A model builder's guide to cosmological collider physics

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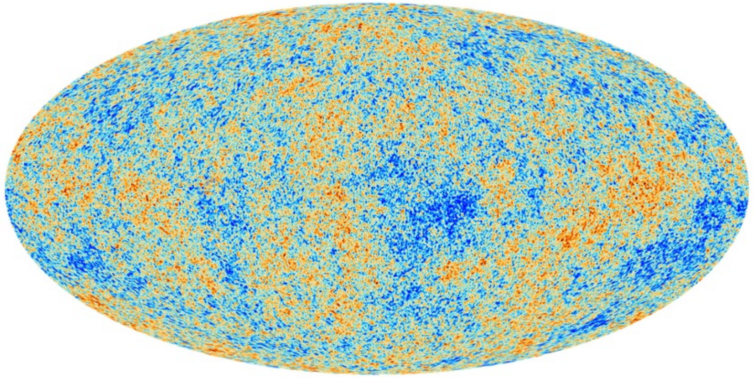
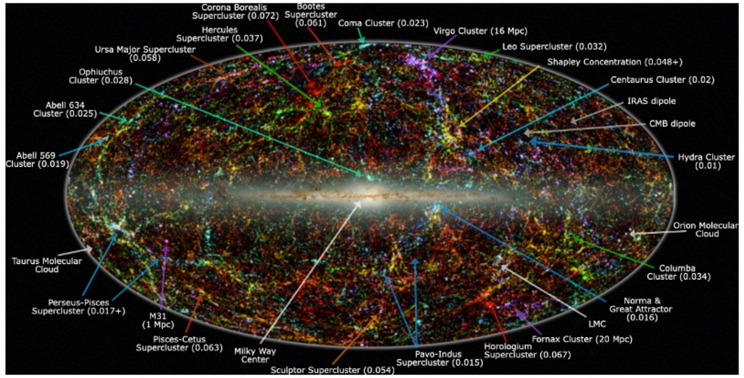
What is the cosmological collider?



CMB (Planck 2018)



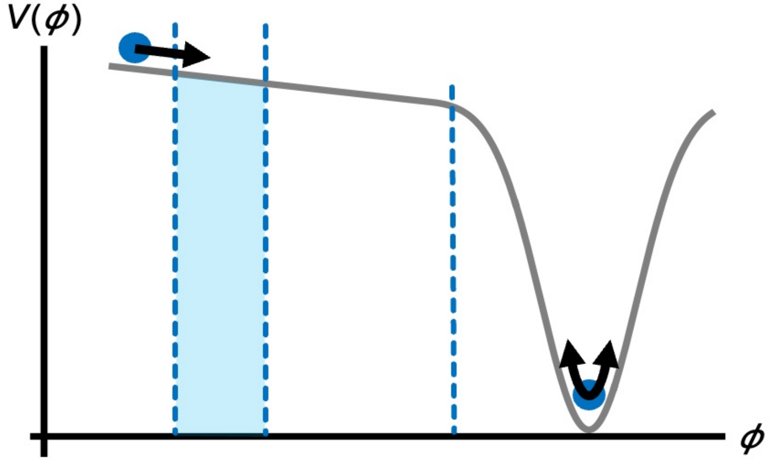
LSS [astro-ph/0405069]

$$\langle \delta T_1 \delta T_2 \rangle$$

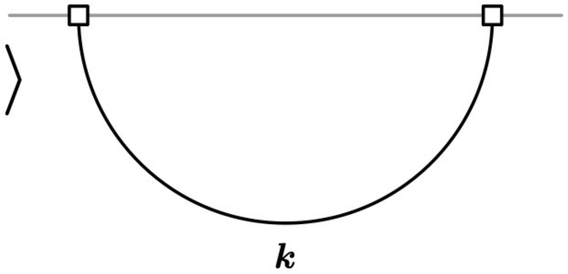
$$\langle \delta \rho_1 \delta \rho_2 \rangle$$

$$\langle \zeta_1 \zeta_2 \rangle$$

$$\langle \delta \phi_1 \delta \phi_2 \rangle$$

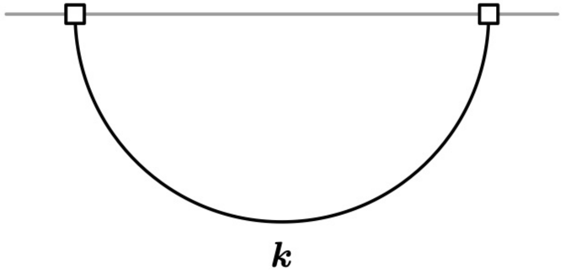


$$\phi(t, \boldsymbol{x}) = \phi_0(t) + \delta\phi(t, \boldsymbol{x})$$



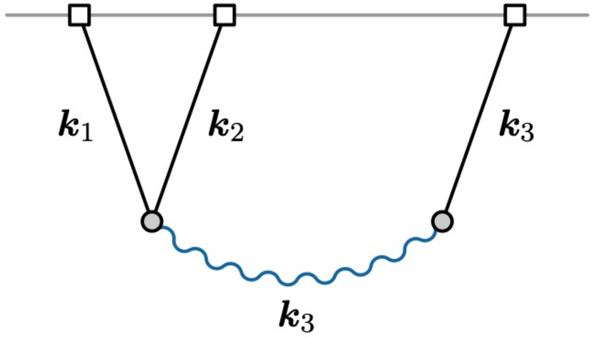
Inflaton fluctuation

What is the cosmological collider?



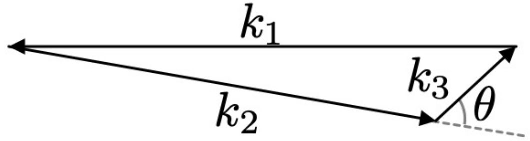
Inflaton fluctuation

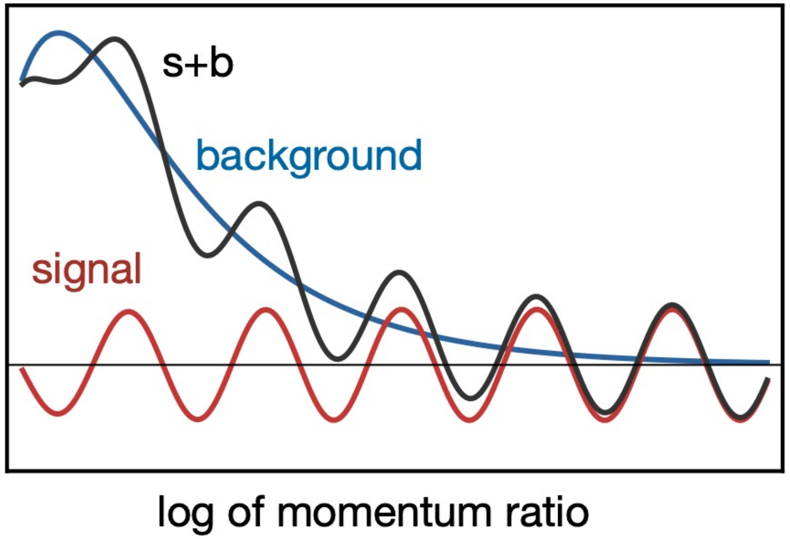
$$\langle \delta\phi_1 \delta\phi_2 \rangle$$



Heavy partice

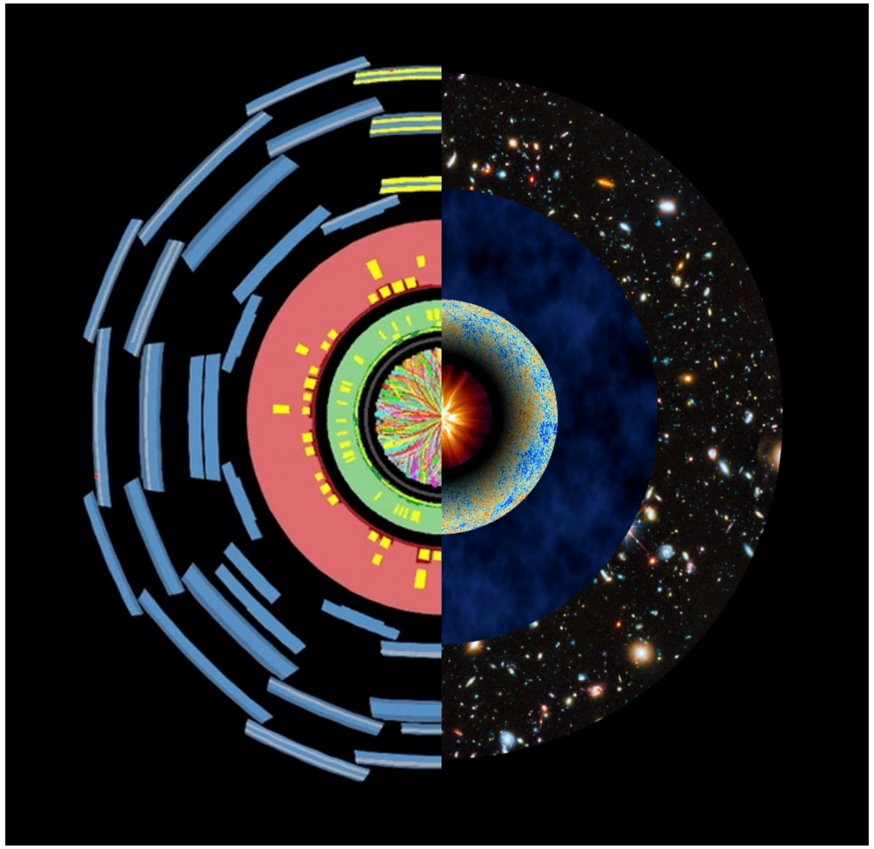
$$\langle \delta\phi_1 \delta\phi_2 \delta\phi_3 \rangle$$





"Frequency ~ mass, angular dep ~ spin"
-- Wrong but convenient

What is the cosmological collider?



What's the energy scale of this machine? – Hubble

$$r \sim 0.1 \left(\frac{H}{10^{14}\mathrm{GeV}} \right)^2 \sim 0.1 \left(\frac{E}{10^{16}\mathrm{GeV}} \right)^4$$

r $\lesssim$ 0.03 (Planck 2018) => H $\lesssim$ 10^{14} GeV

Happen to be around the GUT scale!

H (or E) could be much lower, at the expense of a more tuning potential: $r = 16\epsilon$

We have good reasons to consider high-scale inflation models

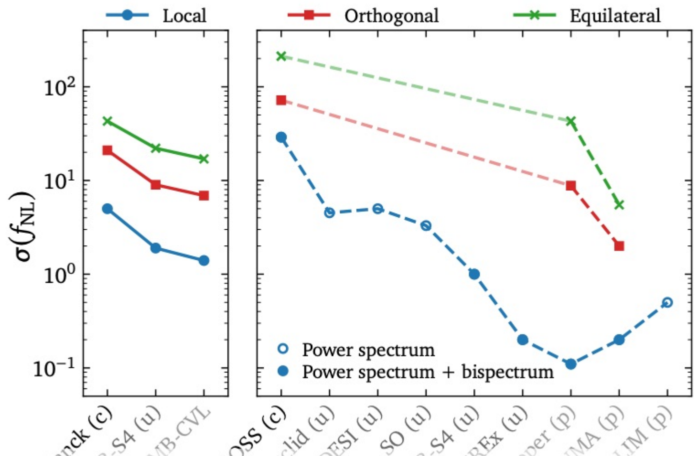
[Caveat: all formulae here true only for SFSR models]

Current status

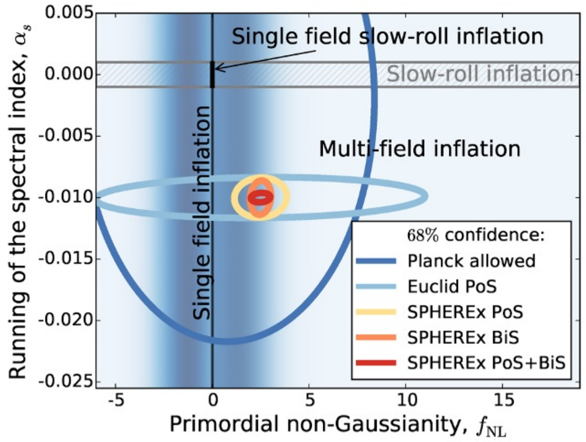
The size of the 3-point correlator is measured by a dimensionless parameter f_{NL} .

[Caveat: the f_{NL} parameter is defined shape by shape. Strictly speaking, f_{NL}'s of differential shapes cannot be compared directly, although people often do this if only for convenience.]

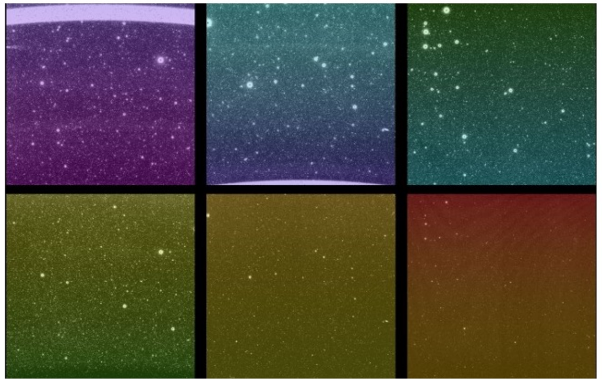
Best constraint from Planck: f_{NL}(local) $\lesssim$ O(5); LLS down to O(0.1) and 21cm to O(0.01)?



Snowmass 2021: 2203.08128



SPHEREx, 1412.4872



Credit: SPHEREx

People have started to seriously search for CC signals from real data!

Constraint from Planck (CMB) [Sohn et al., 2404.07203]; BOSS (LSS) [Cabass et al. 2404.01894]

Constraint of realistic particle models [Bao, Wang, ZX, Zhong, 2504.02931]

Timely to study more systematically the CC phenomenology of BSM new physics!

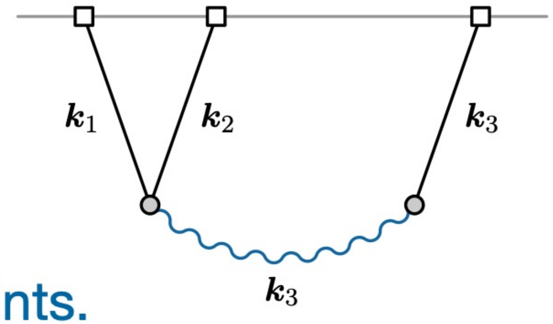
Plan for the rest of the talk

Starting point: You have a nice new-physics model probably at a very high scale.
Goal: Let's work out CC signals of your model.

1) Three questions

1. Where are large-scale fluctuation from? (What are external modes?)
2. How does your model couple to the external modes?
3. How to estimate the size of CC signals of your model?

With them answered, you can already do forecast and put rough constraints.



2) An example and more examples

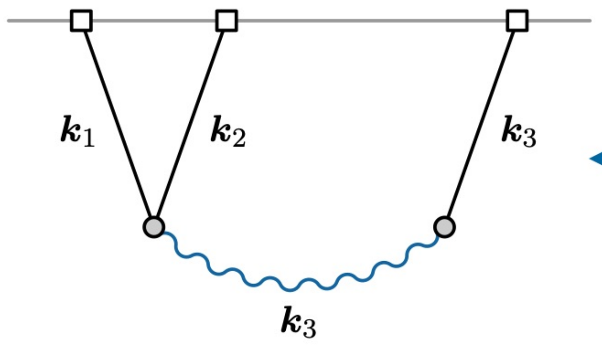
Quasi-single-field inflation / axion couplings / modulated reheating / more …

3) Computing full shapes, a quick summary

Diagrammatic expansion / Numerical strategies / Analytical methods / Signal extraction

Full shapes ultimately important for signal searches in the real data

Question 1: Where is the curvature fluctuation from?



Asked in another way:
What are these external lines?

- Vanilla models: Single-field slow-roll inflation $\mathscr{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)$
  CC physics is largely model independent; the specific form of $V(\phi)$ is irrelevant;
  Relevant is that the inflaton background has a slow-roll solution at the observable scales:

$$\phi_0(t) \simeq \phi_{0*} + \dot{\phi}_0 t \qquad \dot{\phi}_0 \simeq 3600H^2$$

- Multifield inflation: the external mode being a particular linear combination of fields.

- Not from an inflaton?
  Curvaton [Kumar, Sundrum, 1908.11378] / modulated reheating [Lu, Wang, ZX, 1907.07390]

Question 2: How does your model talk to the inflaton?

- You need to couple your model to the inflaton. Two options.

- **Option 1:** Couple your model to the full inflaton $\phi(t, \boldsymbol{x})$ in a Lorentz invariant Lagrangian
  Pros: Relativistic invariant, more in line with the normal practice of particle model building; Coupling sizes follow from dimension counting [RG] arguments, easier to estimate
  Cons: Not completely model independent; a "fundamental" inflaton is assumed:

$$\phi(t, \boldsymbol{x}) = \phi_0(t) + \delta\phi(t, \boldsymbol{x})$$

- **Option 2:** "EFT of inflation" Only the fluctuation $\delta\phi(t, \boldsymbol{x})$ is experimentally confirmed.
  => An EFT for the fluctuation only; A Goldstone of broken time diffeomorphism
  => Couple your model only to the fluctuation [Cheung et al., 0709.0293]
  Pros: (Seemingly) more model independent
  Cons: Since $\delta\phi(t, \boldsymbol{x})$ is the Goldstone of broken time diff, the couplings are no longer Lorentz invariant; uncontracted time indices allowed; power-counting obscured

- I'll go with Option 1: Couple your model to the inflaton $\phi(t, \boldsymbol{x})$
  The inflaton has an approximate shift symmetry [In plain words: it has a very flat potential] which you don't want to spoil
  => Either the inflaton is derivatively coupled or the coupling is slow-roll suppressed

- Then, expand the full inflaton as $\phi(t, \boldsymbol{x}) = \phi_0(t) + \delta\phi(t, \boldsymbol{x})$
  At the fluctuation level, this introduces both boost-breaking interactions and corrections to Lagrangian parameters

- This means, in particular, that the mass spectrum of your model can be significantly distorted by the inflaton couplings.

Example

Lorentz invariant coupling

$$\frac{1}{\Lambda^2}\sigma^2(\partial_\mu\phi)^2$$

Boost breaking coupling

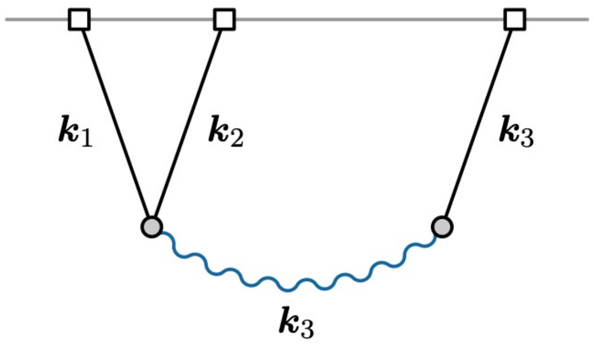
$$\frac{\dot{\phi}_0}{\Lambda^2}\sigma^2\delta\phi'$$

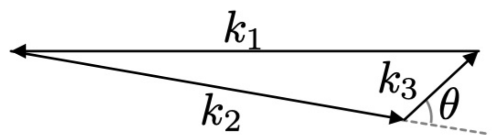
Mass correction

$$\frac{\dot{\phi}_0^2}{\Lambda^2}\sigma^2$$

Question 3: how large is the CC signal in your model?

- You have all ingredients (internal & external lines & their couplings). Then you can compute Feynman diagrams.
- But wait: The calculation is quite hard. Probably you don't want to do it if your CC signal is actually very tiny.
- Can we roughly understand the signal without really computing it?
- Here I only consider 3pt correlators and also ignore spins.
  [For full kinematics with 4pt correlators, see Qin, ZX, 2205.01692]





The dimensionless shape function:

$$S(k_1, k_2, k_3) = -\frac{(k_1 k_2 k_3)^2}{(2\pi)^4 P_\zeta^2}\left(\frac{H}{\dot{\phi}_0}\right)^3 \langle \delta\phi_{k_1}\delta\phi_{k_2}\delta\phi_{k_3}\rangle'$$

$P_\zeta \simeq 2\times 10^{-9}$ is the measured 2pt function

For the 3pt function, the signal part can be described by 4 parameters (ignoring spin info)

Obviously, most crucial is the overall size *B* (the dimensionless f_{NL} for the CC signal)
[See Wang, ZX, 1910.12876 for details]

Other parameters very useful as well. L tells the scaling of the particle (tree or loop), ω tells the mass
[Wang, ZX, Zhong, 2109.14635]

A subtler but very telling parameter is the phase δ. [See Qin, ZX, 2205.01692]

signal size — scaling exponent

$$\mathcal{S}_{\rm signal}(k_1,k_2,k_3) \sim B\left(\frac{k_1}{k_3}\right)^L \sin\left[\omega\log\left(\frac{k_1}{k_3}\right)+\delta\right]$$

frequency — phase

$$B \sim \frac{1}{2\pi P_\zeta^{1/2}} \times \text{loop factors} \times \text{vertices} \times \text{propagators}$$

		B	L	ω
$s=0,\, m>\frac{3}{2},\, \mu=0$ [7]	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-\frac{9}{4}}$
$s=0,\, 0<m<\frac{3}{2},\, \mu=0$ [7]	tree	–	$\frac{1}{2}-\sqrt{\frac{9}{4}-m^2}$	0
$s>0,\, m>s-\frac{1}{2},\, \mu=0$ [13]	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-(s-\frac{1}{2})^2}$
$s>0,\, 0<m<s-\frac{1}{2},\, \mu=0$ [13]	tree	–	$\frac{1}{2}-\sqrt{(s-\frac{1}{2})^2-m^2}$	0
$s=0,\, m>\frac{3}{2},\, \mu=0$ [7]	1-loop	$e^{-2\pi m}$	2	$2\sqrt{m^2-\frac{9}{4}}$
Dirac fermion, $m>0,\, \mu=0$ [8]	1-loop	$e^{-2\pi m}$	3	$2m$
Dirac fermion, $m>0,\, \mu>0$ [8]	1-loop	$e^{2\pi\mu-2\pi\sqrt{m^2+\mu^2}}$	2	$2\sqrt{m^2+\mu^2}$
$s=1,\, m>\frac{1}{2},\, \mu\geq 0$ [10]	1-loop	$e^{2\pi\mu-2\pi m}$	2	$2\sqrt{m^2-\frac{1}{4}}$

From 2109.14635

Subtleties and pitfalls

- It's not so trivial to get an observably large cosmological collider signal
  [-- unless you arbitrarily fine-tune parameters; But you don't, because you are a good model builder.]

$$B \sim \frac{1}{2\pi P_\zeta^{1/2}} \times \text{loop factors} \times \text{vertices} \times \text{propagators}$$

$\frac{1}{2\pi P_\zeta^{1/2}}$: ~3600

propagators: $\sim e^{-\pi m}$

← You probably want this larger than ~O(0.1) to be seen w/ LSS
[or O(0.01) for 21cm]

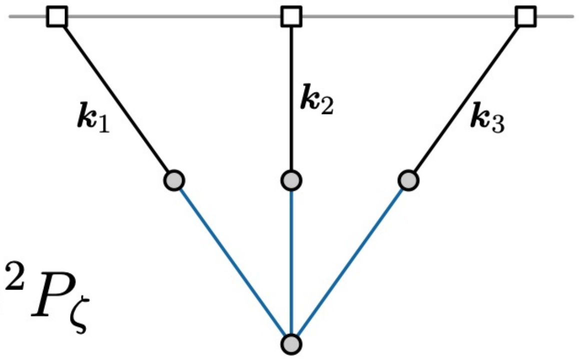
- Take our example $\frac{1}{\Lambda^2}\sigma^2(\partial_\mu\phi)^2$: it produces a coupling $\frac{\dot\phi_0}{\Lambda^2}\sigma^2\delta\phi'$ and a mass $\frac{\dot\phi_0^2}{\Lambda^2}\sigma^2$
- A large vertex prefers a low cutoff Λ but a not-too-large mass prefers the opposite
- For an enumeration of all "simple" couplings that could or could not easily produce large signals, see [Wang, ZX, 1910.12876]

Example: (A refined version of) quasi-single-field inflation

- Quasi-single-field inflation: the earliest CC model, a single heavy scalar [Chen, Wang, 0911.3380]
- A refined version [Wang, ZX, 1910.12876]:

$$\mathscr{L} = \underbrace{-\tfrac{1}{2}(\partial_\mu\phi)^2 - V(\phi)}_{\text{Inflaton sector}} - \underbrace{\tfrac{1}{2}(\partial_\mu\sigma)^2 - \tfrac{1}{2}\mu^2\sigma^2 - \lambda\sigma^4}_{\text{Heavy scalar}} - \frac{1}{\Lambda^2}\sigma^2(\partial_\mu\phi)^2$$

- The dim-6 coupling gives a wrong-sign mass term to σ : $\dfrac{\dot\phi_0^2}{\Lambda^2}\sigma^2$
- σ picks up a vev, which further induces a 2pt mixing: $\dfrac{\sigma_0\dot\phi_0}{\Lambda^2}\sigma\delta\phi'$
  So we have a 3pt function with triple σ-exchanges



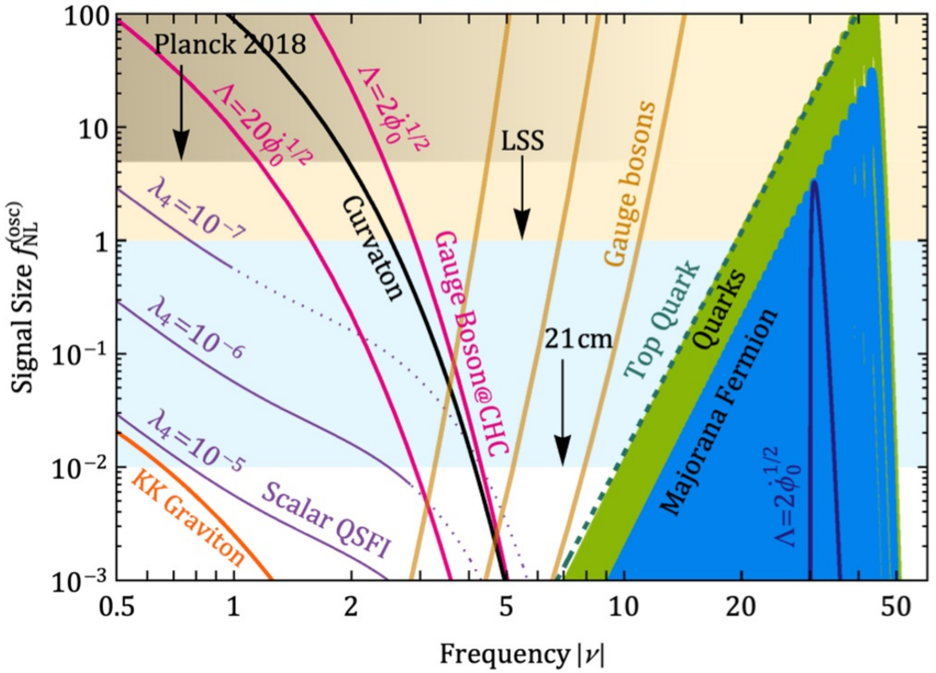
- Applying our simple formula for an estimate, we get: $B \sim \lambda^{-1}(2\pi)^2 P_\zeta$
- So we get a large signal if λ is small [technically natural]

More examples

- Axion couplings [Wang, ZX, 1910.12876, 2004.02887]
  Boltzmann factors $e^{-\pi m}$ are annoying! – Within the vanilla SFSR models, the suppression is naturally relieved by axion couplings:

$$\frac{1}{\Lambda}(\partial_\mu \phi) J_5^\mu \longrightarrow \frac{\dot{\phi}_0}{\Lambda} J_5^0$$

- Different external states:
  cosmo Higgs collider [Lu, Wang, ZX, 1907.07390]
  cosmo graviton collider [Tong, ZX, 2203.06349]
  DM/axion isocurvature collider [Lu 2103.05958, etc]



Wang, ZX, 1910.12876

- More BSM new phyics:
  GUT with extra dim [Kumar, Sundrum: 1811.11200] Neutrino seesaw and leptogenesis [Chen, Wang, ZX, 1805.02656, Cui, ZX, 2112.10793] Flavor [Pinol et al., 2112.05710], BCS phase transition [Tong, Wang, Zhang, Zhu, 2304.09428] Parity violation [Liu, Tong, Wang, ZX, 1909.01819]

Full shape: QFT correlators in dS

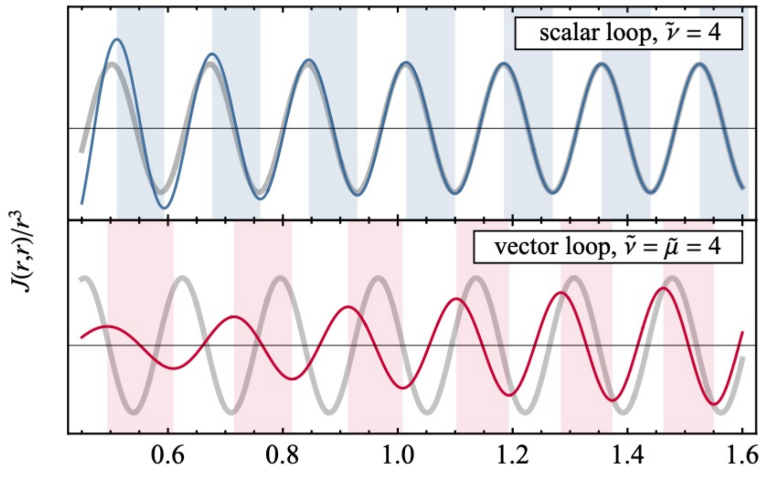
- The signal is sinusoidal only in the squeezed limit
  Also, there are background contributions.

- For real signal searches, the full shape is important.

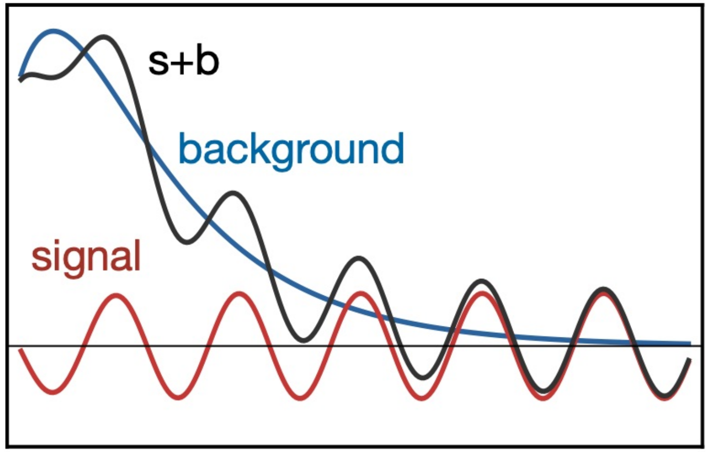
- Correlators are equal-time expectation values, not scattering amplitudes!

- A standard method: Schwinger-Keldysh path integrals
  New features: 1) field number doubled; 2) space-time treated separately; 3) modes in special functions

- For a pedagogical introduction to the SK path integral, Feynman rules for correlators, and numerical strategies, see [Chen, Wang, ZX, 1703.10166]
  Numerical packages for tree graphs also available
  [Werth, Pinol, Renaux-Petel, 2302.00655]



Qin, ZX, 2205.01692



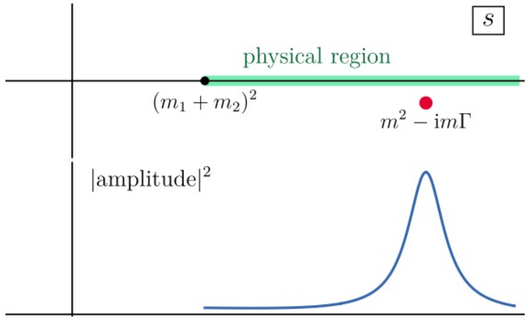
log of momentum ratio

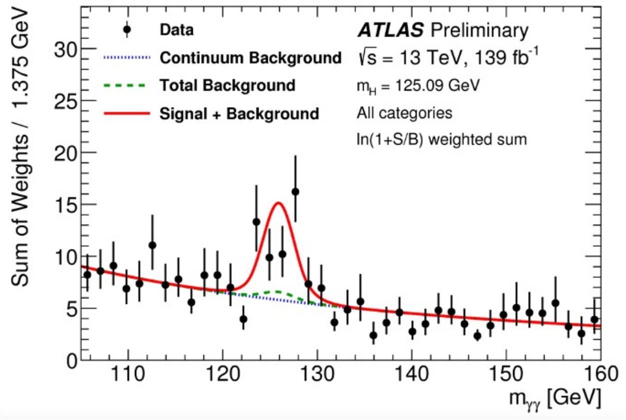
Full shape: QFT correlators in dS

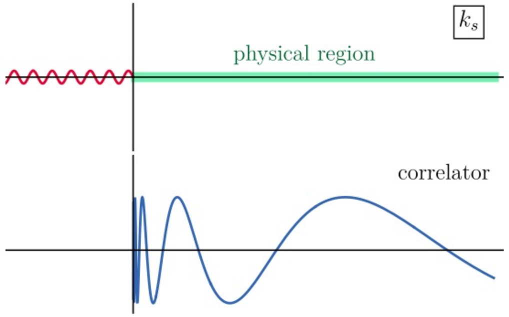
- Analytical frontier: many unexpected progress in recent years, an exciting new direction
- Full shapes for single exchange solved in 2018 [Arkani-Hamed et al. 1811.00024], double exchange in 2023 [ZX, Zang, 2309.10849, Aoki et al., 2404.09547]
- Arbitrary massive tree graphs finally solved in 2024 [Liu, ZX, 2412.07843]
  The full shape has to be high-transcendental-weight hypergeometric functions: identified, expanded, and anlytically continued
- Simple loops computable w/ spectral or dispersion techniques
  [ZX, Zhang, 2211.03810; Liu, Qin, ZX, 2407.12299]
- As a model builder, probably you only need the explicit results, without diving into all the amplitude techniques. Then you could find:
  Full expressions for 3-point tree graphs of sufficient generality in [Qin, ZX, 2301.07047], 4-point tree graphs in [Qin, ZX, 2208.13790], and 1-loop signals in [Qin, ZX, 2205.01692, 2304.13295]

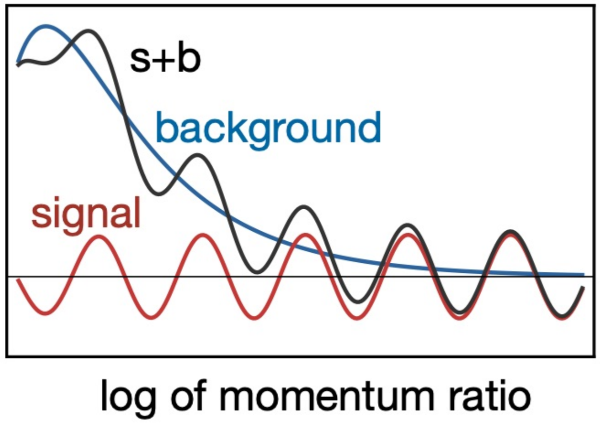
Signal extraction

- The signal is intimately related to the (non)analytic properties of amplitudes:

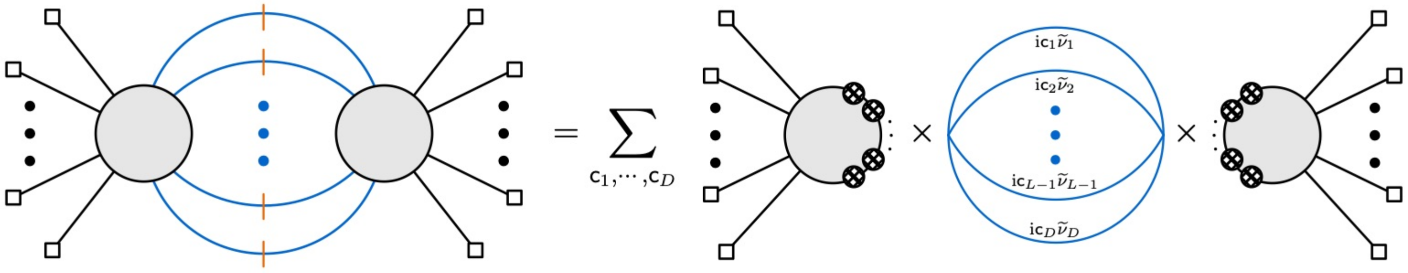








- For CC: (nonlocal) signals in arbitrary correlators factorize in the soft limits
  => Factorization theorems & cutting rules => signals analytically computable to all loop orders
  [Qin, ZX, 2304.13295; 2308.14802]



Summary

- Data are coming: a new window to particle physics at the inflation scale

- Signals manifest as oscillations in the correlation functions: quite different from S-matrix
  Easy to estimate, hard to get the full shape, but we have done most of hard works for you!

- Not trivial but not impossible to get large signals from arbitrary new physics models
  There are fun challenges for you model builders. We believe in your ingenuity!

Thank you!