Illustrating the liquid gas transition of nuclear matter through functional QCD approach

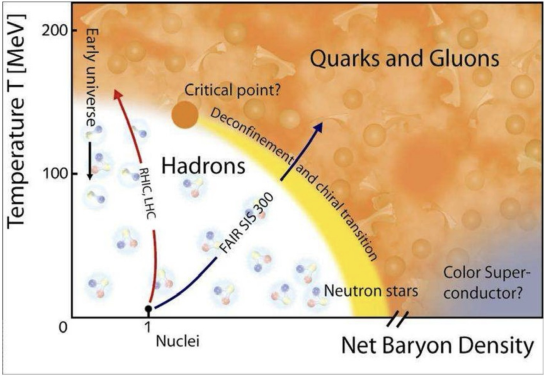
Fei Gao

Beijing Institute of Technology, BIT

fQCD collaboration :

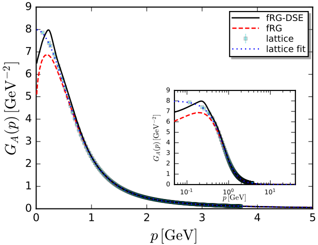
Braun, *Chen*, *Fu*, *Gao*, *Ihssen*, *Geissel*, *Huang*, *Lu*, *Pawlowski*, *Rennecke*, *Sattler*, *Schallmo*, *Stoll*, *Tan*, *Toepfel*, *Turnwald*, *Wen*, *Wessely*, *Wink*, *Yin*, *Zorbach*

- Chiral phase transition with a CEP located at $(T, \mu_B) \sim (100 - 120, 600 - 700)$ MeV from
  functional QCD approaches(DSE and fRG),
  holographic QCD,
  extrapolation from lattice data....
- The equation of state of QCD builds the bridge between the theory and experiment;
  Quantitative analysis at $\mu_B/T < 4$ from fQCD
  Fu et al, PRD 111, L031502; Y. Lu, FG et al, arXiv:2504.05099
- For extremely high density, liquid gas transition of nuclear matter



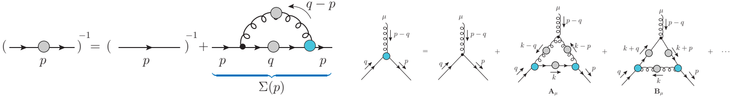
A minimal scheme for solving QCD

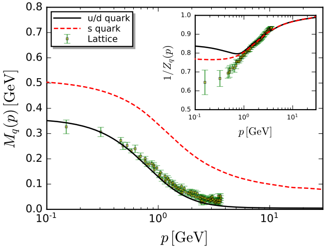
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Solve the DSEs of quark propagator and quark gluon vertex:





lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p)=\sum_{i=1}^{8} t_i(q,p)P^{\mu\nu}(q-p)\mathcal{T}_i^{\nu}(q,p)\,,$$

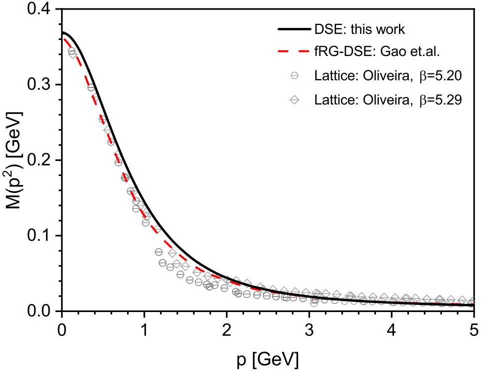
The dominant structures are Dirac and Pauli term:

$$\mathcal{T}_1(p,q)=-i\gamma^{\mu}\,,\mathcal{T}_4^{\mu}(p,q)=\sigma_{\mu\nu}(p-q)^{\nu}\,,$$

$$t_1(p,q)=F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

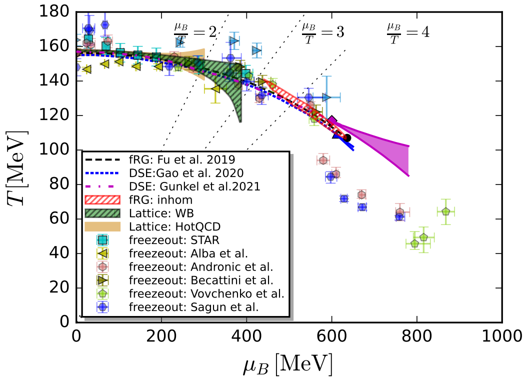
$$t_4(p,q)=\left[Z(k^2)\right]^{-1/2}\frac{B(p^2)-B(q^2)}{p^2-q^2}$$

All quantities are expressed by the running of two point functions.
The Quark Mass function:



FG, J. Papavassiliou, J. Pawlowski, PRD 103.094013 (2021).
Y, Lu, **FG**, YX Liu, J. Pawlowski, PRD 110, 014036 (2024)

Chiral Phase diagram for 2+1 flavour QCD can be directly obtained from quark propagator



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\text{MeV}$ and $\kappa \sim 0.016$
- Estimated range of CEP:
  $T \in (100, 120)$ MeV
  $\mu_B \in (600, 700)$ MeV

W.-j. Fu et al, PRD 101, 054032 (2020)

FG and J. Pawlowski, PRD 102, 034027 (2020)

FG and J. Pawlowski, PLB 820, 136584(2021)

P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

QCD thermodynamic properties

The quark propagator is the central element to compute the EoS of QCD in the functional QCD approaches.

The number density is calculated from the propagator by:

$$n_q^f(T,\mu_B) \simeq -N_c Z_2^f\, T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr}_D \left[\gamma_4 S^f(p)\right]$$

One may then incorporate the lattice QCD simulation at $\mu = 0$ here to combine the advantages of the two methods. The pressure is given by:

$$P(T,\boldsymbol{\mu}) = P_{Latt.}(T,\mathbf{0}) + \sum_q \int_0^{\mu_q} n_q(T,\mu)\, d\mu$$

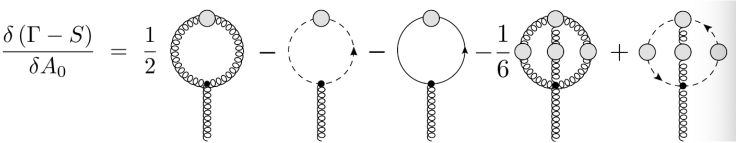
---

[1] P. Isserstedt, C.S. Fischer and T. Steinert, PRD103 (2021) 054012
[2] **FG**, Yuxin Liu, PRD 94 (2016) 9, 094030
[3] H. Chen, M. Baldo, G. F. Burgio, and H.-J. Schulze, PRD86(2012)045006

The quark propagator contains: dynamical quark mass and gluonic background condensate:

$$S_q^{-1}(p) = \mathrm{i}(\tilde{\omega}_n + gA_0)\gamma_4\, Z_q^E(\boldsymbol{p}, \tilde{\omega}_n) + \mathrm{i}\gamma \cdot \boldsymbol{p}\, Z_q^M(\boldsymbol{p}, \tilde{\omega}_n) + Z_q^E(\boldsymbol{p}, \tilde{\omega}_n)\, M_q(\boldsymbol{p}, \tilde{\omega}_n),$$

One can obtain A_0 condensate by solving the The DSE of A_0^a as $\frac{\delta(\Gamma - S_A)}{\delta A_0} = 0$. The diagrammatic representation is:



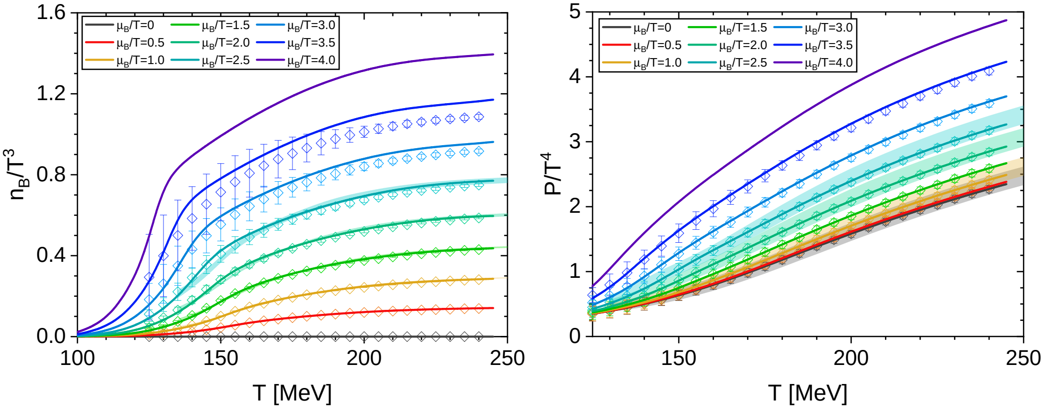
Polyakov loop in background field approach is related to A_0^a condensate as:

$$\mathcal{L}(A_0) = \frac{1}{N_c}\mathrm{tr}\,\mathcal{P} e^{\mathrm{i}g\int dx_0 A_0} = \frac{1}{3}\left[1 + 2\cos\left(g\beta A_0/2\right)\right]$$

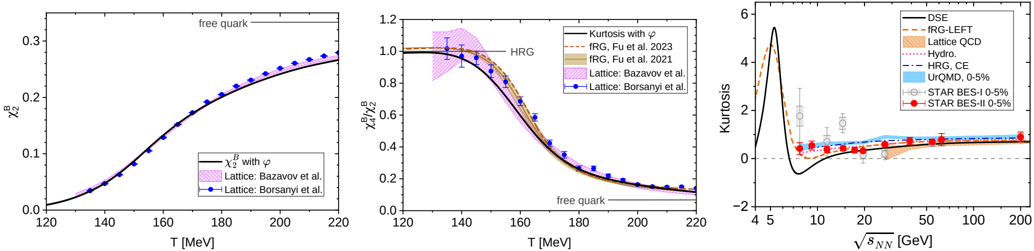
Y. Lu, **FG**, Y. Liu and J. Pawlowski, arXiv:2504.05099 an details in **Dr. Yi Lu's poster**

Number density and Pressure of QCD

The minimal scheme of DSE together with gluonic background condensate can describe the QCD thermodynamic properties at $\mu_B/T < 4$ with high precision.



Fluctuations and Beam Energy Scan



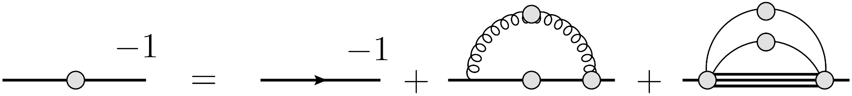
- Fluctuations at zero chemical potential is consistent with lattice results
- Kurtosis at freeze out line is consistent with fRG results (Fu et al, PRD 111, L031502) and other computations, and also BESII
- The results depend on the parametrization of the freeze out line, a further check is required.

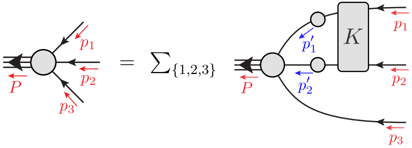
The hadron resonance channel becomes important at high density and the Faddeev equation for hadron is required.

The quark gap equation can be written as:

$$S^{-1}(p;\mu_q) = S_0^{-1}(\tilde{p}) + \Sigma^{\mathrm{G}}(p;\mu_q) + \Sigma^{\mathrm{N}}(p;\mu_q),$$

with diagrammatic representation as:





[1]FG, Yi Lu, Si-xue Qin, Bai Zhan, Lei Chang, Yu-xin Liu, arXiv:2504.00539

QCD phase transition at zero temperature

At zero temperature:

- Silver Blaze property at low chemical potential;
- Liquid gas transition of nuclear matter at $\mu_B \approx 922$ MeV;
- Chiral phase transition of QCD at $\mu_B \approx 1000$ MeV .

Silver Blaze property:
QCD matter observables remain unchanged for different chemical potentials up to the liquid gas transition

The gap equation and the Faddeev equation does not contain singularities and keep the analytic continuation from the vacuum:

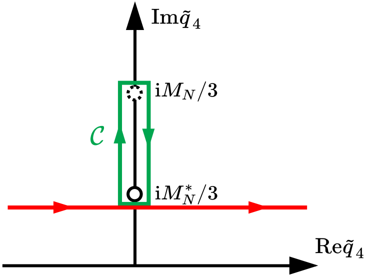
$$S(p;\mu_q) = S_{\rm vac}(\tilde{p}), \quad \Gamma^{(3)}(p_1, p_2, P; \mu_q) = \Gamma^{(3)}_{\rm vac}\left(\tilde{p}_1, \tilde{p}_2, \tilde{P}\right).$$

Liquid gas transition

The exploded view:

- **the happening of the liquid gas transition**:
  when μ_B exceeds the nucleon mass pole $M_N = 938$ MeV, the gap equation contains the first singularity and the analytic continuation is broken;
- **the nucleon mass shifting**:
  At the phase transition point, the Faddeev equation changes through the quark propagator in its kernel;
- **the finite shifting causing a first order phase transition**:
  This new "liquid solution" contains a new singularity at M_N^*, representing the in-medium nucleon pole mass and corresponding to the nuclear liquid-gas phase transition chemical potential at zero temperature.
- **the true phase transition at $\mu_B = M_N^* = 922$ MeV**:
  Essentially, the liquid solution represents a self-consistent solution of both the quark gap and the Faddeev equations in medium

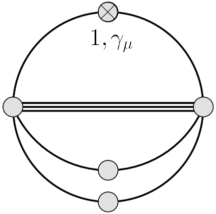
The new solution is the vacuum solution with in medium nucleon mass M_N^* plus the contour contribution of the pole jumping from M_N to M_N^*:



The constructed liquid solution is:

$$S_{\text{liq}}^{-1}(p;\mu_q) = S_{\text{vac},M_N^*}^{-1}(\tilde{p}) - \delta f_{\mathcal{C}}(\tilde{p})\, S_{\text{vac},M_N^*}^{-1}(\tilde{p})$$

One can prove that the above construction satisfies the gap equation.



The scalar and vector charge are:

$$\mathcal{S} = \frac{2M_N}{3}\int \frac{d^4q}{(2\pi)^4}\frac{g_s(q^2,0)}{q^2+M_N^2}, \quad g_s(-M_N^2) = \frac{\sigma_N}{2m_q}$$

$$\mathcal{V} = \frac{2M_N}{3}\int \frac{d^4q}{(2\pi)^4}\frac{g_v(q^2,0)}{q^2+M_N^2}, \quad g_v(-M_N^2) = 1$$

$$\delta_{\mathcal{C}}\mathcal{S} = \frac{2M_N^*}{3}\oint_{\mathcal{C}}\frac{dq_4}{2\pi}\int\frac{d^3q}{(2\pi)^3}\frac{g_s(-M_N^2,0)}{q^2+(M_N^*)^2} = \frac{1}{2}\oint_{\mathcal{C}}\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[S_{\mathrm{vac},M_N^*}\Sigma(q;\mu_q)S_{\mathrm{vac},M_N^*}]$$

$$= \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[S_{\mathrm{vac},M_N^*}\delta f_{\mathcal{C}}(q;\mu_q)] \cong \frac{1}{2}\langle\bar{q}q\rangle\delta\bar{f}$$

$$\delta_{\mathcal{C}}\mathcal{V} = \frac{2M_N^*}{3}\oint_{\mathcal{C}}\frac{dq_4}{2\pi}\int\frac{d^3q}{(2\pi)^3}\frac{g_v(-M_N^2,0)}{q^2+(M_N^*)^2} \cong \frac{1}{2}\langle\bar{q}\gamma_0 q\rangle\delta\bar{f} = \frac{1}{6}n_B^0\delta\bar{f}$$

From Faddeev equation, one has:

$$\lambda^{\text{vac}}(p^2 = -M_N^{*2}) = 1 + Z\frac{M_N^{*2} - M_N^2}{M_N^2},$$

With the in-medium nucleon eigenvalue $\lambda(p^2 = -M_N^{*2}) = 1$ we have:

$$M_N^* = (1 - \delta\bar{f}/Z)M_N.$$

and one has the binding energy and also the location of the liquid gas transition of nucleon as:

$$\delta\bar{f} = \frac{81\pi^4 Z^3 M_\pi^4 f_\pi^4}{8M_N^6\sigma_N^2} = 0.0164, \quad \mu_B^* = M_N^* = 939 - 15.9 = 923.1\mathrm{MeV},$$

The saturation density is:

$$n_{\mathrm{B}}^0 = \frac{\sqrt{8\delta f/Z^3}}{27\pi^2}M_N^3 = \frac{M_\pi^2 f_\pi^2}{3\sigma_N} = 0.15\,\mathrm{fm}^{-3}.$$

These results are in precise agreement with the experiments.

Summary

The conclusions and discussions:

- The functional QCD approaches **calculate** the chiral PT and give the CEP located at $\mu_B \approx 600 - 700$ MeV.
- **Quantitative** description for QCD thermodynamic quantities at $\mu_B/T < 4$. A combined analysis with BESII data will result in a better understanding of heavy ion collision experiments.
- Towards the large chemical potential for determining CEP and also neutron star physics: **hadron resonance channel** in DSE.
- For the first step: **liquid gas transition** of nuclear matter can be described.

Thank you!