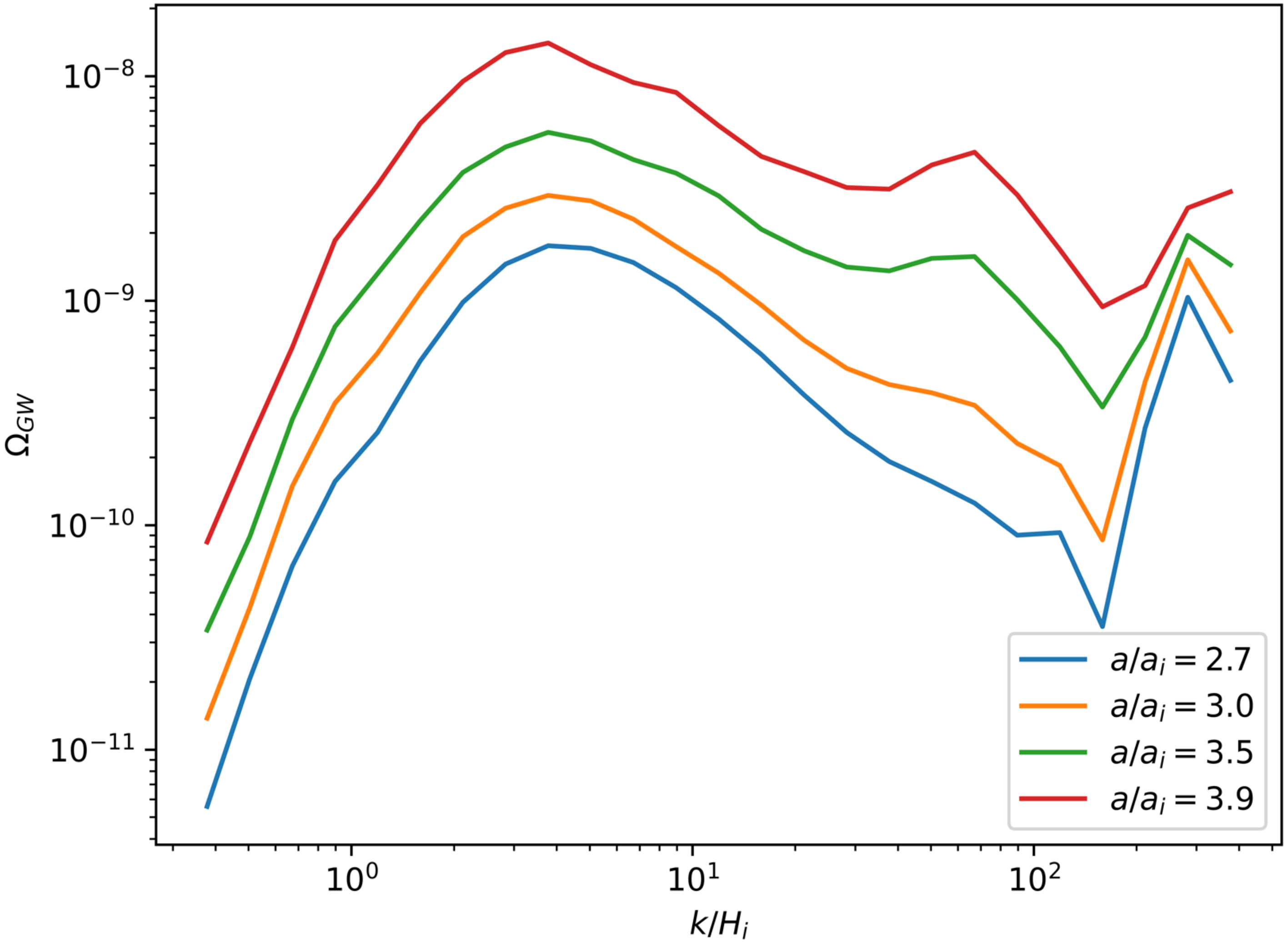
We use AMReX to build our phase transition code and impose two layers of grids

The effective grid size is 3072^3 .



AMR method

GW power spectrum for $\frac{\Delta\rho}{\rho} = 0.1$



Summary and future work

Summary:

- in dark sector first order phase transition, the GW production can be scalar-dominant
- the boost factor of the bubble walls can be extremely large and can easily exceed the ability of numerical simulation
- the T_{ij} is dominated by the UV mode of the field
- we can treat the field as a free field when calculating the GW
- the far IR region of GW power spectrum scales as $k^3 log^2 k$

Future work:

- when will the free field approximation ends
- a more analytic model to deal with the UV mode

Thank you

GW calculation

in RD
$$\partial_\eta^2 h_{ij}^{TT}(\eta,\mathbf{k}) + \frac{2a'}{a}\partial_\eta^2 h_{ij}^{TT}(\eta,\mathbf{k}) + k^2 h_{ij}^{TT}(\eta,\mathbf{k}) = 16\pi G T_{ij}^{TT}(\eta,\mathbf{k})$$

$$\mathrm{h}_{ij}^{TT}(\eta,\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_i}^{\eta} \mathrm{d}\eta' \sin\left[k\left(\eta-\eta'\right)\right] a\left(\eta'\right) T_{ij}^{TT}\left(\eta',\mathbf{k}\right)$$

for a complex field
$$T_{ij}(\mathbf{x},t) = T_{ij}^{\rho}(\mathbf{x},t) + T_{ij}^{\theta}(\mathbf{x},t) = \partial_i\rho\partial_j\rho + \rho^2\partial_i\theta\partial_j\theta$$

the energy density of GW
$$\begin{aligned}\rho_{GW} &= \frac{1}{32\pi G a^4(\eta) V}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\left|\partial_\eta \mathrm{h}_{ij}^{TT}(\eta,\mathbf{k})\right|^2 \\ &= \frac{4\pi G}{a^4(\eta) V}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\left[\left|\int_{\eta_i}^{\eta_f} \mathrm{d}\eta' a\left(\eta'\right)\cos k\eta' T_{ij}^{TT}\left(\eta',\mathbf{k}\right)\right|^2 + \left|\int_{\eta_i}^{\eta_f} \mathrm{d}\eta' a\left(\eta'\right)\sin k\eta' T_{ij}^{TT}\left(\eta',\mathbf{k}\right)\right|^2\right]\end{aligned}$$

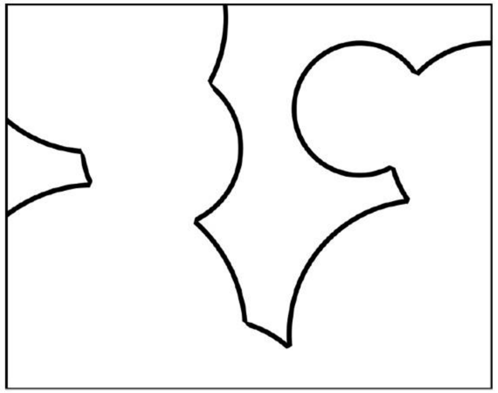
GWs from bubble collisions

envelope approximation:

thin wall approximation

$+$ T_{ij} of the bubble walls vanishes once they collide with others

$=$ the IR part of GW power spectrum scales as k^3



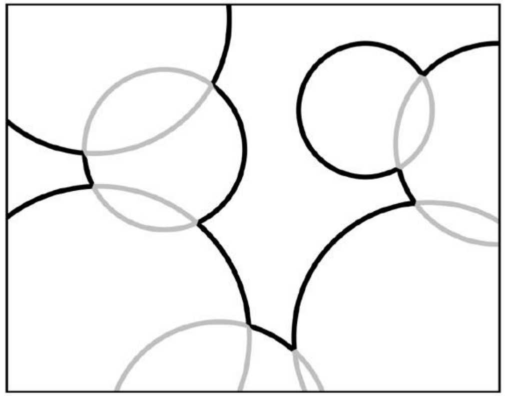
envelope case, arXiv: 1605.01403v2

beyond envelope approximation:

thin wall approximation

$+$ the bubble wall surface energy density drops as R^{-2} with a damping term after collision

$=$ the IR part of GW power spectrum transit from k^1 to k^3



beyond envelope, arXiv: 1707.03111

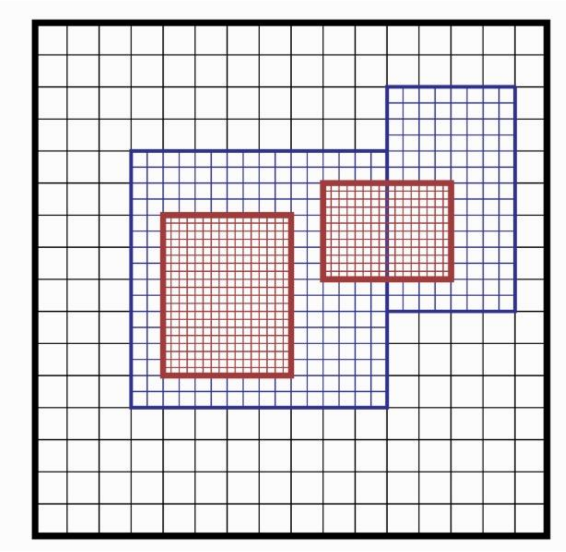
we want to achieve this in field language

Adaptive Mesh Refinement method

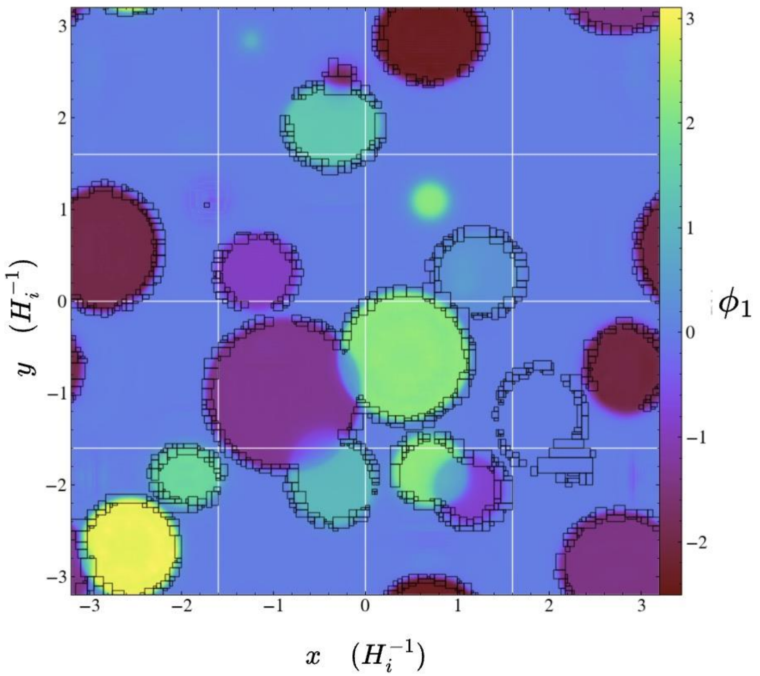
we extend the ability of 3+1 D simulation by using AMR method

bubble is a codimensional-1 object

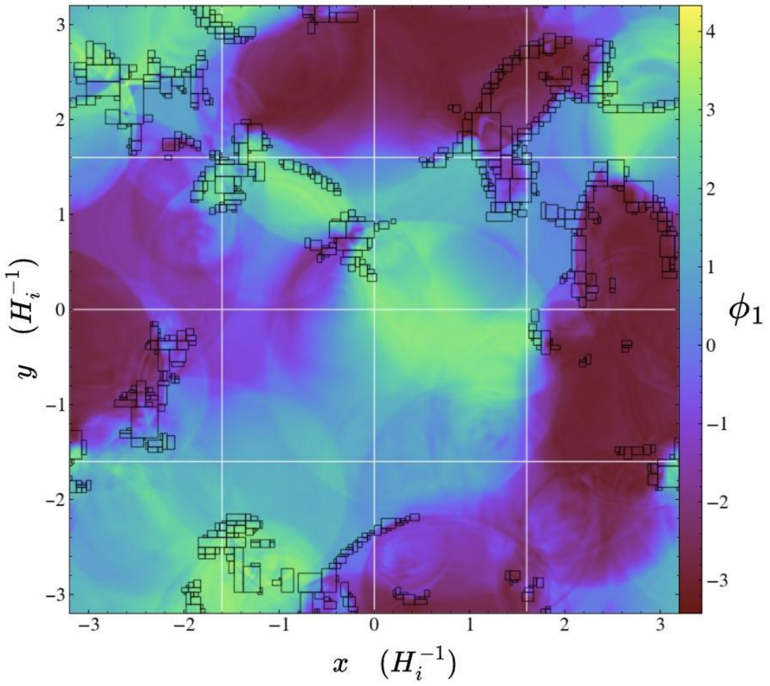
we use AMReX to build our phase transition code



grids with three levels



$a/a_i = 1.5$



$a/a_i = 3$

Adaptive Mesh Refinement method

the condition for building thinner grid level:

$$|\nabla\Phi| > C(\gamma)mv$$

we fix the model parameter and do several simulations to determine $C(\gamma)$