Characteristics and Influences of Nonequilibrium Evolution near the Phase Boundary

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References:

[1] Xiaobing Li, Mingmei Xu, Yanhua Zhang, Zhiming Li, Yu Zhou, Jinghua Fu, Yuanfang Wu, Phys. Rev. C 105, 064904 (2022).
[2] Xiaobing Li, Ranran Guo, Mingmei Xu, Jinghua Fu, Lizhu Chen, Yu Zhou, Yuanfang Wu, arXiv:2412.18909.

第二十届全国中高能核物理大会，2025年4月24日至28日，上海

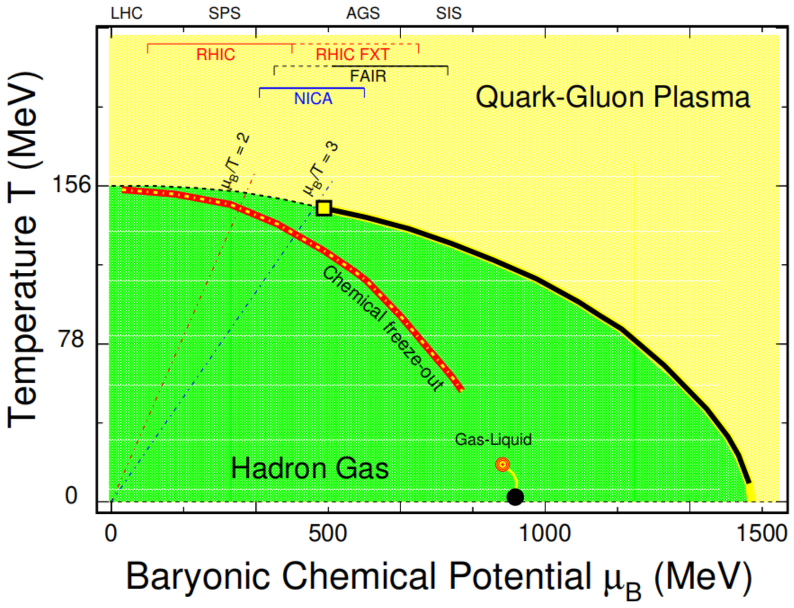
OUTLINE

- Motivation
- Ising Model and Metropolis algorithm
- Characteristics of nonequilibrium evolution
  - near the CP
  - near the 1st order Phase Transition Line (1st-PTL)
- Influences of nonequilibrium evolution
- Summary and discussions

Introduction

Nonequilibrium is inevitable in HIC

due to finite evolution time: $\approx$20 fm/c



(1) Initial states

J. Adams et al. (STAR Collaboration), Nucl. Phys. A 757, 102 (2005).

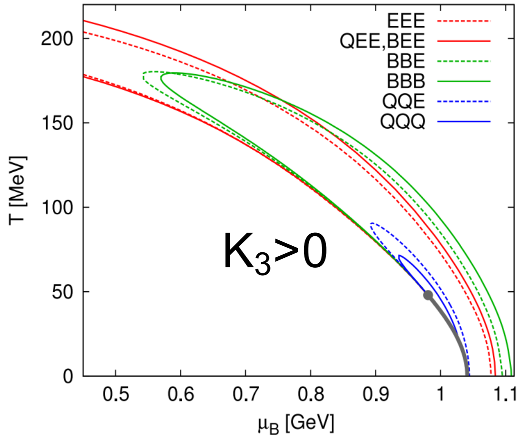
(2) Critical slowing down

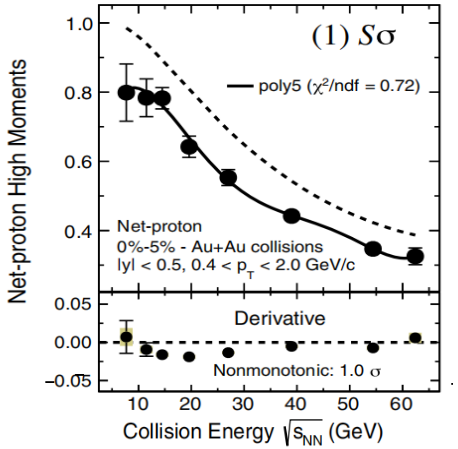
$$\tau \propto \xi^z$$

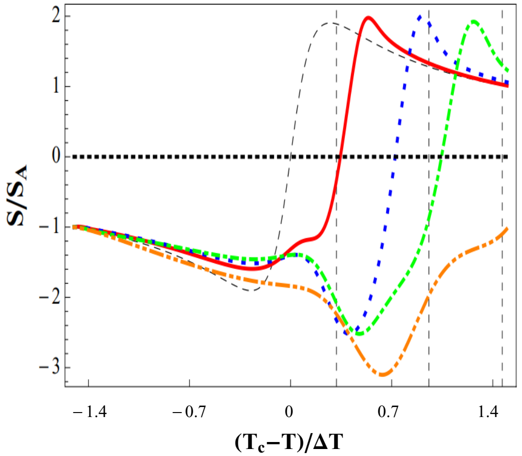
Critical sensitive observables:

high cumulants of conserved charges, e.g. $S\sigma$

sign change of $S\sigma$ $\xleftarrow{?}$ nonequilibrium evolution







S. Mukherjee, R. Venugopalan and Y. Yin, Phys. Rev.C 92, 034912 (2015).
M. Asakawa, S. Ejiri, and M. Kitazawa, Phys.Rev.Lett. 103,262301 (2009)
S. Wu, Z. Wu and H. Song, Phys. Rev. C 99, 064902 (2019).
K. Rajagopal, G. Ridgway, R. Weller and Y. Yin, Phys.Rev. D 102, 094025 (2020).
J. Adam et al. (STAR Collaboration), Phys. Rev. Lett. 126, 092301 (2021).

Current Status:

- No ready nonequilibrium statistics

- Current analytical methods – dynamical models
  - Relaxational models, e.g. Fokker-Planck equation

    S. Mukherjee, R. Venugopalan and Y. Yin, Phys. Rev.C 92, 034912 (2015).

  - Langevin dynamics

    S. Wu, Z. Wu and H. Song, Phys. Rev. C 99, 064902 (2019).
    Lijia Jiang, Jingyi Chao, Eur. Phys. J. A 59, 30 (2023).

Belong to dynamic universality class “model A”

Limited to crossover, then extended to 1st-PT

Relaxation time:

- Autocorrelation time

  Autocorrelation function

$$\chi(t) = \int [m(t') - \langle m \rangle] [m(t' + t) - \langle m \rangle] \, dt'$$

$$\chi(t) \sim \mathrm{e}^{-t/\tau_{\mathrm{auto}}}$$

  equilibrium evolution (from an equilibrium to another equilibrium)

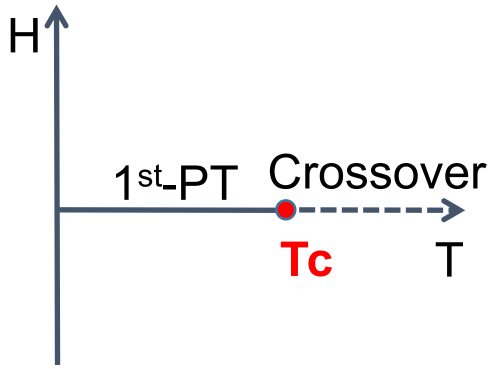
- Equilibrium time: the magnetization reach stable value

  nonequilibrium evolution (from an initial state to an equilibrium)

Studies on equilibration time remain limited.

MC of 3D Ising model by Metropolis algorithm

- **Why MC simulation?**
  - Closer to real HIC experiments
- **Why 3D Ising model?**
  - The same static universality class with the QCD CP
  - The relaxation processes can be easily realized on the entire phase boundary



Why Metropolis algorithm?

- Single spin-flipping dynamics, a local dynamics of Glauber type

  N. Menyhard and G. Odor, Brazilian Journal of Physics 30, 113 (2000).

- Suitable for studying evolutions from nonequilibrium to equilibrium

  Phys. Rev. B 89, 054307 (2014) . Phys. Rev. E 56, 2407 (1997).

- Belong to dynamic universality class “model A”

  Phys. Rev. E 101, 022126 (2020).

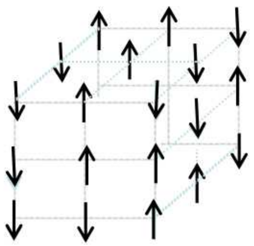
- Swendsen-Wang algorithm or Wolff algorithm, cluster algorithm, suitable for equilibrium properties

3D Ising model

Constant nearest-neighbor interactions J

Uniform external field H

$$E_{\{s_i\}} = -J \sum_{\langle ij \rangle} s_i s_j - H \sum_{i=1}^{N} s_i$$



$N = L^3$

$s_i = +1, -1$

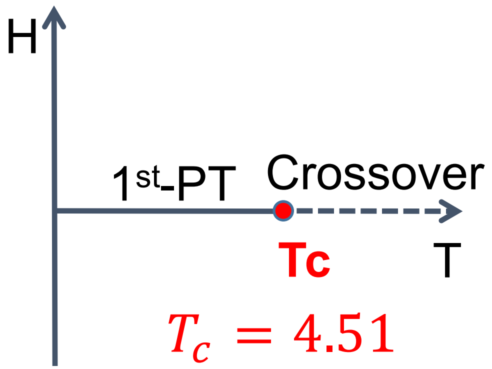
Partition function

$$Z(T, H) = \sum_{\{s_i\}} \exp(-E_{\{s_i\}}/k_B T)$$

Per-spin magnetization

$$m = \frac{1}{N} \sum_{i=1}^{N} s_i$$

(order parameter)



$T_c = 4.51$

Metropolis algorithm

- Initial configuration
- Test a spin for flipping (a Monte Carlo step)

  Acceptance probability $A(\boldsymbol{u} \to \boldsymbol{v})$,

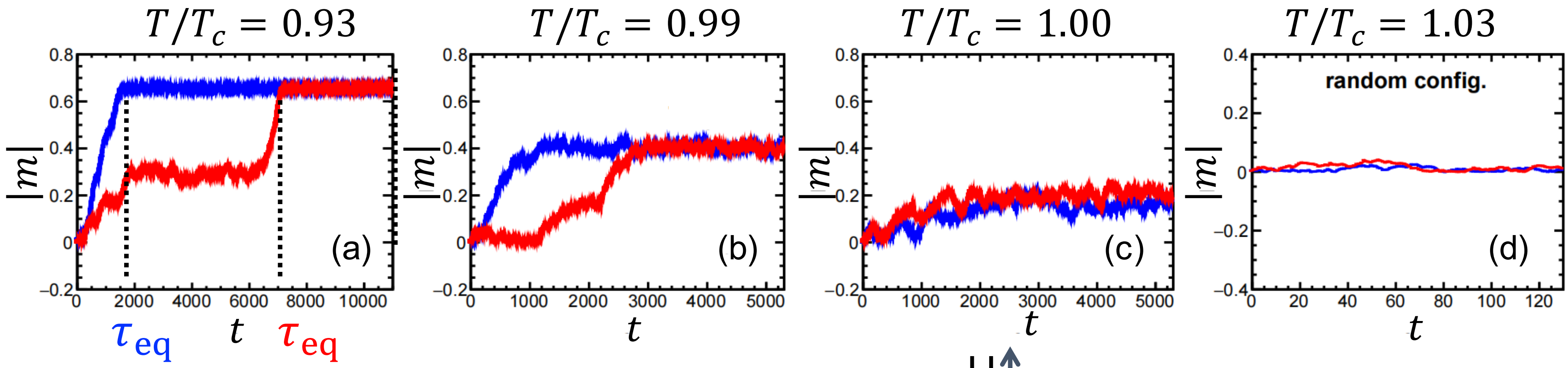
$$A(\boldsymbol{u} \to \boldsymbol{v}) = \begin{cases} \exp\left[-\dfrac{E_{\boldsymbol{v}} - E_{\boldsymbol{u}}}{k_B T}\right] & \text{if } E_{\boldsymbol{v}} > E_{\boldsymbol{u}} \\ 1 & \text{otherwise.} \end{cases}$$

  an increasing function of T

- Every spin in the lattice has been tested (one sweep)

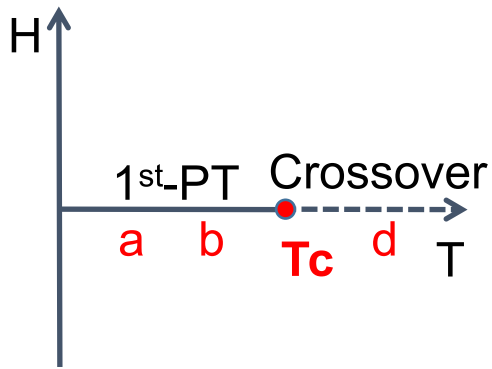
Time t is defined as the number of sweeps.

Evolution of order parameter



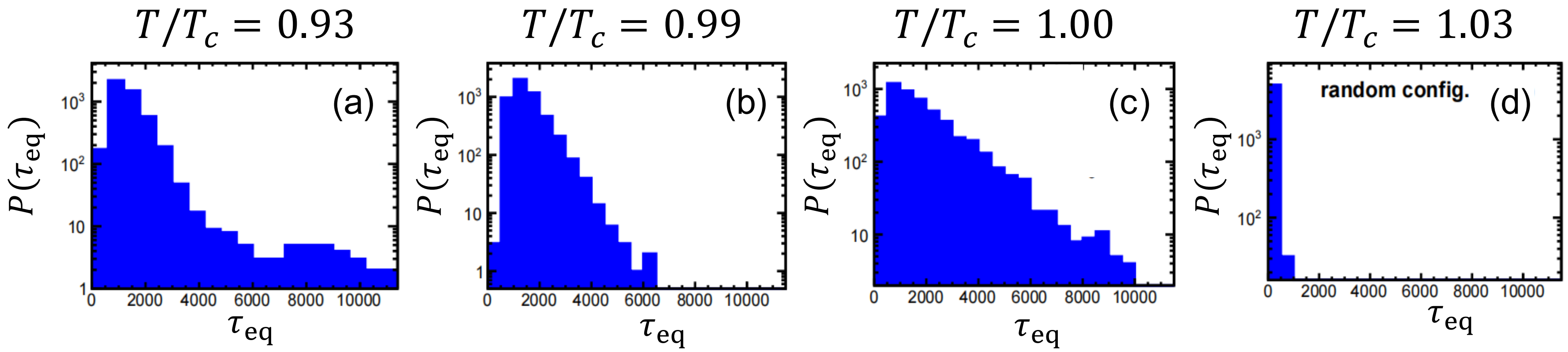
τ_{eq}: the equilibration time

- Random initial config.
- Difference in τ_{eq} is significant at low temperature



Xiaobing Li, Mingmei Xu, Yanhua Zhang, Zhiming Li, Yu Zhou, Jinghua Fu, Yuanfang Wu, Phys. Rev. C 105, 064904 (2022).

Distribution of equilibration time $\boldsymbol{\tau_{eq}}$

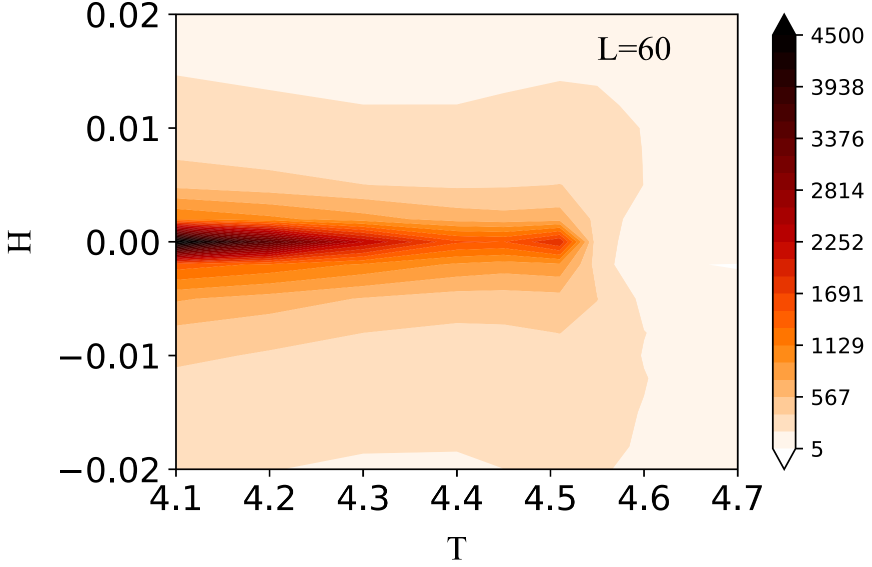


- At $T > T_c$, the width of the distribution is the smallest.
- At T_c, the distribution gets wide.
- At $T < T_c$, the distribution has a long tail.

Define the average equilibration time as

$$\bar{\tau}_{\rm eq} = \frac{1}{n}\sum_{i=1}^{n} \tau_{\rm eq}^{i}$$

$\bar{\tau}_{\text{eq}}$ near the entire phase boundary

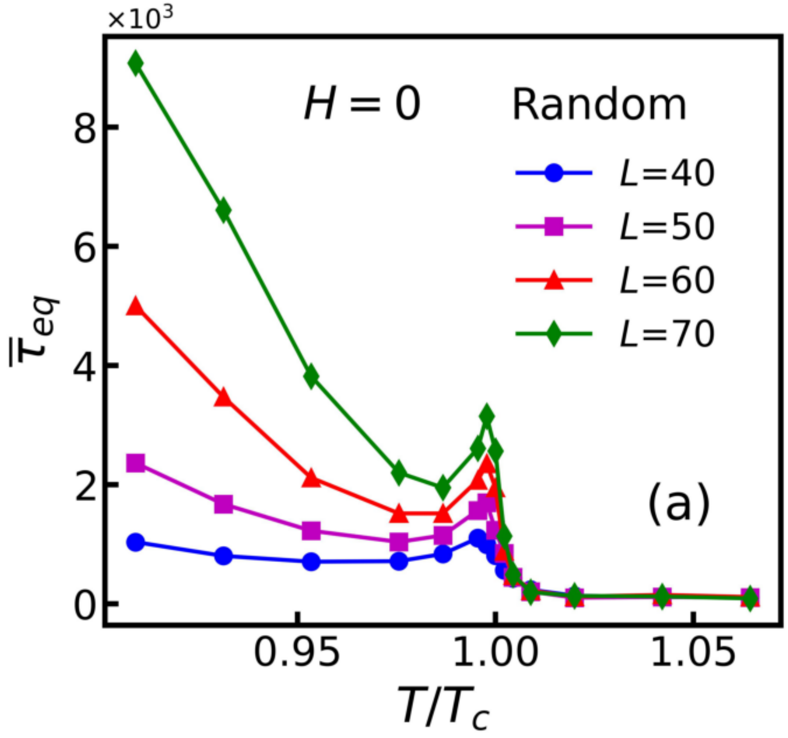


$\bar{\tau}_{\text{eq}}$ near the 1st-PTL $\gg$ $\bar{\tau}_{\text{eq}}$ near the CP

Ultra-slowing relaxation on the 1st-PTL !

Xiaobing Li, Ranran Guo, Mingmei Xu, Jinghua Fu, Lizhu Chen, Yu Zhou, Yuanfang Wu, arXiv:2412.18909.

Characteristics of nonequilibrium evolution



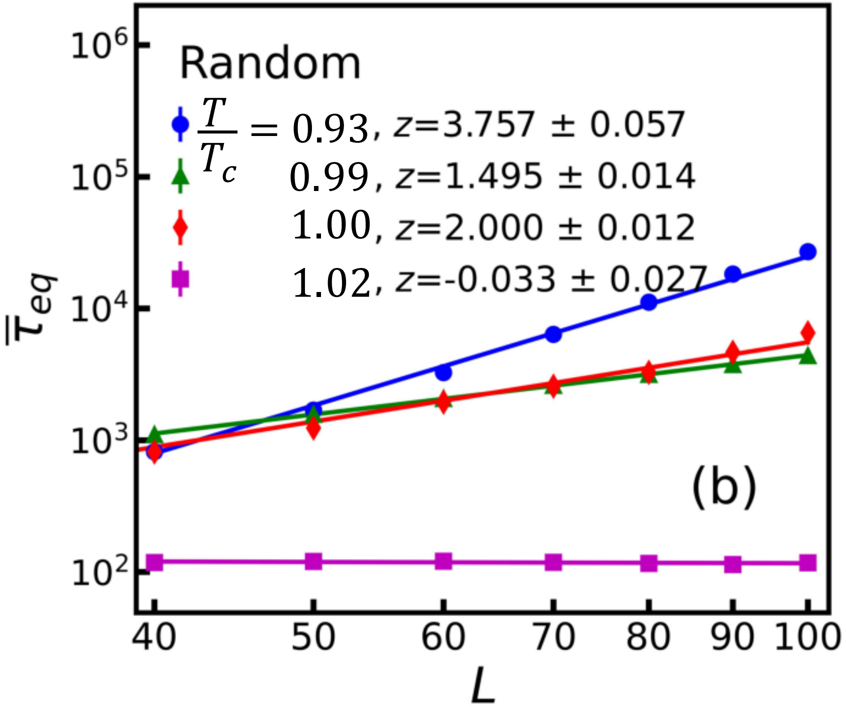
(1) $\bar{\tau}_{\mathrm{eq}}$ shows a decreasing trend with T: the acceptance probability is an increasing function of T.

(2) For $T > T_c$, $\bar{\tau}_{\mathrm{eq}}$ is short.

For $T < T_c$, $\bar{\tau}_{\mathrm{eq}}$ is large.

(4) Near T_c and $T \ll T_c$
strong dependences on system size

The dynamic scaling of $\bar{\tau}_{\text{eq}}$ on the entire phase boundary



- At T_c, **critical slowing down**

$$\bar{\tau}_{\text{eq}} \sim L^z$$

The dynamic exponent z is consistent with model A ($z = 2.0245$).

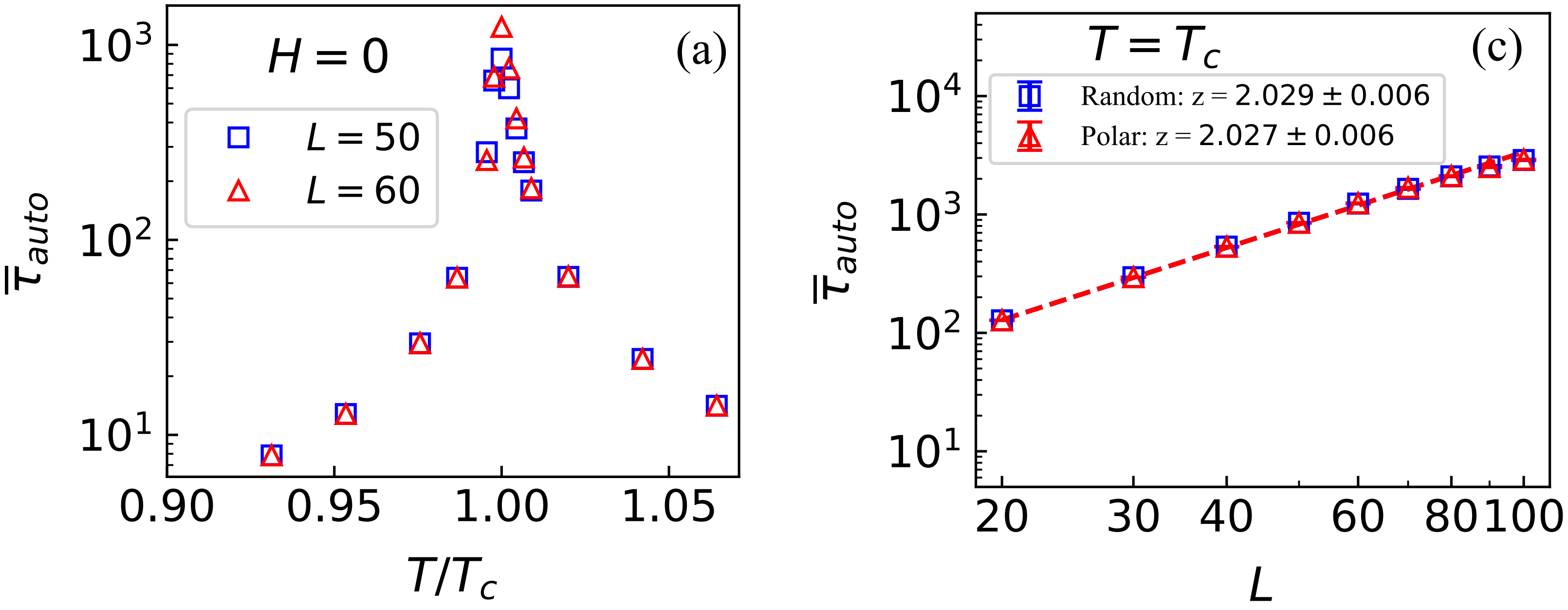
- $\bar{\tau}_{\text{eq}}$ is equivalent to $\tau_{\text{eq}}^{\text{dyn}}$, the same size scaling

- Power law also holds at 1st-PTL

No such scaling for auto-corr. time!

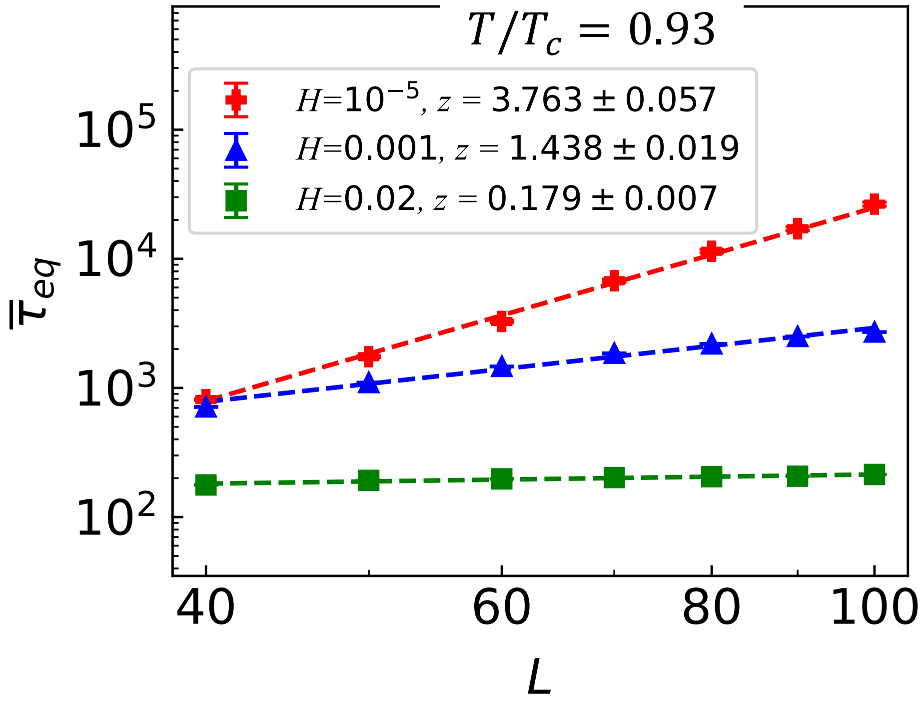
- z on the 1st-PTL > z at CP

Ultra-slowing relaxation on the 1st-PTL !



No size scaling for auto-correlation time at 1st-PT.

The dynamic scaling of $\bar{\tau}_{\mathrm{eq}}$ near the 1st-PTL



Approaching the 1st-PTL, the value of z increases.

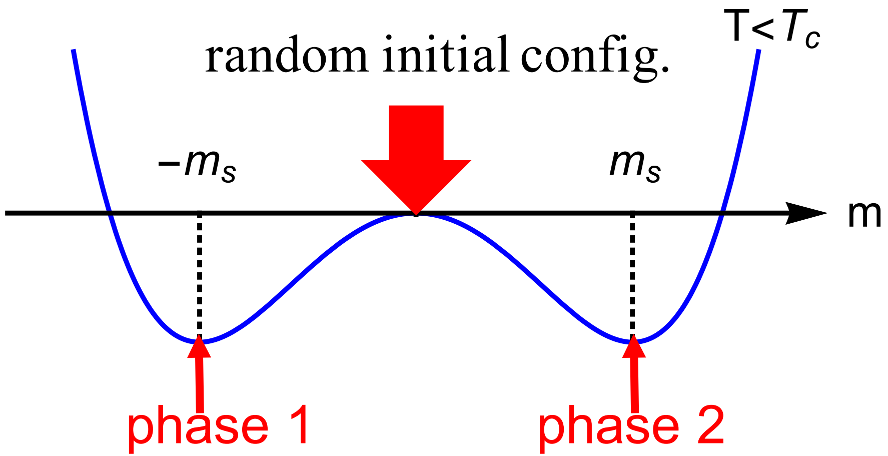
Ultra-slowing relaxation near the 1st-PTL !

Ultra-slowing relaxation understood through the perspective of free energy

An equilibrium state: $$\frac{\partial F}{\partial m} = 0$$

If this condition is not satisfied, the relaxation rate:

$$\frac{dm}{dt} = -\gamma \frac{\partial F}{\partial m}$$



located at the barrier top $\frac{\partial F}{\partial m} = 0$

relax extremely slowly

The free energy landscape at lower temperatures, see poster ID 172 by Ranran Guo.

Influences of nonequilibrium evolution on observables

Corresponding observables in Ising

$$C_1 = \langle X \rangle$$ the mean

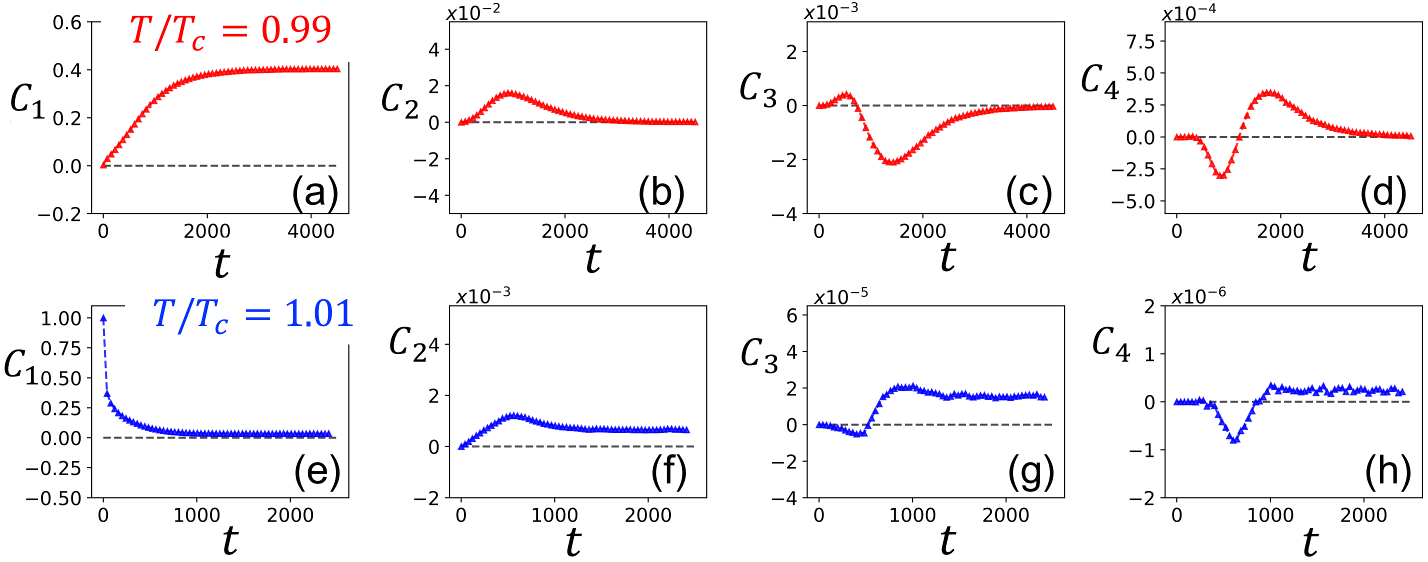
$$C_2 = \langle (\delta X)^2 \rangle$$ the variance

$$C_3 = \langle (\delta X)^3 \rangle$$ ~ the skewness

$$C_4 = \langle (\delta X)^4 \rangle - 3\langle (\delta X)^2 \rangle^2$$ ~ the kurtosis

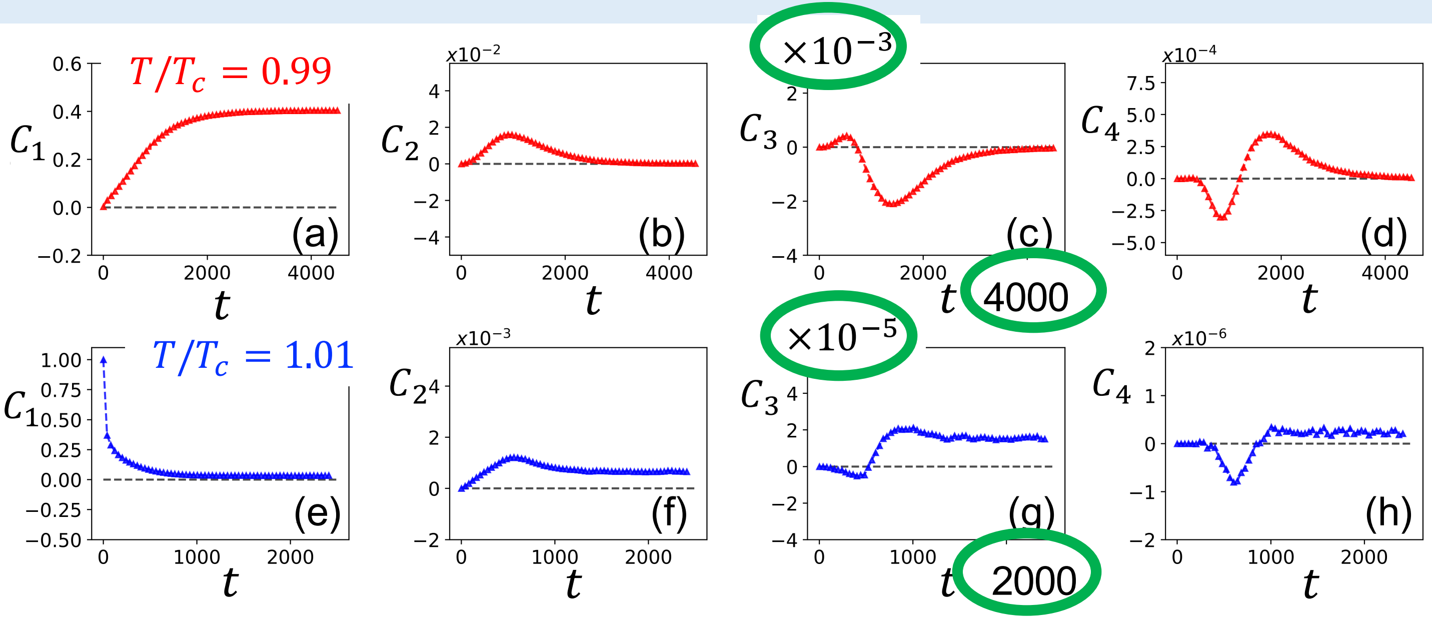
where $X = |m|, \delta X = |m| - \langle |m| \rangle$

Time evolution of cumulants of order parameter near CP



- C_1: a similar trend with Langevin dynamics
- C_3 and C_4: oscillations and sign changes consistent with STAR measurement and dynamical equations

Time evolution of cumulants of order parameter near CP



- $\bar{\tau}_{\rm eq}$ at T/T_c=0.99 ~ 2$\bar{\tau}_{\rm eq}$ at T/T_c=1.01
- The magnitude of C_3: higher by two orders at T/T_c=0.99

Summary and discussions

The MC simulation by Metropolis algorithm is an effective method for studying nonequilibrium.

(1) The trend of the order parameter, consistent with Langevin dynamics

(2) $\bar{\tau}_{\mathrm{eq}} \sim L^z$, consistent with critical slowing down

$$\left.\begin{array}{l} \bar{\tau}_{\mathrm{eq}} \text{ in numerical simulations} \\ \tau_{\mathrm{eq}}^{\mathrm{dyn}} \text{ in dynamical equations} \end{array}\right\} \text{equivalent}$$

(3) The sign change at T>Tc, consistent with dynamical models and STAR measurement.

Summary and discussions

On 1st-PTL, more difficult to achieve equilibrium, and the influence of nonequilibrium on observables is much stronger than that at the crossover side.

(1) $\bar{\tau}_{\mathrm{eq}}$ at $T < T_c$ $\gg$ $\bar{\tau}_{\mathrm{eq}}$ at $T > T_c$

(2) influences at $T < T_c$ $\gg$ influences at $T > T_c$

Nonequilibrium effects near 1st-PTL need more attention!

Thank you!