青島理工大學
QINGDAO UNIVERSITY OF TECHNOLOGY

QCD phase structure and properties of compact stars

QCD 相结构与致密星体性质

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April 24-28, 2025

相关文章:

[1] H. Liu, K.-J. Sun, and P.-C. Chu, arXiv:2412.17621
[2] H. Liu, Y.-H. Liu, Y.-H. Yang, M. Ju, X.-H. Wu, H.-M. Liu, and P.-C. Chu, Phys. Rev. D **111**, L051501 (2025).
[3] H. Liu, Y.-H. Yang, C. Yuan, M. Ju, X.-H. Wu, and P.-C. Chu, Phys. Rev. D **109**, 074037 (2024).
[4] H. Liu, Y.-H. Yang, Y. Han, and P.-C. Chu, Phys. Rev. D **108**, 034004 (2023).
[5] H. Liu, X.-M. Zhang, and P.-C. Chu, Phys. Rev. D **107**, 094032 (2023).

第二十届全国中高能核物理大会
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Outline

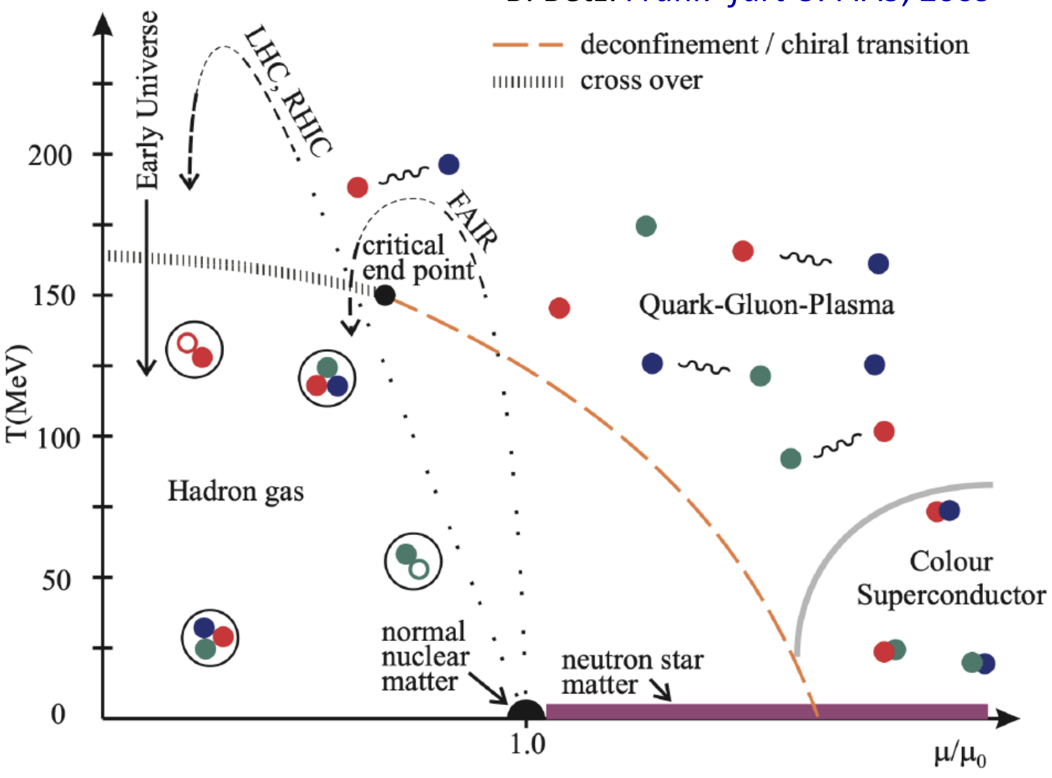
1. Introduction
2. Speed of sound and polytropic index in phase diagram
3. Density fluctuations and light nucleus yield ratios
4. Axion effects on quark matter and quark cores in massive hybrid stars
5. Conclusions

1. Introduction

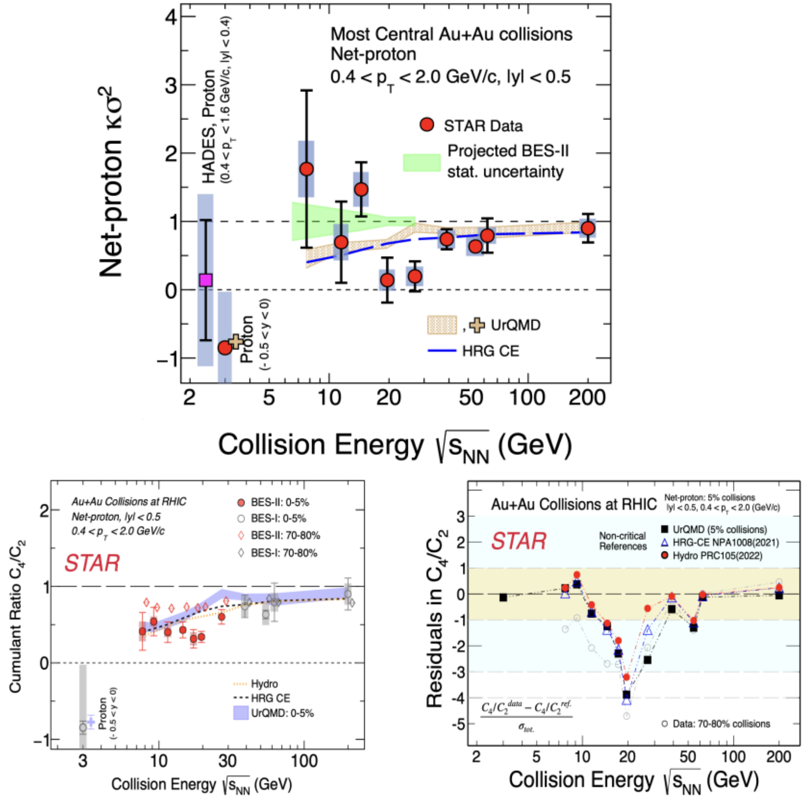
M. S. Abdallah et al.(STAR Collaboration) Phys. Rev. Lett. 128, 202303 (2022)

J. Adam et al.(STAR Collaboration) Phys. Rev. Lett. 126, 092301 (2021)

B. Betz. *Frank- furt U. FIAS, 2009*



Schematic depiction of QCD phase diagram



arXiv:2504.00817v1

CPOD2024_Pandav_STAR Collaboration.pdf

To search for CEP, the net-proton fluctuation, baryon direct flow, pion HBT radius and light nucleus yield ratio have been measured at BES program.

Observed hint of non-monotonic trend in BES-I

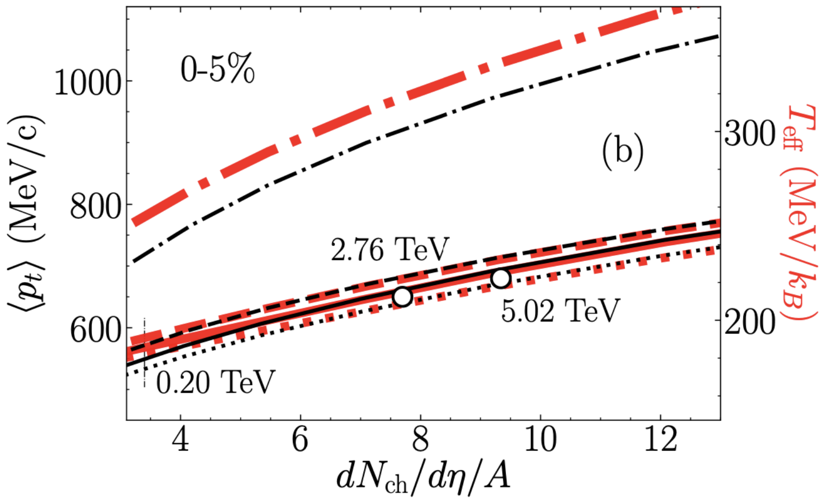
C_4/C_2 shows minimum around ~20 GeV in BES-II

2. Speed of sound and polytropic index

$$c_s^2 = \frac{\partial P}{\partial \varepsilon} \qquad \gamma = \frac{\partial \ln P}{\partial \ln \varepsilon}$$

Conformal limit: $c_s^2 = 1/3 \qquad \gamma = 1$

F. G. Gardim et al. Nat. Phys. 16, 615 (2020)



$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \frac{sdT}{Tds}\bigg|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}.$$

$$T_{\rm eff} = 222 \pm 9 \text{ MeV}$$

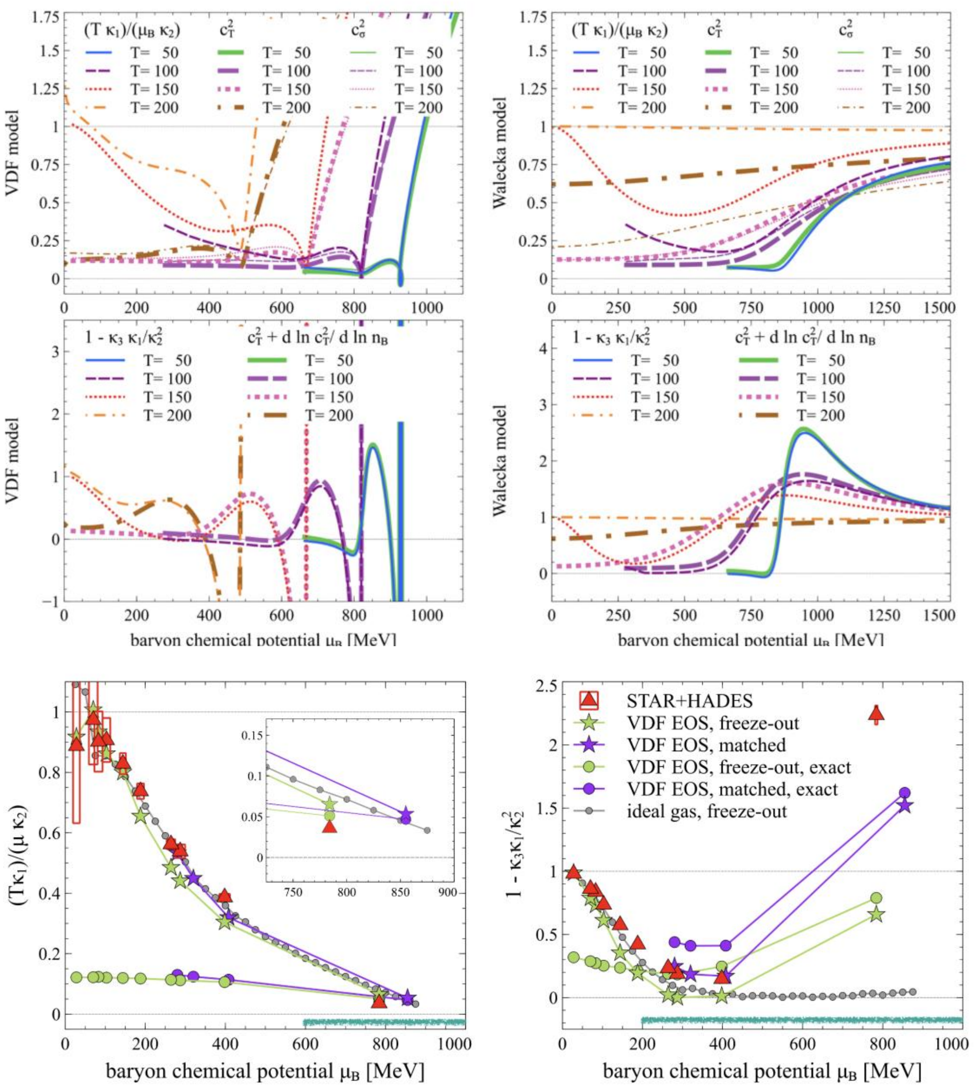
$$c_s^2(T_{\rm eff}) = 0.24 \pm 0.04$$

>> Hydrodynamic model indicate the speed of sound can be written as a function of charge particle multiplicity.

A. Sorensen et al. Phys. Rev. Lett. 127, 042303 (2021)

$$c_T^2 = \frac{\left(\frac{dP}{dn_B}\right)_T}{T\left(\frac{ds}{dn_B}\right)_T + \mu_B} \qquad c_T^2 = \left[\left(\frac{\partial \log \kappa_1}{\partial \log T}\right)_{\mu_B} + \frac{\mu_B}{T}\frac{\kappa_2}{\kappa_1}\right]^{-1}$$

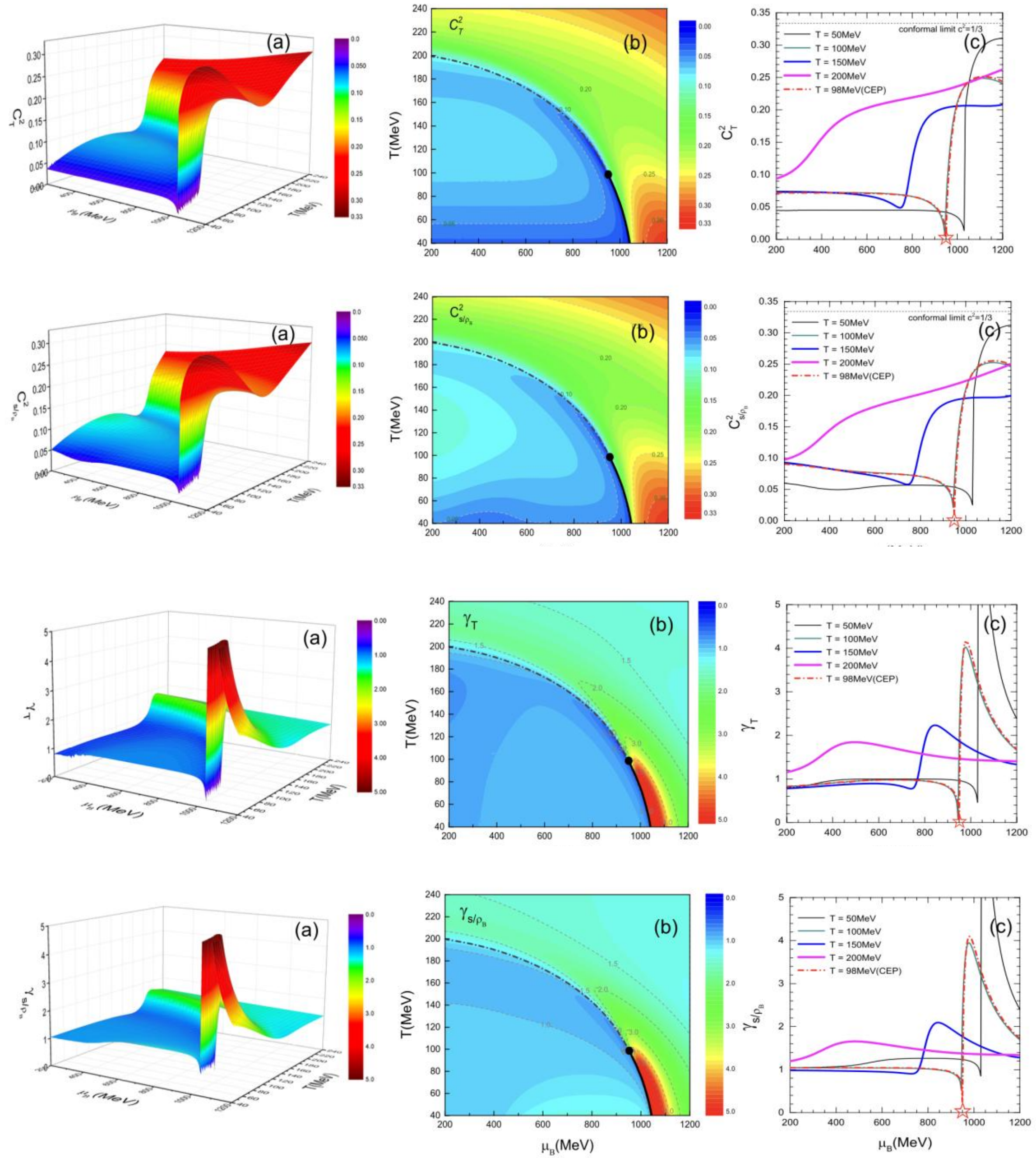
if $(\mu_B/T) \gg 1$, $\quad c_T^2 \approx c_\sigma^2 \qquad c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2} \qquad \left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$



W. B He et al. Phys. Rev. C 107, 014903 (2023)

He Liu et al. Phy. Rev. D 109, 074037 (2024)

2. Speed of sound and polytropic index（PNJLmodel）



$$c^2 = \frac{\partial P}{\partial \varepsilon}$$

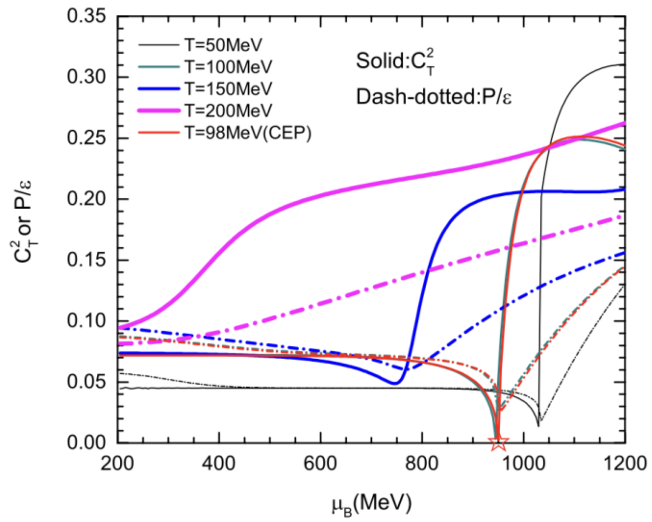
$$c_T^2 = \frac{\rho_B}{T(\frac{\partial s}{\partial \mu_B})_T + \mu_B(\frac{\partial \rho_B}{\partial \mu_B})_T}$$

$$c_{s/\rho_B}^2 = \frac{s\rho_B(\frac{\partial s}{\partial \mu_B})_T - s^2(\frac{\partial \rho_B}{\partial \mu_B})_T - \rho_B^2(\frac{\partial s}{\partial T})_{\mu_B} + \rho_B s(\frac{\partial \rho_B}{\partial T})_{\mu_B}}{(sT + \mu_B\rho_B)\left[(\frac{\partial s}{\partial \mu_B})_T(\frac{\partial \rho_B}{\partial T})_{\mu_B} - (\frac{\partial s}{\partial T})_{\mu_B}(\frac{\partial \rho_B}{\partial \mu_B})_T\right]}$$

$$\gamma = \frac{\partial \ln P}{\partial \ln \varepsilon}$$

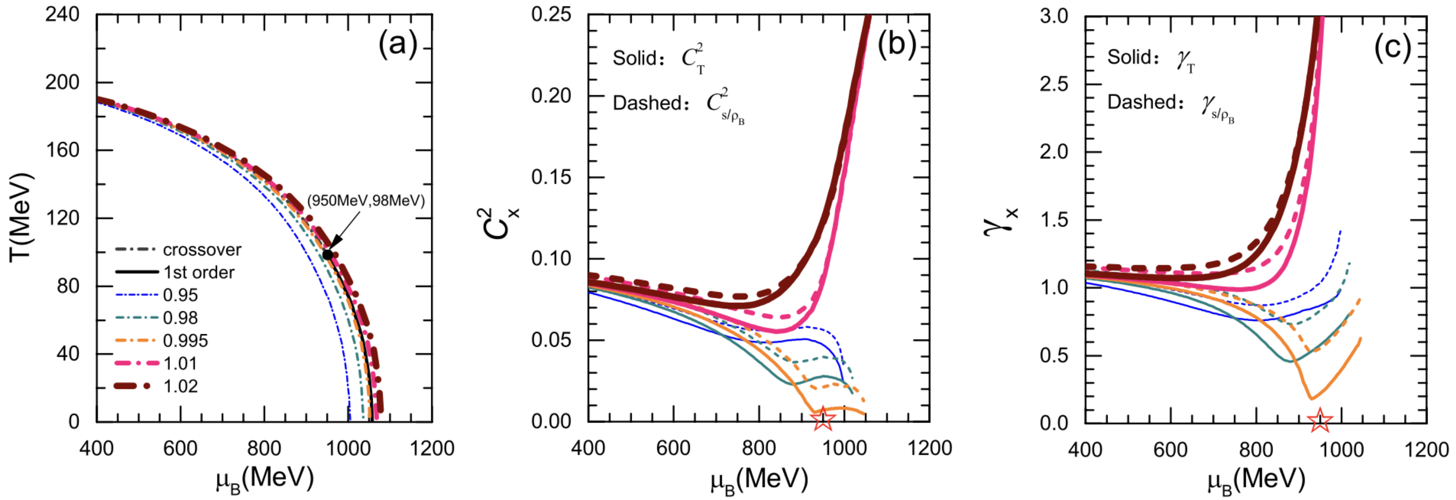
$$\gamma_T = \left(\frac{\partial P}{\partial \varepsilon}\right)_T \Big/ \frac{P}{\varepsilon} = \frac{\varepsilon}{P} c_T^2$$

$$\gamma_{s/\rho_B} = \left(\frac{\partial P}{\partial \varepsilon}\right)_{s/\rho_B} \Big/ \frac{P}{\varepsilon} = \frac{\varepsilon}{P} c_{s/\rho_B}^2$$



2. Speed of sound and polytropic index（PNJLmodel）

He Liu et al. Phys. Rev. D 109, 074037 (2024)

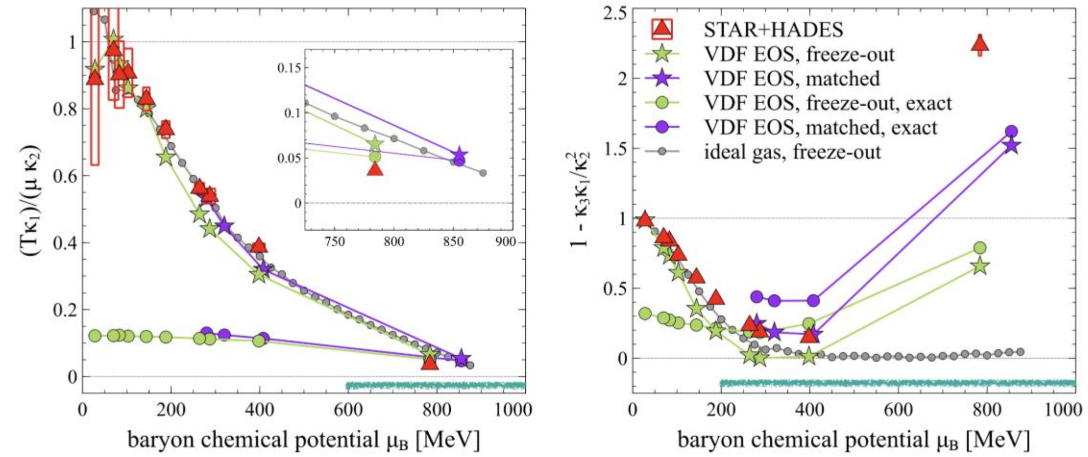


A. Sorensen et al. Phys. Rev. Lett. 127, 042303 (2021)

$$c_T^2 = \frac{\left(\frac{dP}{dn_B}\right)_T}{T\left(\frac{ds}{dn_B}\right)_T + \mu_B} \qquad c_T^2 = \left[\left(\frac{\partial \log \kappa_1}{\partial \log T}\right)_{\mu_B} + \frac{\mu_B}{T}\frac{\kappa_2}{\kappa_1}\right]^{-1}$$

if $(\mu_B/T) \gg 1$,

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2} \qquad \left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$$

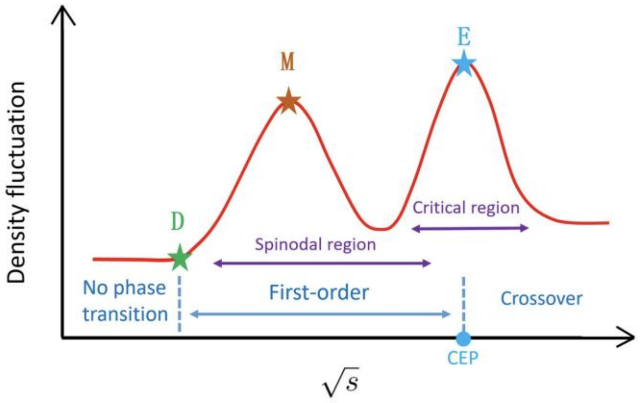


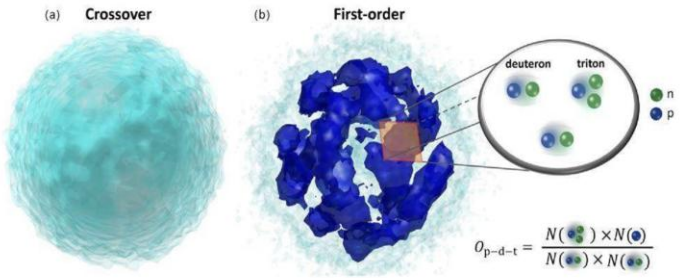
>> The speed of sound rapidly decreases near the CEP, followed by a small spinodal behavior and eventually continuing to decrease.

>> The polytropic index, especially γ_T, exhibits a more pronounced and nearly close to zero dip structure as it approaches the CEP.

>> How can we find the observations in experiment related to polytropic index?

3. Density fluctuations and light nucleus yield ratios





$$N_d = g_d \int dx_1 dx_2 dp_1 dp_2 f_{np}(x_1, p_1; x_2, p_2) \times W_d\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}\right)$$

$$N_t = g_t \int dx_1 dx_2 dx_3 dp_1 dp_2 dp_3 f_{nnp}(x_1, p_1; x_2, p_2; x_3, p_3)$$
$$\times W_t\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}, \frac{x_1 + x_2 - 2x_3}{\sqrt{6}}, \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)$$

Wigner function: $W_d(r,k) = 8\exp(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 k^2)$ $\sigma_d \approx 2.26\text{fm}$

$W_t(\rho,\lambda,k_\rho,k_\lambda) = 8^2 \exp(-\frac{\rho^2}{\sigma_t^2} - \frac{\lambda^2\sigma_d^2}{\sigma_t^2} - \sigma_t^2 k_\rho^2 - \sigma_t^2 k_\lambda^2)$ $\sigma_t \approx 1.59\text{fm}$

$$N_d = \frac{3}{\sqrt{2}}\left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p \langle\rho_n\rangle\left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

$$N_t = \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p \langle\rho_n\rangle^2\left[1 + \underline{2C_{np} + \Delta\rho_n} + \frac{3\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right) + O(G^2)\right]$$

$\rho_n(x) = \langle\rho_n\rangle + \delta\rho_n(x)$ $C_{np} = \langle\delta\rho_n(x)\delta\rho_p(x)\rangle / (\langle\rho_n\rangle\langle\rho_p\rangle)$

$\rho_p(x) = \langle\rho_p\rangle + \delta\rho_p(x)$ $\Delta\rho_n = \langle\delta\rho_n(x)^2\rangle / \langle\rho_n\rangle^2$

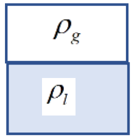
K.J. Sun et al. Phys. Lett. B 781, 499 (2018)
K.J. Sun et al. Eur. Phys. J. A 57, 313 (2021)
K.J. Sun et al. Phys. Lett. B 816, 136258 (2021)
K.J. Sun et al. arxiv.org/abs/2205.11010

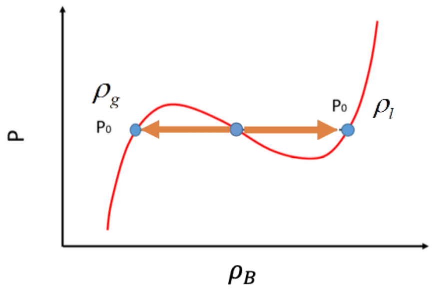
3. Density fluctuations and light nucleus yield ratios

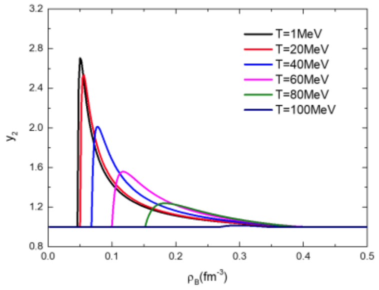
$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1+\Delta\rho_n + \frac{\lambda}{\sigma}G\left(\frac{\xi}{\sigma}\right)\right]$$

$$y_2 \approx 1 + \frac{\int d\mathbf{x}(\delta\rho(\mathbf{x}))^2}{\int d\mathbf{x}\rho_0^2} \equiv 1+\Delta\rho$$

$$y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int d\mathbf{x}\rho^3(\mathbf{x})]}{[\int d\mathbf{x}\rho^2(\mathbf{x})]^2} = \frac{(\rho_g\lambda_g+\rho_l\lambda_l)(\rho_g^3\lambda_g+\rho_l^3\lambda_l)}{(\rho_g^2\lambda_g+\rho_l^2\lambda_l)}$$

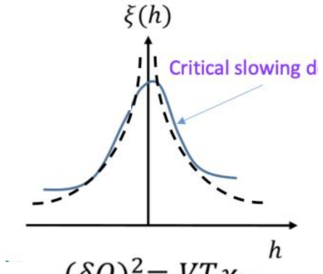






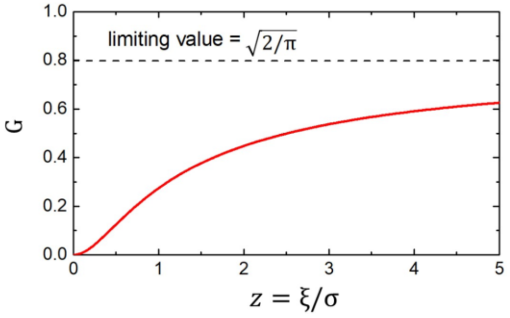
K.J. Sun et al. Phys. Lett. B 781, 499 (2018)

$$G(z) = \sqrt{\frac{2}{\pi}} - \frac{1}{z}e^{\frac{1}{2z^2}}\operatorname{erfc}\left(\frac{1}{\sqrt{2}z}\right)$$



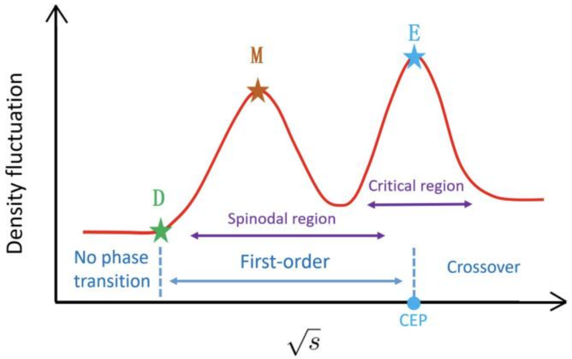
$(\delta Q)^2 = VT\chi_T$

$\chi_T \propto \xi^{2-\eta}$

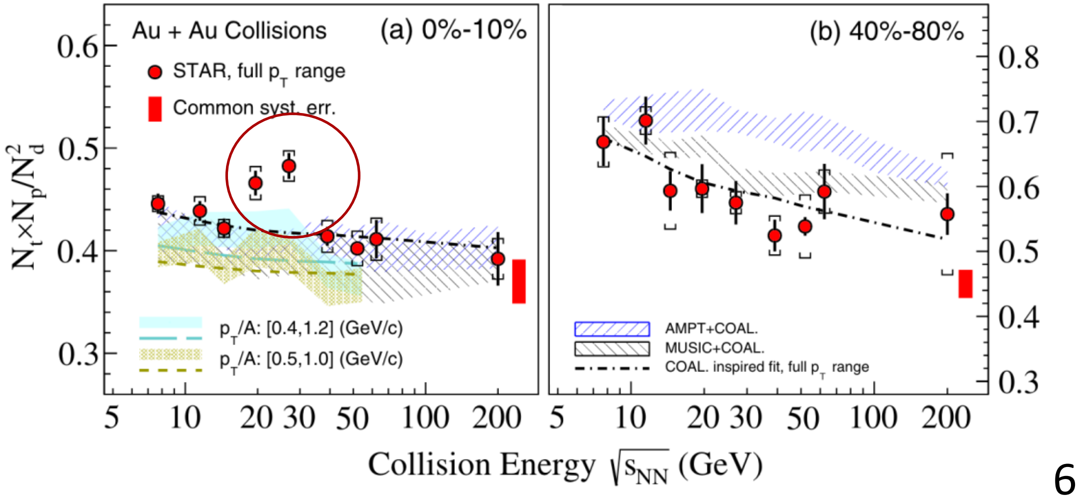


B. Berdnikov et al. Phys. Rev. D 61 105017 (2000)

K.J. Sun et al. Phys. Lett. B 816, 136258 (2021)



M. I. Abdulhamid et al. (STAR Collaboration) Phys. Rev. Lett. 130, 202301 (2023)



Which factor leads to the enhancements in the yield ratios: the first-order phase transition or CEP?

3. Density fluctuations and light nucleus yield ratios

$$\mathcal{L}_{\rm NJL}^{\rm SU(3)} = \bar\psi(i\partial\!\!\!/ - \hat m)\psi + G_S\sum_{a=0}^{8}[(\bar\psi\lambda^a\psi)^2 + (\bar\psi i\gamma_5\lambda^a\psi)^2] - K\{\det[\bar\psi(1+\gamma_5)\psi] + \det[\bar\psi(1-\gamma_5)\psi]\} + \mathcal{L}_1^{8q} + \mathcal{L}_2^{8q} + \mathcal{L}_{SV}^{8q}, \quad (1)$$

$$\mathcal{L}_1^{8q} = \frac{G_1}{2}\{[\bar\psi_i(1+\gamma_5)\psi_j][\bar\psi_j(1-\gamma_5)\psi_i]\}^2, \quad (2)$$

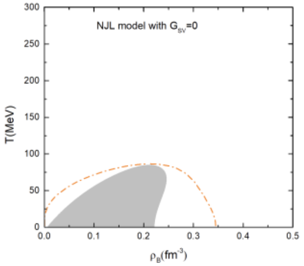
$$\mathcal{L}_2^{8q} = G_2\{[\bar\psi_i(1+\gamma_5)\psi_j][\bar\psi_j(1-\gamma_5)\psi_k] \times [\bar\psi_k(1+\gamma_5)\psi_l][\bar\psi_l(1-\gamma_5)\psi_i]\}. \quad (3)$$

A. Bhattacharyya. Phys. Rev. D 95, 054005 (2017)

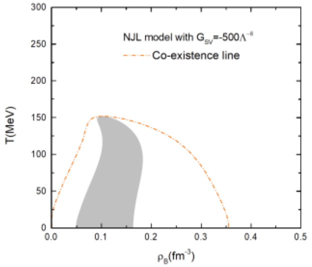
K.J. Sun et al. Phys. Rev. D 103.014006 (2021)

$$\mathcal{L}_{SV}^{8q} = G_{SV}\{\sum_{a=1}^{3}[(\bar\psi\lambda^a\psi)^2 + (\bar\psi i\gamma_5\lambda^a\psi)^2]\} \times \{\sum_{a=1}^{3}[(\bar\psi\gamma^\mu\lambda^a\psi)^2 + (\bar\psi\gamma_5\gamma^\mu\lambda^a\psi)^2]\}. \quad (4)$$

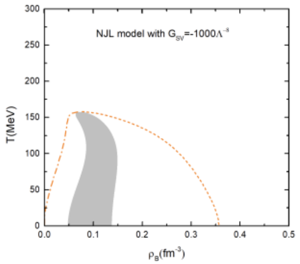
G_{SV}=0



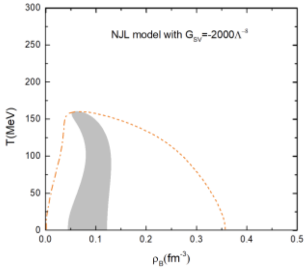
G_{SV}=-500 Λ^{-8}

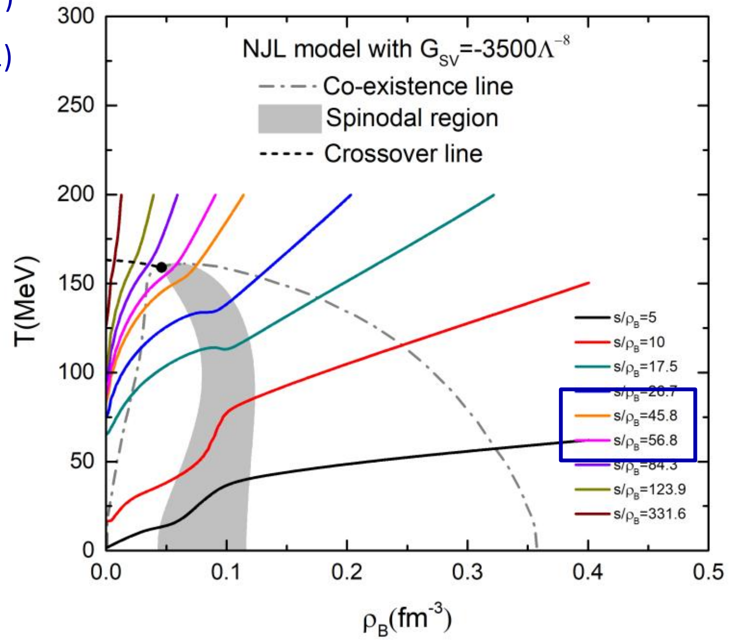


G_{SV}=-1000 Λ^{-8}

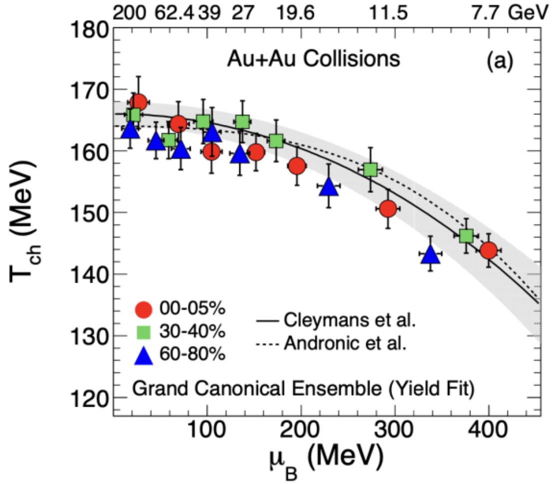


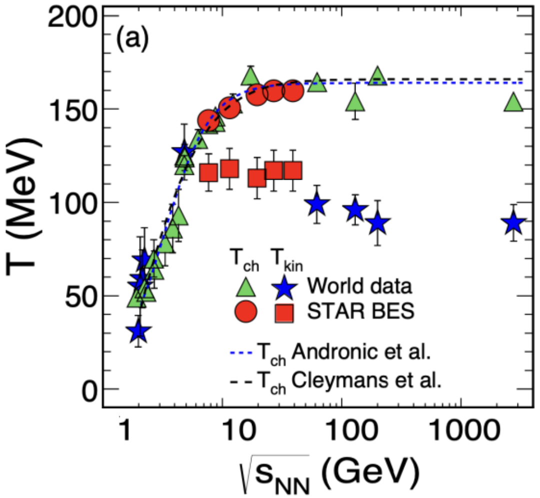
G_{SV}=-2000 Λ^{-8}





L. Adamczyk et al., (STAR), Phys. Rev. C **96** 044904 (2017)

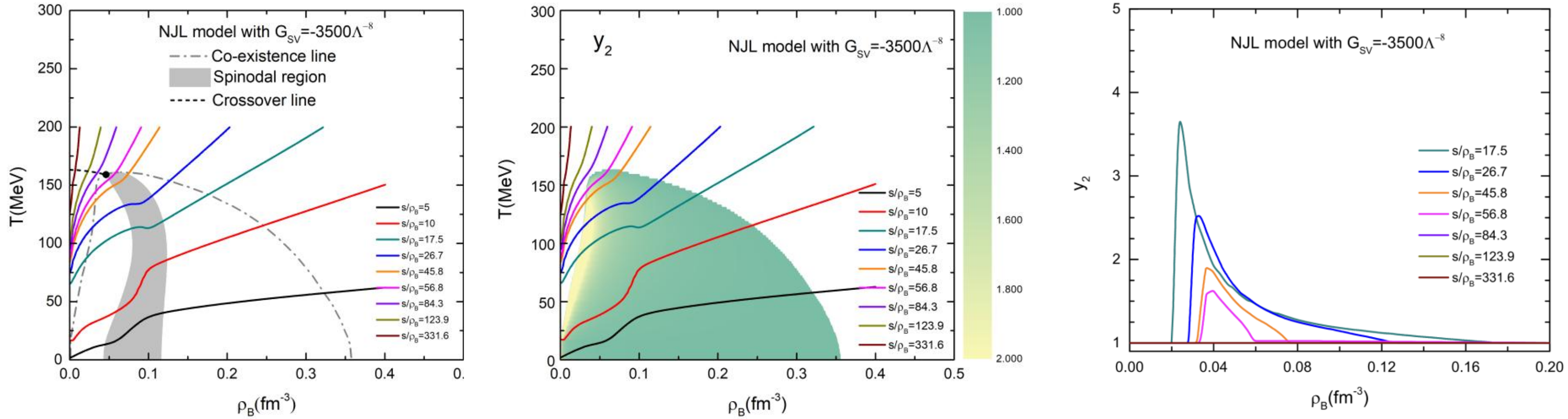




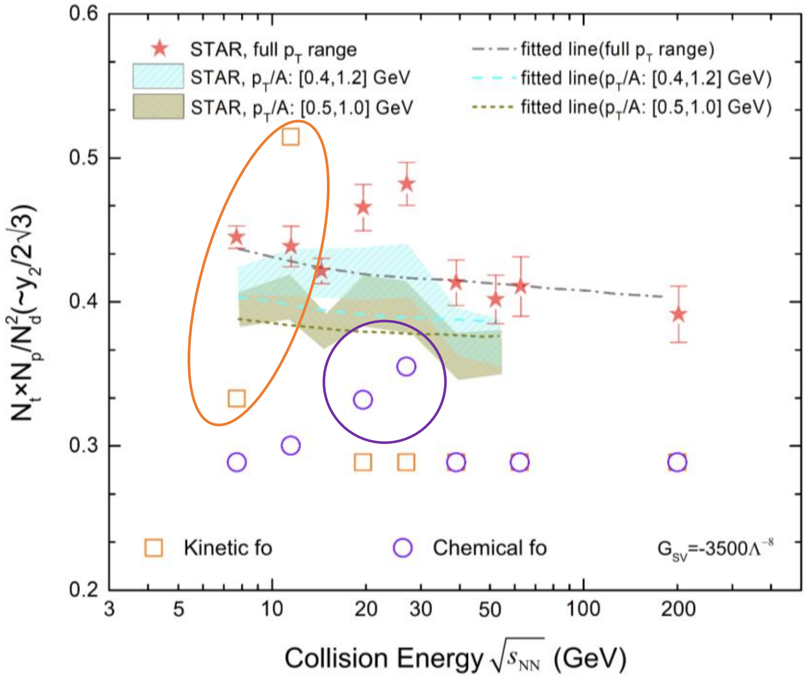
M. Motta et al. Eur. Phys. J. C 80,770 (2020)

$\sqrt{s_{NN}}$(GeV)	s_0(fm^{-3})	s/ρ_B	$T_{\rm kin}^{\rm fo}$	$T_{\rm ch}^{\rm fo}$
7.7	29.6	17.5	116	143
11.5	35.3	26.7	118	151
19.6	43.0	45.8	113	158
27.0	45.8	56.8	117	160
39.0	47.6	84.3	117	160
62.4	60.2	123.9	99	161
200	84.0	331.6	89	162

3. Density fluctuations and light nucleus yield ratios



M. Motta et al. Eur. Phys. J. C 80,770 (2020)



$\sqrt{s_{NN}}$(GeV)	s_0(fm^{-3})	s/ρ_B	T_{kin}^{fo}	T_{ch}^{fo}
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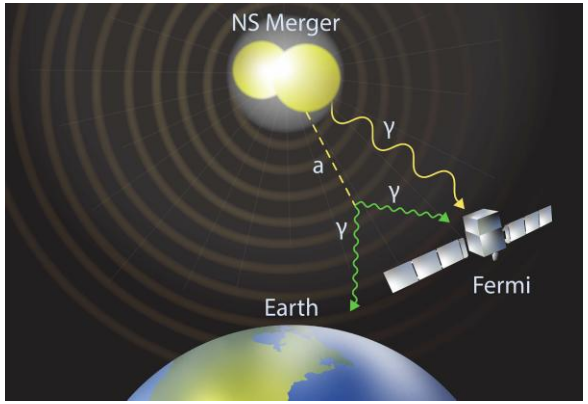
62.4	60.2	123.9	99	161
200	84.0	331.6	89	162

Could the first-order phase transition lead to the enhancements in the yield ratios?

4. Axion effects on quark matter and quark cores in massive hybrid stars

QCD *CP-violation* problem:

$$\mathcal{L}_\theta = \frac{\theta}{64\pi^2} g^2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$



S. R. Coleman, Subnucl. Ser. 15, 805 (1979)

Spontaneous CP violation for $\theta \neq 0$

R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977)

The axion as a pseudo-Goldstone boson from spontaneous breaking of the Peccei-Quinn (PQ) symmetry.

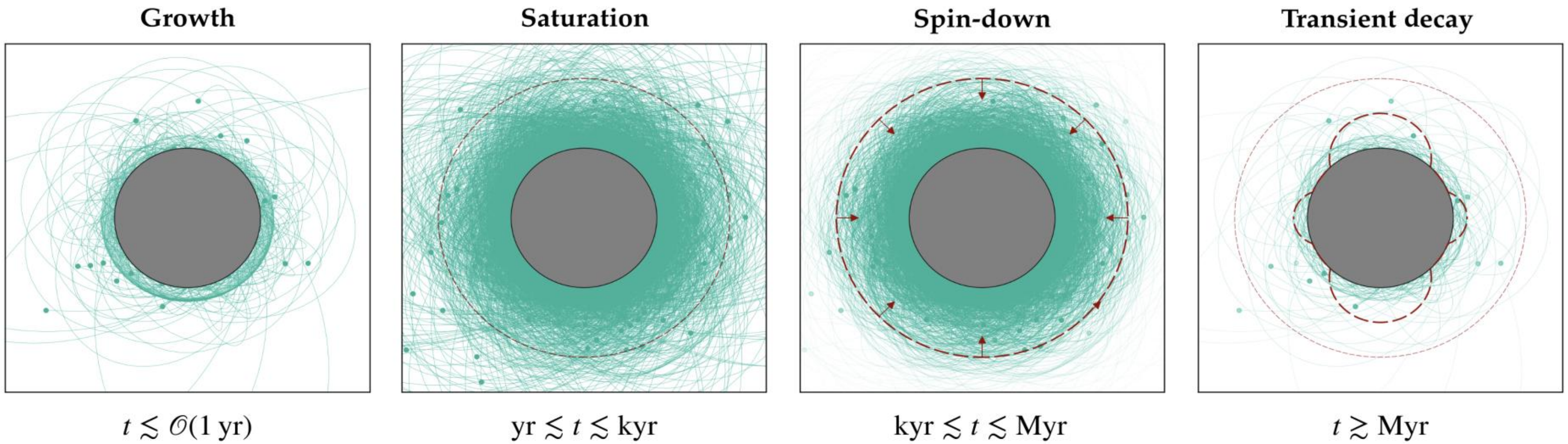
P. S. Bhupal De et al. Phys. Rev. Lett. 132, 101003 (2024)

$$\mathcal{L} \supset \frac{1}{2}\partial^\mu a \partial_\mu a - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}g_{a\gamma\gamma} a F^{\mu\nu}\tilde{F}_{\mu\nu}$$

Primakoff process: $\gamma + p \longrightarrow a + p$

Photon coalescence: $\gamma + \gamma \longrightarrow a$

Dion Noordhuis et al. Phys. Rev. X 14, 041015 (2024)



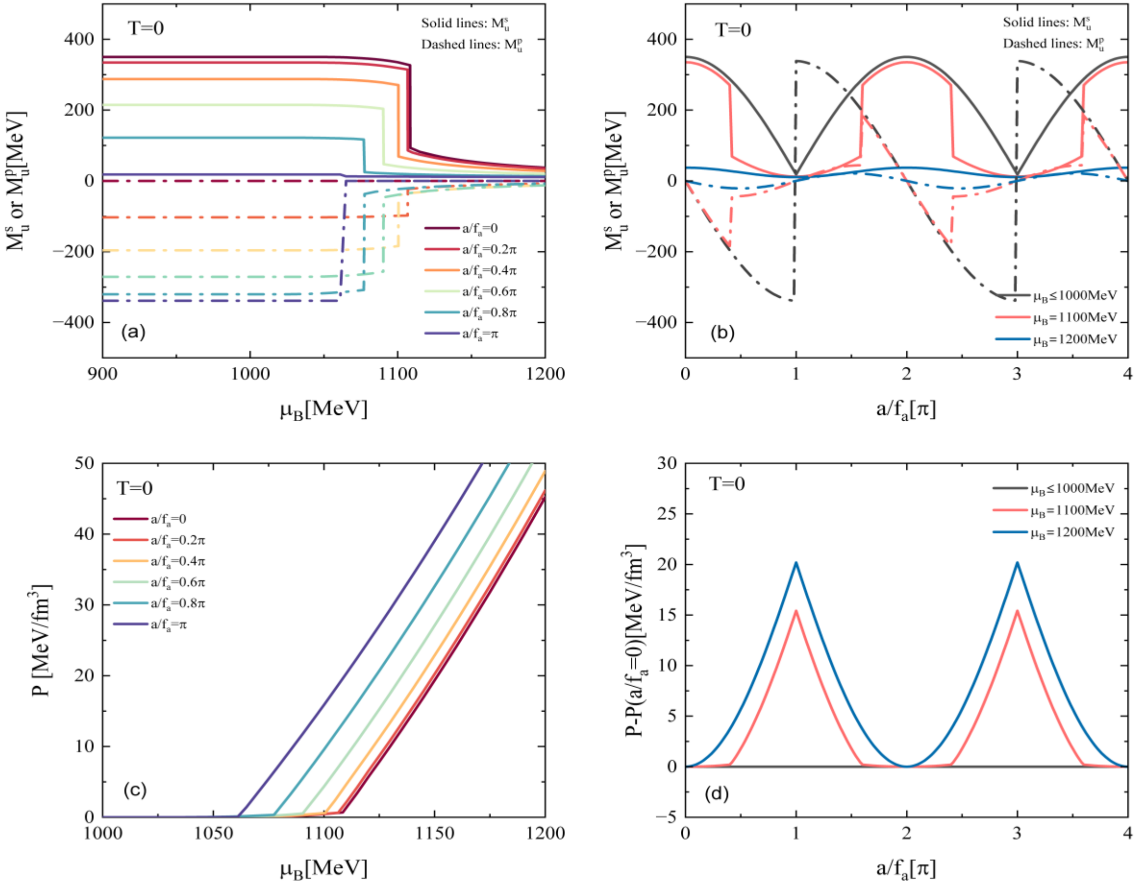
4. Axion effects on quark matter and quark cores in massive hybrid stars

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2]$$
$$- K\{e^{i\frac{a}{f_a}}\det[\bar{\psi}(1+\gamma_5)\psi] + e^{-i\frac{a}{f_a}}\det[\bar{\psi}(1-\gamma_5)\psi]\}$$
$$- \frac{G_V}{2}\sum_{a=0}^{8}[(\bar{\psi}\gamma_\mu\lambda_a\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\lambda_a\psi)^2],$$

$$M_i^s = m_i + 2G_S\sigma_i + 2K\Big[\cos\frac{a}{f_a}(\sigma_j\sigma_k - \eta_j\eta_k) + \sin\frac{a}{f_a}(\sigma_j\eta_k + \eta_j\sigma_k)\Big],$$
$$M_i^p = 2G_S\eta_i + 2K\Big[\cos\frac{a}{f_a}(\eta_j\sigma_k + \sigma_j\eta_k) + \sin\frac{a}{f_a}(\eta_j\eta_k - \sigma_j\sigma_k)\Big]$$

$$\sigma_i = \langle\overline{\psi_i}\psi_i\rangle \qquad \eta_i = \langle\overline{\psi_i}i\gamma_5\psi_i\rangle$$

He Liu et al. Phys. Rev. D 111, L051501 (2025)

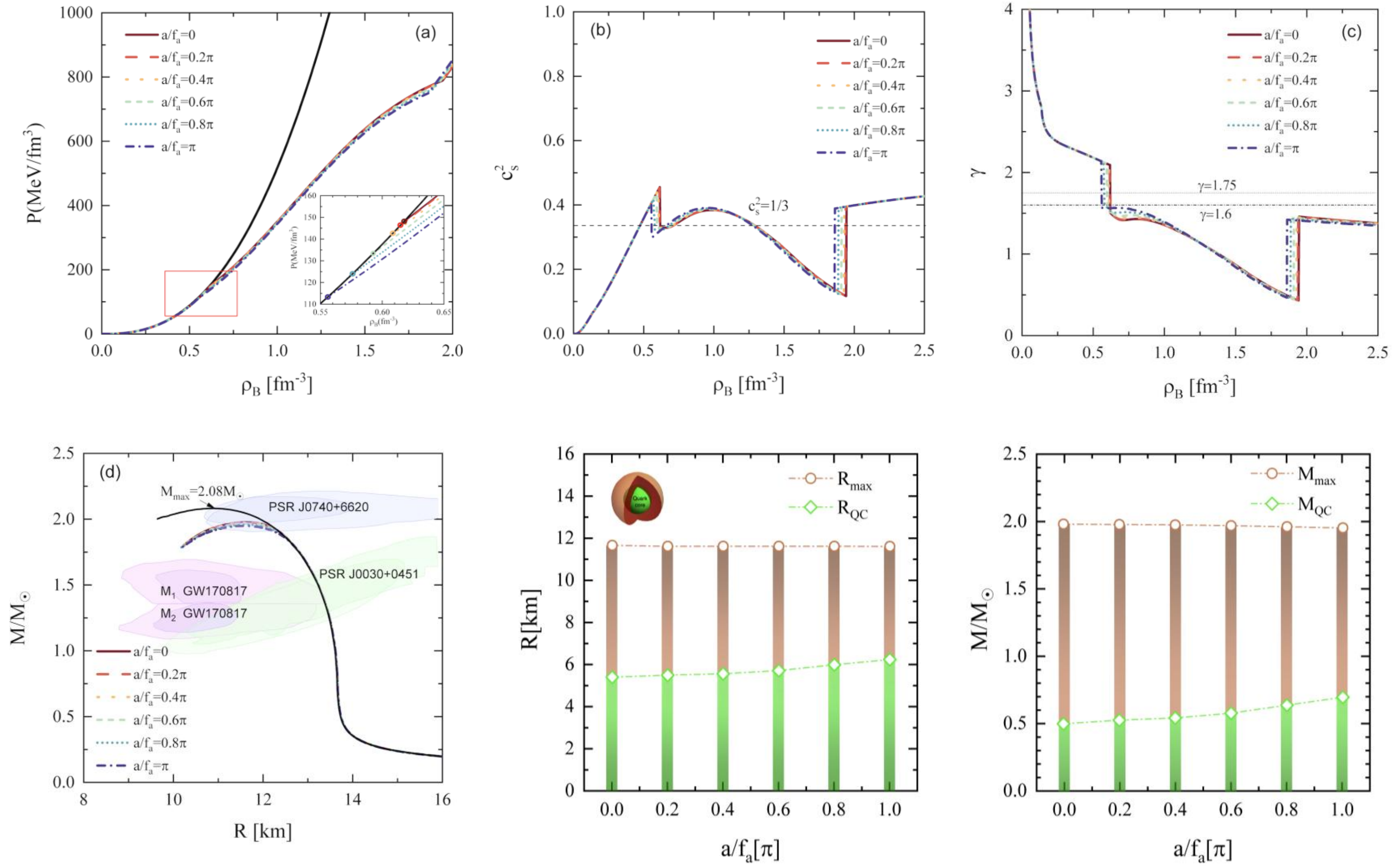


>> The first-order chiral phase transition is sensitive to the scaled axion field.

>> For $\mu_B \leq 1000$ MeV, spontaneous *CP* violation is clearly evident when $a/f_a = \pi + 2k\pi$, with M^p_u exhibiting two different sign solutions.
For $\mu_B = 1100$ MeV, we can see the disco- ntinuous changes in the magnitudes of M^s_u and M^p_u, which are the results of the first-order chiral phase transition.
For $\mu_B = 1200$ MeV, the restoration of chiral symmetry and CP symmetry leads to continuous changes in the magnitudes of M^s_u and M^p_u.

4. Axion effects on quark matter and quark cores in massive hybrid stars

He Liu et al. Phys. Rev. D 111, L051501 (2025)



>> Axion field shifts the mixed phase onset to lower densities but slightly softens the EOS for the mixed phase matter, which marginally reduces hybrid star maximum mass and radius but significantlyenlarges the quark-matter core's mass and radius.

5. Conclutions

1. The non-monotonic behaviors were observed in speed of sound and polytropic index around the phase transition boundary. Along the hypothetical chemical freeze-out lines below the chiral phase transition boundary, the speed of sound rapidly decreases near the CEP, followed by a small spinodal behavior, while the polytropic index, especially γ_T, exhibits a more pronounced and nearly close to zero dip structure as it approaches the CEP.
2. Could the first-order phase transition lead to the enhancements in the yield ratio?
3. The axion field shifts the onset of the hadron-quark mixed phase to lower densities and slightly softens the equation of state of the mixed phase matter, which also results in a slight decrease in the maximum mass and corresponding radius of the hybrid stars. However, we also find that the lowering of the onset of the mixed phase significantly increases the radius and mass of the quark-matter core in the hybrid star.

Thanks for your attention!

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BACKUP

1. NJL model

$$\mathcal{L}_{NJL}^{SU(3)} = \mathcal{L}_0 + \mathcal{L}_S + \mathcal{L}_{KMT} + \mathcal{L}_V + \mathcal{L}_{IV}$$

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi$$

$$\mathcal{L}_S = \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]$$

$$\mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi]$$

$$\mathcal{L}_V = -\frac{G_V}{2}\sum_{a=0}^{8}[(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]$$

$$\mathcal{L}_{IV} = -G_{IV}\sum_{a=1}^{3}[(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]$$

$$\begin{aligned}\mathcal{L}_{\rm NJL}^{\rm SU(3)} &= \bar{\psi}(i\not{\partial} - \hat{m})\psi + G_S\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ &- K\{\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\} \\ &+ \mathcal{L}_1^{8q} + \mathcal{L}_2^{8q} + \mathcal{L}_{SV}^{8q}, \qquad (1)\end{aligned}$$

$$\mathcal{L}_1^{8q} = \frac{G_1}{2}\{[\bar{\psi}_i(1+\gamma_5)\psi_j][\bar{\psi}_j(1-\gamma_5)\psi_i]\}^2, \qquad (2)$$

$$\begin{aligned}\mathcal{L}_2^{8q} &= G_2\{[\bar{\psi}_i(1+\gamma_5)\psi_j][\bar{\psi}_j(1-\gamma_5)\psi_k] \\ &\times [\bar{\psi}_k(1+\gamma_5)\psi_l][\bar{\psi}_l(1-\gamma_5)\psi_i]\}. \qquad (3)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{SV}^{8q} &= G_{SV}\{\sum_{a=1}^{3}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]\} \\ &\times \{\sum_{a=1}^{3}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2]\}. \qquad (4)\end{aligned}$$

TABLE I. Parameters in the extended NJL model

$m_{u,d}$(MeV)	m_s(MeV)	Λ(MeV)	$G_S\Lambda^2$
5.5	183.468	637.720	3.02
$K\Lambda^5$	G_1(MeV^{-8})	G_2(MeV^{-8})	$G_{SV}\Lambda^8$
9.496	2.193×10^{-21}	-1.890×10^{-22}	-3500.00

2. Hybrid Star

(1) Nuclear matter(ImMDI model)

$$U_\tau(\rho,\delta,\vec{p}) = A_u\frac{\rho_{-\tau}}{\rho_0} + A_l\frac{\rho_\tau}{\rho_0} + B\frac{\rho}{\rho_0}^{\sigma}(1-x\delta^2) - 4x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{-\tau} + \frac{2C_l}{\rho_0}\int d^3\vec{p}'\frac{f_\tau(\vec{r},\vec{p})}{1+(\vec{p}-\vec{p}')^2/\Lambda^2} + \frac{2C_u}{\rho_0}\int d^3\vec{p}'\frac{f_{-\tau}(\vec{r},\vec{p})}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}.$$

$$V_H = V_{HP} + V_{HK} + V_{HM}$$

$$V_{HK} = \sum_b\sum_{\tau_b}\frac{p^5_{F\tau_b}}{10\pi^2 m_{\tau_b}},$$

$$V_{HM} = \sum_b\sum_{\tau_b}\rho_{\tau_b}m_{\tau_b}.$$

(2) Quark matter(NJL model)

$$\varepsilon_Q = -2N_c\sum_{i=u,d,s}\int_0^\Lambda\frac{d^3p}{(2\pi)^3}E_i(1-f_i-\bar{f}_i) - \sum_{i=u,d,s}(\tilde{\mu}_i-\mu_i)\rho_i + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s - G_V(\rho_u^2+\rho_d^2+\rho_s^2) + G_{IS}(\sigma_u-\sigma_d)^2 - G_{IV}(\rho_u-\rho_d)^2 - \varepsilon_0.$$

(3) Lepton

$$V_L = V_e + V_\mu,$$
$$V_e = \frac{p^4_{Fe}}{4\pi^2},$$
$$V_\mu = \frac{1}{4\pi^2}\left[p_{F\mu}\mu_\mu^3 - \frac{1}{2}m_\mu^2 p_{F\mu}\mu_\mu - \frac{1}{2}m_\mu^4\ln\left(\frac{p_{F\mu}+\mu_\mu}{m_\mu}\right)\right].$$

where

$$\mu_l = (m_l^2 + p_{Fl}^2)^{1/2} \text{ with } p_{Fl} = (3\pi^2\rho_l)^{1/3}$$

(4) Equilibrium conditions in mixed phase

$$\mu_i = \mu_B b_i - \mu_c q_i, \qquad P^H = P^Q,$$
$$\rho_B = (1-Y)(\rho_n+\rho_p) + \frac{Y}{3}(\rho_u+\rho_d+\rho_s),$$
$$0 = (1-Y)\rho_p + \frac{Y}{3}(2\rho_u-\rho_d-\rho_s) - \rho_e - \rho_\mu$$

3. TOV Function

(1)TOV Function

$$\frac{dP(r)}{dr} = -\frac{M(r)[\varepsilon(r) + P(r)]}{r^2}\left[1 + \frac{4\pi P(r) r^3}{M(r)}\right] \times \left[1 - \frac{2M(r)}{r}\right]^{-1},$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r).$$

(2) tidal deformability

$$k_2 = 3/2\Lambda\beta^5$$

$$\begin{aligned} k_2 = & \frac{8}{5}(1-2\beta)^2[2-y_R+2\beta(y_R-1)] \\ & \times\{2\beta[6-3y_R+3\beta(5y_R-8)] \\ & +4\beta^3[13-11y_R+\beta(3y_R-2)+2\beta^2(1+y_R)] \\ & +3(1-2\beta)^2[2-y_R+2\beta(y_R-1)]\ln(1-2\beta)\}^{-1} \end{aligned}$$

$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0$$

with

$$F(r) = \frac{r - 4\pi r^3[\varepsilon(r) - P(r)]}{r - 2M(r)},$$

$$Q(r) = \frac{4\pi r[5\varepsilon(r) + 9P(r) + \frac{\varepsilon(r)+P(r)}{\partial P(r)/\partial\varepsilon(r)} - \frac{6}{4\pi r^2}]}{r - 2M(r)} - 4\left[\frac{M(r) + 4\pi r^3 P(r)}{r^2(1 - 2M(r)/r)}\right]^2.$$

4. QCD Axion

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - \hat{m})\psi + \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2]$$
$$- K\{e^{i\frac{a}{f_a}}\det[\bar{\psi}(1+\gamma_5)\psi] + e^{-i\frac{a}{f_a}}\det[\bar{\psi}(1-\gamma_5)\psi]\}$$
$$- \frac{G_V}{2}\sum_{a=0}^{8}[(\bar{\psi}\gamma_\mu\lambda_a\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\lambda_a\psi)^2], \quad (1)$$

$$M_i^s = m_i + 2G_S\sigma_i + 2K\left[\cos\frac{a}{f_a}(\sigma_j\sigma_k - \eta_j\eta_k) + \sin\frac{a}{f_a}(\sigma_j\eta_k + \eta_j\sigma_k)\right], \quad (3)$$

$$M_i^p = 2G_S\eta_i + 2K\left[\cos\frac{a}{f_a}(\eta_j\sigma_k + \sigma_j\eta_k) + \sin\frac{a}{f_a}(\eta_j\eta_k - \sigma_j\sigma_k)\right] \quad (4)$$

$$\Omega = -2N_c\sum_i\left\{\int_0^\Lambda \frac{d^3p}{(2\pi)^3}E_i + \int\frac{d^3p}{(2\pi)^3}[T\ln(1+e^{-\beta(E_i-\tilde{\mu}_i)}) + T\ln(1+e^{-\beta(E_i+\tilde{\mu}_i)})]\right\} + \sum_i[G_S(\sigma_i^2+\eta_i^2) - G_V\rho_i^2]$$
$$-4K[\cos\frac{a}{f_a}(\sigma_u\sigma_d\sigma_s - \sigma_s\eta_u\eta_d - \sigma_u\eta_d\eta_s - \sigma_d\eta_u\eta_s)$$
$$-\sin\frac{a}{f_a}(\eta_u\eta_d\eta_s - \sigma_u\sigma_s\eta_d - \sigma_d\sigma_s\eta_u - \sigma_u\sigma_d\eta_s)], \quad (2)$$

$$\sigma_i = -2N_c\left[\int_0^\Lambda\frac{d^3p}{(2\pi)^3}\frac{M_i^s}{E_i} - \int\frac{d^3p}{(2\pi)^3}\frac{M_i^s}{E_i}(f_i + \bar{f}_i)\right], \quad (5)$$

$$\eta_i = 2N_c\left[\int_0^\Lambda\frac{d^3p}{(2\pi)^3}\frac{M_i^p}{E_i} - \int\frac{d^3p}{(2\pi)^3}\frac{M_i^p}{E_i}(f_i + \bar{f}_i)\right], \quad (6)$$