Transport model study of conserved charge fluctuations and QCD phase transition in heavy-ion collisions

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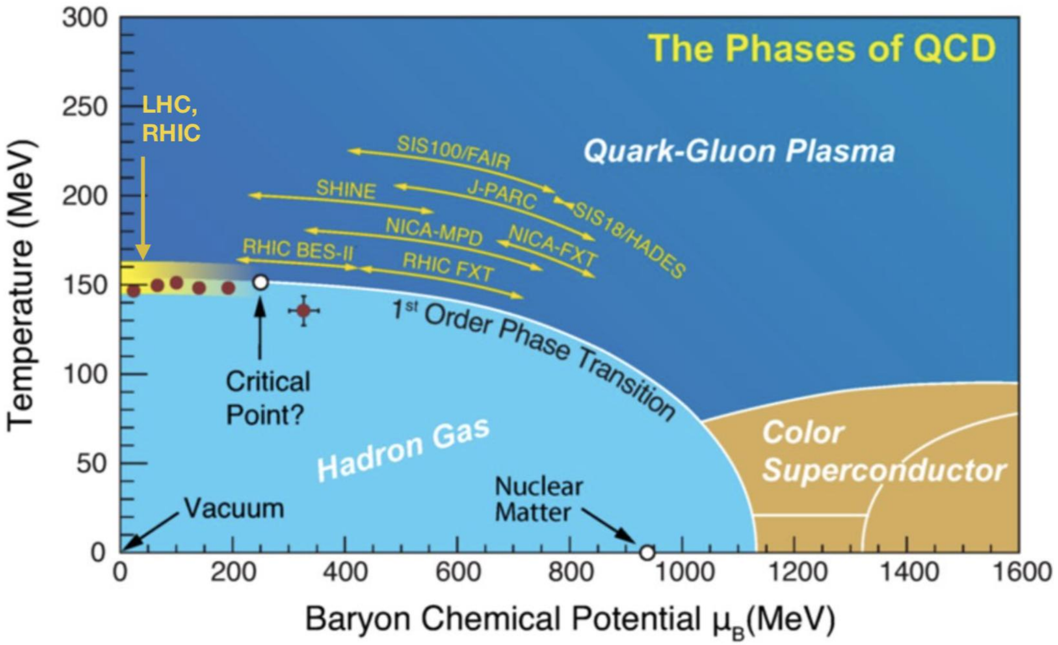
Outline

- Introduction
  - QCD Phase Diagram
  - Cumulants of Conserved Charges
  - Beam Energy Scan (BES) at RHIC
- Method
  - A Multi-Phase Transport Model
- Results
  - Fluctuations of Conserved Charges
  - Incorporating FRG Into AMPT Model
- Summary and Outlook

QCD Phase Diagram

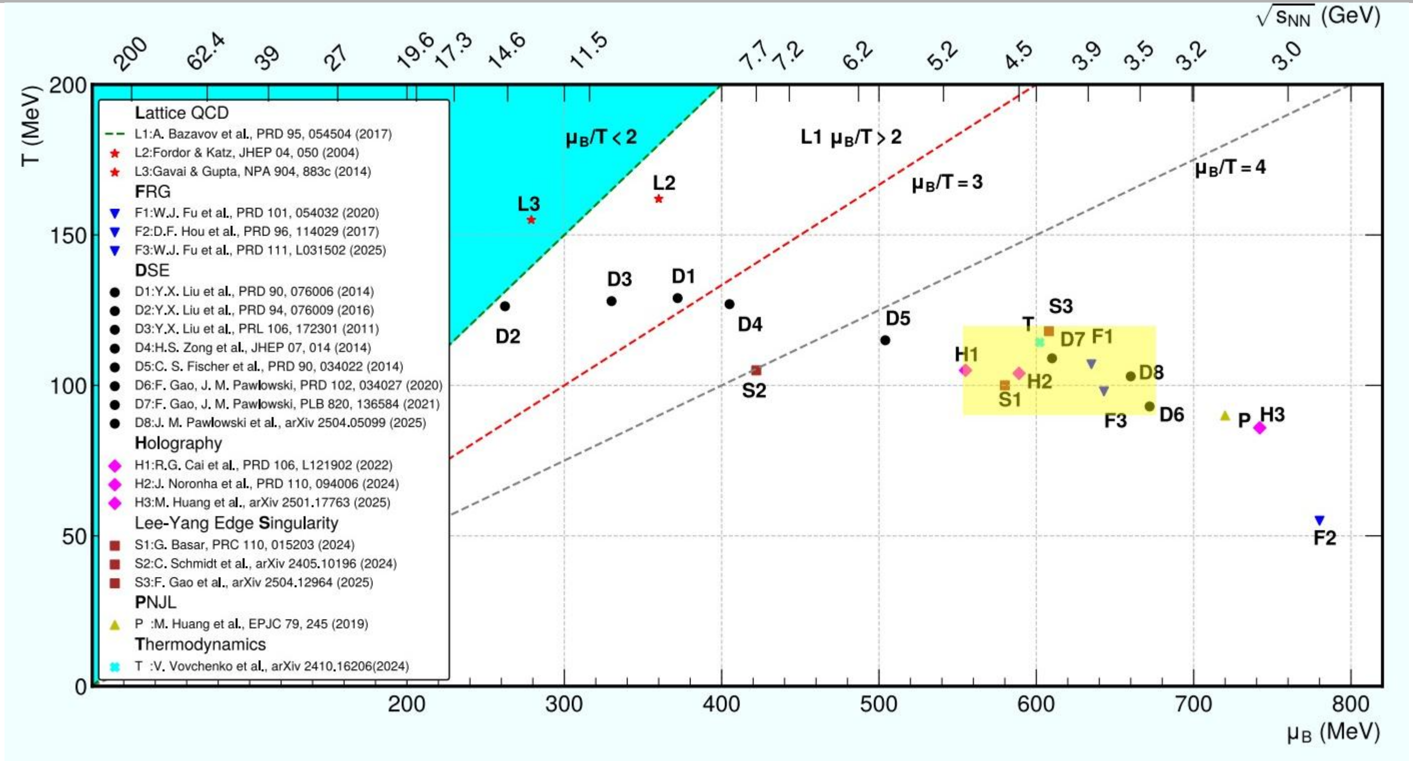


Y. Aoki et al, Nature 443, 675(2006)
A. Bzdak et al, Physics Reports 853,1-87(2020)
X. Luo, N. Xu, Nucl. Sci. Tech. 28, 112 (2017)
X. An et al., Nucl.Phys.A 1017 (2022), 122343

Predictions:

- Smooth crossover at $\mu_B = 0$ MeV by Lattice QCD
- 1st-order phase transition at large μ_B by various models
- QCD critical point (CP)?

◆ Theoretical Predictions



μ_B^{CEP}: ~ 550 – 650 MeV, T^{CEP}: ~ 90 – 118 MeV

[1].https://indico.ihep.ac.cn/event/23976/contributions/187309/attachments/90115/117000/QCD临界点和相边界实验研究进展_final.pdf

Cumulants of Conserved Charges

Cumulant

$$\delta N = N - \langle N \rangle$$
$$C_1 = \langle N \rangle = M$$
$$C_2 = \langle (\delta N)^2 \rangle = \sigma^2$$
$$C_3 = \langle (\delta N)^3 \rangle$$
$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

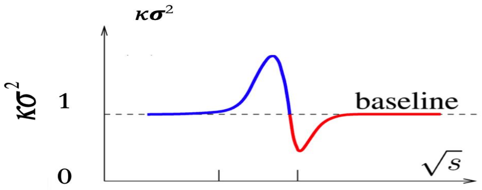
N: event-wise net-particle multiplicity

$$\frac{C_2}{C_1} = \frac{\sigma^2}{M}, \quad \frac{C_3}{C_2} = S\sigma$$
$$\frac{C_4}{C_2} = \kappa\sigma^2$$

Net-proton: a proxy for the net-baryon (B)
Net-kaon: a proxy for the net-strangeness (S)
Net-electric charge (Q)

Factorial cumulant (correlation functions)

$$\kappa_1 = C_1$$
$$\kappa_2 = -C_1 + C_2$$
$$\kappa_3 = 2C_1 - 3C_2 + C_3$$
$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$$



$\kappa\sigma^2 = 1$ (Poisson Fluctuations)

1.Sensitive to correlation length ξ

Near CP $\rightarrow$ $\xi \uparrow$

$$C_3 = \langle (\delta N)^3 \rangle \sim \xi^{4.5}$$
$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \sim \xi^7$$

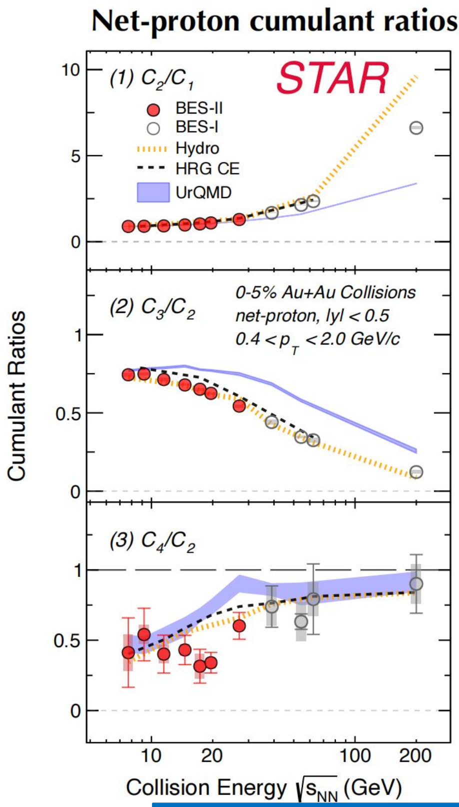
2.Related to susceptibility

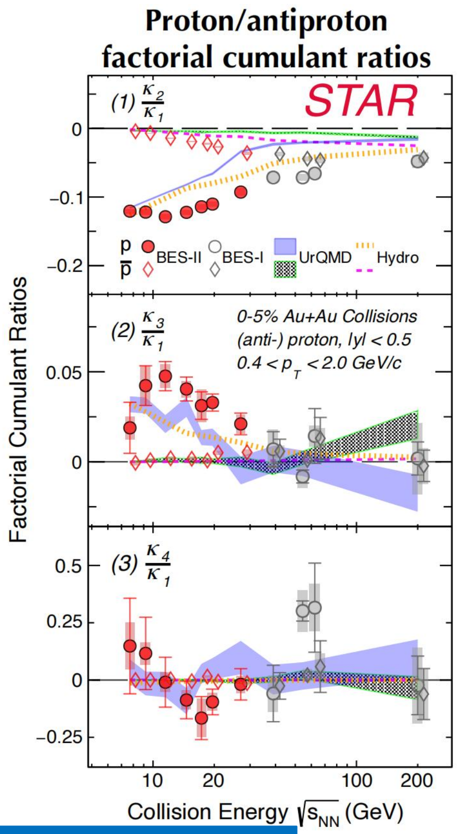
$$\frac{\chi_4^q}{\chi_2^q} = \kappa\sigma^2 = \frac{C_4^q}{C_2^q}, \quad \frac{\chi_3^q}{\chi_2^q} = S\sigma = \frac{C_3^q}{C_2^q}$$
$$\chi_n^q = \frac{1}{VT^3} \cdot C_n^q = \frac{\partial^n (p/T^4)}{\partial(\mu^q)^n}, \quad q = B, Q, S$$

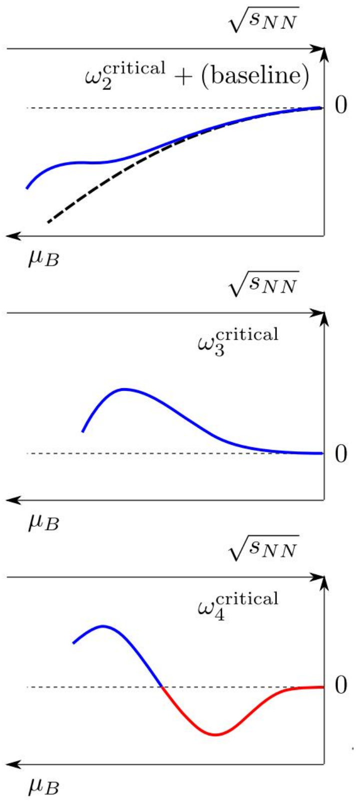
3. Non-monotonic energy dependence of $\kappa\sigma^2$ (C_4/C_2) $\rightarrow$ existence of a critical point

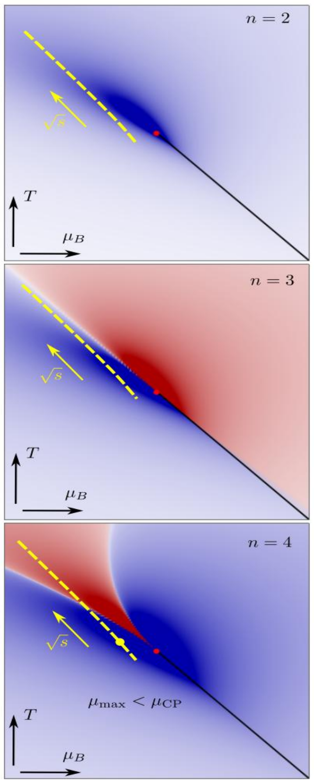
M. A. Stephanov, PRL 102, 032301 (09);
M. Asakawa, S. Ejiri and M. Kitazawa, PRL 103, 262301 (09)
S.Ejiri et al, PLB 633, 275(06);
M. A. Stephanov, PRL 107, 052301 (11);
F. Karsch and K. Redlich, PLB 695, 136 (11)

◆ Net-proton Cumulants from STAR BES-II





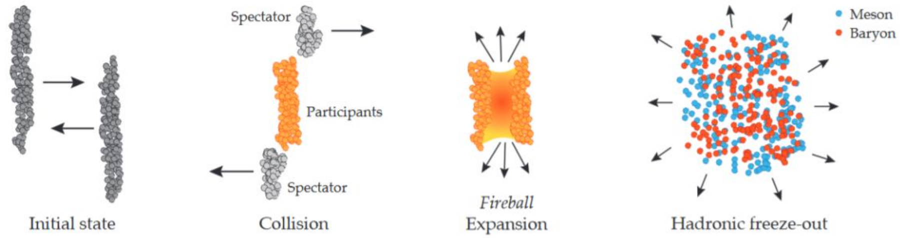


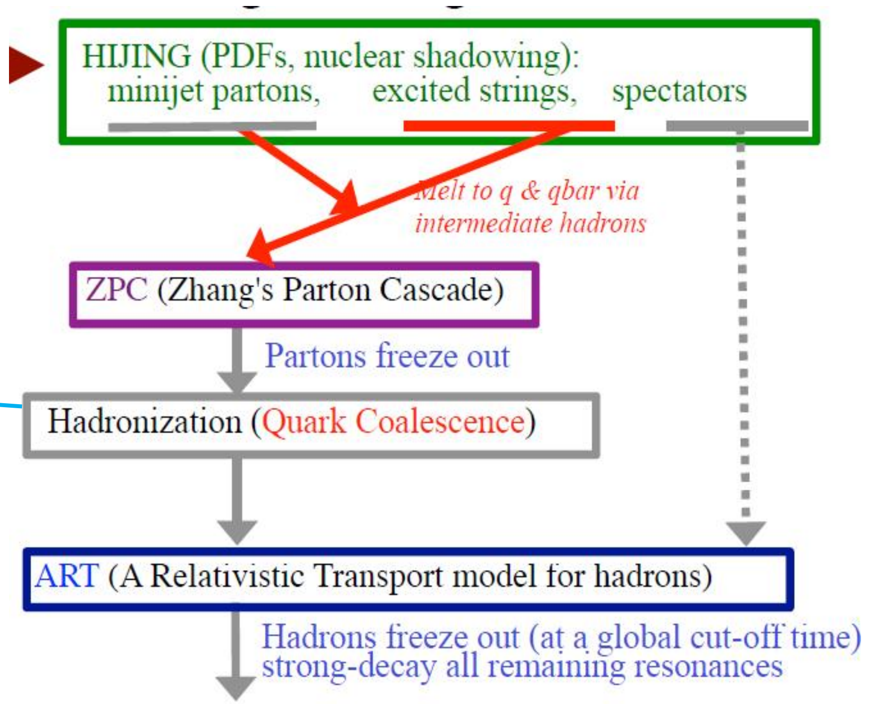


From CPOD2024, SQM2024

arXiv:2410.02861

◆ A Multi-Phase Transport Model （AMPT）





new quark coalescence:

quark to form either a meson or a baryon depending on the distance to its coalescence partner(s)(r_{BM})

$d_B < d_M * r_{BM}$:form a baryon

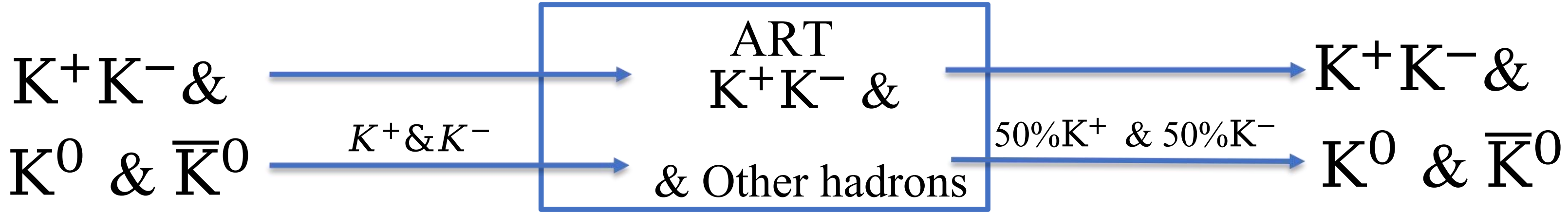
otherwise: form a meson

Y. He and Z.-W. Lin, Phys. Rev. C 96, 014910 (2017).

Extended AMPT model ensures the conservation of various conserved charges (including electric charge, baryon number, and strangeness) for all hadronic reaction channels during the evolution of hadronic phase

◆ A Multi-Phase Transport Model （AMPT）

- In the old version, only K^+ and K^- were introduced in hadron rescatterings as explicit particles, but K^0 and $\overline{K}^0$ were omitted.



- In the old version, some isospin-averaged cross sections were used, and the charge of the final state particles is chosen randomly from all possible charges, independent of the total charge of the initial state.

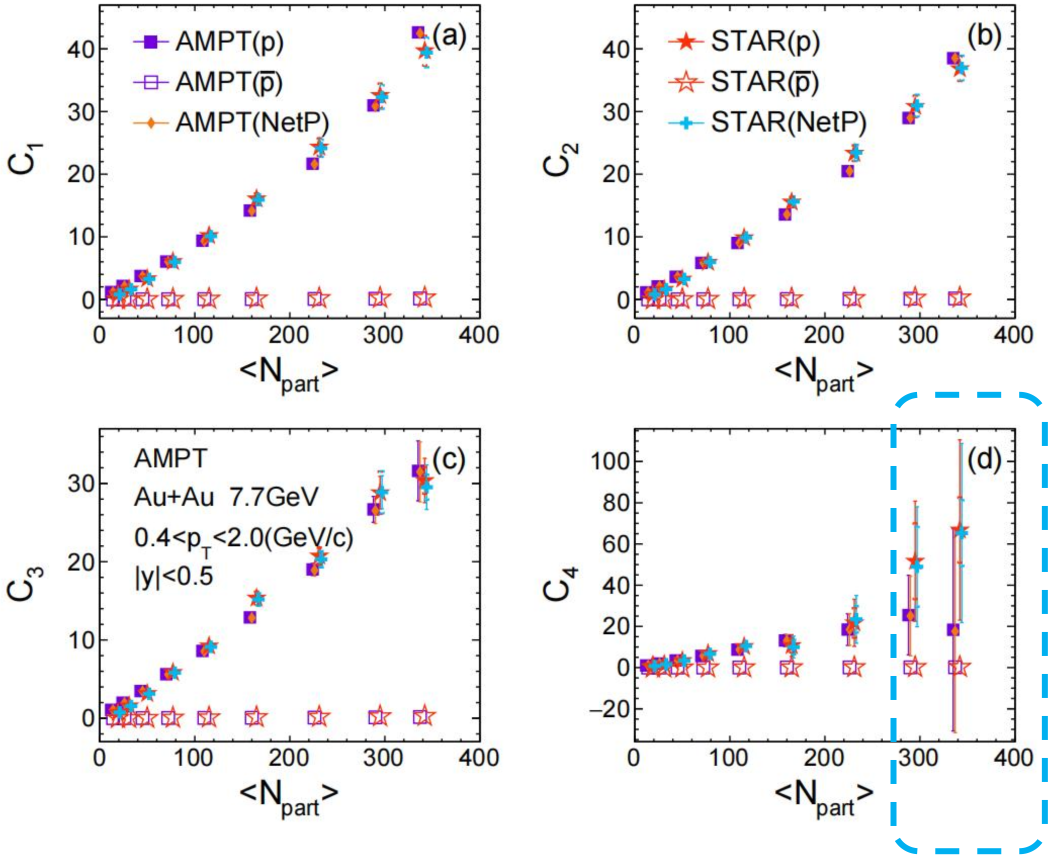
For example:

1) $\pi^+ + \pi^+ \rightarrow \rho^+ + \rho^+$ √

2) $\pi^+ + \pi^+ \rightarrow \rho^+ + \rho^-$ ×

3) $\pi^+ + \pi^+ \rightarrow \rho^- + \rho^-$ ×

Fluctuations of Net-Proton

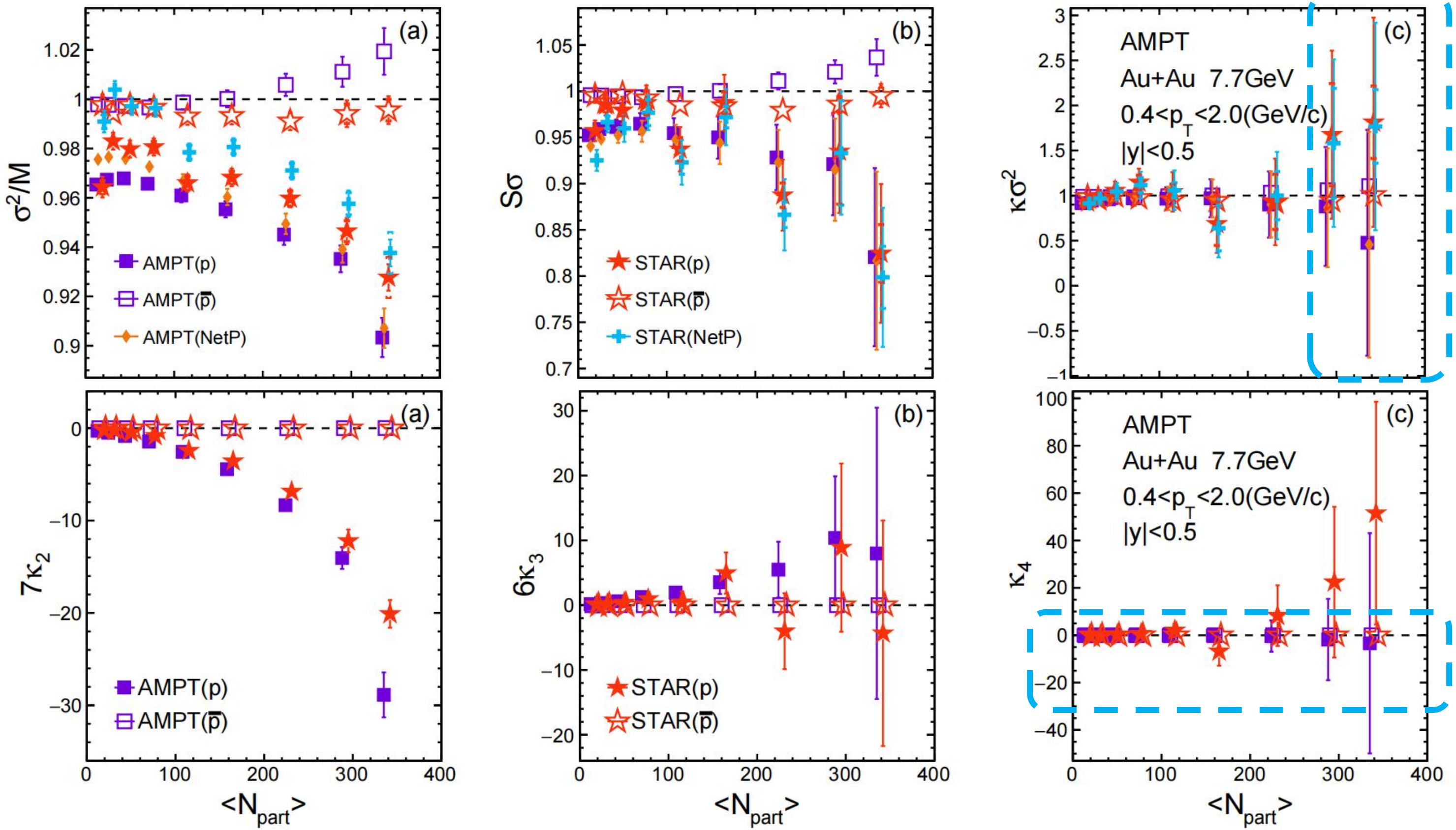


- The cumulants C_n for protons, antiprotons, and net-protons all show a similar increasing dependence on $\langle N_{part} \rangle$
- In the 0-5% and 5-10% centrality ranges, the fourth-order cumulant (C_4) in AMPT notably underestimates STAR's results

Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

◆ Fluctuations of Net-Proton

Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

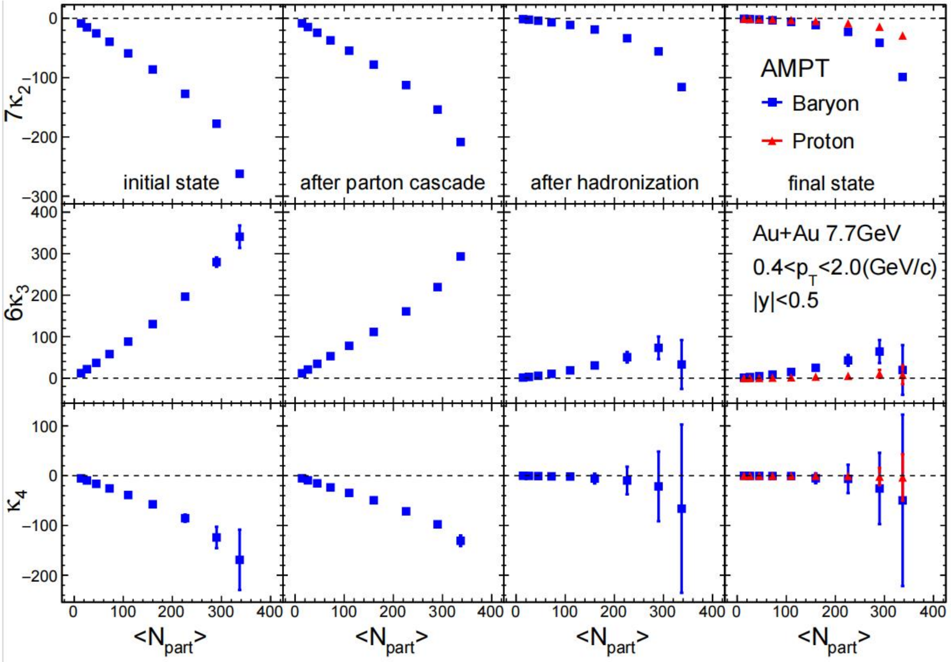


The four-proton correlation from AMPT is very small, consistent with zero.

◆ Fluctuations of Net-Proton

Expectation of baryon number conservation:

$$P(N) = \frac{B!}{N!(B-N)!} p^N (1-p)^{(B-N)}$$



Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

n-baryon correlations:

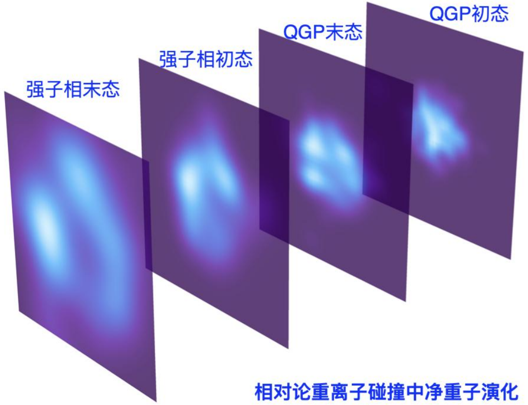
$$\kappa_1 = \langle N \rangle = pB$$

$$\kappa_2 = -\frac{\langle N \rangle^2}{B}$$

$$\kappa_3 = 2\frac{\langle N \rangle^3}{B^2}$$

$$\kappa_4 = -6\frac{\langle N \rangle^4}{B^3}$$

- Multi-baryon correlations are getting weaker with stage evolution of heavy-ion collisions



◆ Fluctuations of Net-Kaon

Cumulants:

$$C_2 = \langle N \rangle + \langle \bar{N} \rangle + \kappa_2^{(2,0)} + \kappa_2^{(0,2)} - 2\kappa_2^{(1,1)}$$

$$C_3 = \langle N \rangle - \langle \bar{N} \rangle + 3\kappa_2^{(2,0)} - 3\kappa_2^{(0,2)} + \kappa_3^{(3,0)} - \kappa_3^{(0,3)} - 3\kappa_3^{(2,1)} + 3\kappa_3^{(1,2)}$$

$$C_4 = \langle N \rangle + \langle \bar{N} \rangle + 7\kappa_2^{(2,0)} + 7\kappa_2^{(0,2)} - 2\kappa_2^{(1,1)} + 6\kappa_3^{(3,0)} + 6\kappa_3^{(0,3)} - 6\kappa_3^{(2,1)} - 6\kappa_3^{(1,2)} + \kappa_4^{(4,0)} + \kappa_4^{(0,4)} - 4\kappa_4^{(3,1)} - 4\kappa_4^{(1,3)} + 6\kappa_4^{(2,2)}$$

Bzdak, Adam et al. Phys.Rev. C86 (2012) 044904

two or more kinds of particles！！！

Factorial moments:

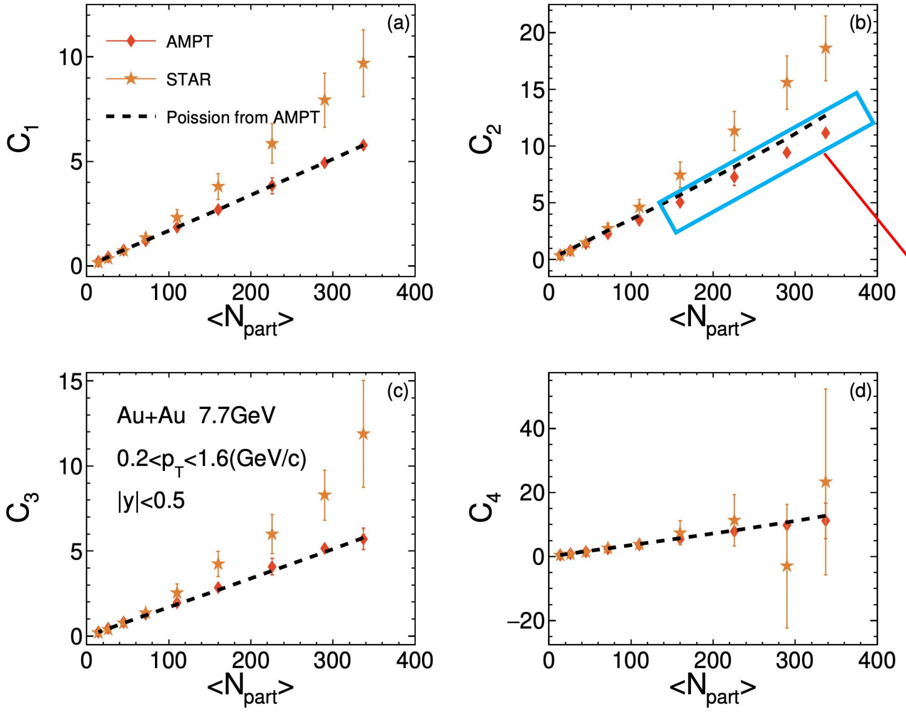
$$F_{i,k} = \left\langle \frac{N!}{(N-i)!} \frac{\bar{N}!}{(\bar{N}-k)!} \right\rangle = \frac{d^i}{dz^i} \frac{d^k}{d\bar{z}^k} H(z,\bar{z})|_{z=\bar{z}=1}$$

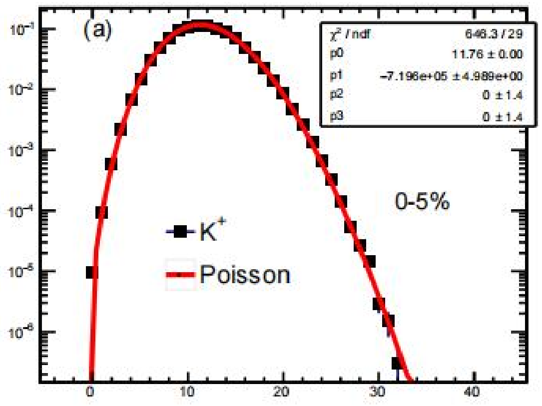
Correlation Functions:

$$\kappa_2^{(2,0)} = -F_{1,0}^2 + F_{2,0},$$

$$\kappa_2^{(1,1)} = -F_{1,0}F_{0,1} + F_{1,1},$$

◆ Fluctuations of Net-Kaon

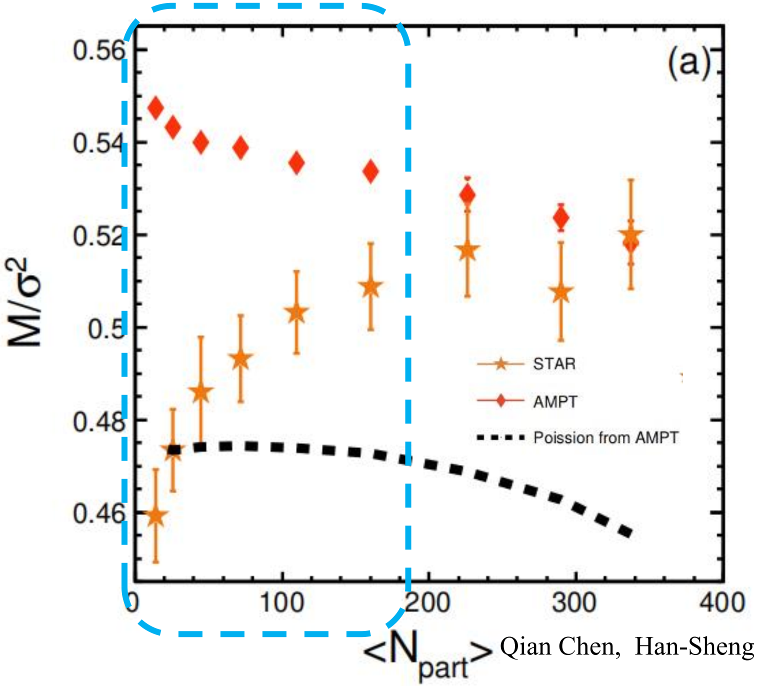


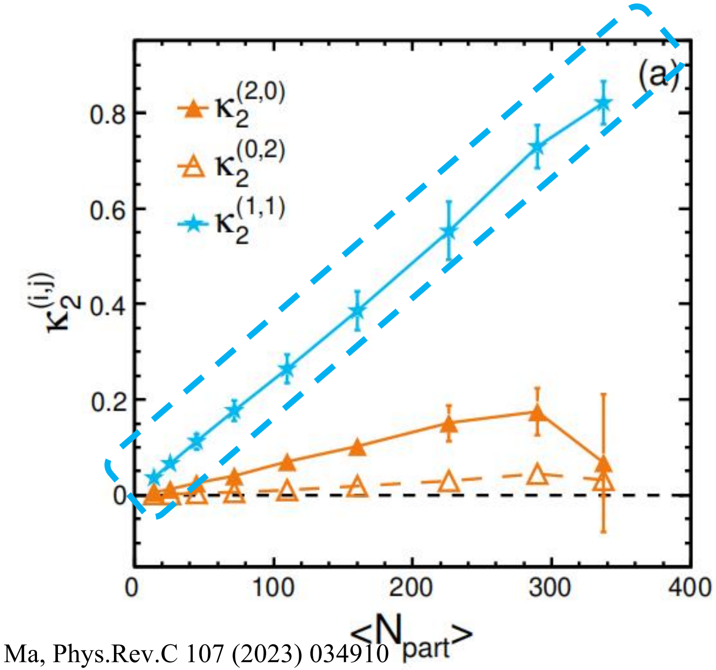


Qian Chen, Han-Sheng Wang, Guo-Liang Ma, Phys.Rev.C 107 (2023) 034910

The C_2 for AMPT is slightly lower than **Poisson baseline** based on its mean multiplicity, suggesting a correlation between K^+ and K^-

◆ Fluctuations of Net-Kaon





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Caused by the **new quarks coalescence mechanism**

Two-particle correlation function between the K^+ and K^-[$\kappa_2^{(1,1)}$]is dominants——**pair production**

◆ Second-order off-diagonal and diagonal cumulants of conserved charges

- Conserved charges susceptibilities in experiment:

$$\chi_\alpha^2 = \frac{1}{VT^3}\kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3}\kappa_{\alpha,\beta}^{1,1}$$

- the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N:

$$\kappa_\alpha^2 = \sigma_\alpha^2 = \left\langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)^2 \right\rangle$$

$$\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)(\delta N_\beta - \langle \delta N_\beta \rangle) \rangle.$$

- The ratios of off-diagonal and diagonal cumulants are defined as follows:

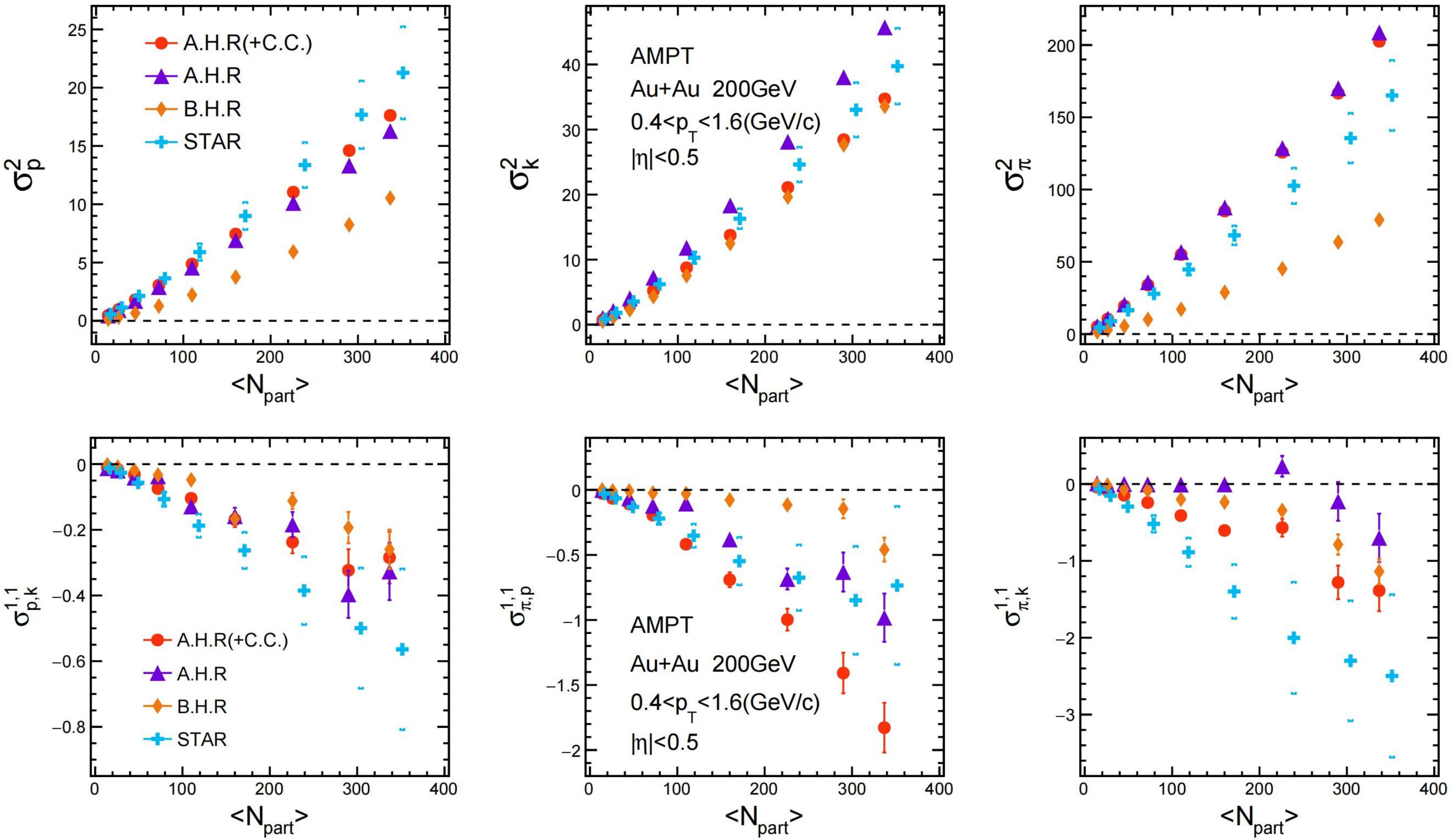
$$C_{p,k} = \frac{\sigma_{p,k}^{1,1}}{\sigma_k^2}, C_{Q,k} = \frac{\sigma_{Q,k}^{1,1}}{\sigma_k^2}, C_{Q,p} = \frac{\sigma_{Q,p}^{1,1}}{\sigma_p^2}.$$

- Avoid autocorrelation problems：

$$C_{Q^{\mathrm{PID}},p} = \frac{\sigma_{Q^{\mathrm{PID}},p}^{1,1}}{\sigma_p^2} = \frac{\sigma_{\pi,p}^{1,1}}{\sigma_p^2} + \frac{\sigma_{k,p}^{1,1}}{\sigma_p^2} + 1,$$

$$C_{Q^{\mathrm{PID}},k} = \frac{\sigma_{Q^{\mathrm{PID}},k}^{1,1}}{\sigma_k^2} = \frac{\sigma_{\pi,k}^{1,1}}{\sigma_k^2} + \frac{\sigma_{k,p}^{1,1}}{\sigma_k^2} + 1.$$

◆ Second-order off-diagonal and diagonal cumulants of conserved charges

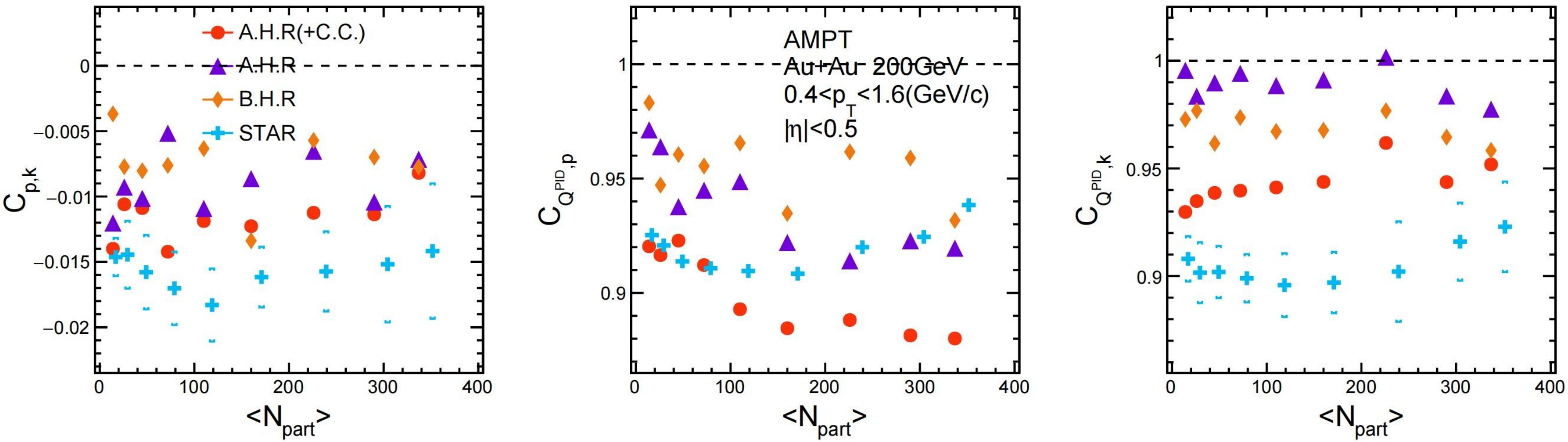


A.H.R(+C.C.): After hadronic rescatterings(with charges conservation)

A.H.R: After hadronic rescatterings

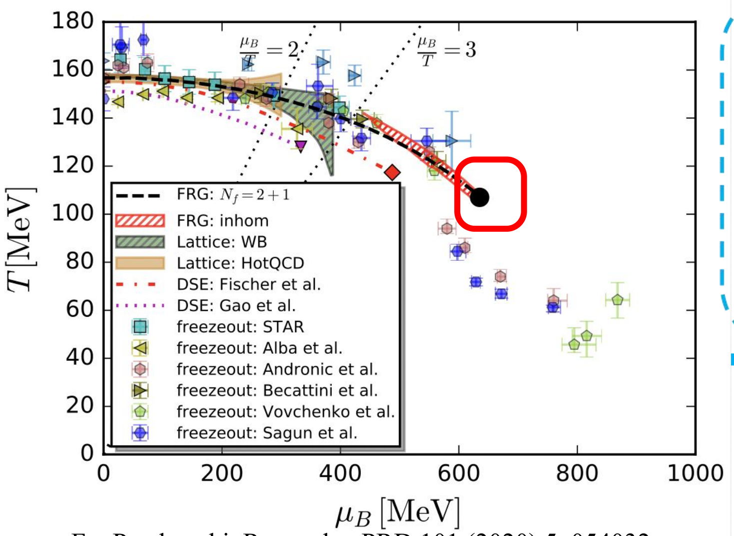
B.H.R: Before hadronic rescatterings

◆ Second-order off-diagonal and diagonal cumulants of conserved charges



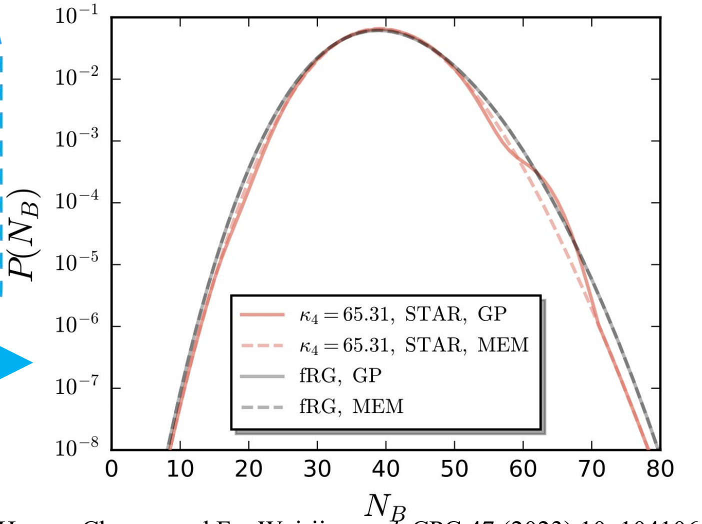
- ➢ The AMPT model, incorporating the conservation of conserved charges, exerts a non-negligible impact on the ratio of off-diagonal and diagonal cumulants.
- ➢ $C_{Q^{PID},K}$ is found to be mostly sensitive to the conservation of conserved charges.

◆ Functional Renormalization Group



Fu, Pawlowski, Rennecke ,PRD 101 (2020) 5, 054032

the net-baryon number distributions are reconstructed from the cumulants of different orders by means of the maximum entropy method



Huang, Chuang and Fu, Wei-jie, *et al.* CPC 47 (2023) 10, 104106

FRG enables the study of equations of state at both high and low baryon chemical potentials.

FRG with critical fluctuations mechanism without interactions between hadrons and decay processes

Incorporating FRG Into AMPT Model

FRG parameter input:

baryon chemical：

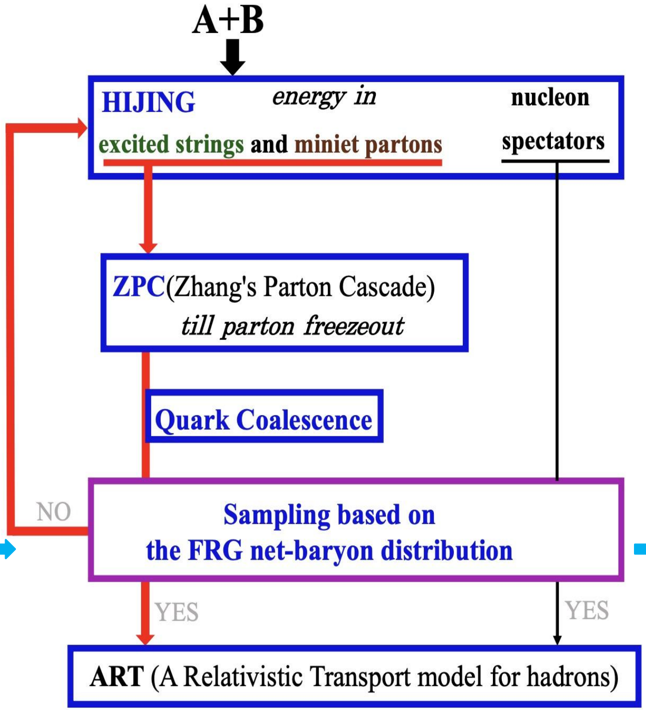
μ_B = 399 MeV;

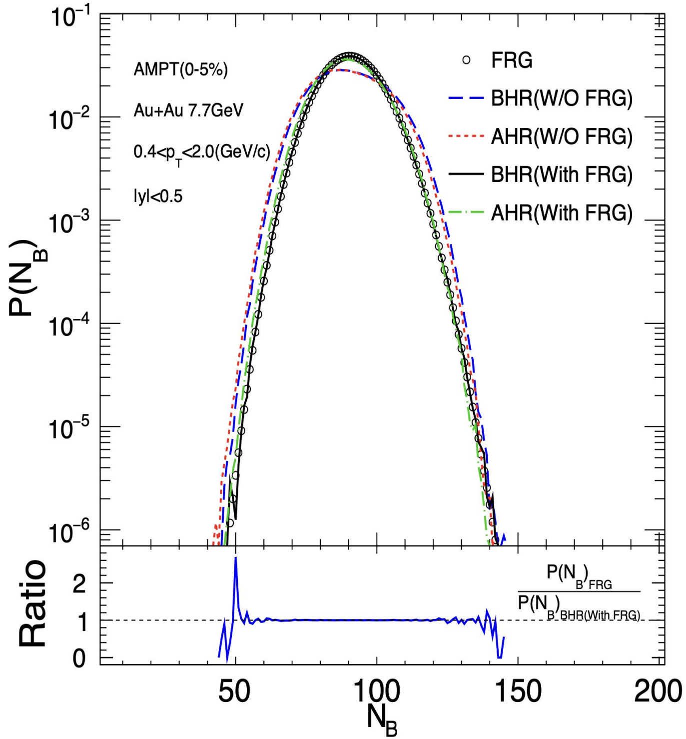
volume of the fire ball：

V = 980 fm^3;

pseudo-critical temperature：

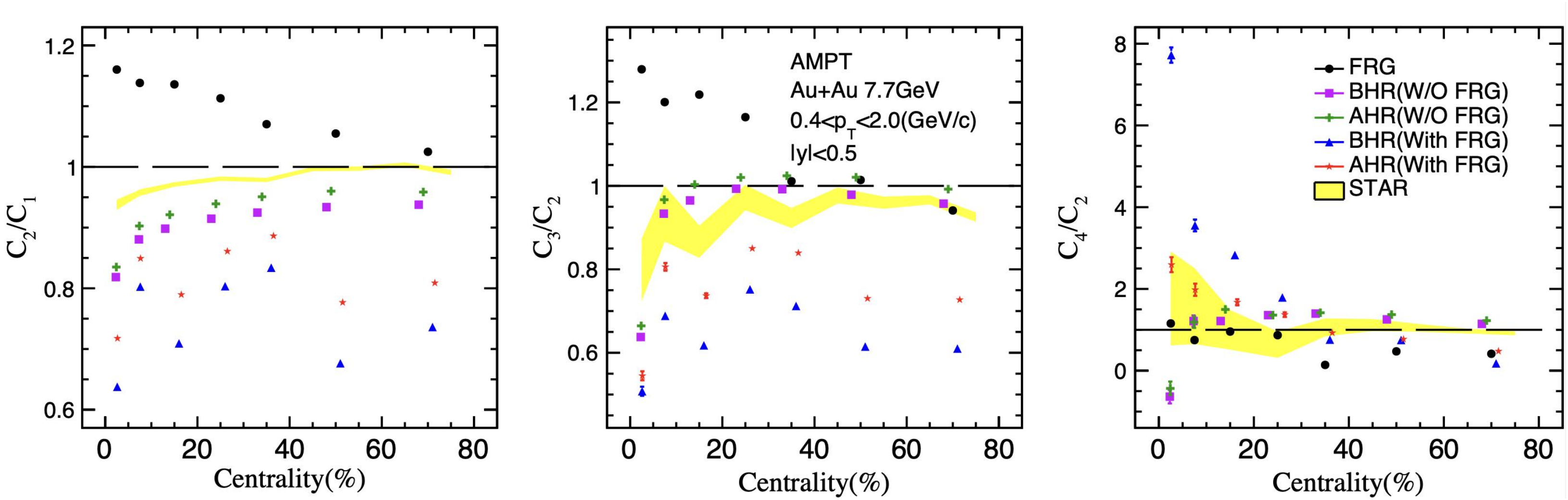
T = 139 MeV;





Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.

◆ Incorporating FRG Into AMPT Model



- ➢ The process of hadronic rescatterings exerts a Poissonization effect on cumulant ratios.
- ➢ The effect of hadronic rescatterings is more significant for **critical fluctuations** than **dynamical fluctuations**.

Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.

Summary and Outlook

Summary

- The AMPT results are consistent with the expectation from baryon number conservation.
- By analyzing the cumulants and correlation functions of net-strangeness and net-kaon, we found that they originate from pair production.
- The incorporation of the FRG into the AMPT model reveals that the hadronic rescatterings process affects different orders of net-baryon cumulant ratios.

Outlook

- Incorporation of critical fluctuation physics into AMPT： FRG、density fluctuations.
- nuclear thickness effects, coalescence mechanisms, different collision systems, ...
- Using the extened AMPT model to the analysis of other energy provides a baseline for experimental study.

THANK You for Your Attention!

The 16th Workshop on QCD Phase Transition and Relativistic Heavy-Ion Physics (QPT 2025)

2025年10月24日至28日
Asia/Shanghai 时区

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经会议顾问委员会讨论决定，“第十六届QCD相变与相对论重离子物理研讨会”将于2025年10月24日至10月28日在广西桂林市召开。本次研讨会由广西师范大学与广西科技大学负责承办。会议将汇聚理论与实验物理学家，围绕相对论重离子碰撞中QCD相变的前沿问题展开深入探讨。会议议题包括但不限于以下方面：

期待金秋十月相聚桂林! Oct.24-28,2025

https://indico.ihep.ac.cn/event/25521/

Back Up

Cumulants:

$$C_1 = \kappa_1,$$

$$C_2 = \kappa_2 + \kappa_1,$$

$$C_3 = \kappa_3 + 3\kappa_2 + \kappa_1,$$

$$C_4 = \kappa_4 + 6\kappa_3 + 7\kappa_2 + \kappa_1.$$

Only for one particle! ! !

Correlation Functions:

$$\kappa_1 = C_1 = \langle N \rangle,$$

$$\kappa_2 = -C_1 + C_2,$$

$$\kappa_3 = 2C_1 - 3C_2 + C_3,$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4.$$

Factorial moments:

$$F_3 = \int dy_1 dy_2 dy_3 \rho_3(y_1, y_2, y_3) = F_1^3 + 3F_1C_2 + C_3$$

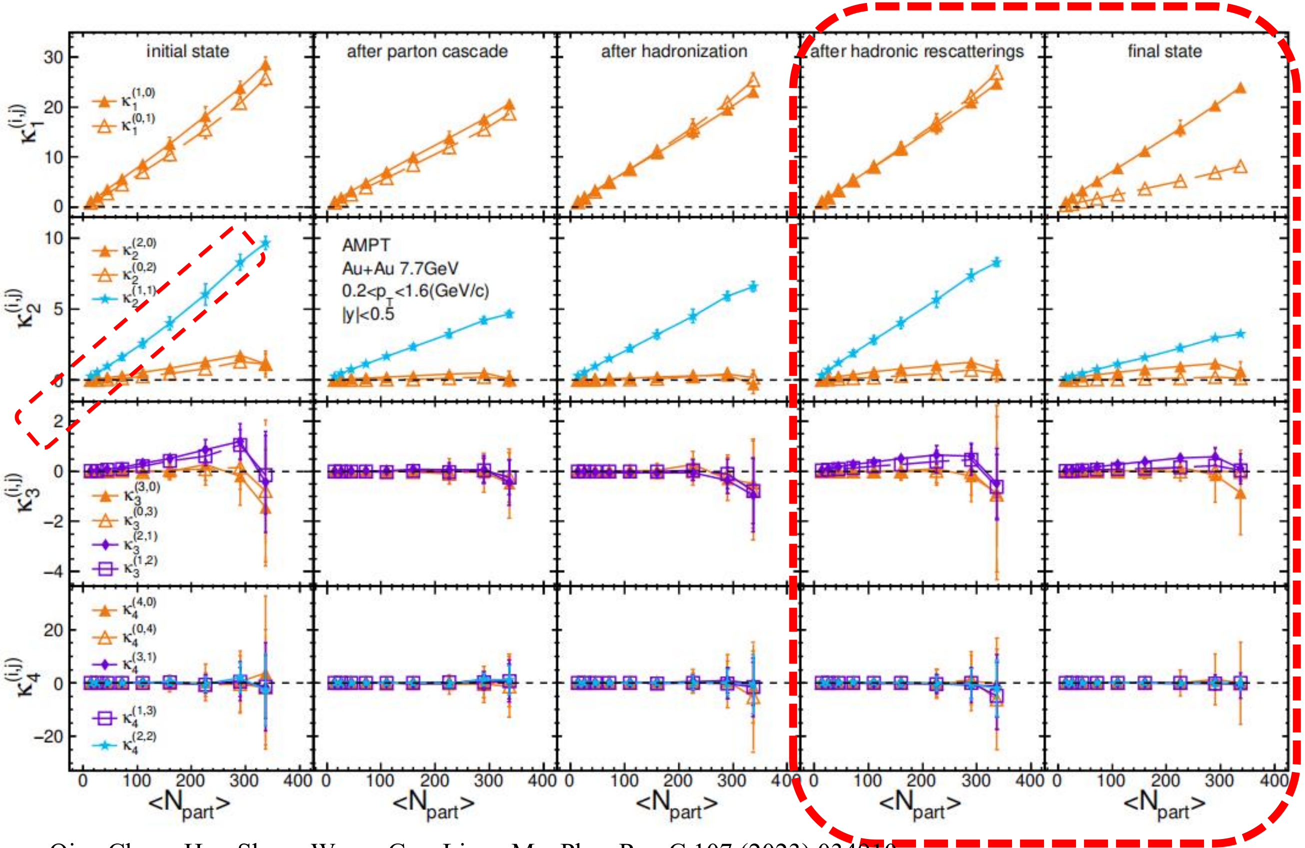
$$\rho_3(y_1, y_2, y_3) = \rho_1(y_1)\rho_1(y_2)\rho_1(y_3) + \rho_1(y_1)C_2(y_2, y_3) + \rho_1(y_2)C_2(y_1, y_3) + \rho_1(y_3)C_2(y_1, y_2) + C_3(y_1, y_2, y_3)$$

Bzdak, Adam et al. Phys.Rev. C95 (2017)5,054906.

Back Up

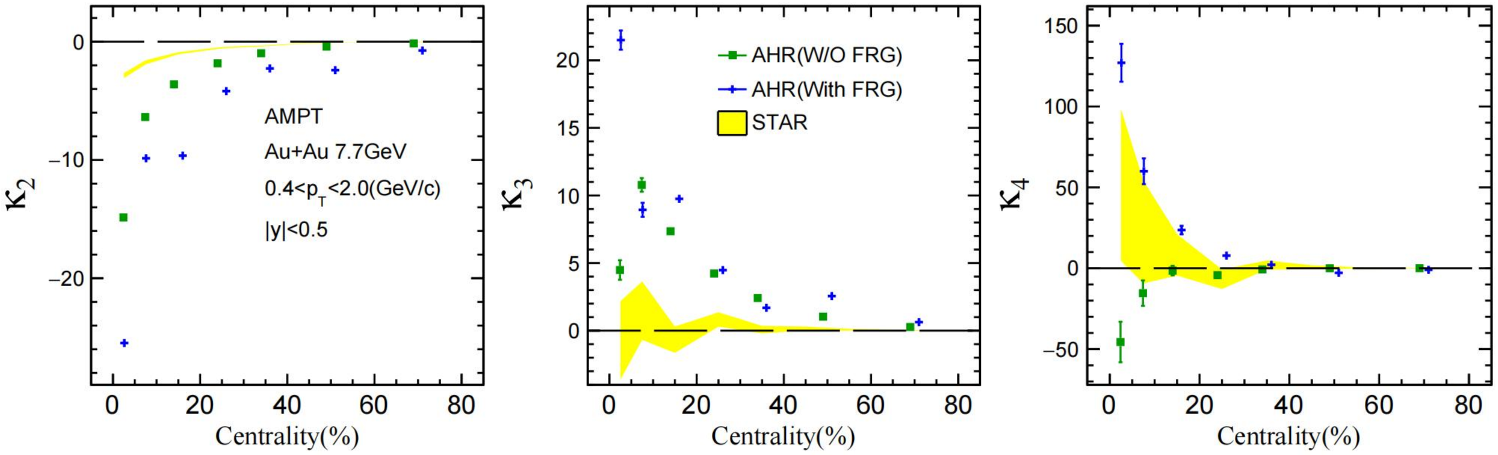
- The fluctuations of strangeness are notably influenced during the weak decay evolution stage

- the two-particle correlation function between the $\bar{s}$ quark and s quark $[\kappa_2^{(1,1)}]$is dominants



Qian Chen, Han-Sheng Wang, Guo-Liang Ma, Phys.Rev.C 107 (2023) 034910

Back Up



- The strengths of the correlation functions κ_2 and κ_3 in the AMPT model without the FRG sampling are smaller than those in the AMPT model with the FRG sampling.
- The correlation functions κ_4 from negative to positive, which would be more consistent with the current experimental measurement.

Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.