Intermittency analysis in relativistic heavy-ion collisions

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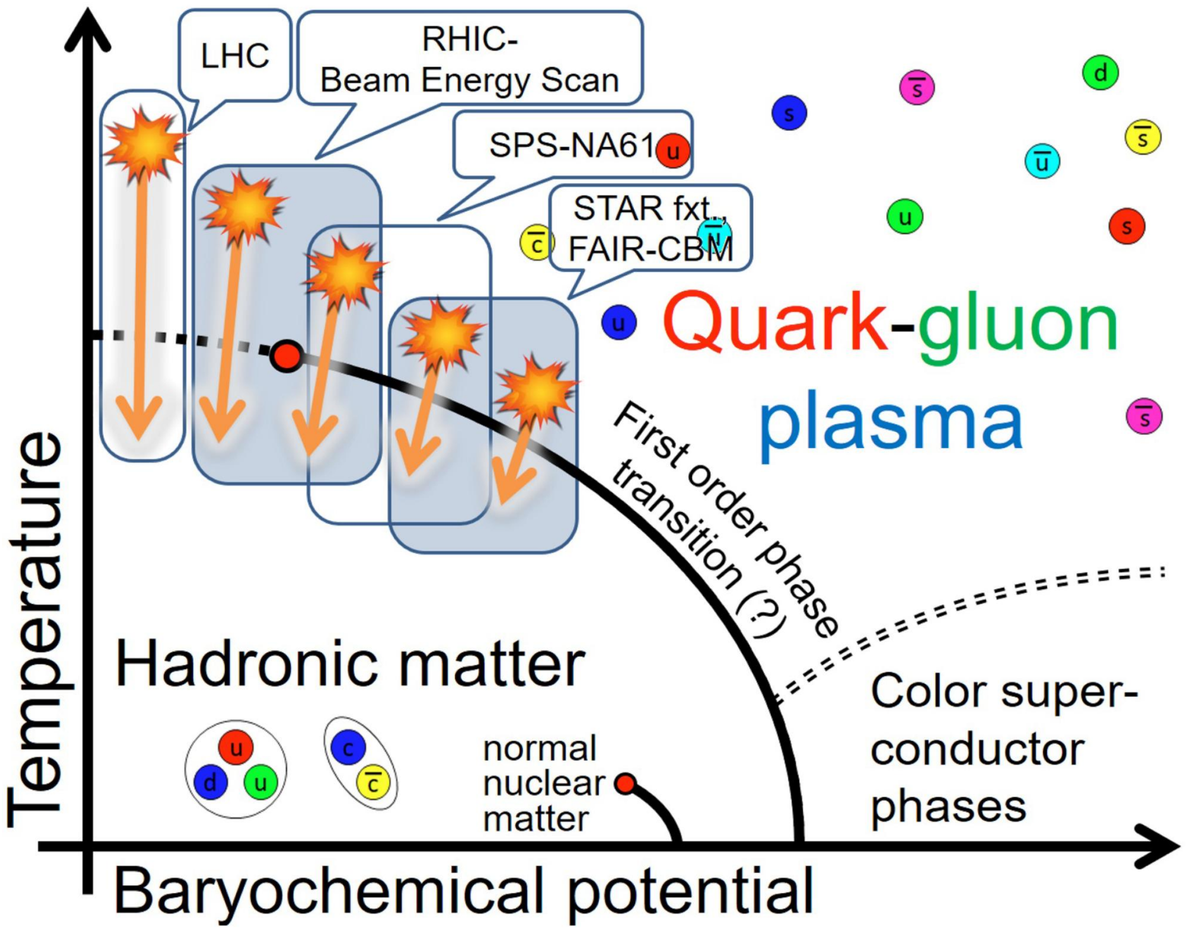
Outline

- Motivation
- Experimental measured intermittency in heavy-ion collisions
- Identifying weak critical fluctuations of intermittency with topological machine learning
- Summary and outlook

Search for the QCD phase diagram

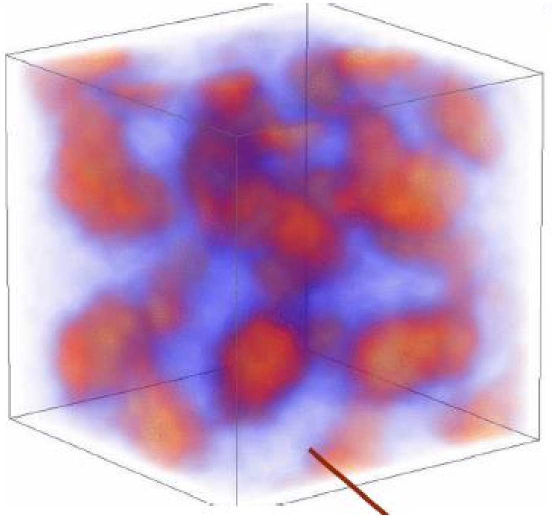


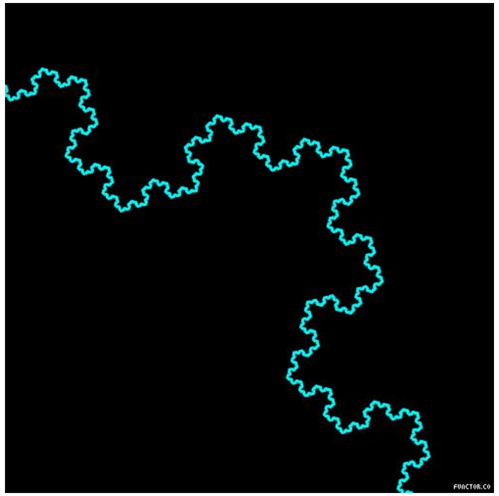
Goal: exploring the QCD phase diagram and Critical Point.

Experimental observables:

Local density fluctuation of conserved charges

- Intermittency：
  fractal, self-similar, scale-invariant





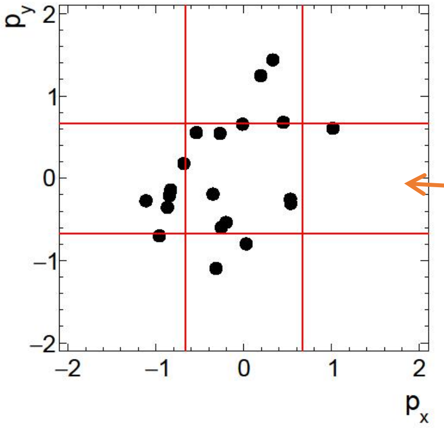
N. Antoniou, et al., Phy. Rev. Lett. 97,032002 (2006)
J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)
Z. Li, Mod. Phys. Lett. A 37, 2230009 (2022)

Intermittency analysis

- Intermittency can be measured via the scaled factorial moments $F_q(M)$,

$$F_q(M) = \frac{< \frac{1}{M^D}\sum_{i=1}^{M^D} n_i(n_i-1)\ldots(n_i-q+1) >}{< \frac{1}{M^D}\sum_{i=1}^{M^D} n_i >^q}$$

A. Bialas and R. B. Peschanski, Nucl. Phys. B 273, 703 (1986)
A. Bialas and R. B. Peschanski, Nucl. Phys. B 308, 857 (1988)

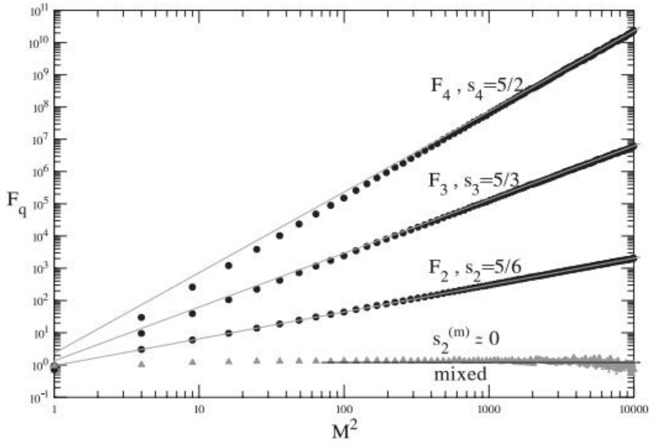


Particle multiplicity in the *i-th* cell n_i

- $\boldsymbol{F_q(M)/M}$ **scaling**:

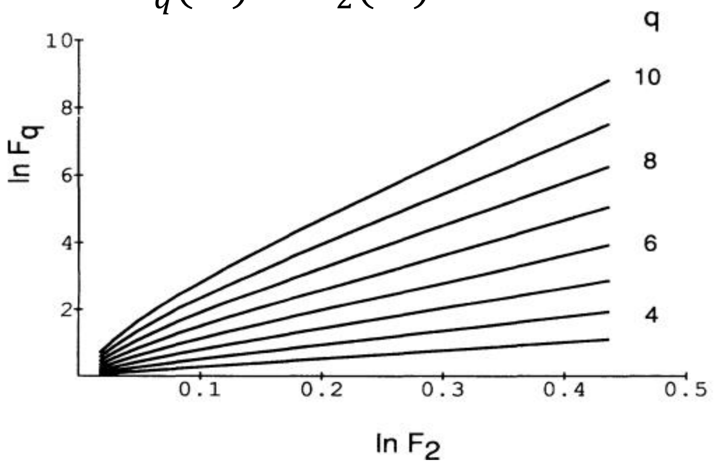
$$F_q(M) \propto (M^2)^{\phi_q}, M \to \infty$$

Intermittency index, ϕ_q



Phy. Rev. Lett. 97,032002 (2006)
Phys. Rev. D 97, 034015 (2018)

$F_q(M) \propto F_2(M)^{\beta_q}$



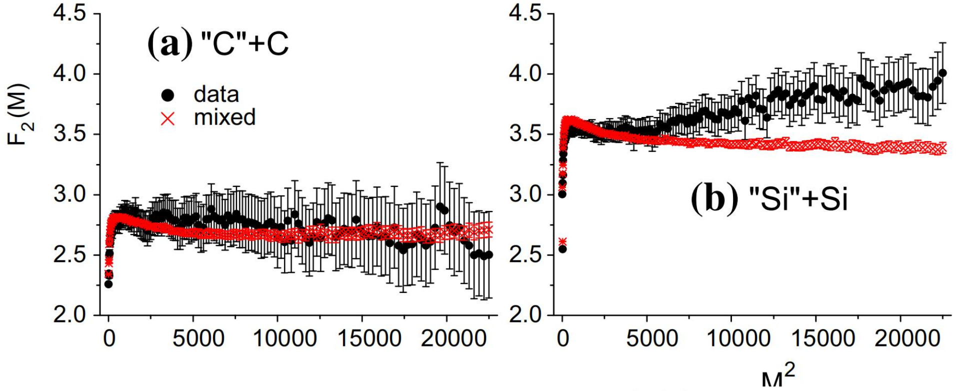
Phys. Rev. Lett. 69, 741 (1992)
Phys. Rev. C 85, 044914 (2012)

- $\boldsymbol{F_q(M)/F_2(M)}$ **scaling:**

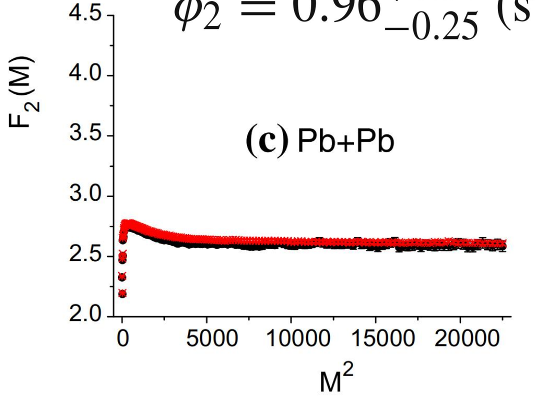
$$F_q(M) \propto F_2(M)^{\beta_q} \qquad \beta_q \propto (q-1)^{\nu}$$

Scaling exponent, ***ν*****,** a critical exponent related to the second-order phase transition

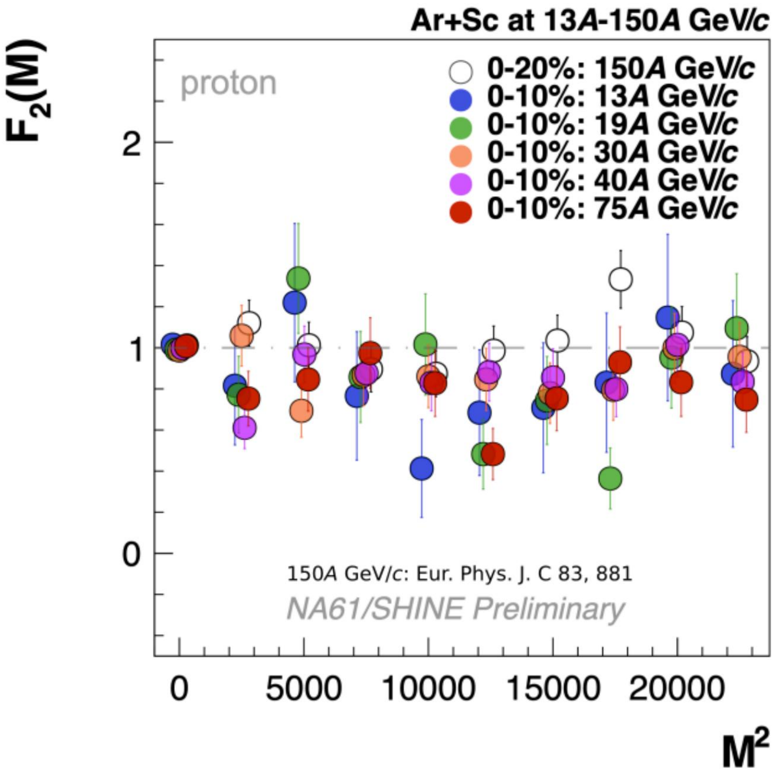
Experimental measured intermittency at SPS



$$\phi_2 = 0.96^{+0.38}_{-0.25}\,(\text{stat.}) \pm 0.16\,(\text{syst.})$$



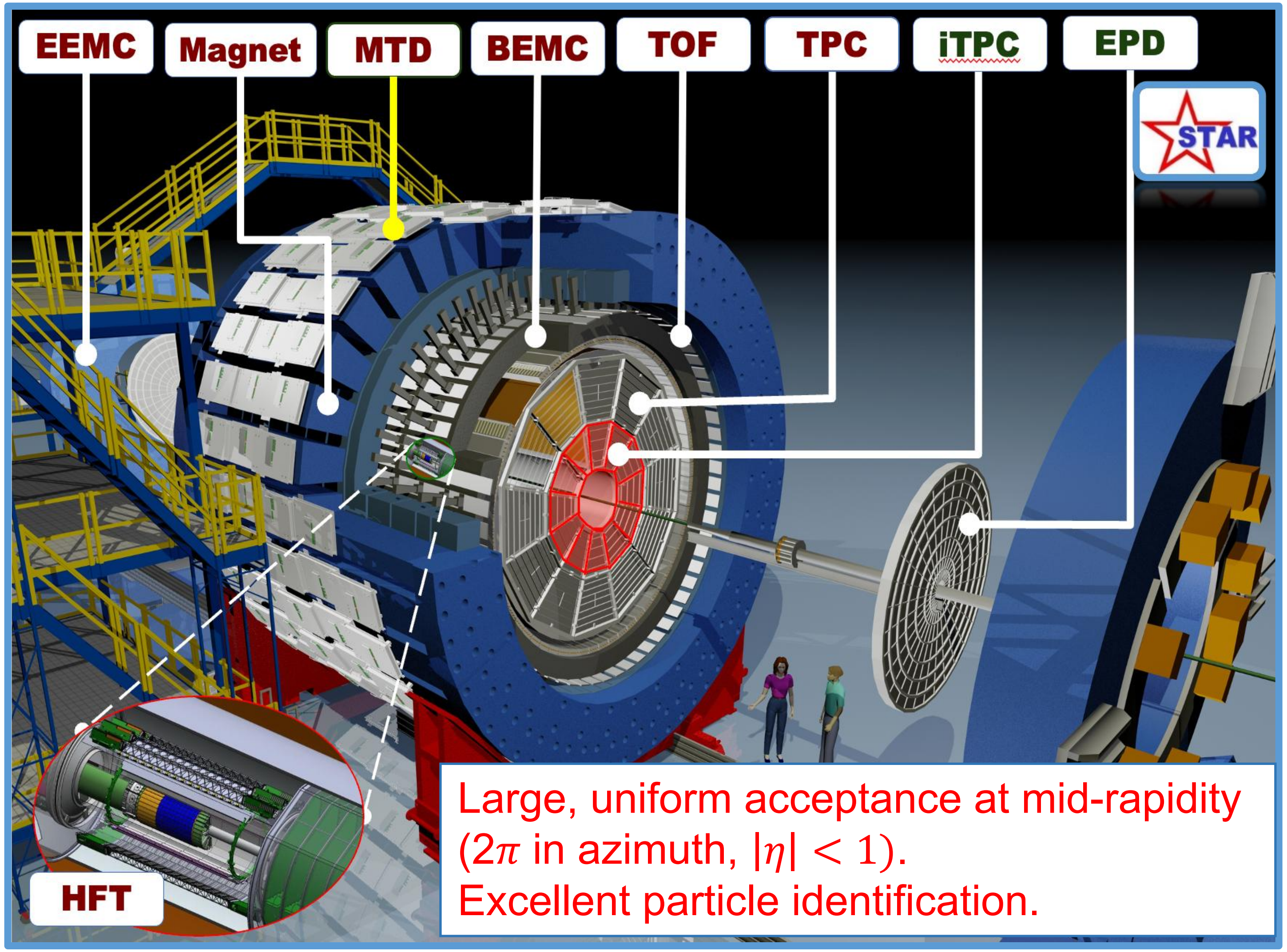
N. Davis, et al. (NA49 Coll.), QM2018
T. Anticic , et al. (NA49 Coll.), Eur. Phys. J. C 75, 587 (2015)



Andrzej Rybicki (for the NA61/SHINE Coll.), arXiv:2409.19763
H. Adhikary *et al.* (NA61/SHINE Coll.), Eur. Phys. J. C 83, 1 (2023)

- Intermittency of NA49 experiment reveals significant power-law fluctuations of proton density in **Si + Si** collisions at $\sqrt{s_{NN}} = 17.3$ GeV. But no intermittency behavior is visible in C+C or Pb + Pb collisions.
- For NA61, F_2(M) is nearly flat with increasing number of division bins for Ar + Sc collisions at 13A-150A GeV/c. No indication of a power-law increase in the number of bins is observed.

RHIC Beam Energy Scan Phase I (2010-2017)

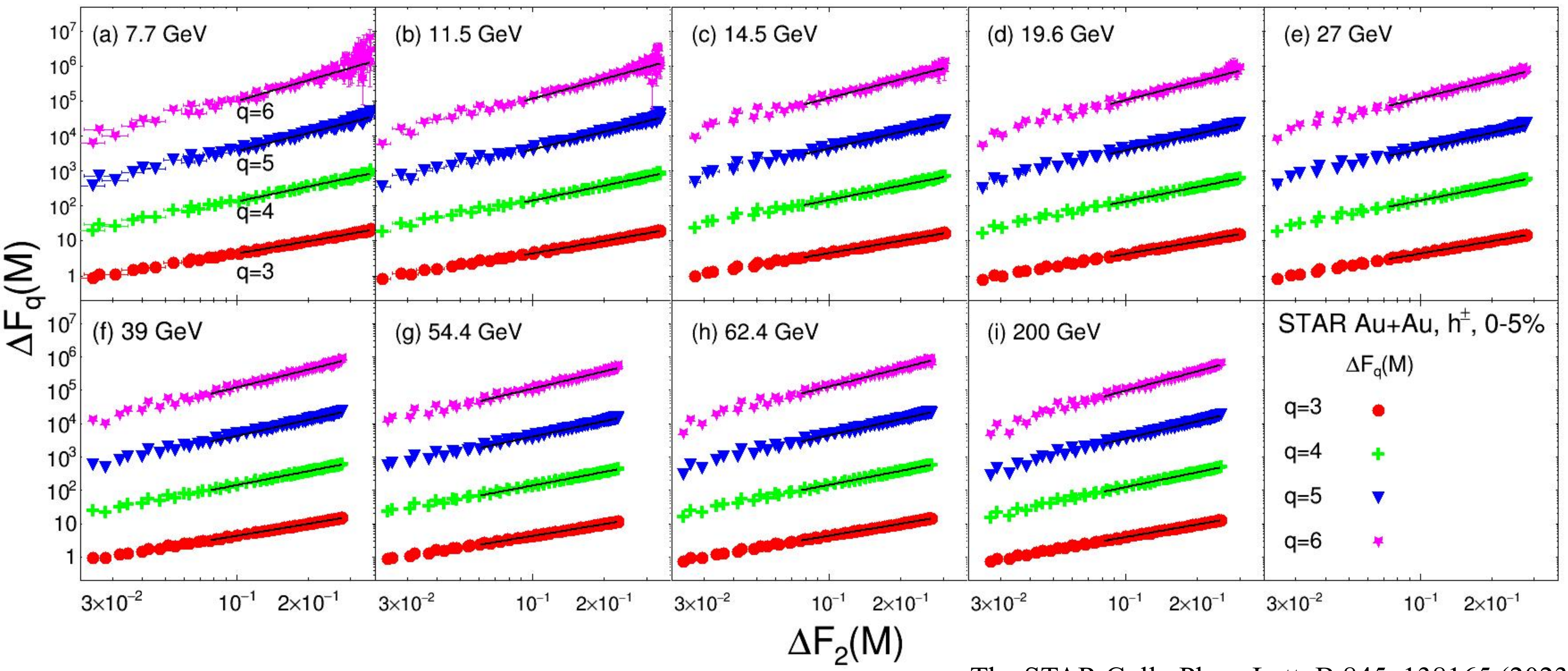


Large, uniform acceptance at mid-rapidity (2π in azimuth, $|\eta| < 1$).
Excellent particle identification.

$\sqrt{s_{NN}}$ (GeV)	Year	$^*\mu_B$ (MeV)	$^*T_{CH}$ (MeV)	Events (Million)
7.7	2010	422	140	3
11.5	2010	316	152	7
14.5	2014	264	156	13
19.6	2011	206	160	16
27	2011	156	162	32
39	2010	112	164	89
54.4	2017	83	165	442
62.4	2010	73	165	47
200	2010	25	166	236

J. Cleymans et. al, PRC 73, 034905 (2006)

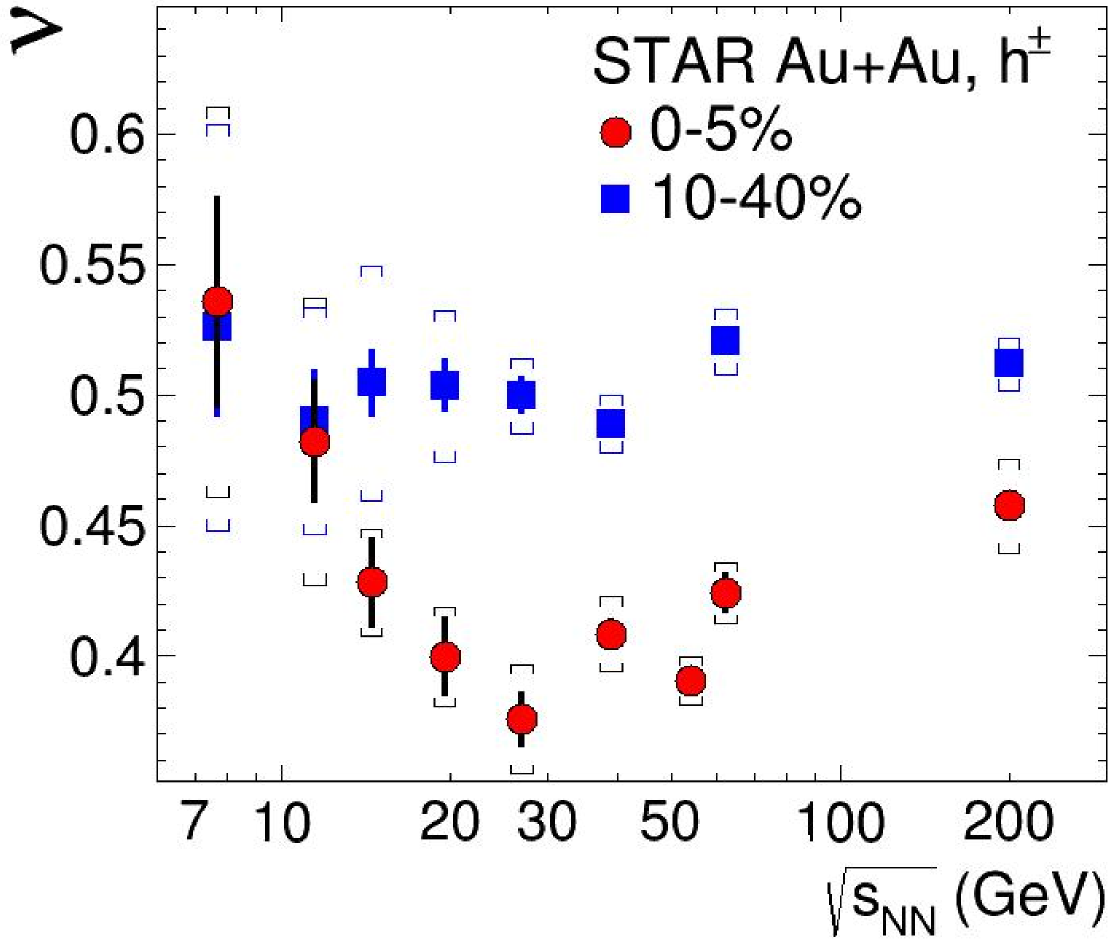
Experimental measured intermittency at RHIC



The STAR Coll., Phys. Lett. B 845, 138165 (2023)

- $\Delta F_q(M)$ (q = 3-6) follow good scaling behaviors with $\Delta F_2(M)$ in the most central Au+Au collisions.
- A clear power-law scaling of $\Delta F_q(M) \propto \Delta F_2(M)^{\beta_q}$ is visible at RHIC BES-I energies.

Experimental measured intermittency at RHIC

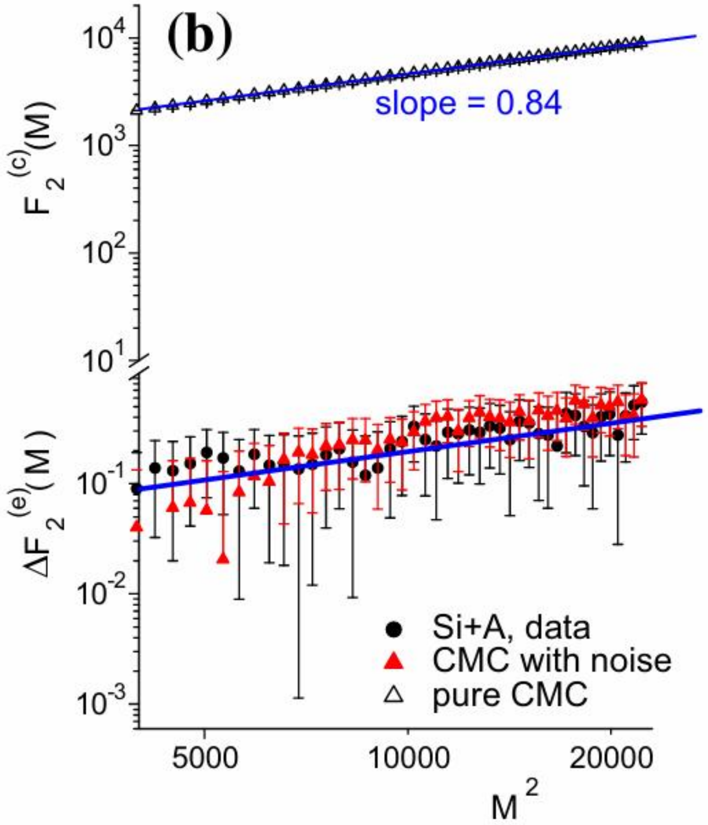


$\Delta F_q(M) \propto \Delta F_2(M)^{\beta_q}$ $\boxed{\beta_q \propto (q-1)^{\nu}}$

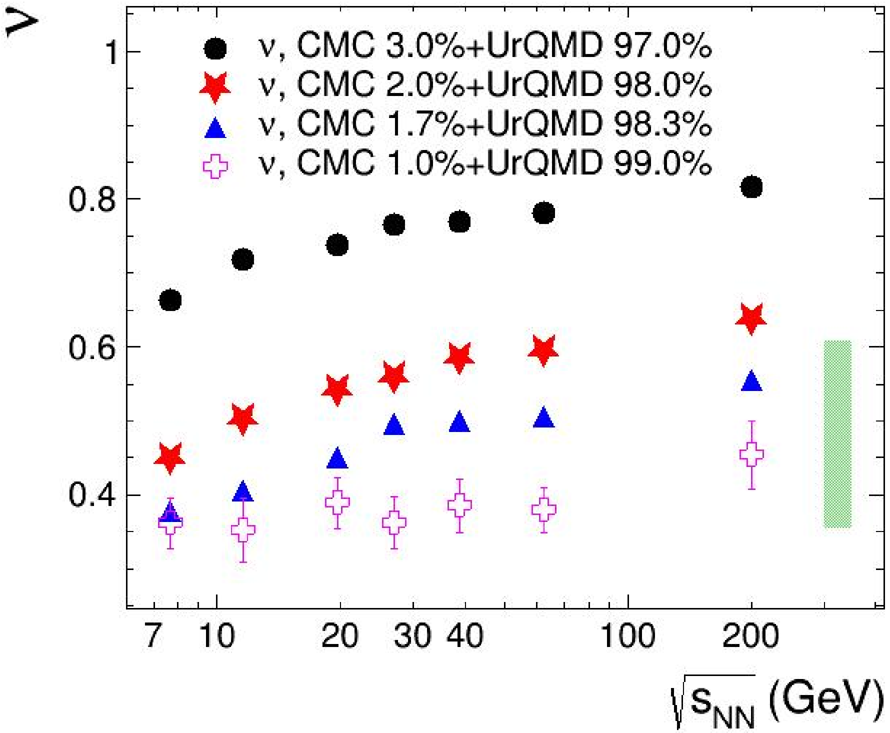
- The scaling exponent (ν) exhibits a non-monotonic behavior on collision energy and seems to reach a minimum around $\sqrt{s_{NN}}$ = 20-30 GeV in the most central collisions.
- However, a flat energy dependence is observed in the mid-central (10-40%) collisions.
- The observed non-monotonic behavior of ν in central collisions needs to be confirmed with BES-II data and to be understood with more theoretical inputs.

The STAR Coll., Phys. Lett. B 845, 138165 (2023)

The observed weak signal of intermittency in heavy-ion collisions



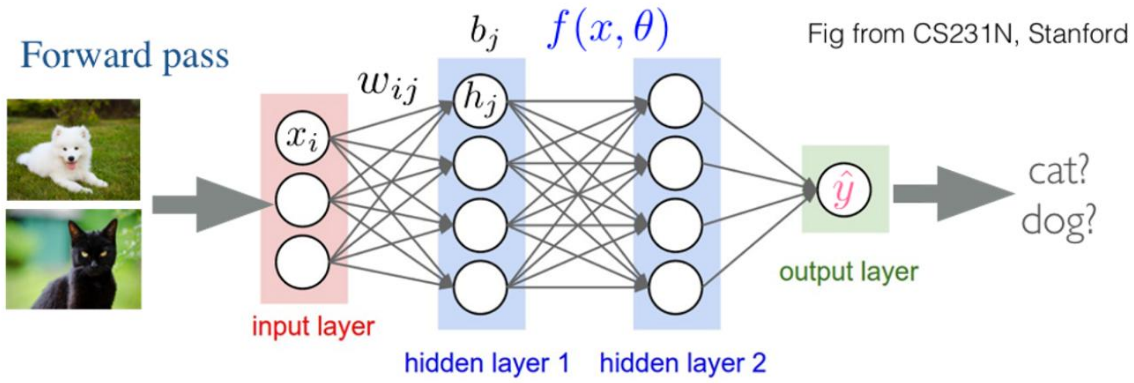
T. Anticic *et al.* (NA49 Coll.), Eur. Phys. J. C **75**, 1 (2015)

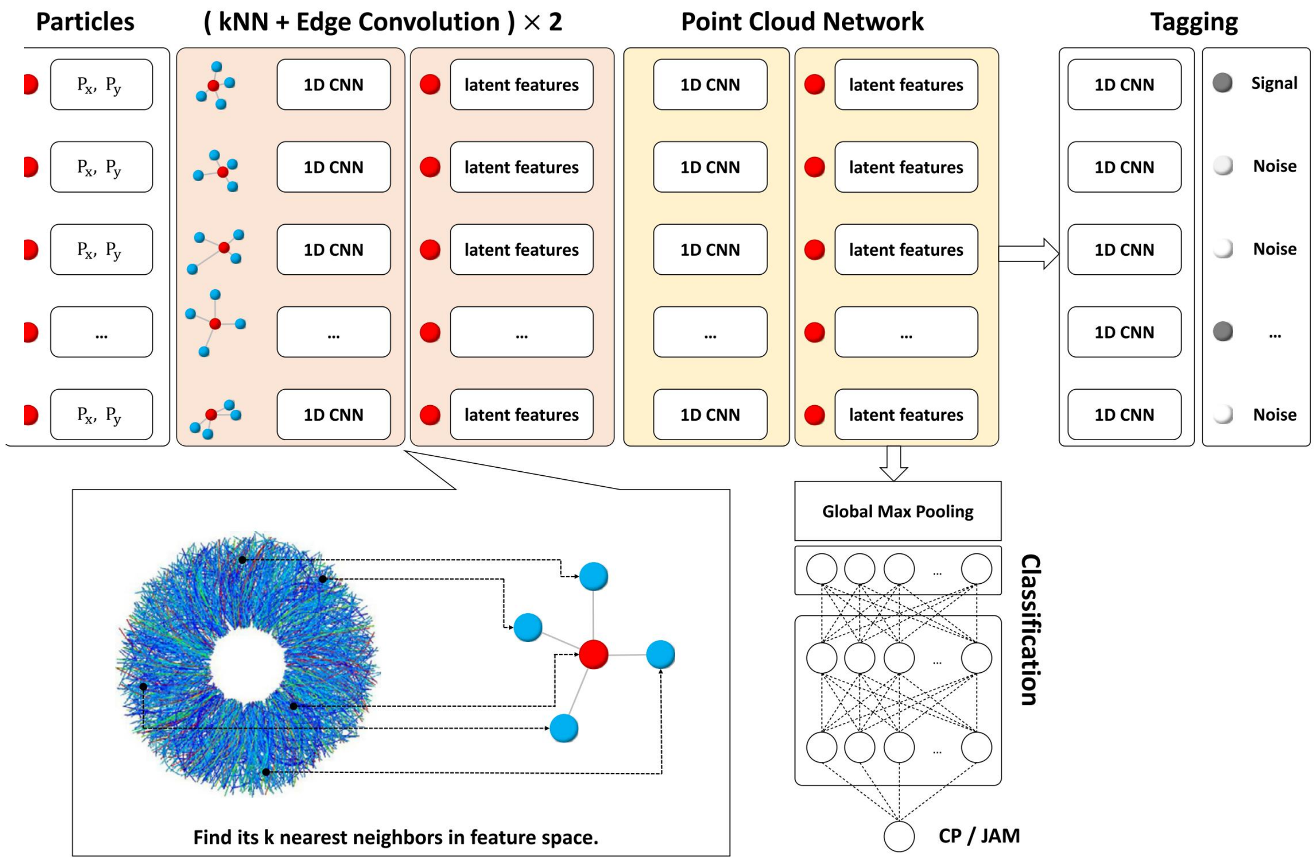


J. Wu, Z. Li, X. Luo, *et al.*, Phys. Rev. C **106**, 054905 (2022)

- The intermittency at NA49 can be reproduced by a mixed sample with **99% background** random tracks and **1% signal particles** generated from a Critical Monte Carlo (CMC) model.
- The STAR experimental results are consistent with a mix of approximately **1-2% signal particles**.

Probing criticality with deep learning





The testing accuracy for dynamical edge convolution network and a simple point cloud network.

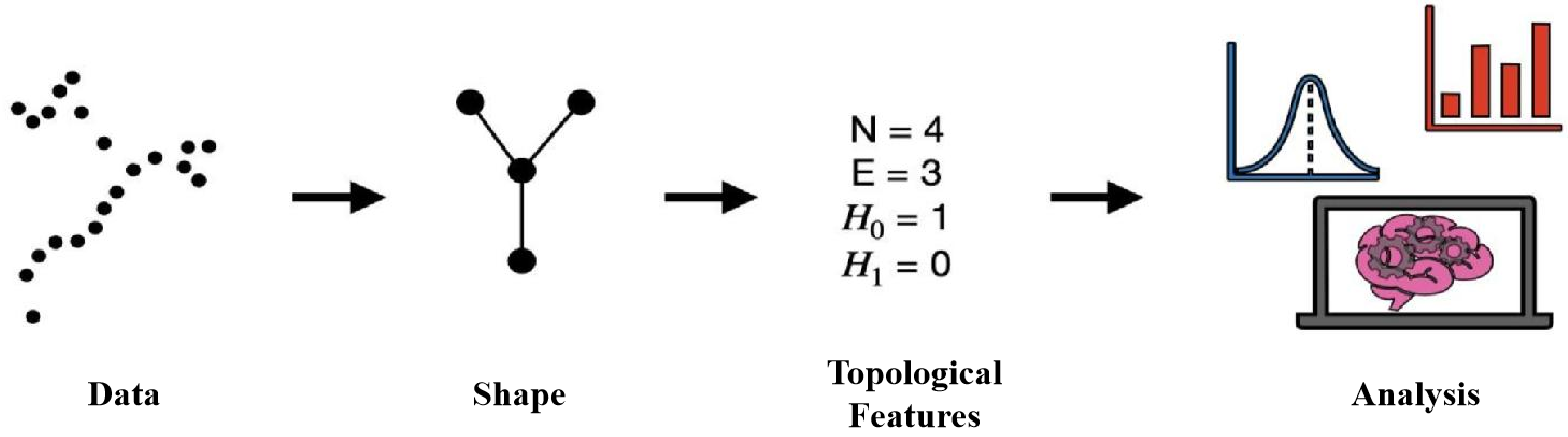
Testing accuracy

η	Edge-Conv	Point-Cloud Net
5%	92.8%	83.4%
10%	97.7%	84.8%

Y. Huang, L. Pang, X. Luo and X. Wang, Phys. Lett. B 827, 137001 (2022)

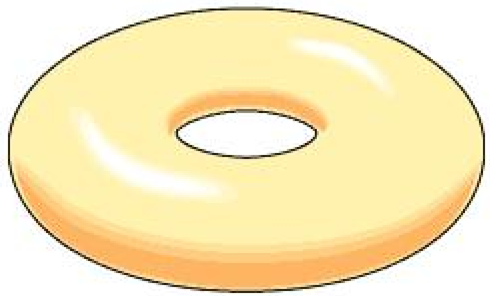
- Deep learning method can identify events with critical fluctuations and pick out a large fraction of signal particles used for decision-making in each single event.

Geometry and topology



A. Cole, G.J. Loges, G. Shiu, Phys. Rev. B 104, 104426 (2021).
N. Sale, J. Giansiracusa, B. Lucini, Phys. Rev. E 105, 024121 (2022).

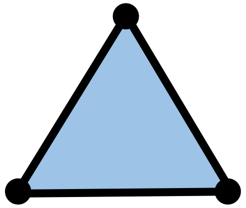
- Geometry and topology have emerged as powerful tools for identifying phase transitions in complex systems



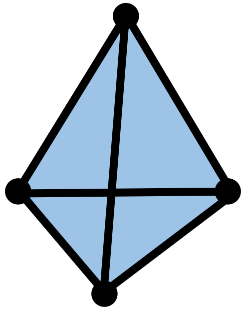
Topological data anylysis (TDA)

0-simplex
(a point)

1-simplex
(a seament)

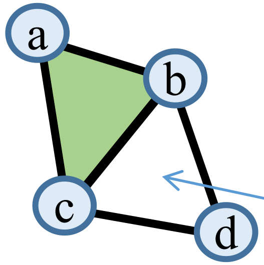


2-simplex
(a triangle)



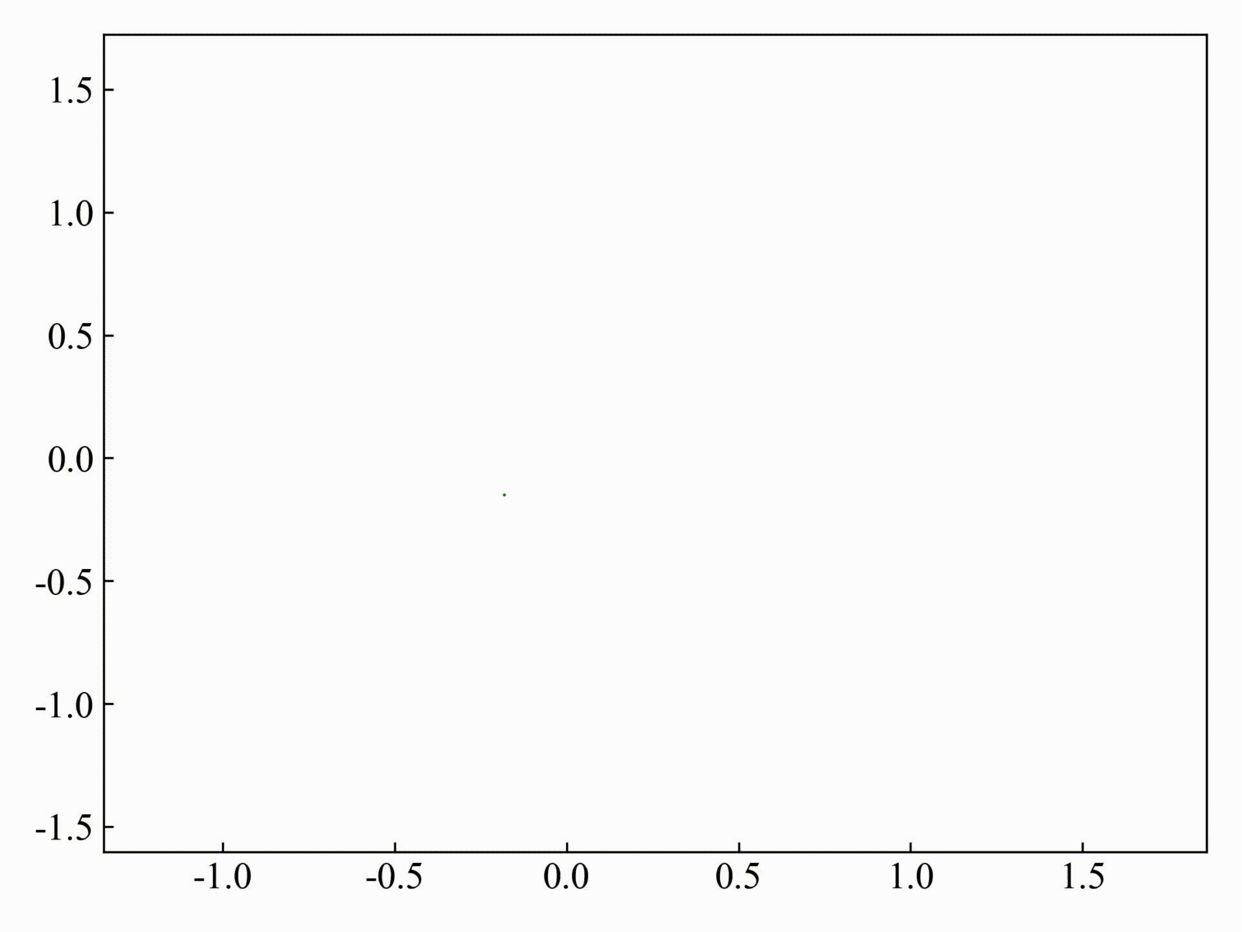
3-simplex
(a tetrahedron)

The n-th Betti number(β_n) counts the number of n-dimensional "holes".



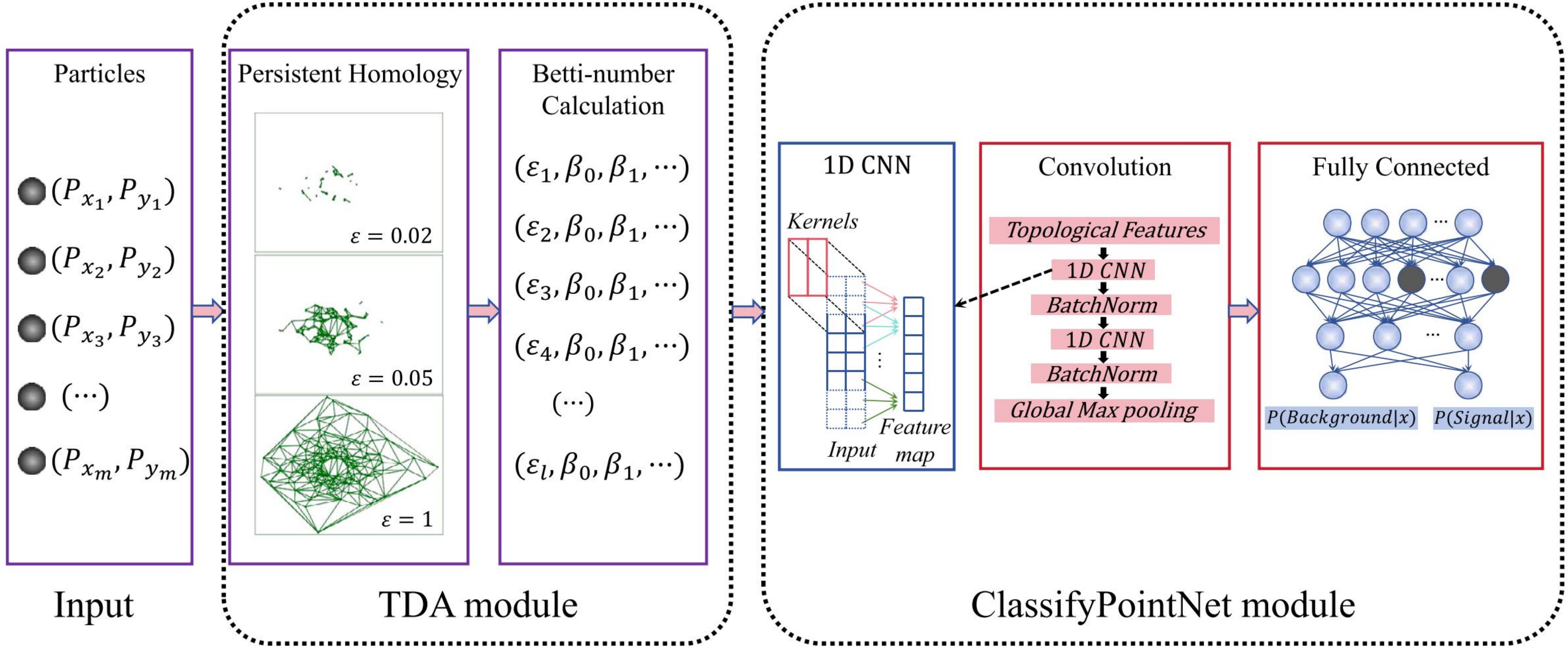
2-complex

β_0:Components count
β_1:holes count



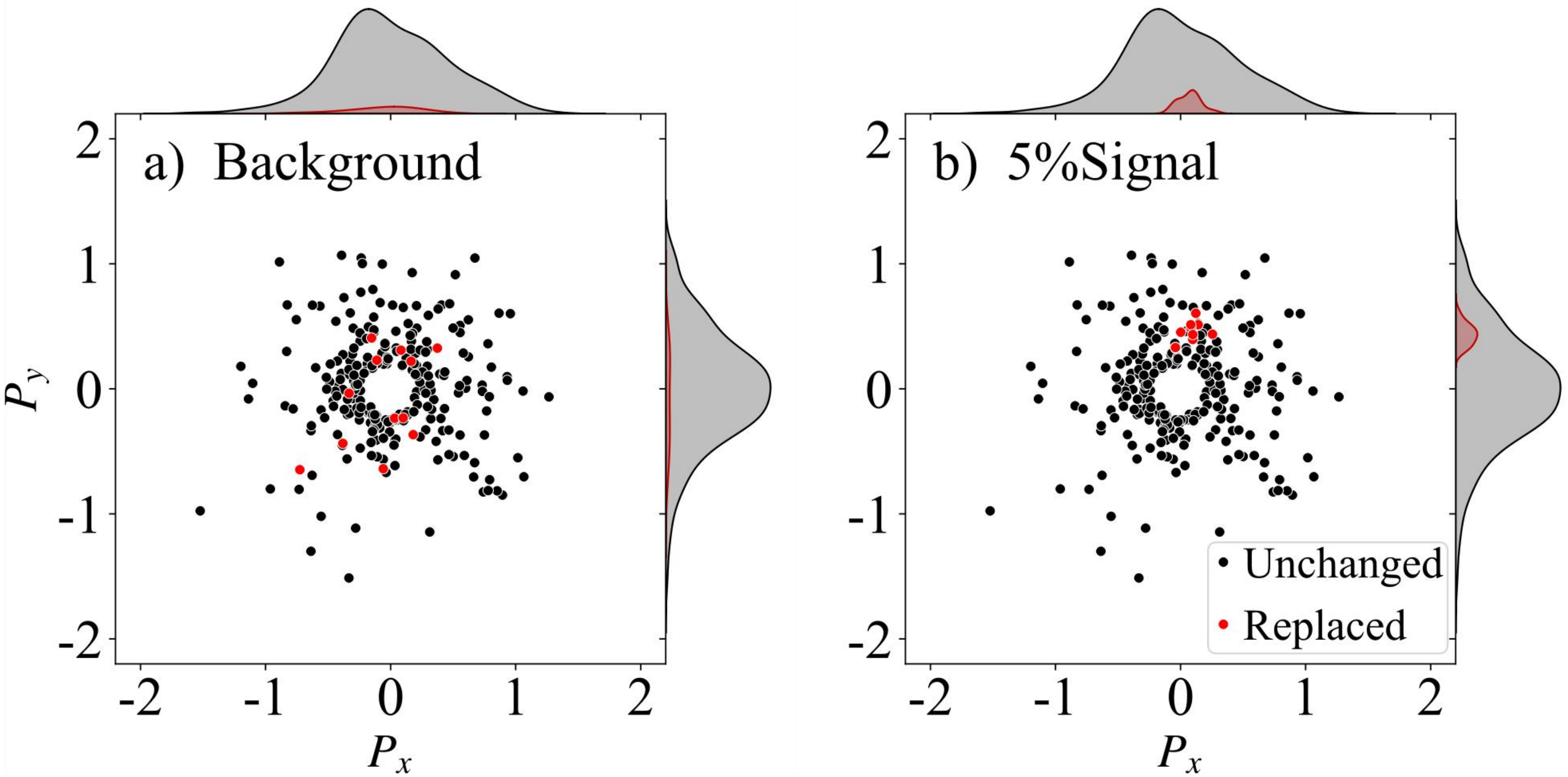
- TDA is a topology-based data analysis method that can capture the structure and shape of data under different data representations.

Topological machine learning



- ➢ By integrating the persistent homology of TDA into a deep learning method, we construct a TopoPointNet to identify and extract weak intermittency signals from the background.

Momentum distributions of different event samples

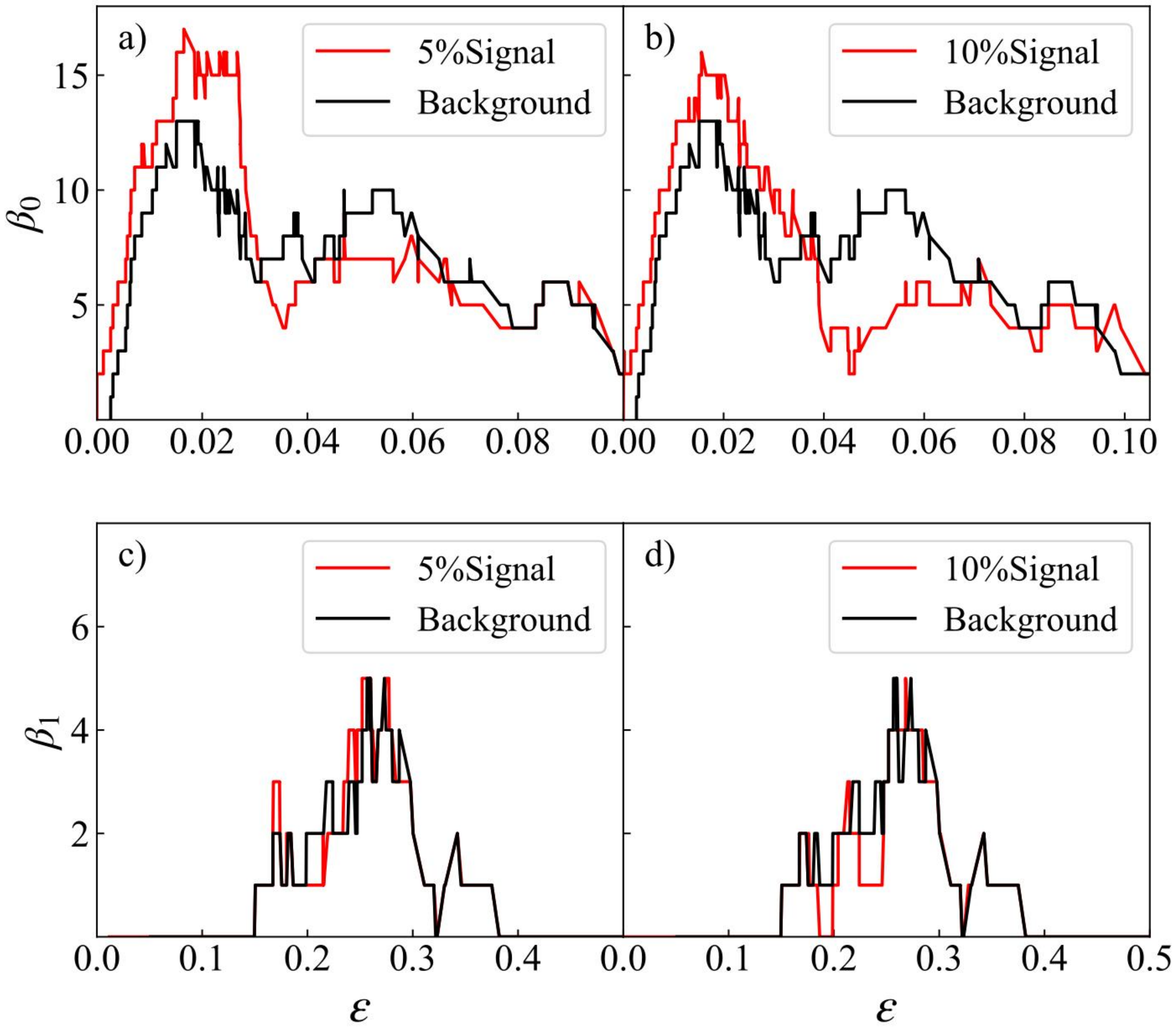


N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis, et al., Phys. Rev. Lett. 97, 032002 (2006)

J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)

- The two-dimensional momentum distributions from (a) background and (b) 5% signal events. The black dots represent the **unchanged UrQMD particles**, while the red dots indicate the **replaced critical particles** introduced by the **CMC model** with a replacement ratio of $\lambda = 5\%$.
- The momentum distribution for the corresponding signal event gather around, exhibiting a formation similar to a cluster.

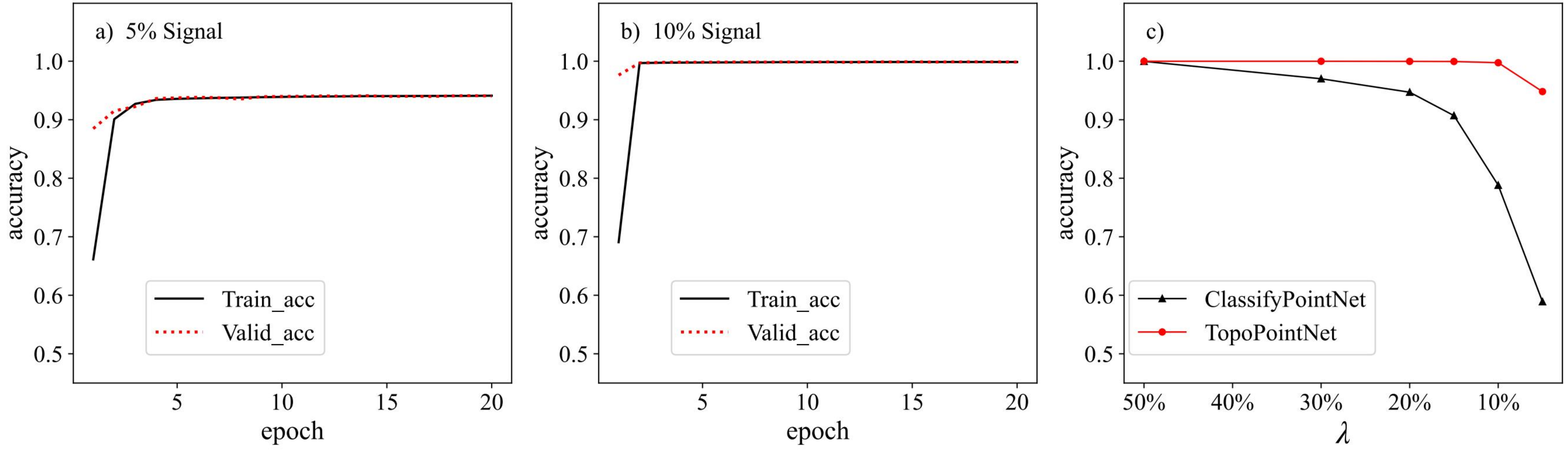
Extracting topological features of Betti Numbers



- At different levels of filtering, there are clear differences of $\boldsymbol{\beta_0}$ between the background event (black curve) and the signal event (red curve).
- However, it is difficult to distinguish distinct characteristics of $\boldsymbol{\beta_1}$ between the background event (black curve) and the signal event (red curve).

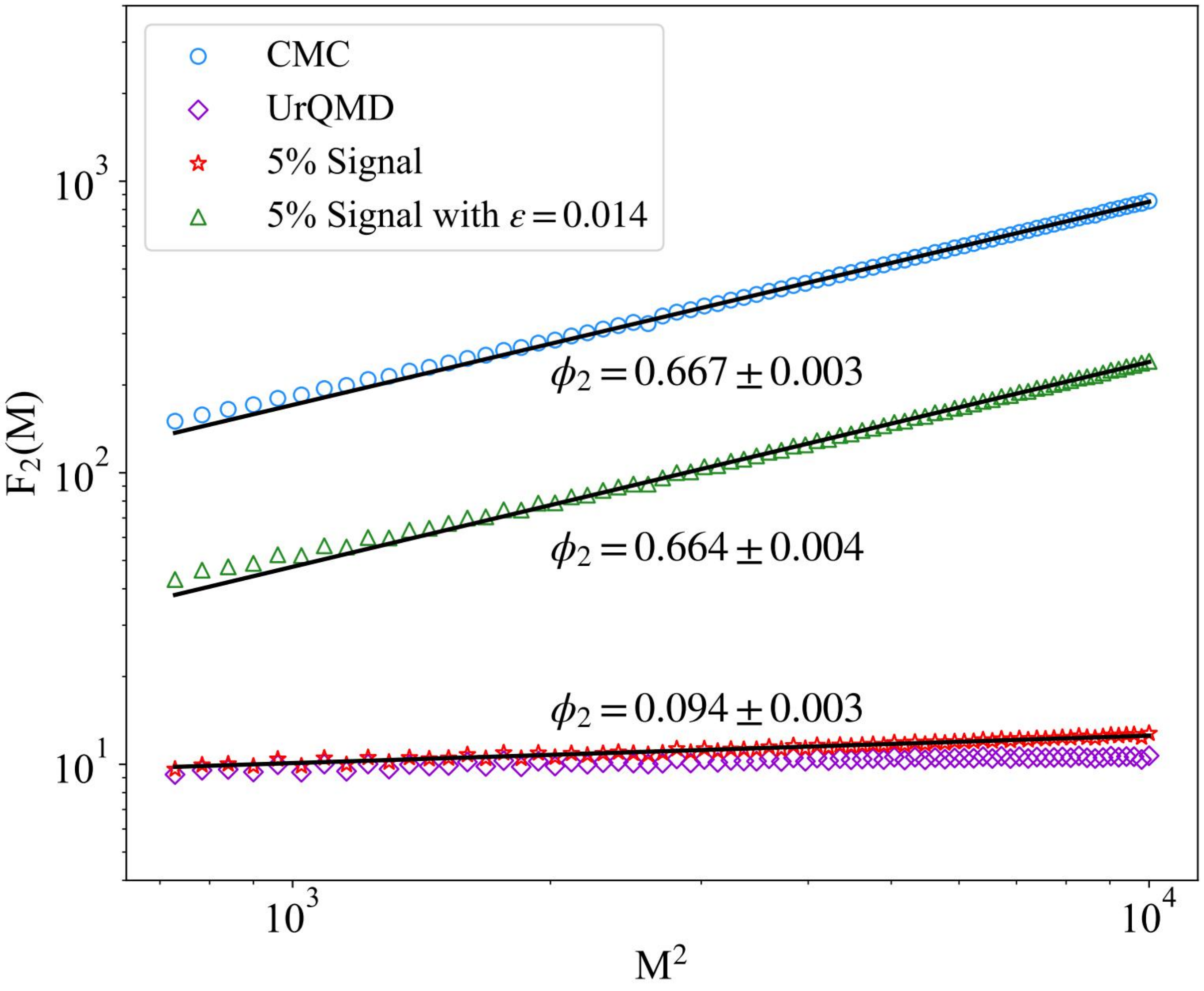
R. Wang, Z. Li, Y. Wu et al., Phys. Lett. B 864, 139405 (2025)

Training and validation accuracy



- The training and validation accuracy as a function of training epochs for 5% and 10% signal events are shown in Fig. (a) and (b), respectively. The testing accuracy is **94.69%** for the **5%** replacement ratio and **99.85%** for the **10%** replacement ratio.
- For comparison, we also train a **pure PointNet** without the TDA module. The testing accuracy of the PointNet without TDA decreases significantly faster with decreasing λ compared to TopoPointNet.

Intermittency analysis on different event samples



R. Wang, Z. Li, Y. Wu et al., Phys. Lett. B 864, 139405 (2025)

- Red stars is the intermittency result of **5%Signal** events, with the fitted intermittency index $\boldsymbol{\phi_2}$**=** **0.094±0.003**.
- Green triangles is the intermittency result of **5%Signal with $\boldsymbol{\varepsilon = 0.014}$**, the calculated intermittency index $\boldsymbol{\phi_2}$**=0.664 ± 0.004**.
- By integrating the topological machine learning method into the traditional intermittency analysis, we can accurately **extract the intermittency index even for weak signal event samples.**

Summary and outlook

- We demonstrate the application of persistent homology from TDA to analyze and extract topological features from event samples containing critical fluctuations of intermittency in heavy-ion collisions. We have observed a clear difference in the 0th Betti number between weak signal and background events.

- By constructing a point cloud neural network with TDA, we have achieved a testing accuracy of 94.69% when only 5% of UrQMD background particles in each event are replaced by CMC signal particles.

- By selecting a filtration level cut of $\varepsilon = 0.014$, the calculated second-order intermittency index for the 5% signal events closely matches that for the pure CMC signal events.

◆ A precise measurement of intermittency in the BES-II data is anticipated.

◆ Extending this study to three-dimensional space and integrating other machine learning techniques, such as unsupervised learning, could broaden the applicability of this approach to complex systems.