Attractor of hydrodynamics with general rapidity distribution

References:

Shile Chen and Shuzhe Shi,
Phys.Rev.C 111 (2025) 2, L021902
Phys.Rev.D 111 (2025) 1, 014001
Phys. Rev. C, 109 (2024), 5, L 051901

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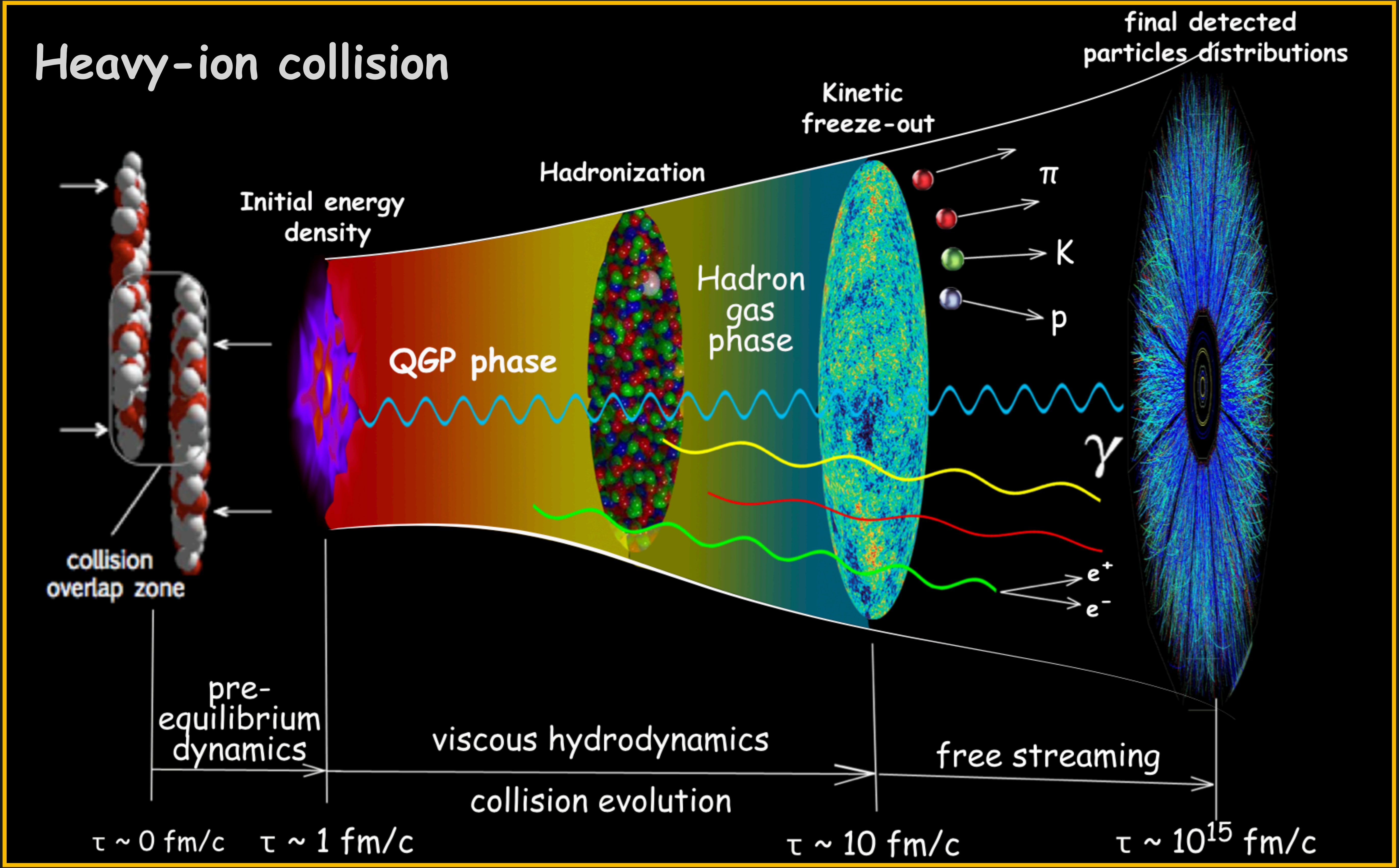
in collaboration with Shuzhe Shi

Quark-Gluon plasma and hydrodynamics



Near equilibrium ?

Large enough in microscopic view and small enough in macroscopic view

✓OK

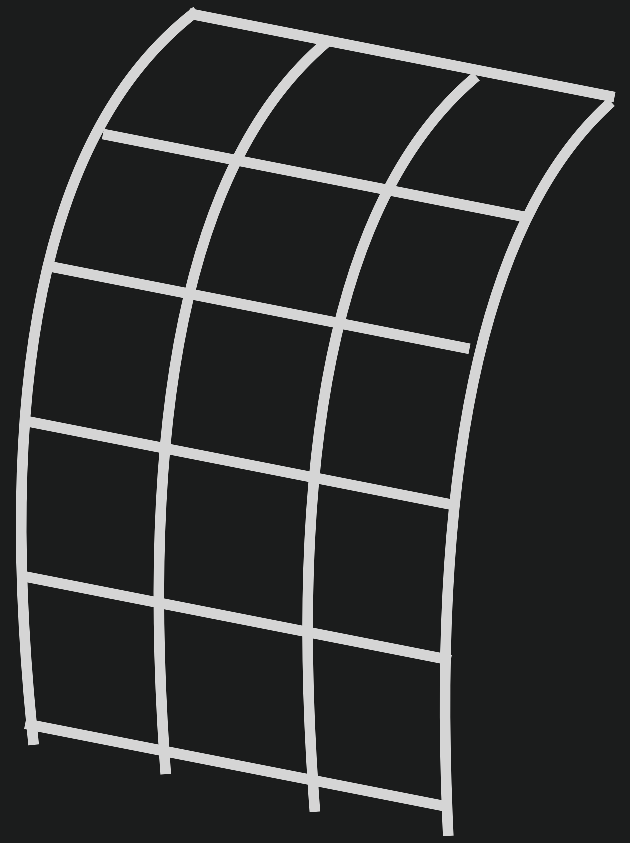
Knudsen number< 0.1

Gradient expansion and lower order counting

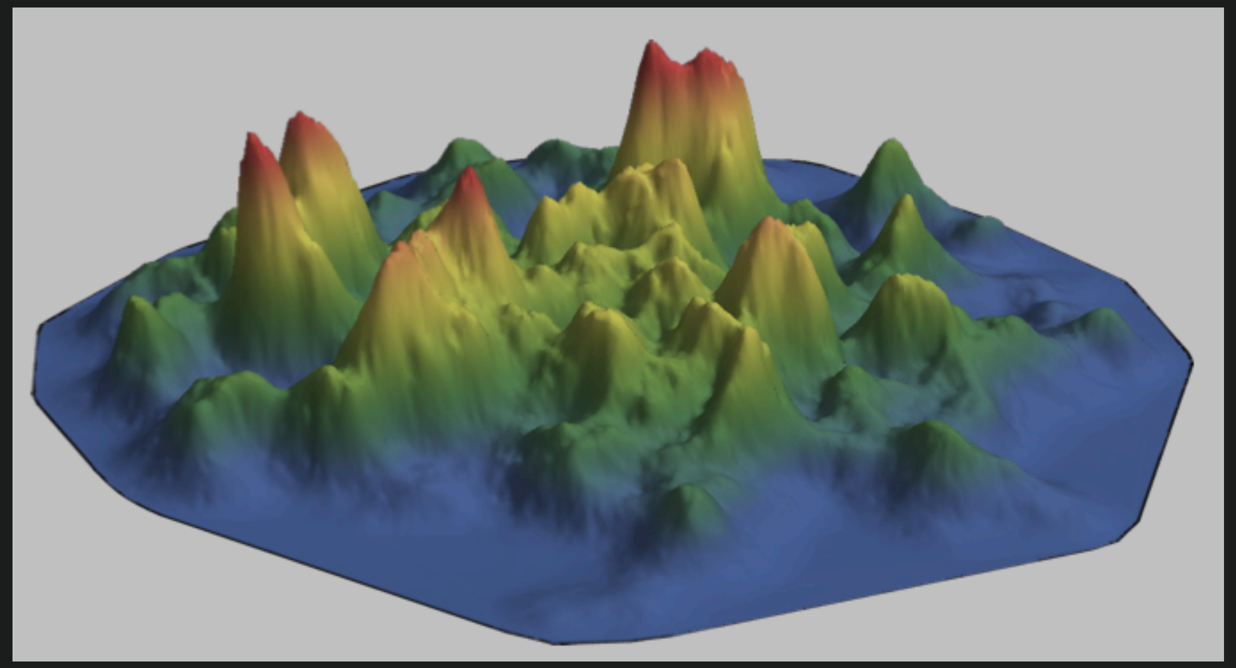
$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \ldots$$

Motivation:

Hydrodynamics: Does it an EFT of the slow(?) evolution of conserved currents in collective media like QGP?



What we expect about the slowing moving fluid frame in the direction of energy



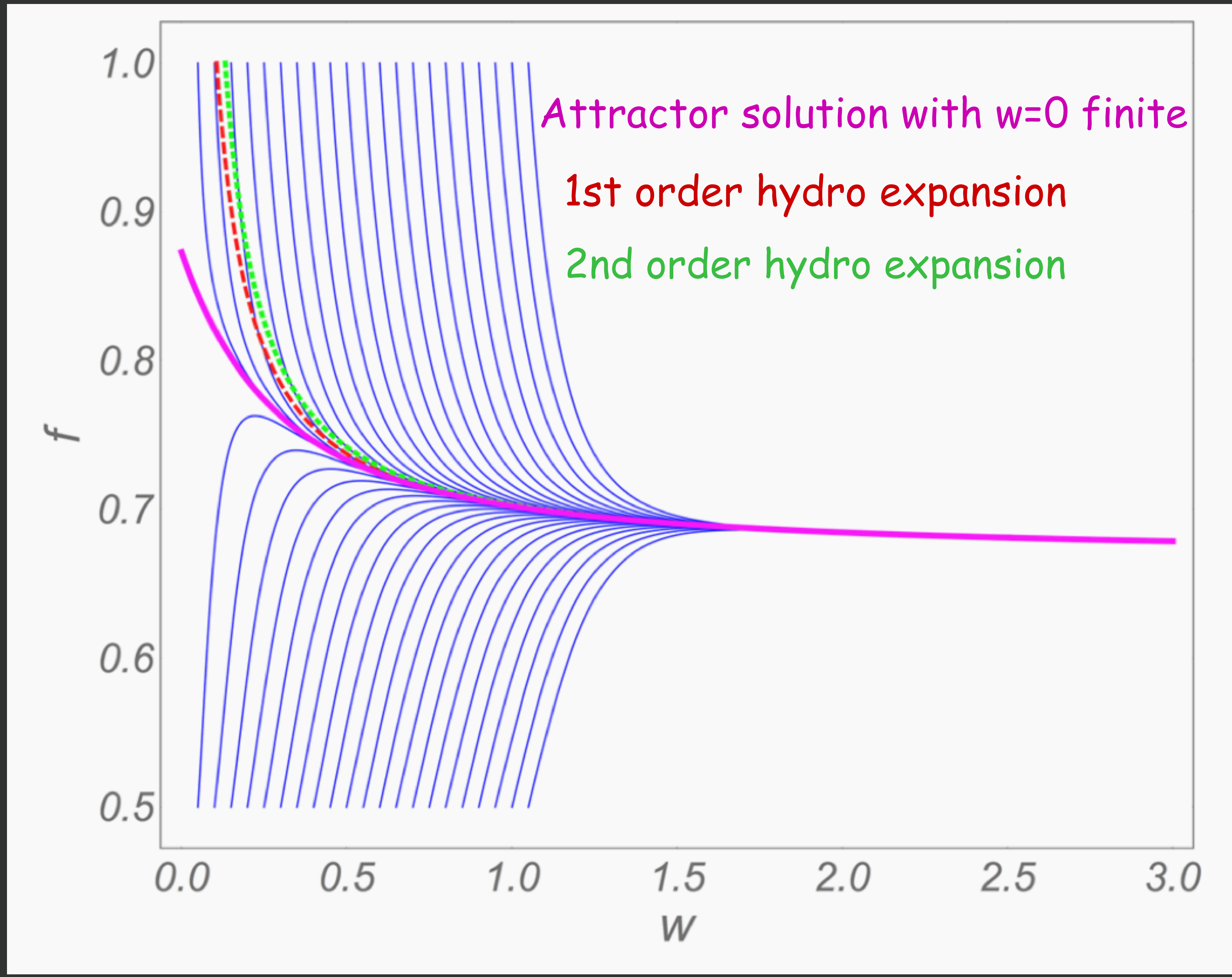
QGP energy density with Knudsen Number~(mfp/L)>>0.1

Still HYDRODYNAMICS works well??

One of the assumptions: The collective media experiences a very fast decrease of the DOFs in the very beginning of the evolution -> Hydro attractor

Attractor behavior in Bjorken expansion

M. P. Heller and M. Spalinski, Phys. Rev. Lett. 115, no. 7, 072501(2015)



$$w = \tau T \propto \tau/\tau_{rel} \qquad f = 1 + \tau\frac{\partial_\tau T}{T}$$

Obtained from MIS theory

$$C_{\tau\Pi} w f f' + 4C_{\tau\Pi} f^2 + (w - \frac{16C_{\tau\Pi}}{3})f - \frac{4C_\eta}{9} + \frac{C_{\tau\Pi}}{9} - \frac{2w}{3} = 0$$

$\frac{1}{w}$ expansion $\quad f = \boxed{\frac{2}{3}} + \frac{4C_\eta}{9w} + \frac{8C_\eta C_{\tau\Pi}}{27w^2} + \mathcal{O}(\frac{1}{w^3})$

Value in global equilibrium

A unique stable solution

Perturbation around this solution $\quad \delta f(w) \sim \exp(-\frac{3}{2C_{\tau\Pi}}w)w^{\frac{C_\eta - 2C_{\lambda 1}}{C_{\tau\Pi}}}(1 + \mathcal{O}(\frac{1}{w}))$

Attractor behavior in different symmetries and hydro theories

Hydro attractors in different theories

MIS & BRSSS	M. P. Heller and M. Spalinski, Phys. Rev. Lett. 115, no. 7, 072501(2015)
HJSW	I. Aniceto and M. Spalinski, Phys. Rev. D 93, no.8, 085008 (2016)
DNMR & aHydro	M. Strickland, J. Noronha and G. Denicol, Phys. Rev. D 97, no.3, 036020 (2018)
rBRSSS	P. Romatschke, Phys. Rev. Lett. 120 no.1, 012301 (2018)
aHydro with collision kernels	D. Almaalol and M. Strickland, Phys. Rev. C 97 no.4, 044911 (2018) D. Almaalol, M. Alqahtani and M. Strickland, Phys. Rev. C 99 no.1, 014903 (2019)
kinetic theory	J. P. Blaizot and L. Yan, Phys. Lett. B 780, 283-286 (2018) M. Strickland, JHEP 12, 128 (2018)

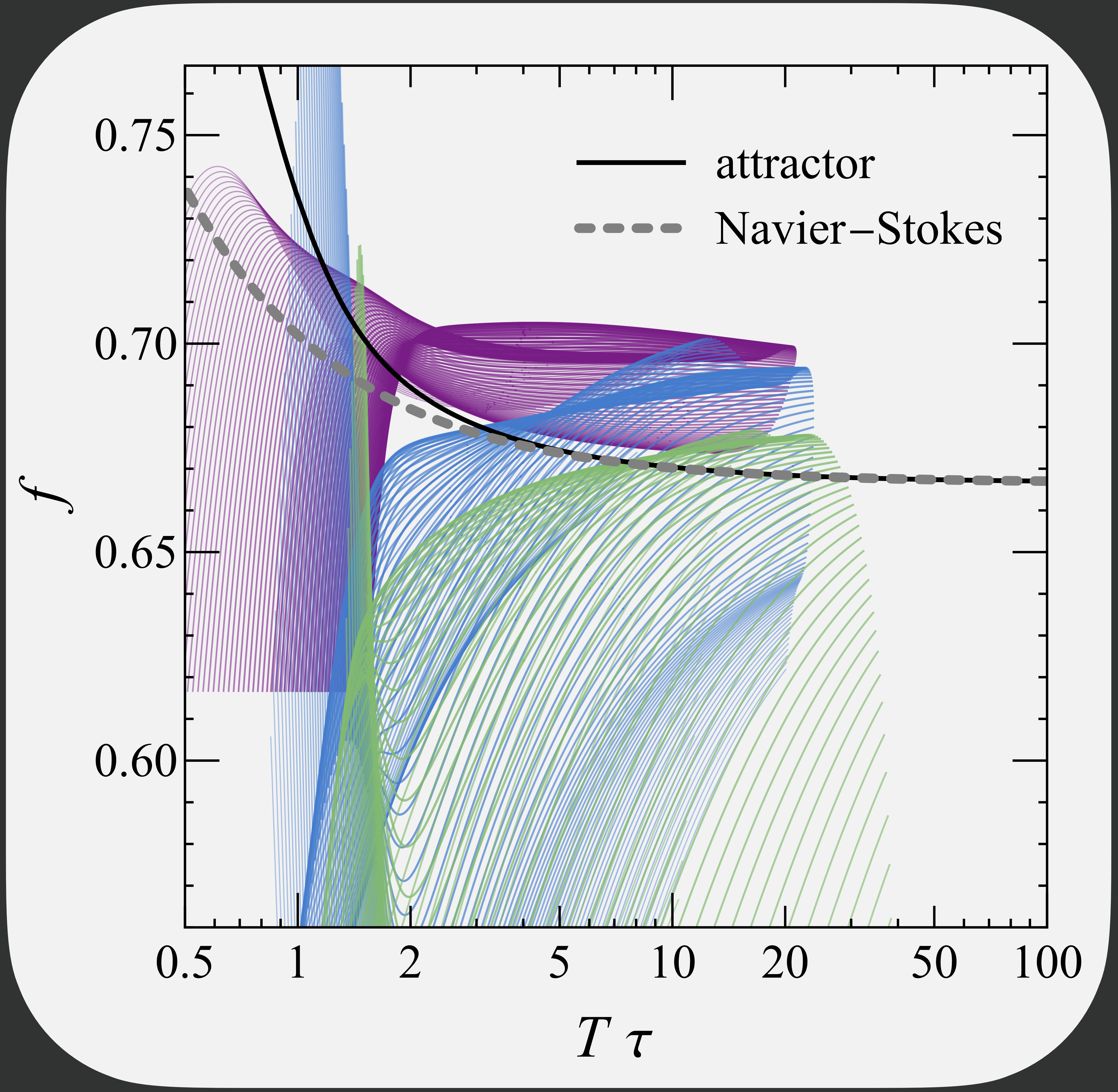
Hydro attractors in different symmetries

Hubble expansion	Z. Du, X. G. Huang and H. Taya , Phys. Rev. D 104, no.5, 056022 (2021)
Gubser flow	A. Behtash, C. N. Cruz Camacho and M. Martinez, Phys. Rev. D 97 no.4, 044041 (2018)
2+1 Conformal	P. Romatschke, JHEP, 12, 079 (2017)

Attractor behavior in different symmetries and hydro theories

Hydro attractors(?) with rapidity dependence

Boost-non invariant variables for f and T tau



Properties of Naiver Stokes equations

Paul Romatschke, JHEP 12, 079 (2017)

Stress tensor conservation $\nabla_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = (\epsilon + \Pi + P)u^\mu u^\nu - (\Pi + P)g^{\mu\nu} + \pi^{\mu\nu}$$

Naiver-Stokes limit $\pi^{\mu\nu} = \eta\sigma^{\mu\nu} = 2\eta\Delta^{\mu\nu}_{\alpha\beta}\nabla^\alpha u^\beta$ $\quad \Pi = \xi\nabla_\lambda u^\lambda$

$\nabla_\lambda u^\lambda$ Expansion rate

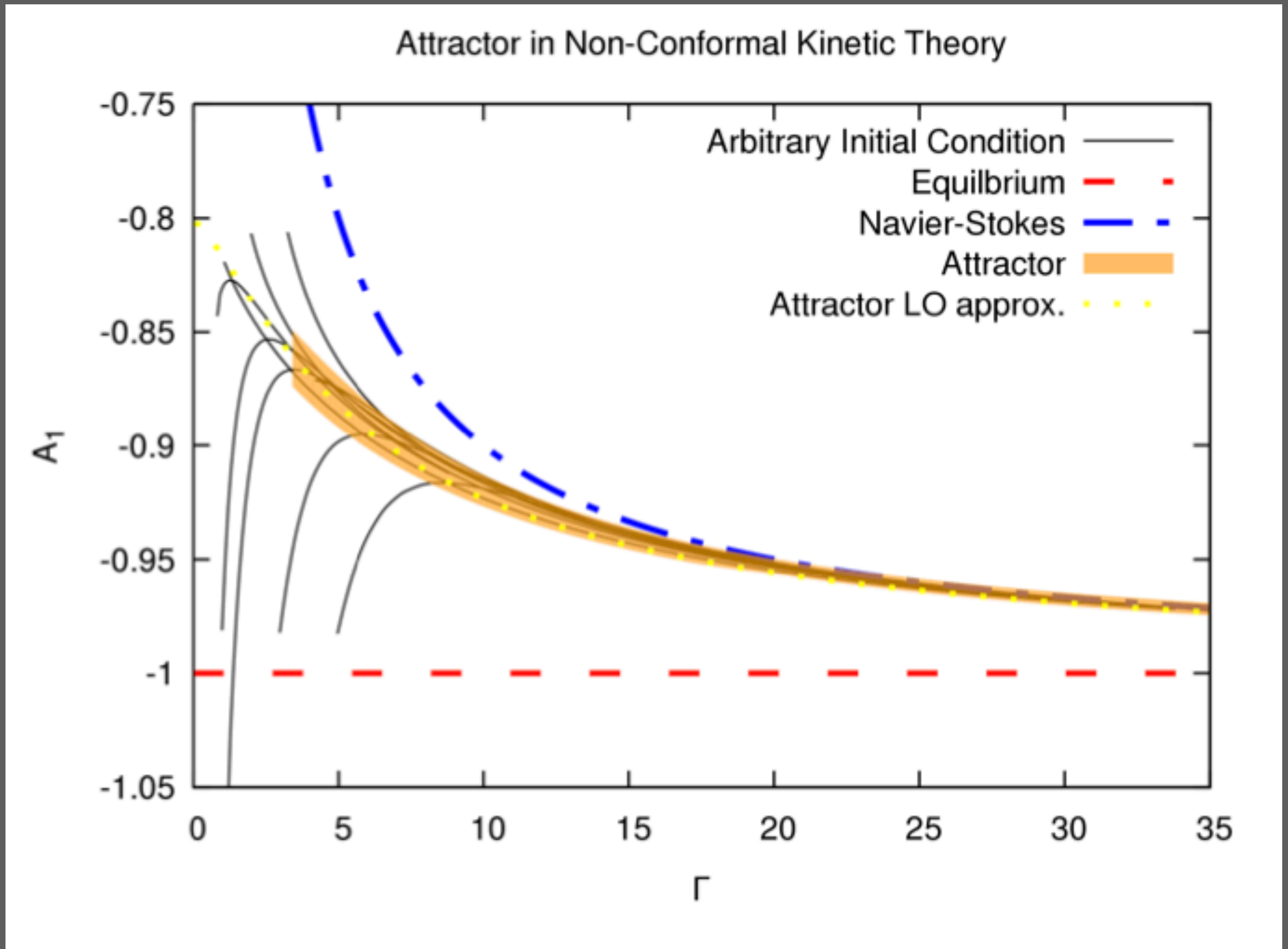
$$\frac{D\epsilon}{(\epsilon + P)\nabla_\lambda u^\lambda} = \frac{D\ln s}{\nabla_\lambda u^\lambda} = -1 + \frac{\eta}{2s}\frac{\sigma^{\mu\nu}\sigma_{\mu\nu}}{T\nabla_\lambda u^\lambda} + \frac{\xi}{s}\frac{\nabla_\lambda u^\lambda}{T}$$

Γ^{-1} 'Inverse time'

Time evolution of the quantity will behave similarly for small gradients irrespective of initial conditions

Properties of Naiver Stokes equations

Paul Romatschke, JHEP 12, 079 (2017)



$- \pi^{\mu\nu}$

$$\Pi = \xi \nabla_\lambda u^\lambda$$

$\nabla_\lambda u^\lambda$ Expansion rate

$\nabla_\lambda u^\lambda / T$

Γ^{-1} 'Inverse time'

or small gradients irrespective of initial conditions

Attractor for 1+1 viscous hydro with general rapidity dependence

Dynamics for MIS $\nabla_\mu T^{\mu\nu} = 0$

Shear $\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} u^\lambda \nabla_\lambda \pi^{\alpha\beta} = -\pi^{\mu\nu} + \eta\sigma^{\mu\nu}$

Bulk $\Pi = 0$

Homogeneous in transverse plane -> A closed set of differential equations as

$$\begin{cases} 0 = \partial_\tau\left(\frac{2\varepsilon}{3}\cosh\zeta + \frac{\varepsilon}{3} + \pi^{\tau\tau}\right) + \frac{4\varepsilon}{3\tau}\cosh\zeta + \partial_\eta\left(\frac{2\varepsilon}{3\tau}\sinh\zeta + \pi^{\tau\eta}\right) + \tau\pi^{\eta\eta} + \frac{\pi^{\tau\tau}}{\tau} \\ 0 = \left(\frac{3}{\tau} + \partial_\tau\right)\left(\frac{2\varepsilon}{3\tau}\sinh\zeta + \pi^{\tau\eta}\right) + \partial_\eta\left(\frac{2\varepsilon}{3\tau^2}\cosh\zeta - \frac{\varepsilon}{3\tau^2} + \pi^{\eta\eta}\right) \\ 0 = u^\lambda\partial_\lambda\pi^{\tau\tau} - \pi^{\tau\tau}\coth\frac{\zeta}{2}u^\lambda\partial_\lambda\zeta - \frac{\eta\,\sigma^{\tau\tau} - \pi^{\tau\tau}}{\tau_\pi} \end{cases}$$

$$u^x = u^y = 0$$

$$\partial_x = \partial_y = 0$$

$$u^\tau \equiv \cosh\frac{\zeta}{2},\ u^\eta \equiv \frac{1}{\tau}\sinh\frac{\zeta}{2}$$

Attractor for 1+1 viscous hydro with general rapidity dependence

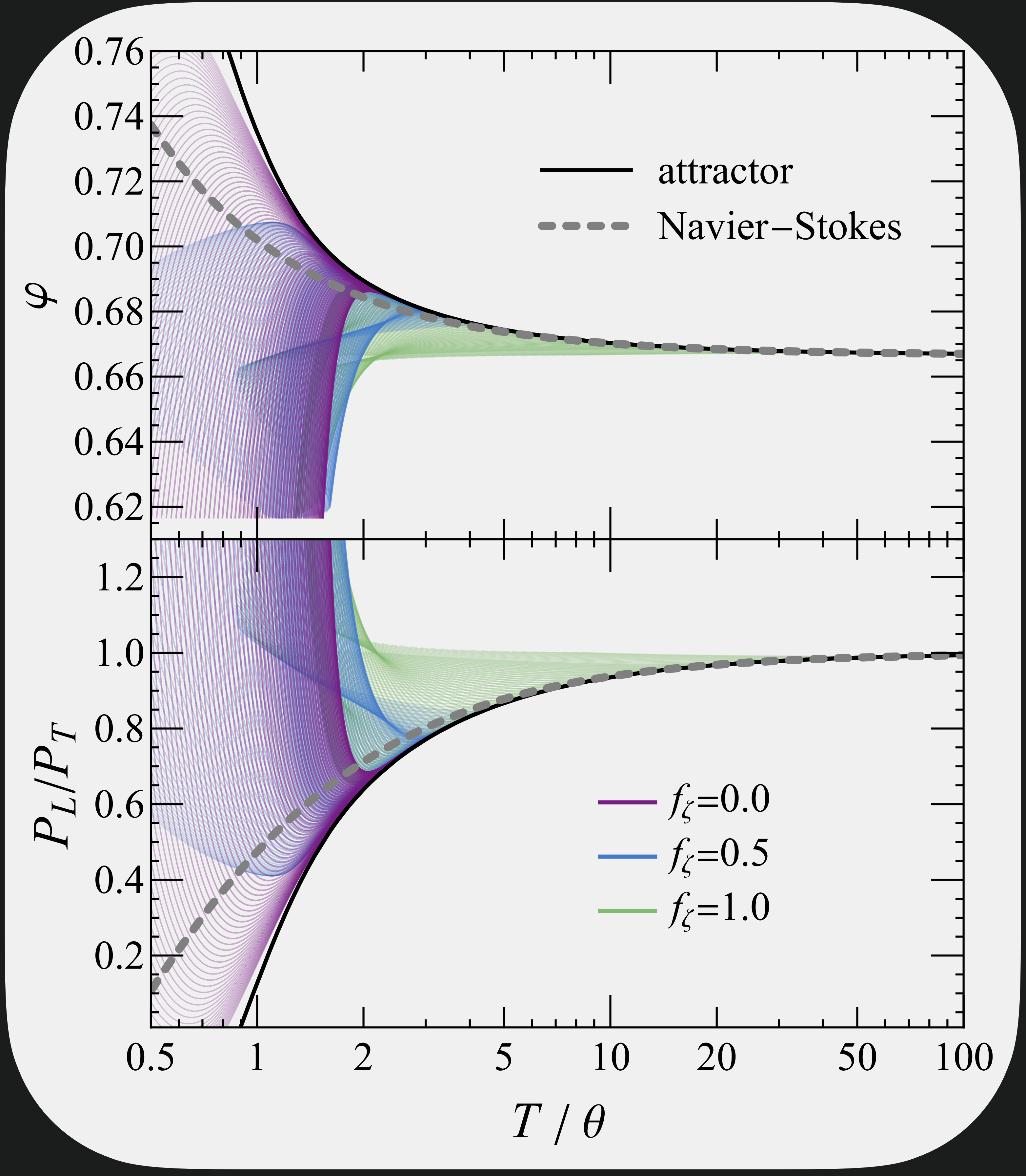
Observables ——— Rebuild Lorentz covariance to f & w

$$\mathscr{D}_\tau \to u^\mu \mathscr{D}_\mu$$

$$\tau \to 1/\theta \equiv 1/(\mathscr{D}_\mu u^\mu)$$

$$w = \tau T \quad \Rightarrow \quad \tilde{w} = T/\theta$$ "Proper time"

$$f = 1 + \tau \frac{\partial_\tau T}{T} \quad \Rightarrow \quad \varphi = 1 + \frac{u^\mu \partial_\mu T}{\theta T}$$ Observable



aHydro and Boltzmann equation

It is difficulty to obtain an analytical formal solution to the Boltzmann equation with general rapidity distribution then we here use a specialized solution with rapidity dependence from Hydro

$$\hat{\tau} = \frac{2a\,\tau_0}{1+a^2}\left(\left(\frac{t_0 + a\,(t+z)}{\tau_0}\right)^{\frac{1}{a}}\left(\frac{t_0 + \frac{t-z}{a}}{\tau_0}\right)^{a}\right)^{\frac{1+a^2}{4a}}$$

$$\hat{\eta} = \frac{1+a^2}{4a}\ln\left(\left(\frac{t_0 + a\,(t+z)}{\tau_0}\right)^{\frac{1}{a}} \Big/ \left(\frac{t_0 + \frac{t-z}{a}}{\tau_0}\right)^{a}\right)$$

$$\hat{g}^{\tau\tau} = 2\frac{\partial\hat{\tau}}{\partial x^+}\frac{\partial\hat{\tau}}{\partial x^-} = e^{2\frac{1-a^2}{1+a^2}\hat{\eta}}$$

$$\hat{g}^{\eta\eta} = 2\frac{\partial\hat{\eta}}{\partial x^+}\frac{\partial\hat{\eta}}{\partial x^-} = -\frac{1}{\hat{\tau}^2}e^{2\frac{1-a^2}{1+a^2}\hat{\eta}}$$

Hydro dynamics solution in co-moving frame

S. Shi, S. Jeon and C. Gale, Phys. Rev. C 105, no.2, L021902 (2022)

$$\Rightarrow \left(\hat{p}^{\tau}\frac{\partial}{\partial\hat{\tau}} + \hat{p}^{\eta}\frac{\partial}{\partial\hat{\eta}} + \frac{a^2-1}{a^2+1}(\hat{p}_x^2+\hat{p}_y^2)\frac{\partial}{\partial\hat{p}_\eta}\right)f(\hat{x},\hat{p}) = \mathscr{C}[f]$$

Obtain ODE without dependence of $\hat{\eta}$ while without losing dependence of real rapidity

aHydro and Boltzmann equation with RTA kernel

With specialized RS distribution

$$f(\hat{x},\hat{p}) = f_{\mathrm{RS}}(\hat{x},\hat{p}) = e^{-\frac{\sqrt{(\frac{\hat{p}_\eta}{\hat{\tau}})^2(\xi(\hat{x})+1) + \hat{g}_{\tau\tau}(\hat{p}_x^2+\hat{p}_y^2)}}{\Lambda(\hat{x})}}$$

aHydro

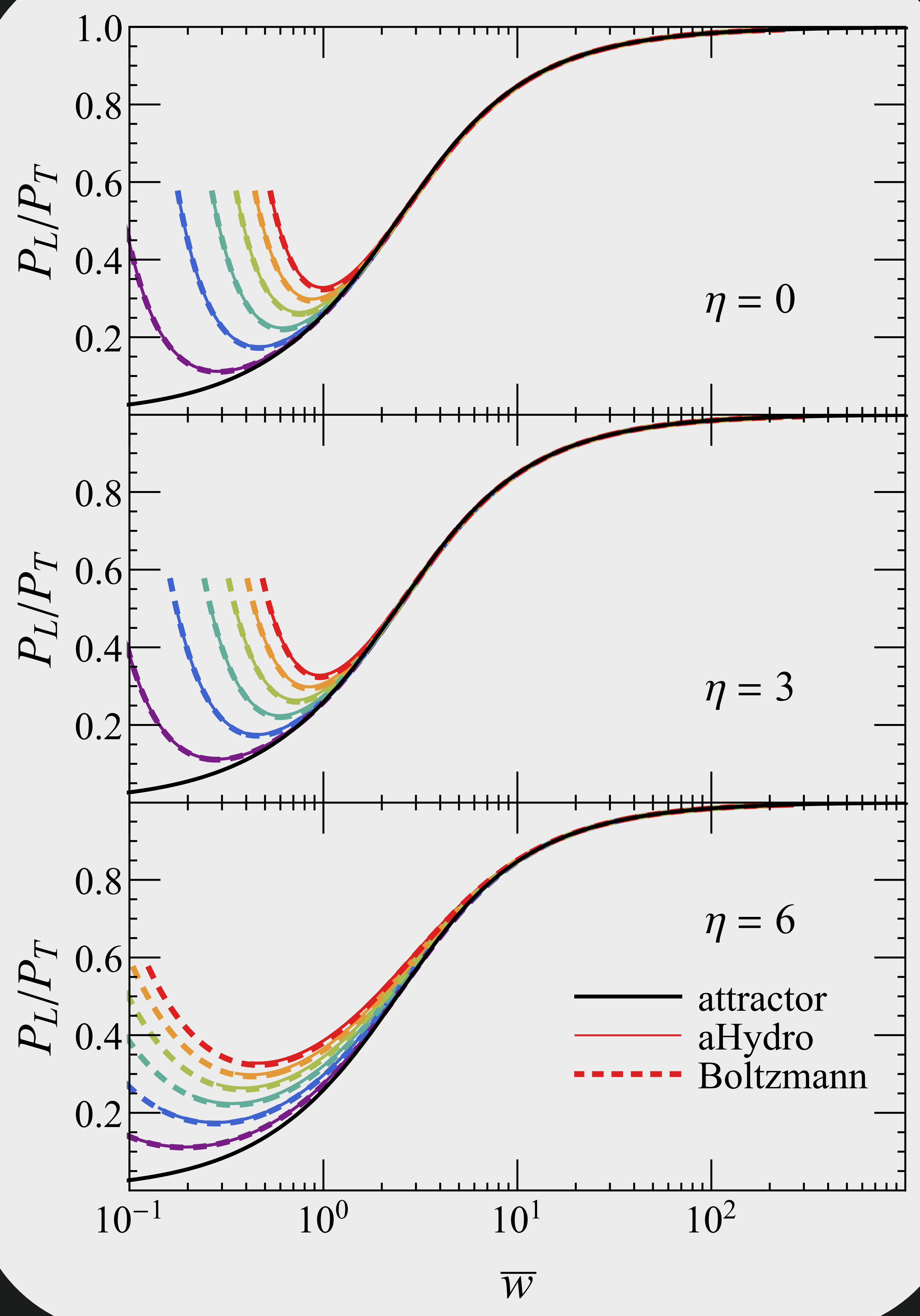
Boltzmann

Building moments equations with RTA

Solving Boltzmann with RTA

Solving differential equations for $\hat{\Lambda}(x)$ $\hat{\xi}(x)$

Building moments to get $\frac{\hat{P}_L}{\hat{P}_T}$



Summary:

Rebuilt Lorentz covariance to observables and scaled time -> rebuilt attractor behavior to general rapidity dependence system with hydrodynamic theory/ specialized rapidity dependent kinetic theory

Indications & questions:

- ✦ Existence of expansion
- ✦ Attractor behavior is the property of EQUATION?
- ✦ Attractor behavior is the property of typical SYMMETRY?
- ✦ Attractor behavior is the choice of scaled time?

Outlook:

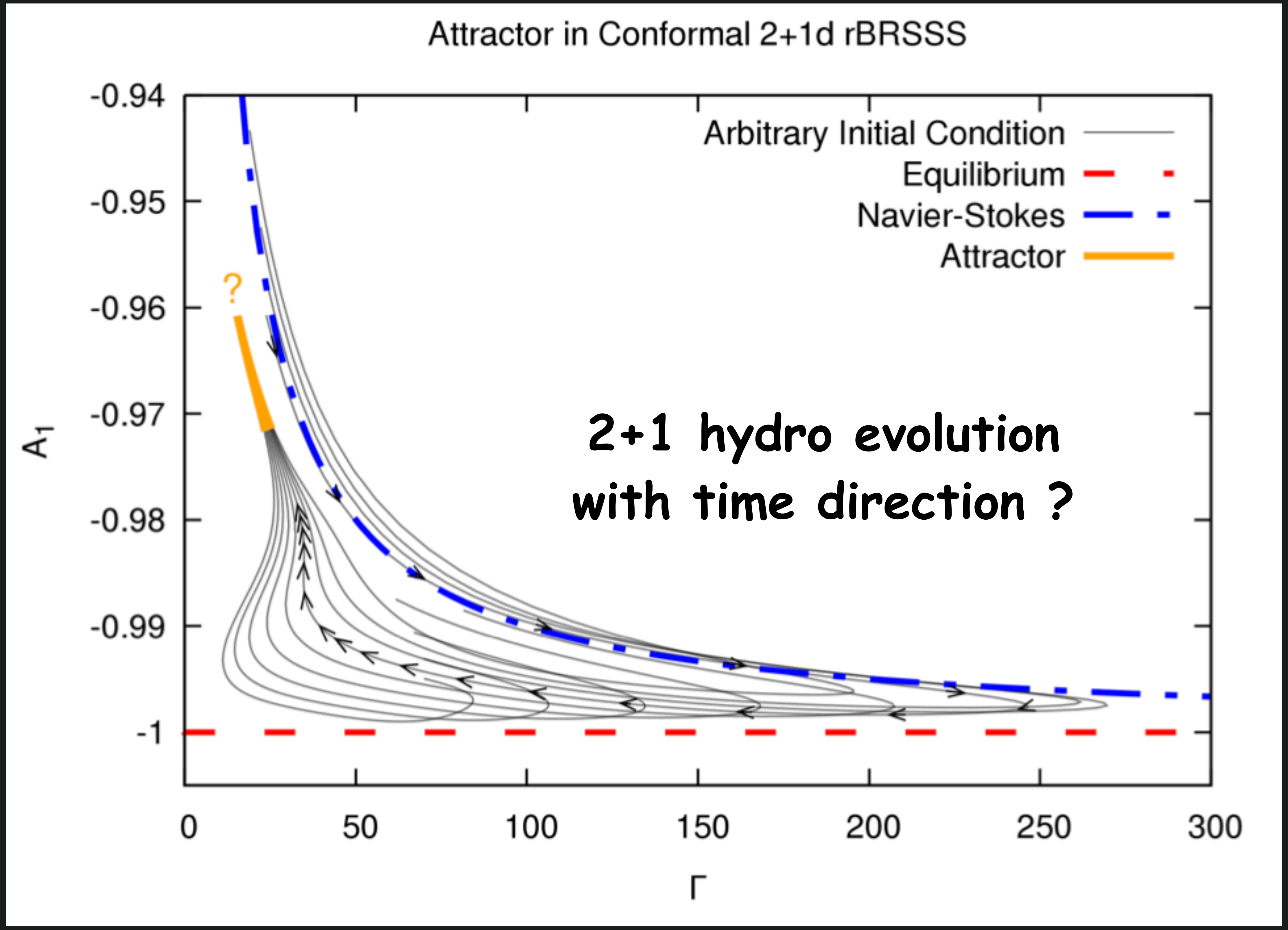
- What kind of requirement is necessary for the appearance of attractor?

Verify more and more theory and symmetries

- Real 3+1 system in HIC
- Non-conformal system?
- Seeing different Observables?

Thank you for your listening!!

Backup



For a expanding system, theta would always be non-zeros, which means time-> inf would not reached

Backup

In ahydro and Boltzmann we here put an typical symmetry to the solution of Boltzmann equation

$$\Rightarrow \left(\hat{p}^{\tau}\frac{\partial}{\partial\hat{\tau}} + \hat{p}^{\eta}\frac{\partial}{\partial\hat{\eta}} + \frac{a^2-1}{a^2+1}(\hat{p}_x^2 + \hat{p}_y^2)\frac{\partial}{\partial\hat{p}_\eta}\right) f(\hat{x},\hat{p}) = \mathscr{C}[f]$$

Assuming no dependence with eta hat(making it ODE) -> special relation of tau and eta

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$$D\epsilon + (\epsilon + P)\nabla_\lambda u^\lambda = \frac{\eta}{2}\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi(\nabla_\lambda u^\lambda)^2$$

Naiver-Stokes equations

$$(\epsilon + P)Du^\alpha + \nabla^\alpha_\perp P = \Delta^\alpha_\nu \nabla_\mu(\eta\sigma + \xi\Delta^{\mu\nu} + \xi\Delta^{\mu\nu}\nabla_\lambda u^\lambda)$$

$\nabla_\lambda u^\lambda$ Expansion rate

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