Hyperon-Nucleon Interaction from Lattice QCD

格点QCD方法研究超子-核子相互作用

上海师范大学 刘航

合作者：王伟，谭金鑫，朱潜腾，刘柳明等

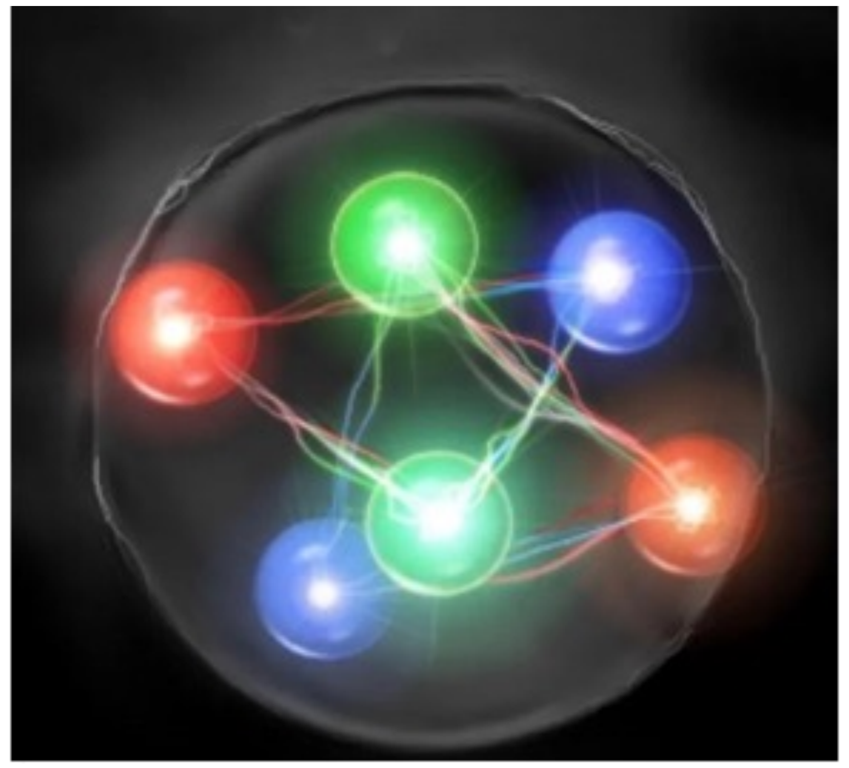
第二十届全国中高能核物理大会

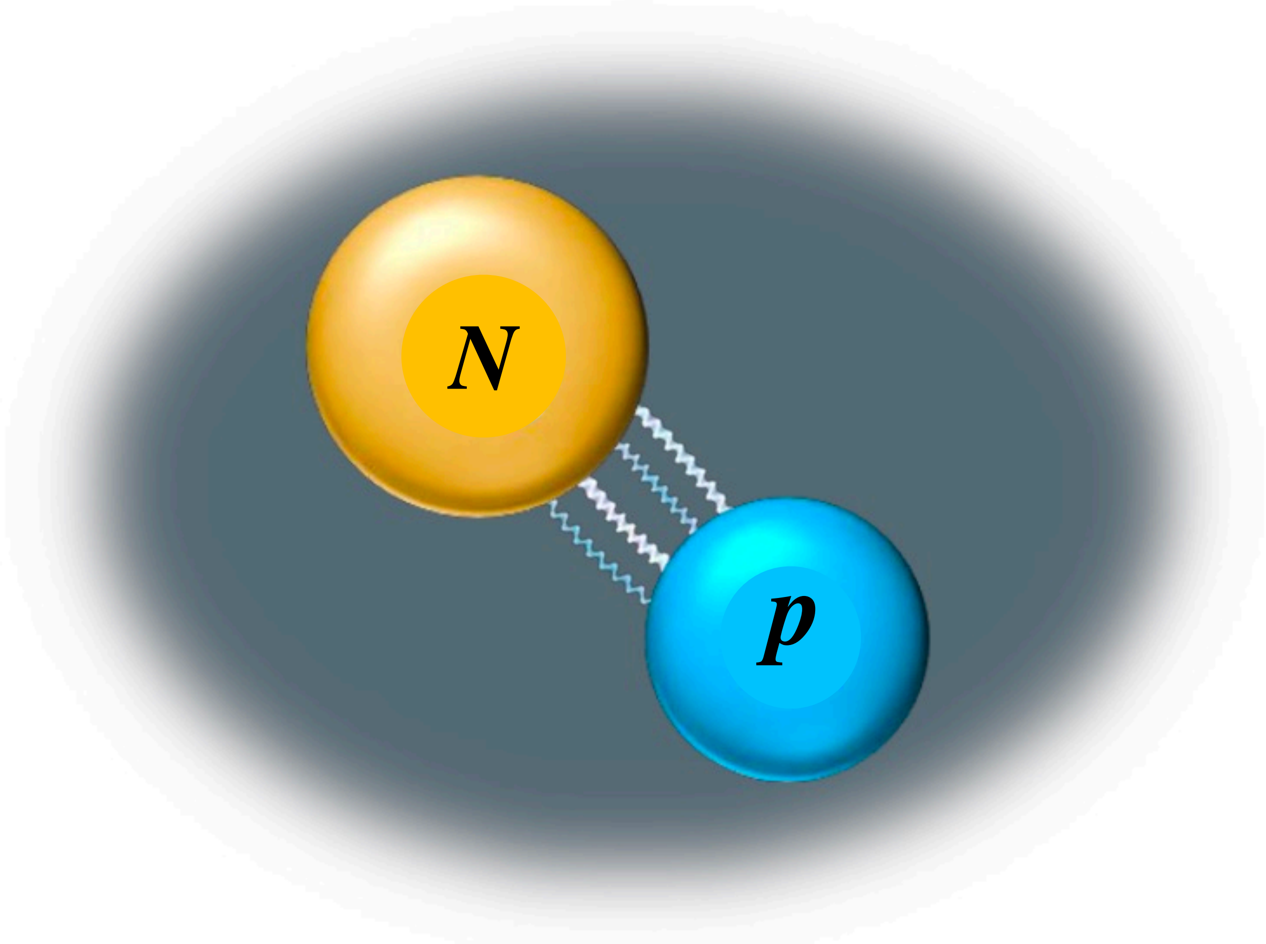
Outline

- Motivation
- proton – Λ interaction from the HALQCD approach
- proton – Λ scattering from the Lüscher's finite volume method
- Summary and Prospect

Motivation

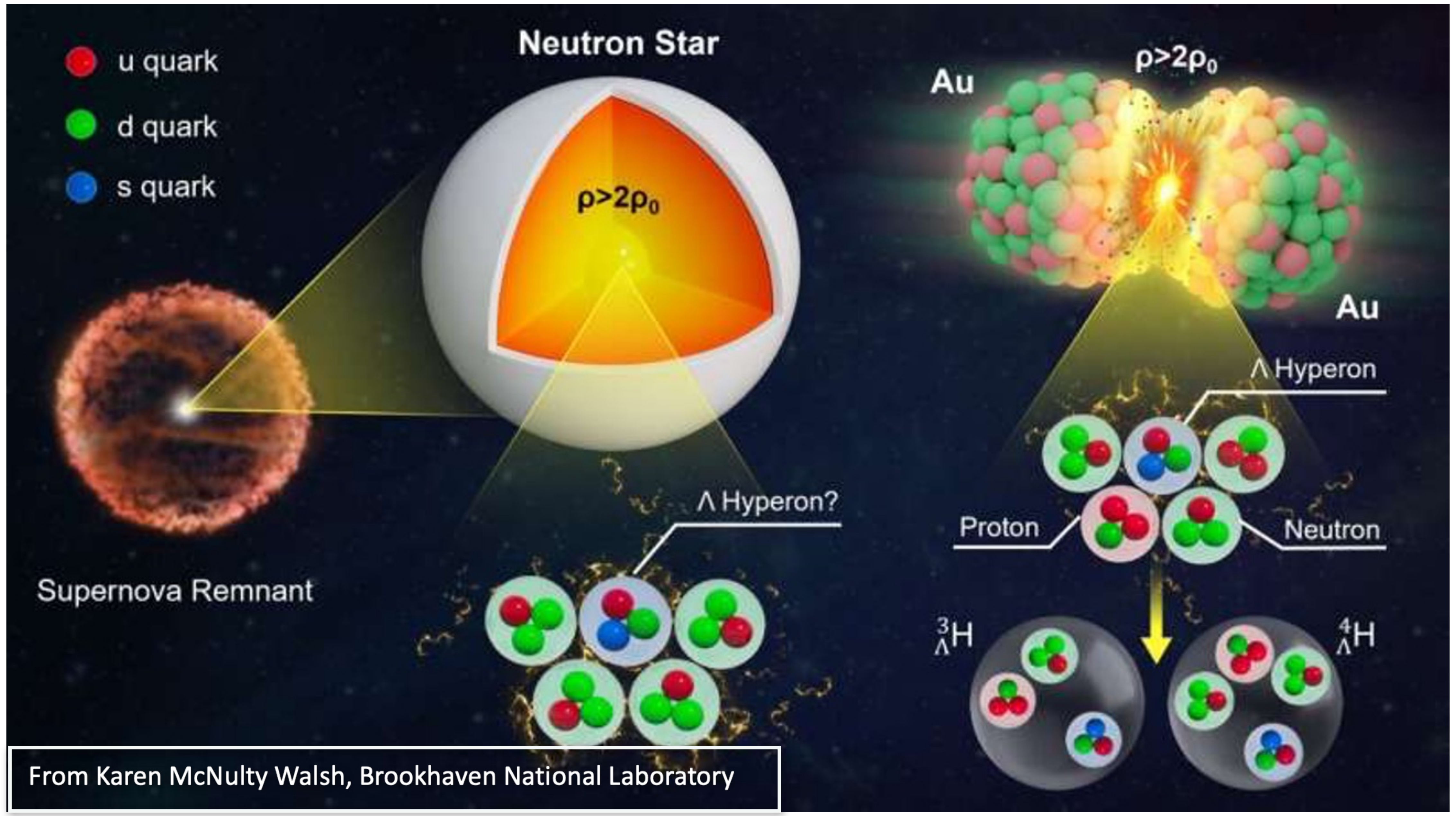
exotic hadron



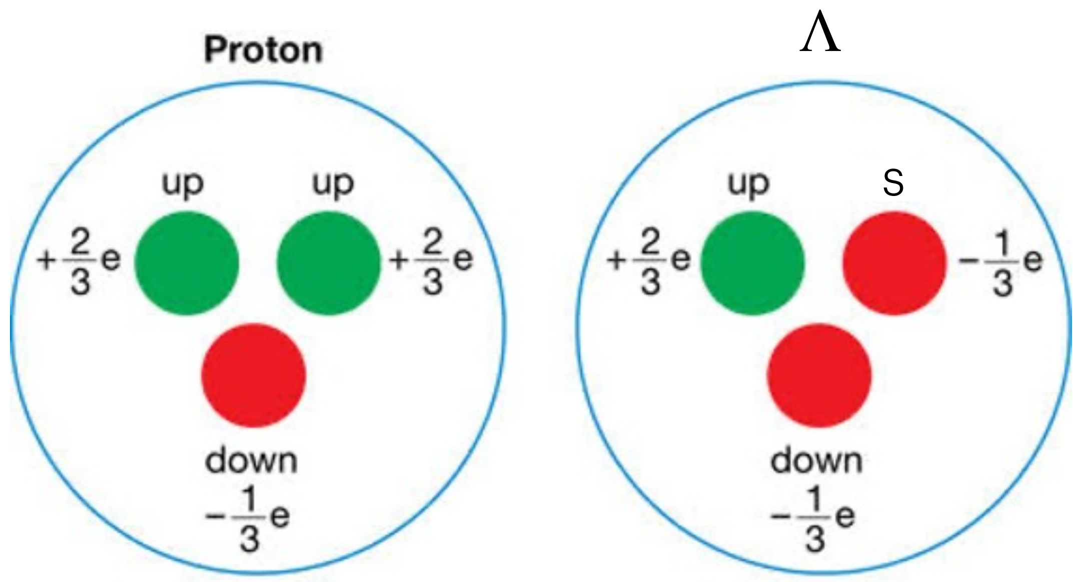


- ✓ hadronic molecule?
- ✓ multiquark state?
- ✓ the effective potential ?
- ✓ the binding energy?

Motivation



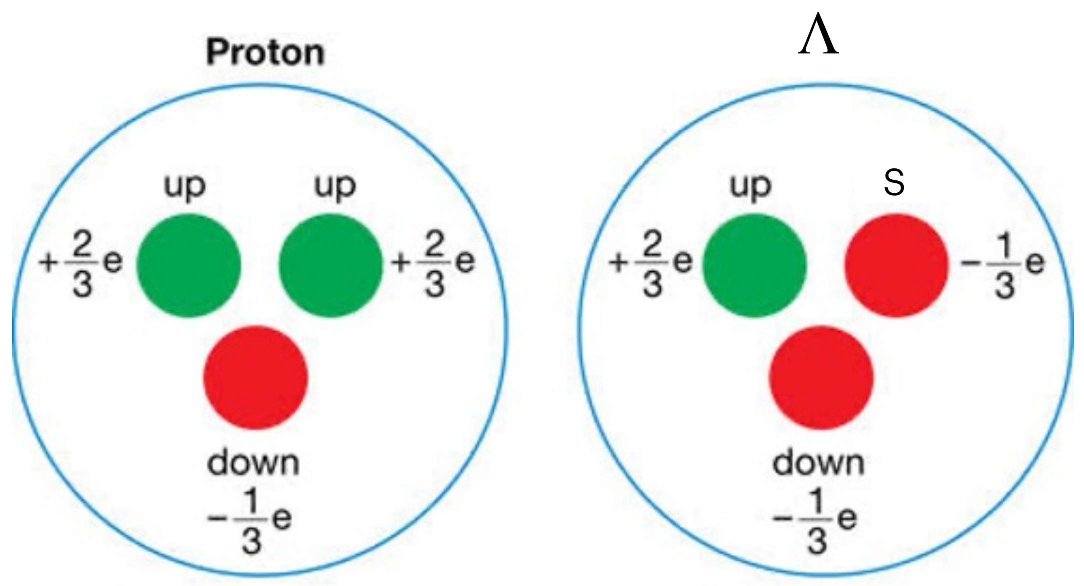
- Hyperon-Nucleon interactions



Also concerned

- ✓ hadronic molecule?
- ✓ multiquark state?
- ✓ the effective potential ?
- ✓ the binding energy?
- ✓ "hyperon puzzle" in neutron stars

Motivation



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)

J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)

S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)

S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

✓ hypernuclei:

[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

✓ YN scattering:

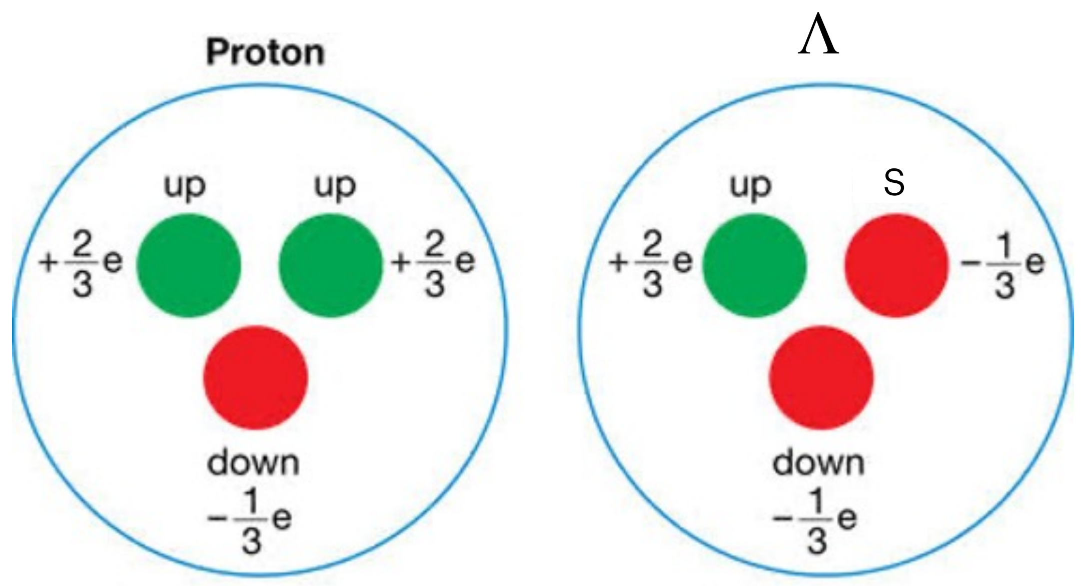
G. Alexander, et al. Phys. Rev. 173, 1452 (1968)

B. Sechi-Zorn, et al. Phys. Rev. 175, 1735 (1968)

J. A. Kadyk, et al. Nucl. Phys. B 27, 13 (1971)

BESIII Collaboration,PhysRevLett.132.231902(2024)

Motivation



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)

J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)

S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)

S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

$$C(\boldsymbol{k}^*) \approx 1 + \frac{|f(k)|^2}{2\boldsymbol{R_G}^2} F(d_0) + \frac{2\mathrm{Re}f(k)}{\sqrt{\pi}\boldsymbol{R_G}} F_1(2kR) - \frac{\mathrm{Im}f(k)}{\boldsymbol{R_G}} F_2(2k\boldsymbol{R_G})$$

$$\frac{1}{f(k)} \approx \frac{1}{\boldsymbol{f_0}} + \frac{\boldsymbol{d_0}k^2}{2} - ik$$

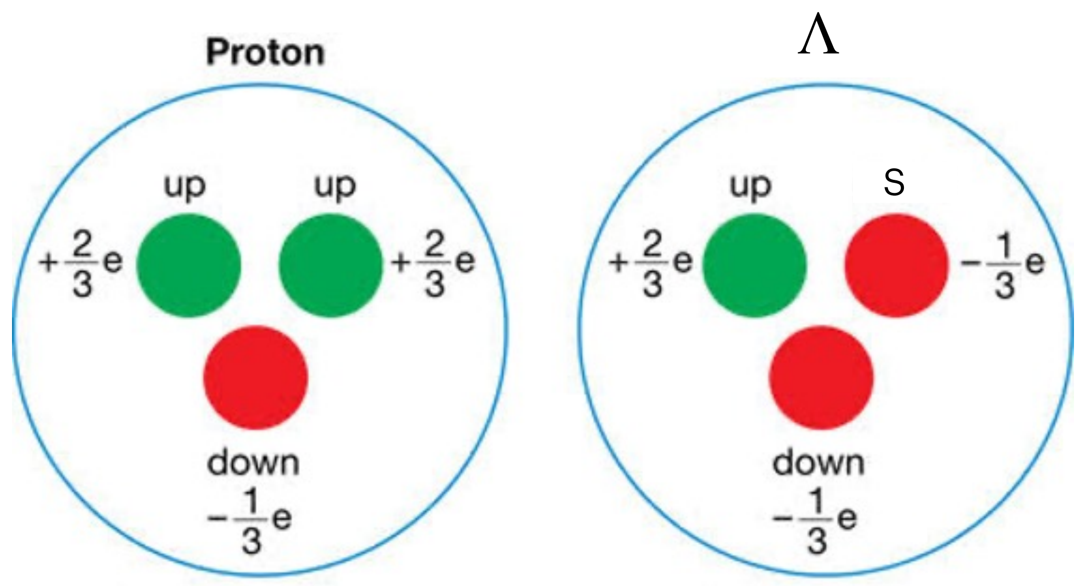
Different f_0 and d_0 for different spin states

B. Sechi-Zorn, et al. Phys. Rev. 175, 1735 (1968)

J. A. Kadyk, et al. Nucl. Phys. B 27, 13 (1971)

BESIII Collaboration,PhysRevLett.132.231902(2024)

Motivation



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)

J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)

S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)

S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

✓ hypernuclei:

[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

PHYSICAL REVIEW LETTERS

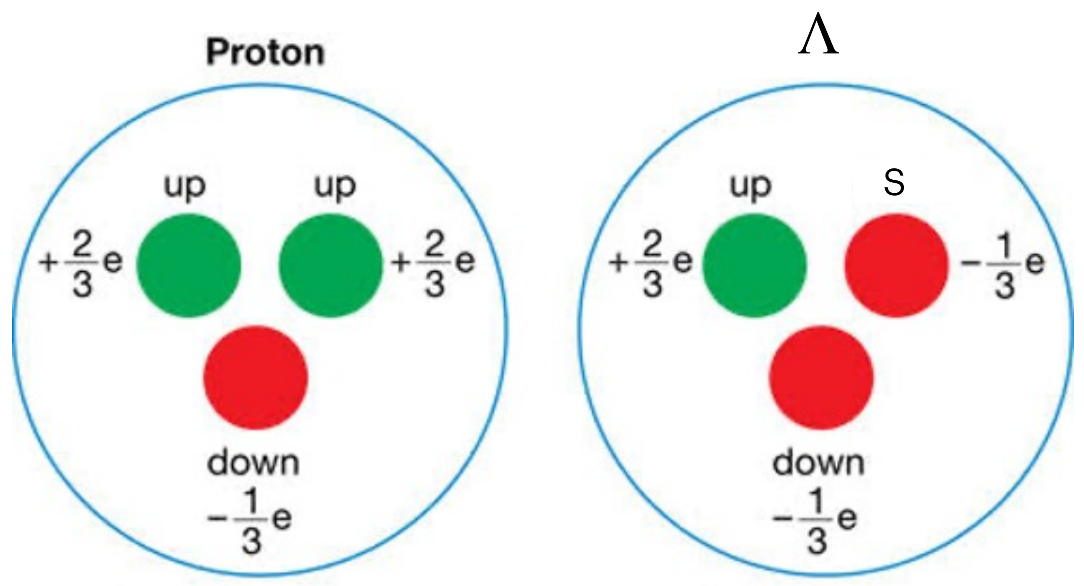
Highlights Recent Accepted Collections Authors Referees Search Press About

Featured in Physics | Editors' Suggestion

Observation of Coulomb-Assisted Nuclear Bound State of $\Xi^-{-}^{14}\mathrm{N}$ System

S. H. Hayakawa *et al.* (J-PARC E07 Collaboration)
Phys. Rev. Lett. **126**, 062501 – Published 11 February 2021

Motivation



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)

J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)

S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)

S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

✓ hypernuclei:

[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

✓ YN scattering:

G. Alexander, et al. Phys. Rev. 173, 1452 (1968)

B. Sechi-Zorn, et al. Phys. Rev. 175, 1735 (1968)

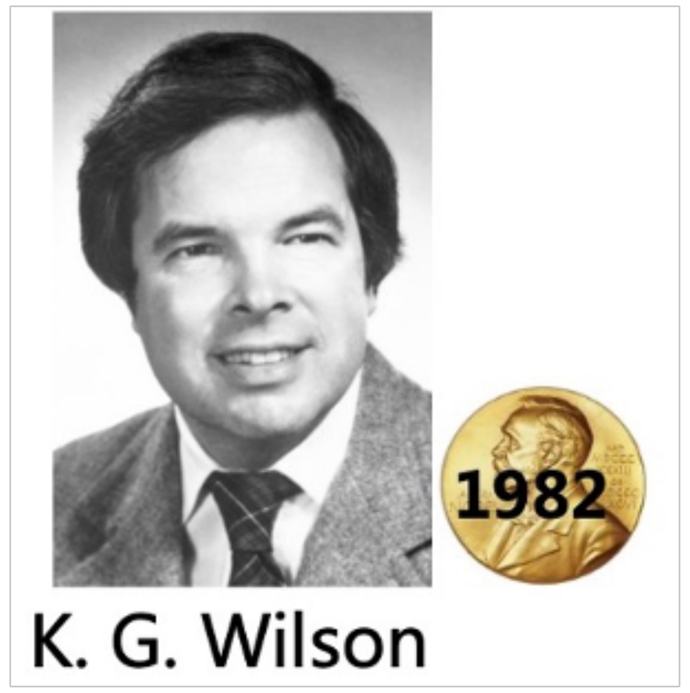
J. A. Kadyk, et al. Nucl. Phys. B 27, 13 (1971)

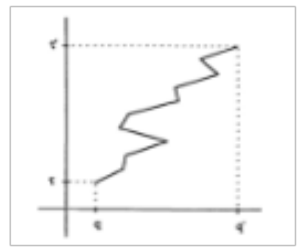
BESIII Collaboration,PhysRevLett.132.231902(2024)

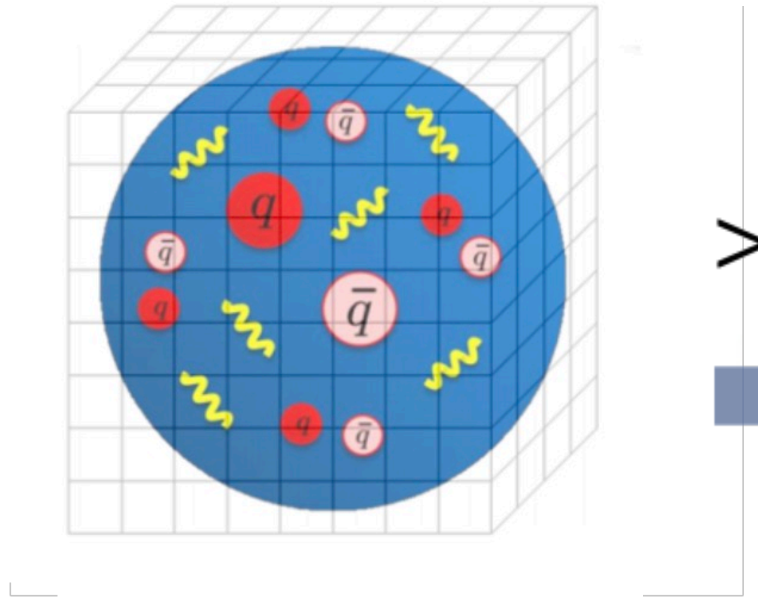
large uncertainty due to short-lifetime of hyperon beams

Lattice QCD

LatticeQCD(Wilson,1974): the ab-initio non-perturbative method





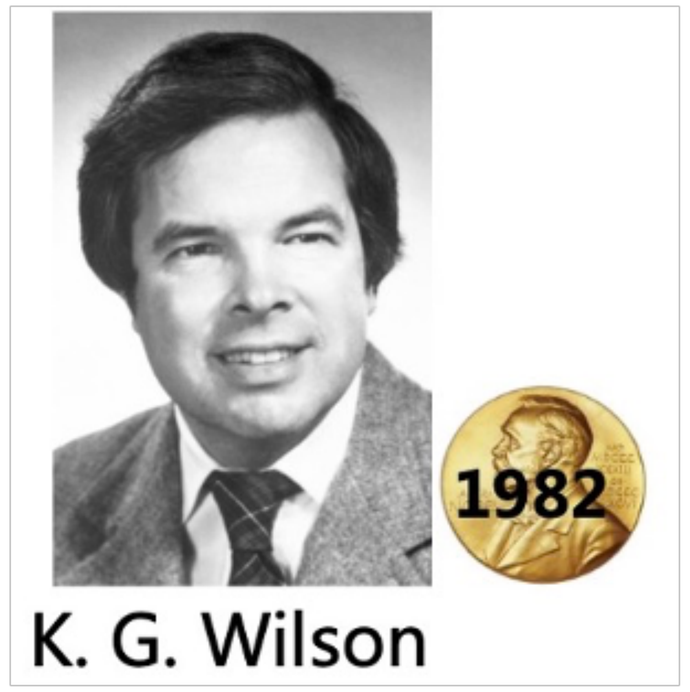


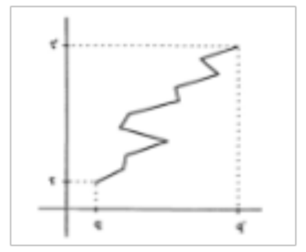
Lattice QCD

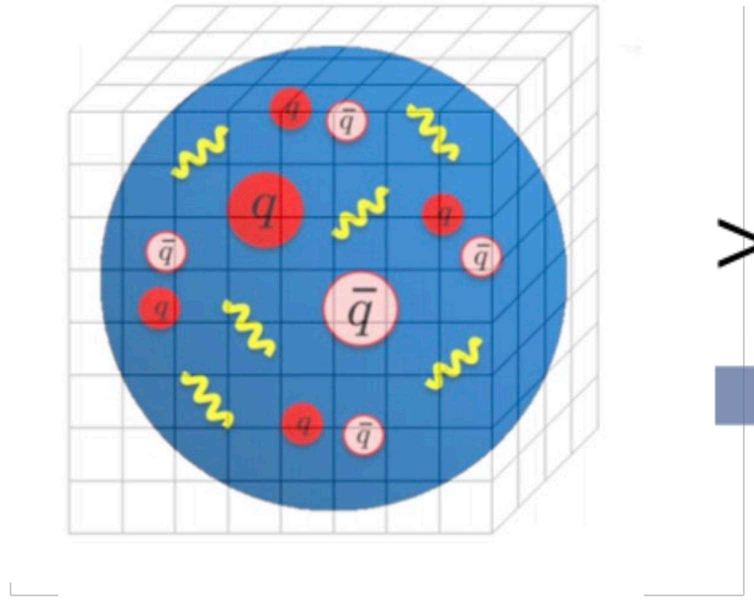
$> 2^{1000000}$

Supercomputer

Lattice QCD



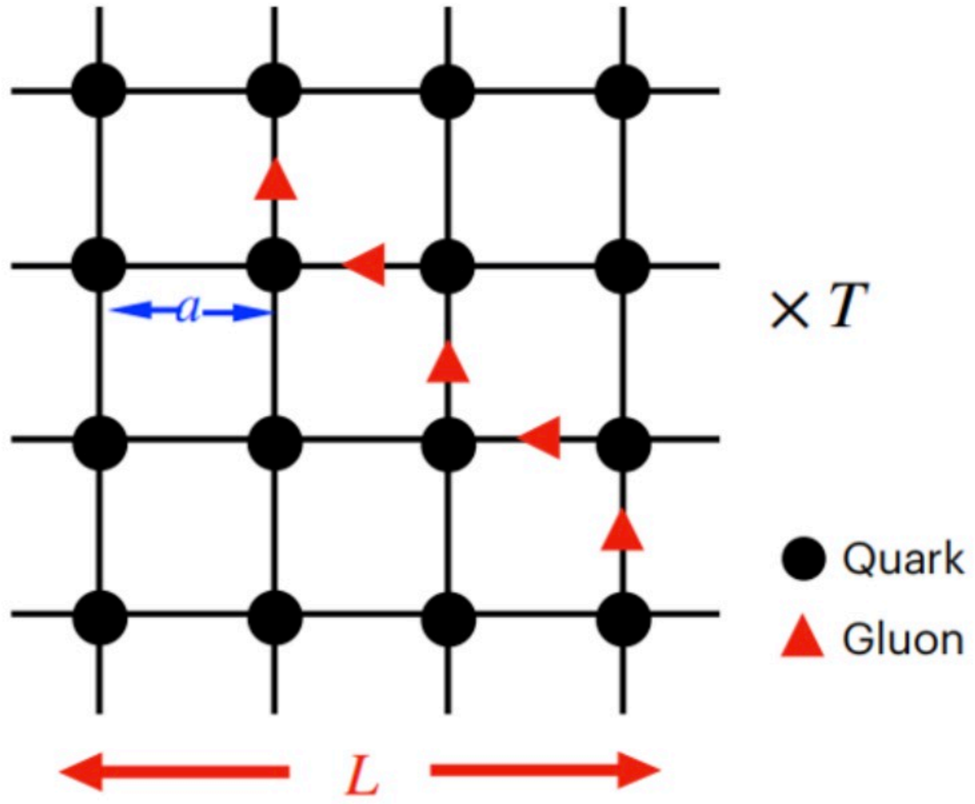




Lattice QCD

$> 2^{1000000}$

Supercomputer

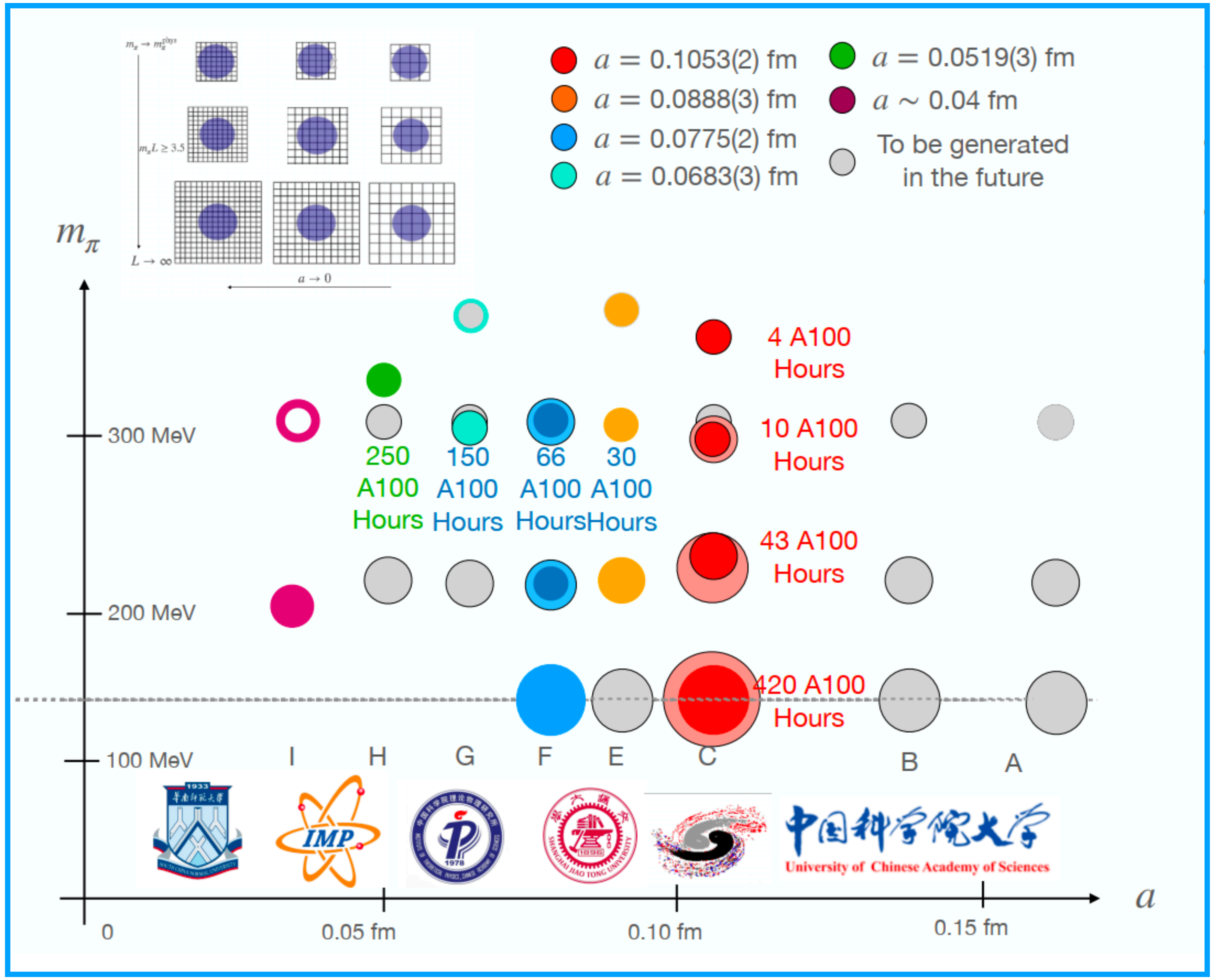


- Quark on discrete lattice:
  consider both IR and UV effects:

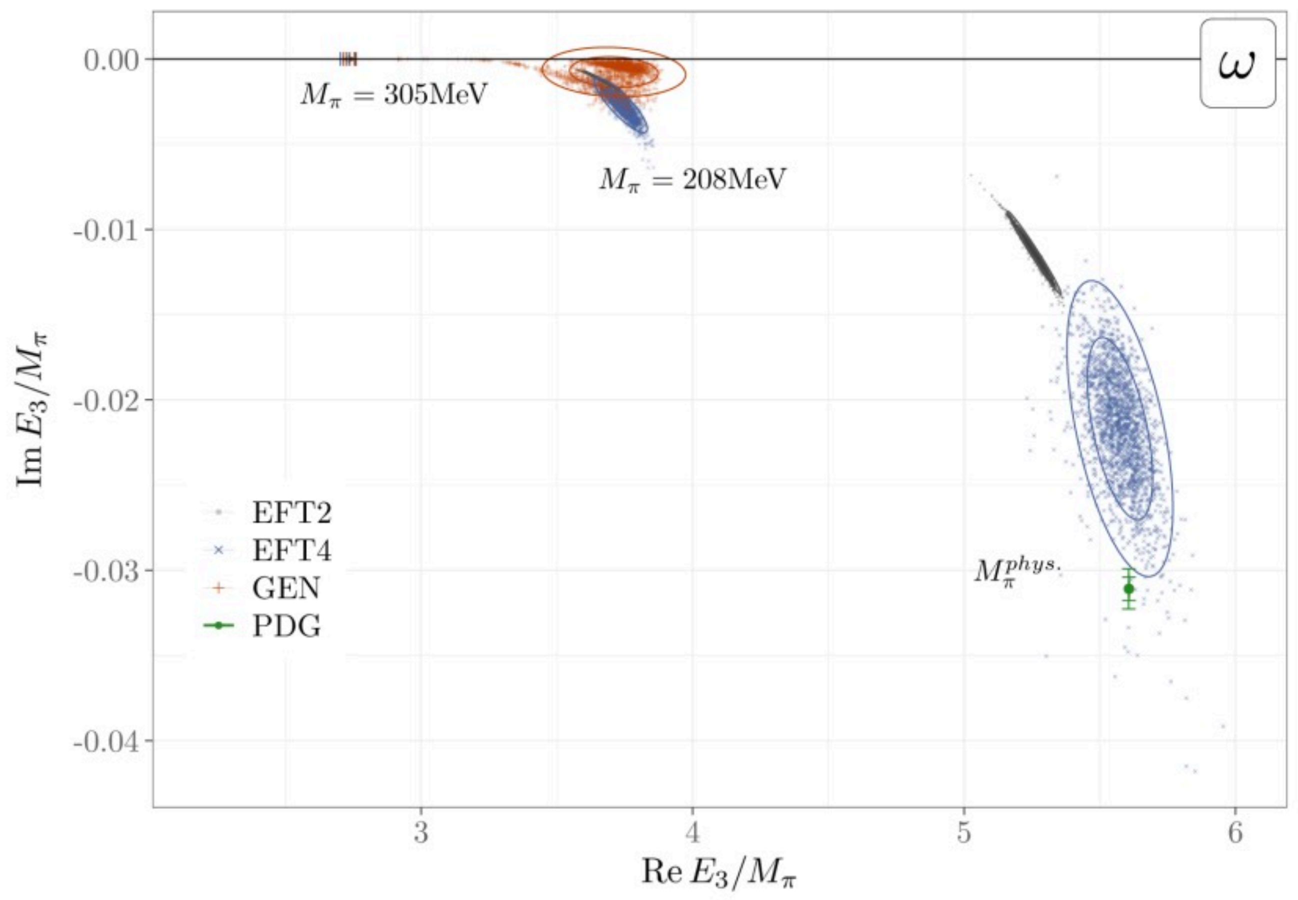
$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

New Lattice QCD configurations

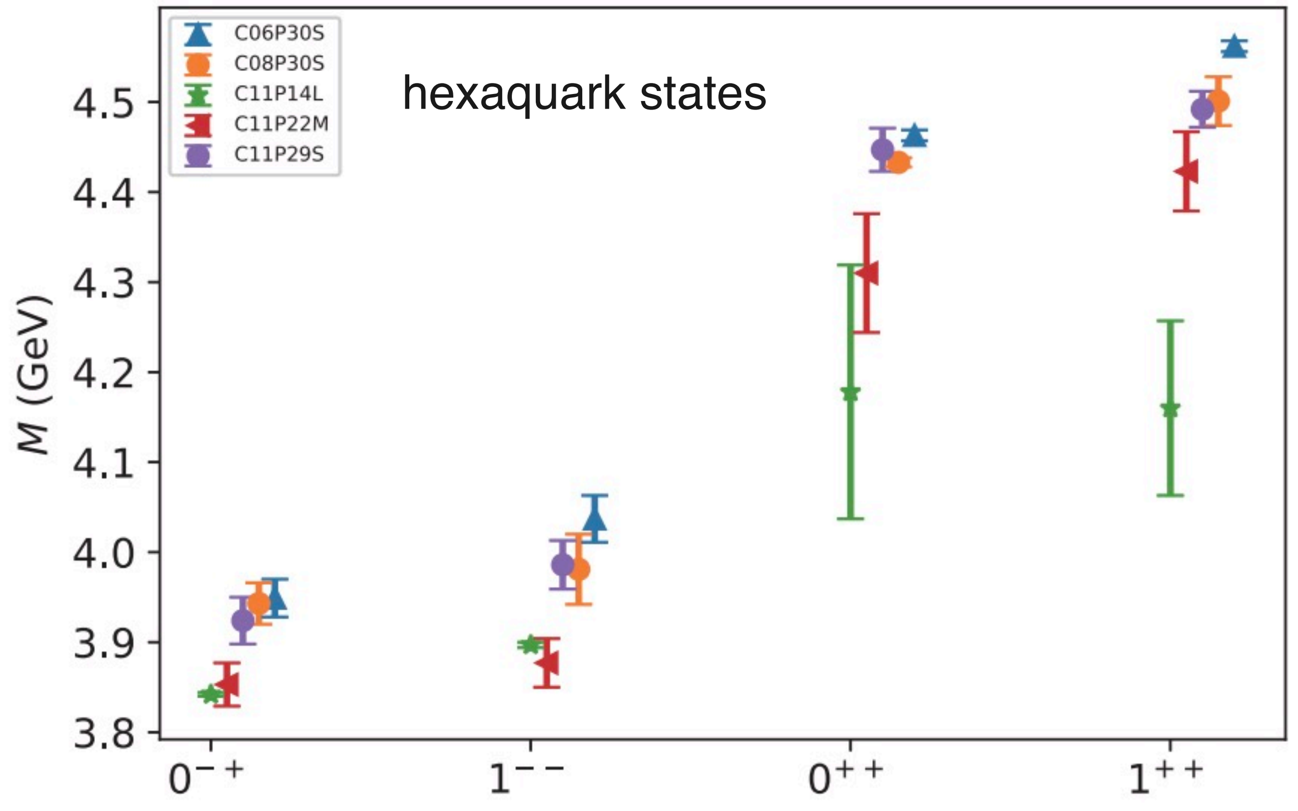
CLQCD



Hu, et.al., PRD 109, 054507 (2024)



Phys.Rev.Lett. 133 (2024) 21, 211906



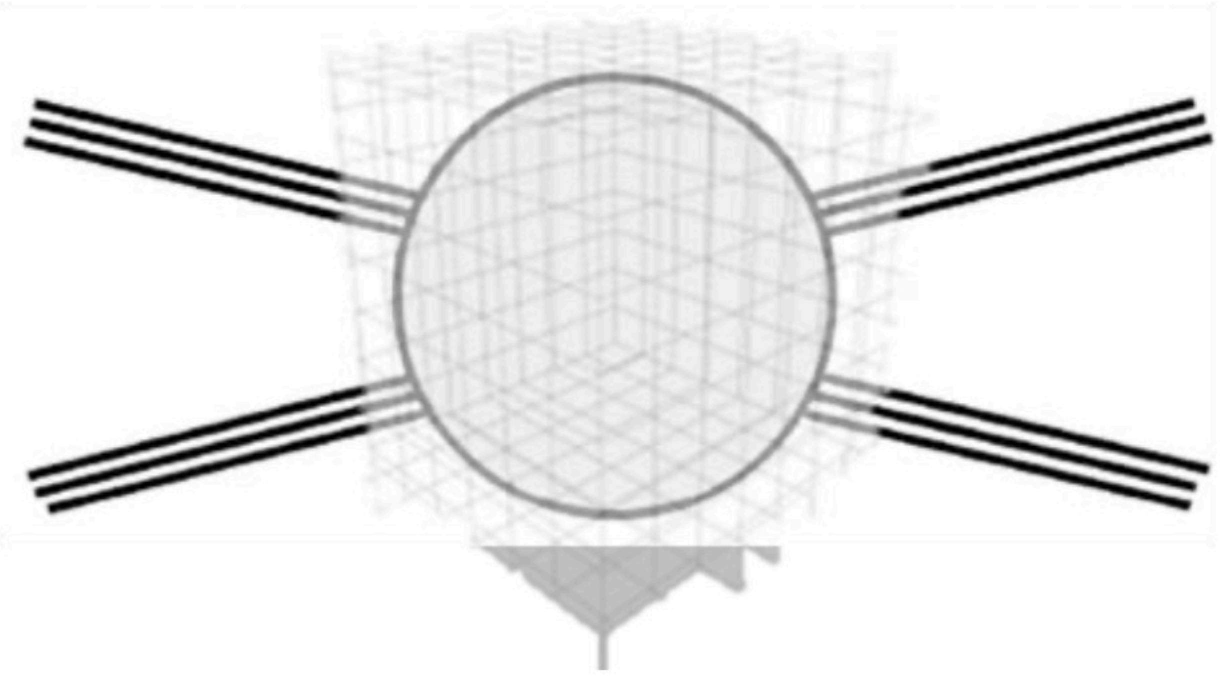
Sci.China Phys.Mech.Astron. 67 (2024) 1, 211011

Outline

- Motivation
- proton – Λ interaction from the HALQCD approach
- proton – Λ scattering from the Lüscher's finite volume method
- Summary and Prospect

The HALQCD method

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0 \rangle$$

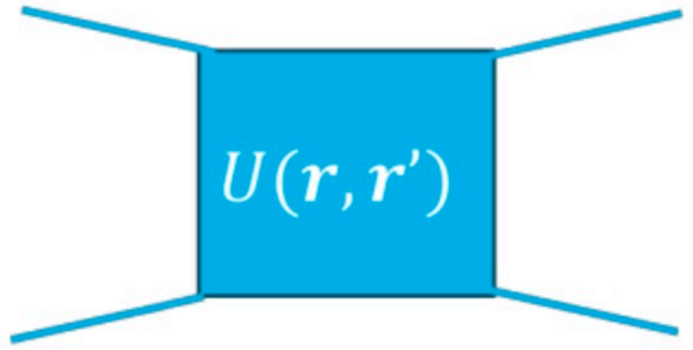


Lattice simulations

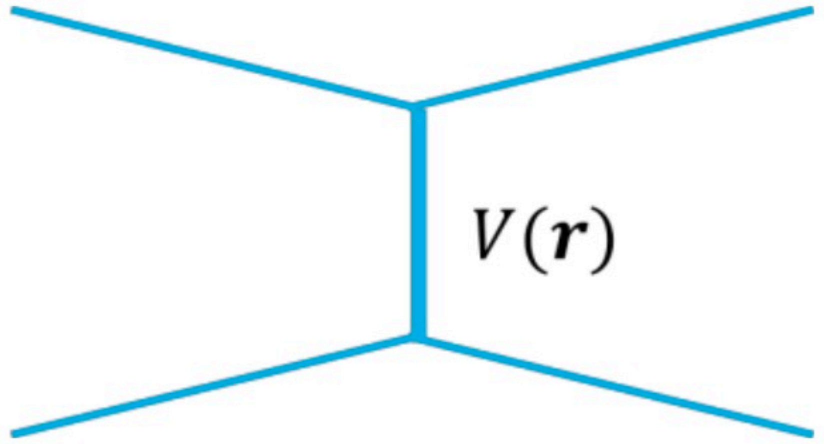
$$\Psi^{W_n}(\vec{r})e^{-W_n t}$$

Nambu-Bethe-Salpeter
wave function

$$(E_k - H_0)\Psi^{W}(\vec{r}) = \int d^3\vec{r'}\, U(\vec{r},\vec{r'})\Psi^{W}(\vec{r'})$$



$$U(\mathbf{r},\mathbf{r}') = V(\mathbf{r},\nabla)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$



The HALQCD method

To enhance the signal, the following ratio was attempted:

$$R_{p\Lambda}(\vec{r},t) = \frac{C_{p\Lambda}(\vec{r},t)}{C_p(t)C_\Lambda(t)} = \sum_n A_n' \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

Assume the nonlocal potential is energy independent

$$\left[-H_0 - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right] R(\vec{r},t) = \int d^3\overrightarrow{r'}\, U(\vec{r},\overrightarrow{r'}) R(\overrightarrow{r'},t)$$

Then the effective potential leads to

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r},t)}{R(\vec{r},t)}$$

Lattice setup for $p - \Lambda$

	β	$L^3 \times T$	a	m_π	N_{confs}
C24P29	6.20	$24^3 \times 72$	0.10530	292.7(1.2)	872
C32P29	6.20	$32^3 \times 64$	0.10530	292.4(1.1)	984
C48P23	6.20	$48^3 \times 96$	0.10530	225.6(0.9)	265
C48P14	6.20	$48^3 \times 96$	0.10530	135.5(1.6)	259
F48P21	6.41	$48^3 \times 96$	0.07746	207.2(1.1)	220
F48P30	6.41	$48^3 \times 96$	0.07746	303.4(0.9)	359
H48P32	6.72	$48^3 \times 144$	0.05187	317.2(0.9)	274

7 different ensembles: 3 different lattice spacings,
pion mass 140MeV~320MeV,
different volume

Results can be extend to the physical point and the continuum limit.

Two-point Correlation Function

One-particle operators and two-particle operators

$$p_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x) - d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}u^c_\rho(x)$$

$$\Lambda_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}s^c_\rho(x)$$

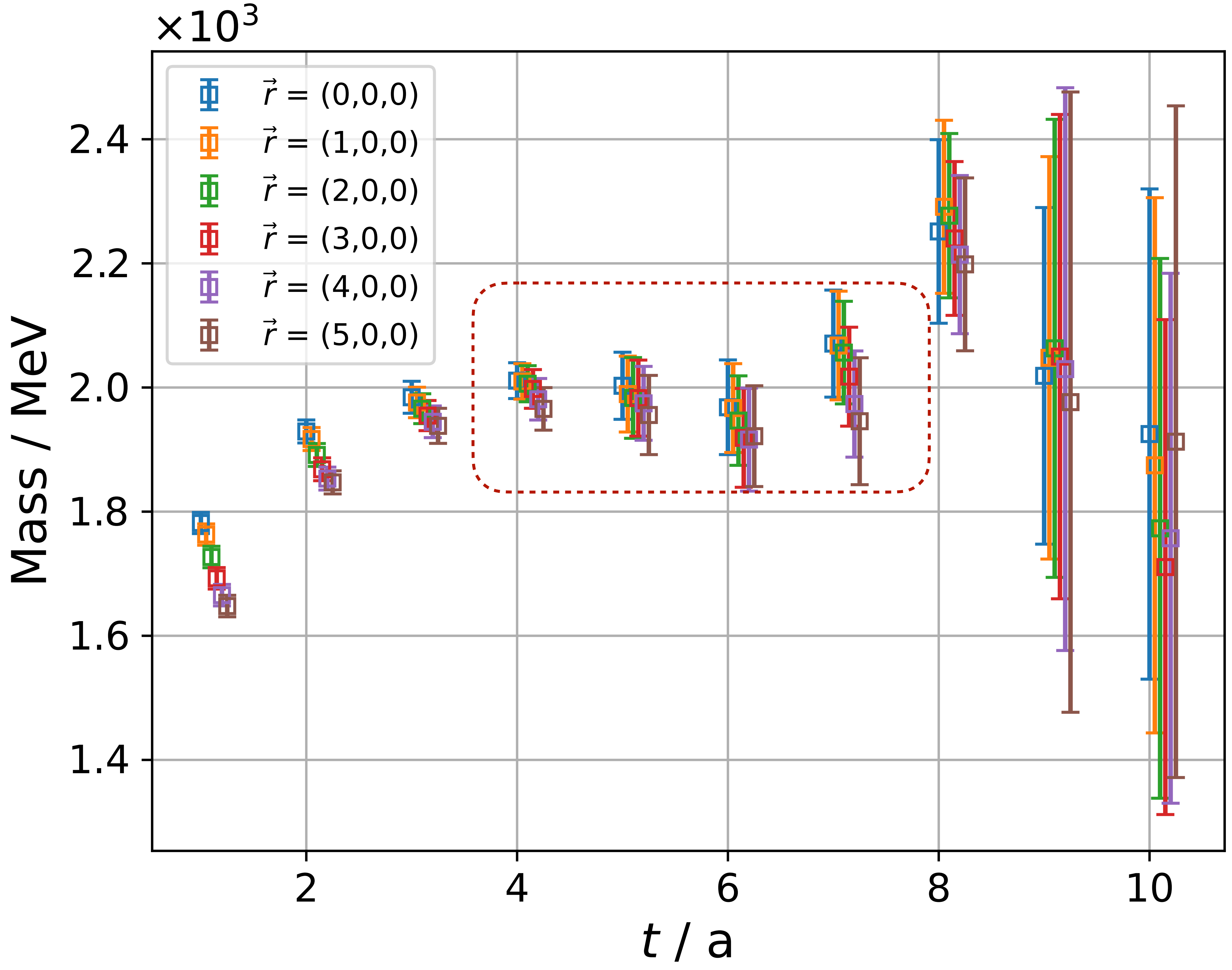
$$p\Lambda_{\rho\mathtt{m}}(t) = \sum_{\vec{x}_1,\vec{x}_2\in\Lambda_S}\psi^{[D]}_{\mathtt{m}}(\vec{x}_1,\vec{x}_2)\sum_{\sigma,\sigma'}v^\rho_{\sigma\sigma'} \frac{1}{\sqrt{2}}[p_\sigma(\vec{x}_1,t)\Lambda_{\sigma'}(\vec{x}_2,t) + (-1)^{1-\delta_{\rho 0}}\Lambda_\sigma(\vec{x}_1,t)p_{\sigma'}(\vec{x}_2,t)],$$

For the p-Λ system, correlation functions can be obtained

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}}\langle 0|(p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t))^D \bar{J}^H_{p\Lambda}(0)|0\rangle$$

The effective mass

We find the appropriate time slice for the ground state saturation of the system.



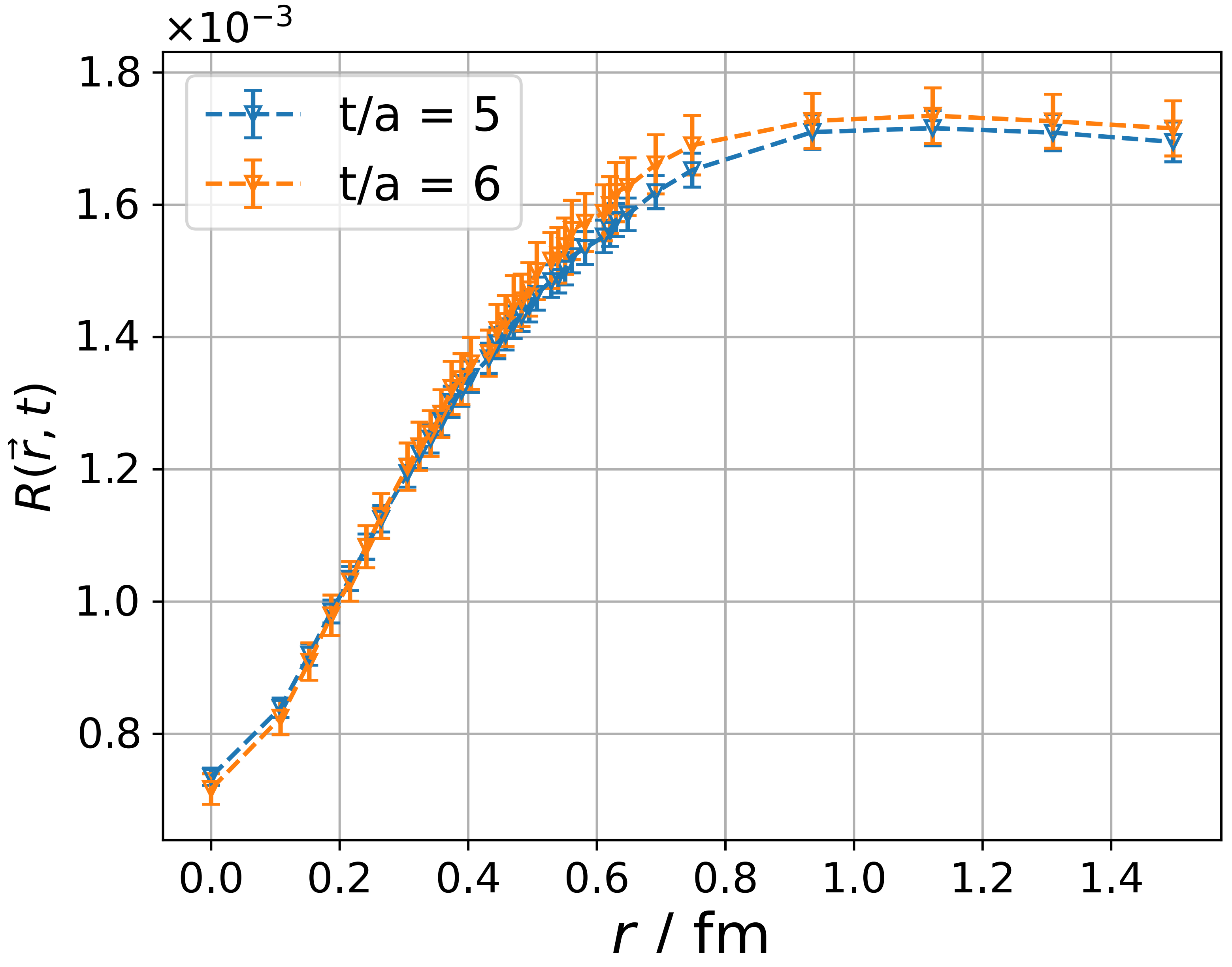
effective mass of
the dibaryon system

The HALQCD method

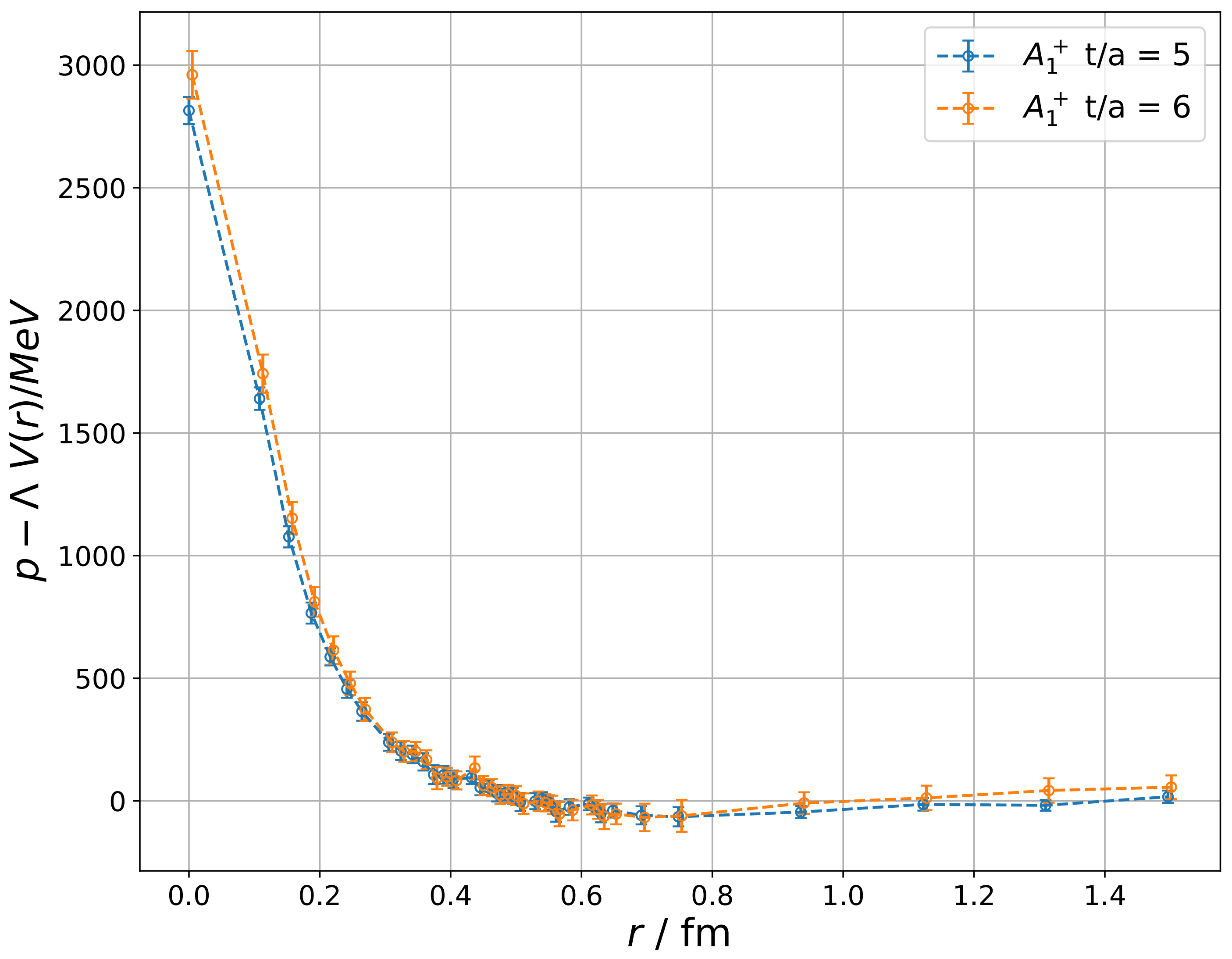
We extract the NBS wave function and the effective potential on the timeslice $t/a = 5{,}6$.

$$R_{p\Lambda}(\vec{r},t) = \frac{C_{p\Lambda}(\vec{r},t)}{C_p(t)C_\Lambda(t)} = \sum_n A_n' \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r},t)}{R(\vec{r},t)}$$



NBS wave function



The effective potential

Phase shift from the HALQCD method

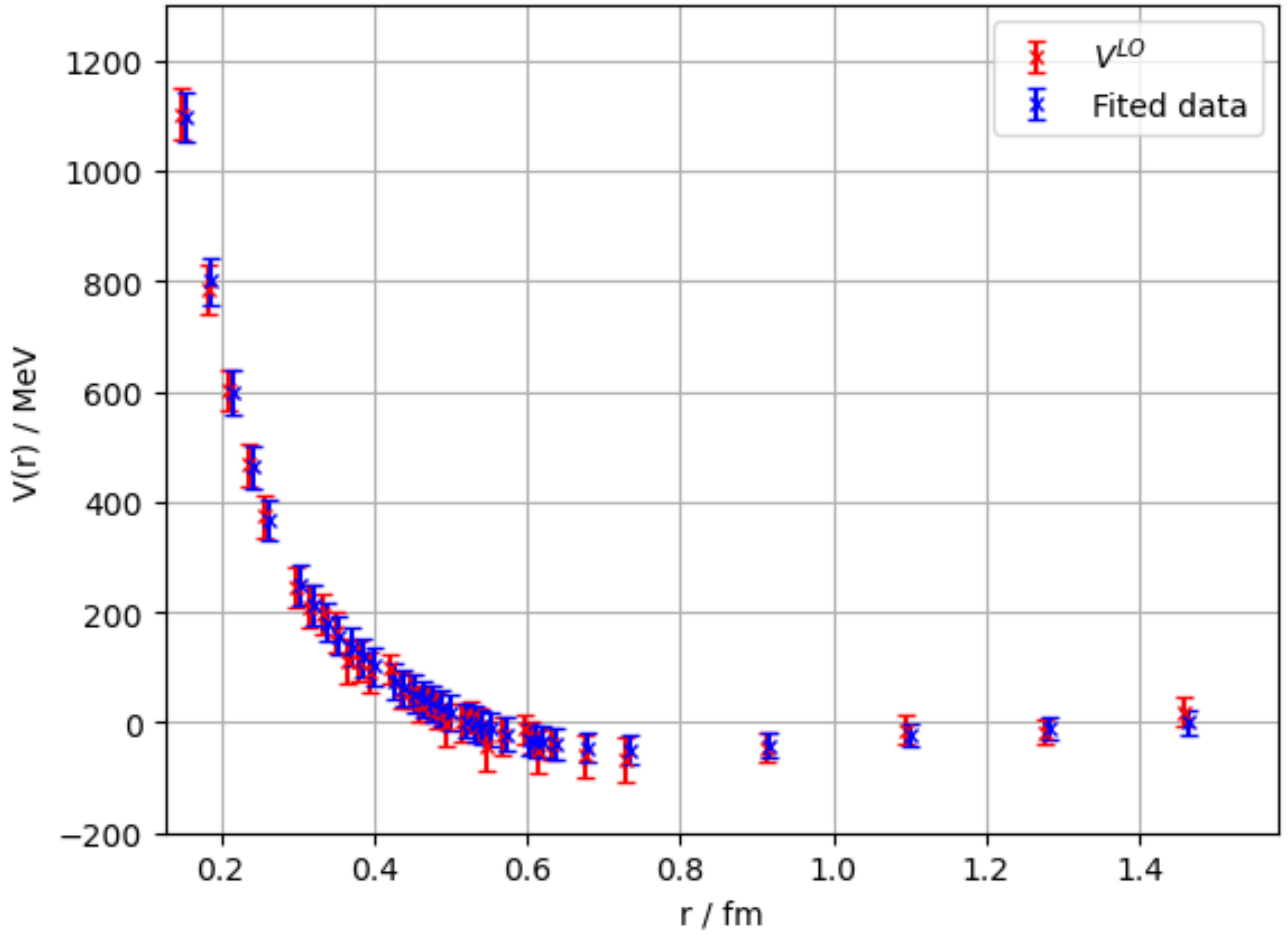
We parameterize the effective potential in this form

Argonne-type form factor

$$V(r) = v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$

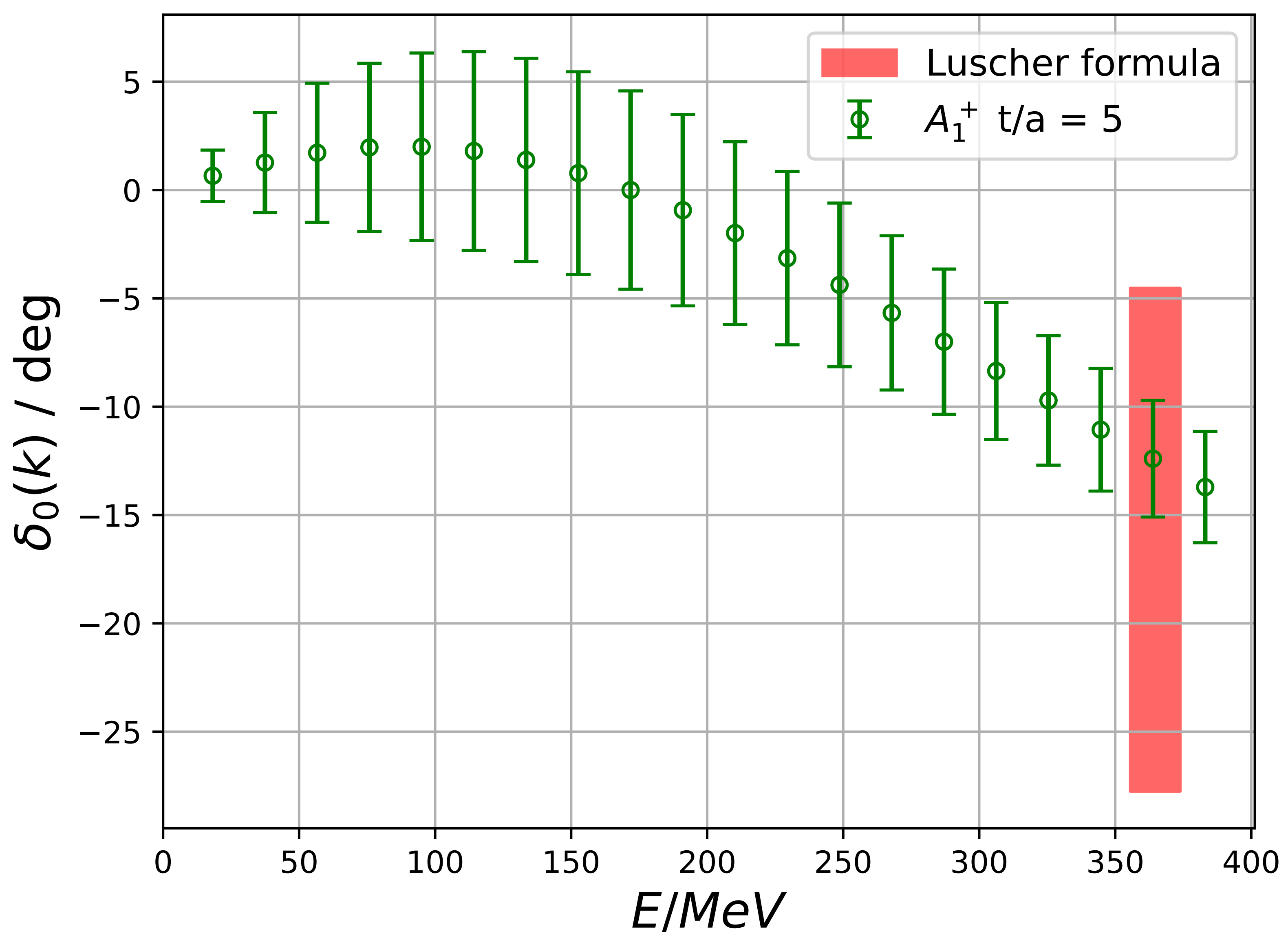
Gaussian form

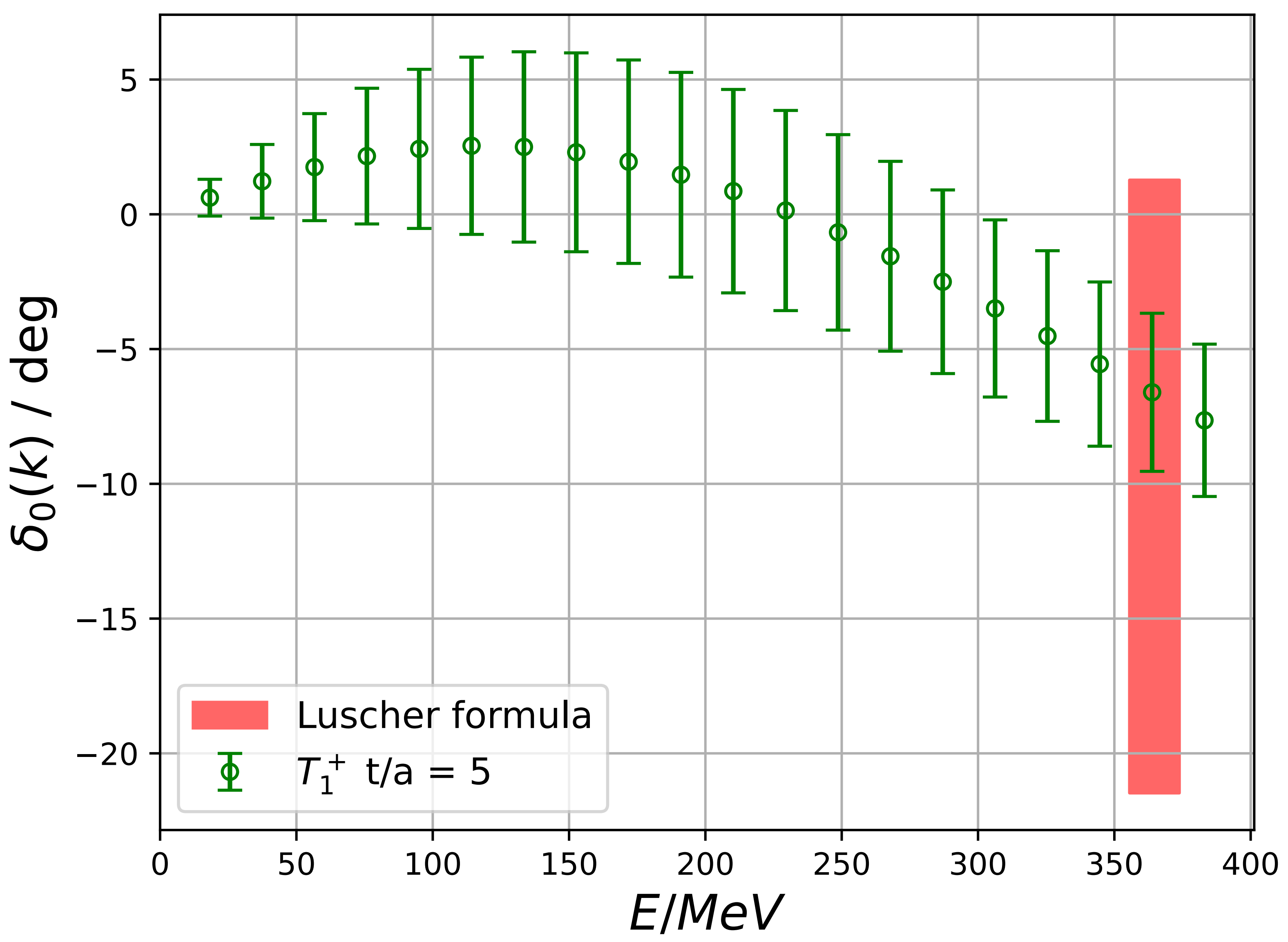
two-pion exchange



Phase shift from the HALQCD method

The scattering phase shift can be obtained by solving the Schrodinger equation





Outline

- Motivation
- proton $-$ Λ interaction from the HALQCD approach
- proton $-$ Λ scattering from the Lüscher′s finite volume method
- Summary and Prospect

Lüscher's **finite volume formula**

The direct method for scattering on the lattice: Lüscher's **finite volume method**

M. Lüscher, Nucl. Phys. B354, 531(1991)

Finite volume energies

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}$$

$$k = n * 2\pi/L$$

格点计算

Lüscher formula

Infinite volume scattering

$$\delta_l \,.\, a_l \,.\, r_l$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length effective range

无限体积中的散射

Lüscher′s **finite volume formula**

The direct method for scattering on the lattice: **Lüscher′s finite volume method**

M. Lüscher, Nucl. Phys. B354, 531(1991)

Finite volume energies

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2}$$

$$k = n * 2\pi/L$$

Due to the interaction, $\bar{k}^2$ differs from its free counter-part k^2

格点计算

Lüscher formula

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

$$q^2 = \bar{k}^2 * \frac{L^2}{(2\pi)^2}$$

$$\begin{cases} q^2 = n^2 \\ q^2 < n^2 \\ q^2 > n^2 \end{cases}$$

Infinite volume scattering

$$\delta_l \,.\, a_l \,.\, r_l$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length effective range

无限体积中的散射

Lattice calculation

The two-particle operators for the ${}^3S_1(T_1^+)$ and ${}^1S_0(A_1^+)$ channels

$$
\begin{aligned}
&A_1^+ : \\
&\mathcal{O}_{A_1^+} = p_{\frac{1}{2}}(0)\Lambda_{-\frac{1}{2}}(0) - p_{-\frac{1}{2}}(0)\Lambda_{\frac{1}{2}}(0) \\
&\mathcal{O}'_{A_1^+} = p_{\frac{1}{2}}(e_x)\Lambda_{-\frac{1}{2}}(-e_x) - p_{-\frac{1}{2}}(e_x)\Lambda_{\frac{1}{2}}(-e_x) + p_{\frac{1}{2}}(-e_x)\Lambda_{-\frac{1}{2}}(e_x) - p_{-\frac{1}{2}}(-e_x)\Lambda_{\frac{1}{2}}(e_x) \\
&\quad + p_{\frac{1}{2}}(e_y)\Lambda_{-\frac{1}{2}}(-e_y) - p_{-\frac{1}{2}}(e_y)\Lambda_{\frac{1}{2}}(-e_y) + p_{\frac{1}{2}}(-e_y)\Lambda_{-\frac{1}{2}}(e_y) - p_{-\frac{1}{2}}(-e_y)\Lambda_{\frac{1}{2}}(e_y) \\
&\quad + p_{\frac{1}{2}}(e_z)\Lambda_{-\frac{1}{2}}(-e_z) - p_{-\frac{1}{2}}(e_z)\Lambda_{\frac{1}{2}}(-e_z) + p_{\frac{1}{2}}(-e_z)\Lambda_{-\frac{1}{2}}(e_z) - p_{-\frac{1}{2}}(-e_z)\Lambda_{\frac{1}{2}}(e_z)
\end{aligned}
$$

The correlation matrix of the form

$$C_i^{\alpha\beta}(t) = \langle 0 | \mathcal{O}_i^{\alpha}(t)\mathcal{O}_i^{\beta\dagger}(0) | 0 \rangle$$

Solving the so-called Generalized Eigenvalue Problem(GEVP)

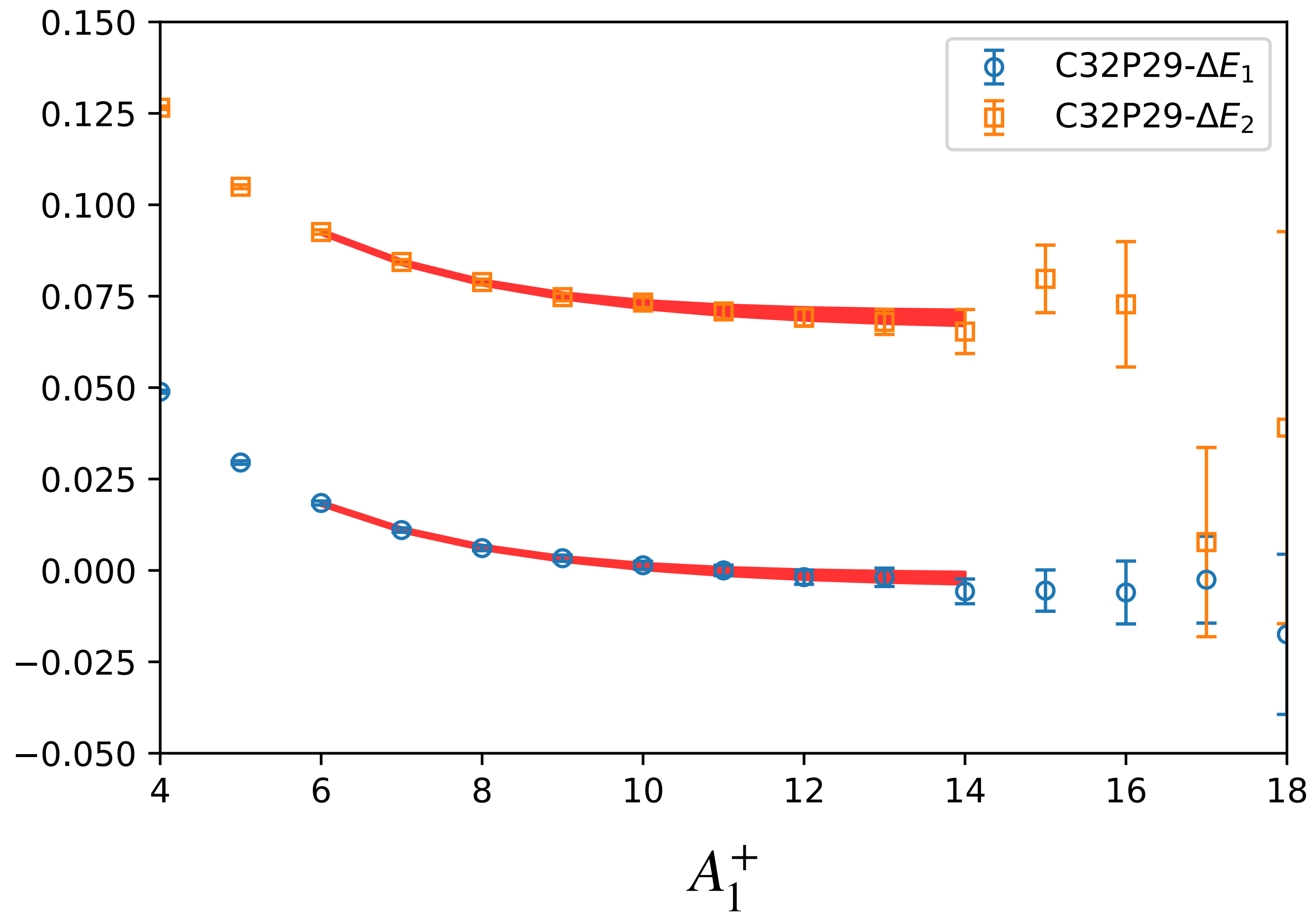
$$C(t)v_n(t) = \lambda_n(t)C(t_r)v_n(t)$$

$$\lambda_n(t) = c_0 e^{-E_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

The Effective mass of the YN system

To enhance the signal, the following ratio was attempted:

$$R_\alpha(t, t_0) = \frac{\lambda_\alpha(t, t_0)}{C^p(t - t_0, \mathbf{0}) C^\Lambda(t - t_0, \mathbf{0})} \propto e^{-\Delta E_\alpha (t - t_0)}$$



Preliminary	C32P29	C48P23	C48P14	F48P30	F48P21	H48P32
Irrep	ΔE_0	ΔE_0	ΔE_0	ΔE_0	ΔE_0	ΔE_0
A_1^+(MeV)	$-5.43(0.56)$	$-2.44(0.56)$	$-1.69(0.56)$	$-2.04(0.51)$	$-1.78(0.76)$	$-2.55(0.38)$

Scattering length & effective range

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

Close to the threshold, a_0 and r_0 can be determined by minimizing the χ^2

$$\chi^2 = \sum_{L,n,n'} [E_n(L) - E_n^{sol.}(L, a_0, r_0)] C_{nn'}^{-1} [E_{n'}(L) - E_{n'}^{sol.}(L, a_0, r_0)],$$

Preliminary	$^1S_0$	
ensemble	a_0/fm	r_0/fm
H48P32	2.40(2.01)	4.48(0.84)

Summary and Prospect

- ✓ HALQCD method: p-Λ NBS wave function, the interaction potential, and phase shift;
- ✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift;

  Effective range expansion: scattering length and effective range;

- ➡ HALQCD method： HEFT
- ➡ Extrapolation: discretization error, pion mass, finite volume effect
- ➡ p-Λ: p-Σ coupled channel
- ➡ ${}^{3}_{\Lambda}H$: three-body problem

Summary and Prospect

- ✓ HALQCD method: p-Λ NBS wave function, the interaction potential, and phase shift;
- ✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift;

  Effective range expansion: scattering length and effective range;

- ➡ HALQCD method： HEFT
- ➡ Extrapolation: discretization error, pion mass, finite volume effect
- ➡ p-Λ: p-Σ coupled channel
- ➡ ${}^{3}_{\Lambda}H$: three-body problem

Thank you!

Back up

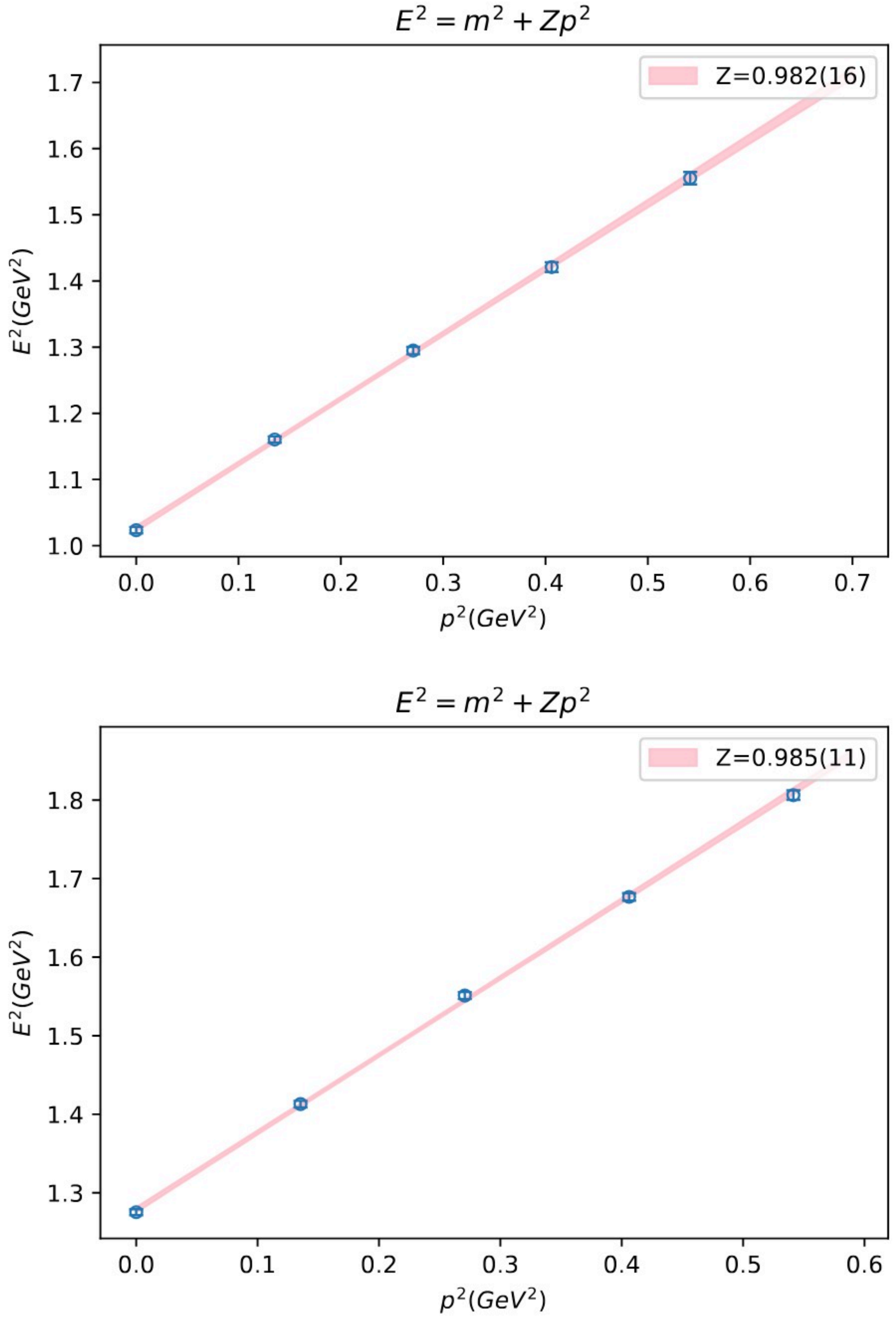
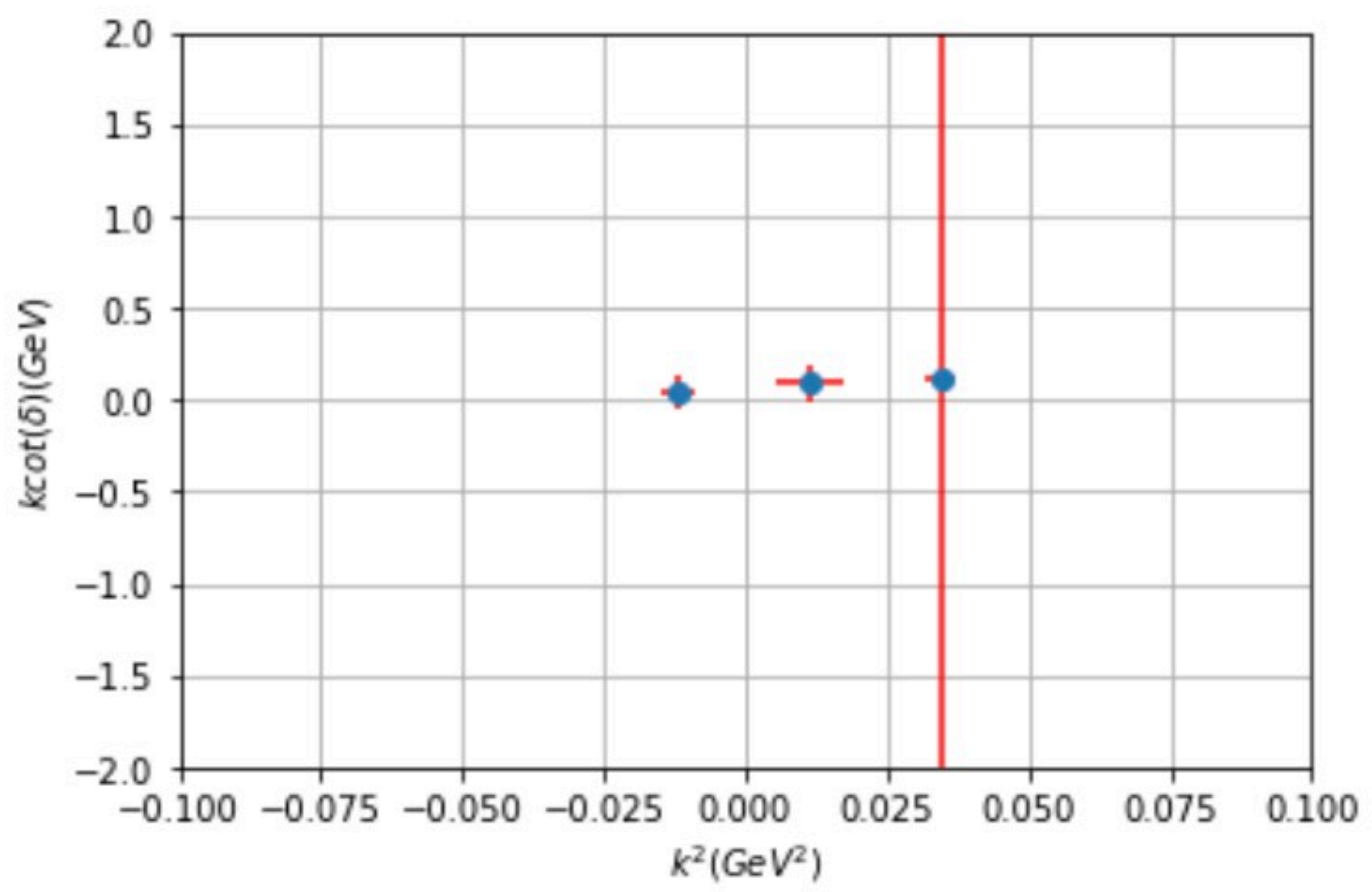


FIG. 1. Dispersion relation for proton (upper panel) and for Λ on the C32P29 ensembles(lower panel).

Back up



2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
$k\cot(\delta)(GeV)$
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
$k^2(GeV^2)$