第二十届全国中高能核物理大会暨第十四届全国中高能核物理专题研讨会

Functional renormalization group study of anomalous magnetic moment in a low energy effective theory

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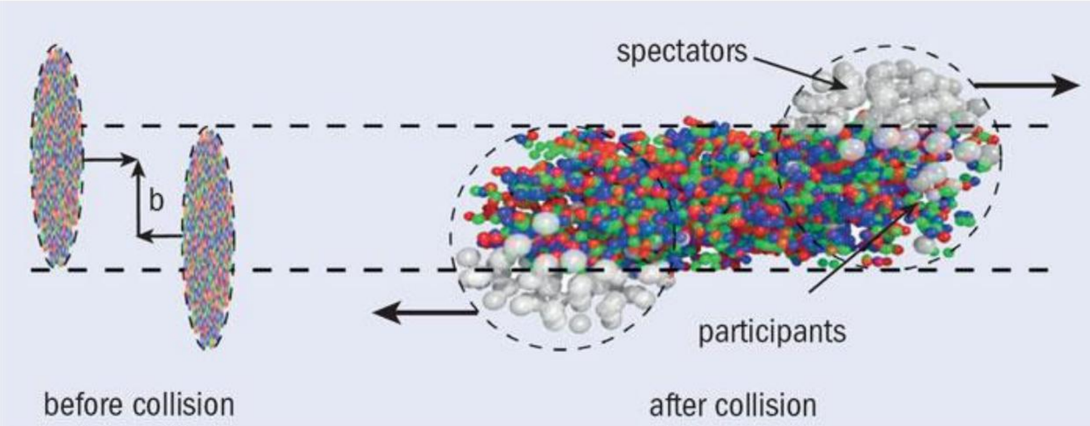
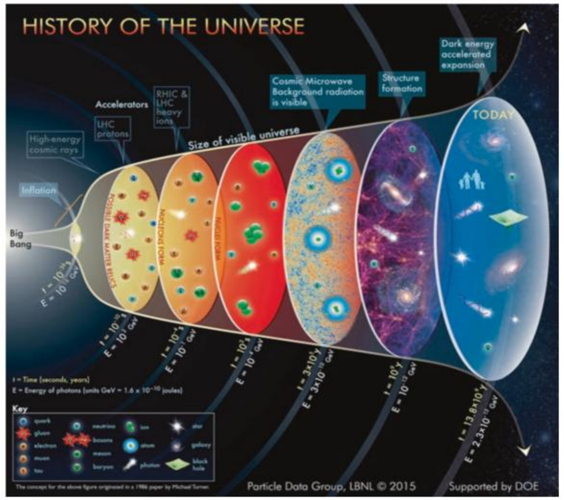
Based on: Rui Wen, Chuang Huang and Mei Huang (2025) in preparation

Outline

- Introduction
- low energy effective theory within FRG
  - The Nambu-Jona-Lasinio type effective action
  - Anomalous magnetic moment
- Numerical results
- Summary

Introduction





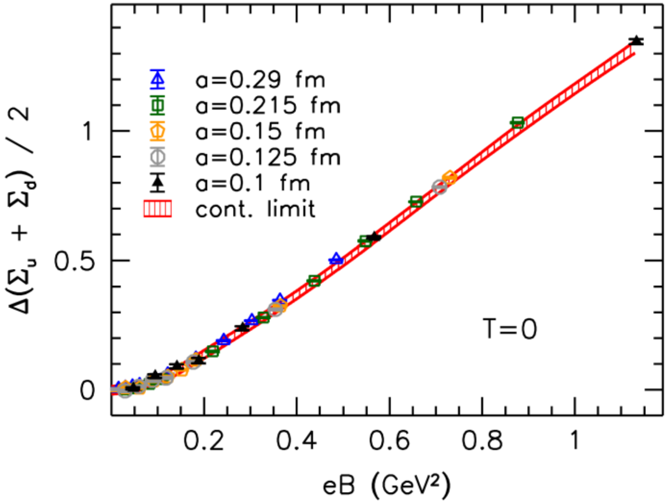
non-central heavy-ion collisions $B \sim 10^{18}$ Gauss

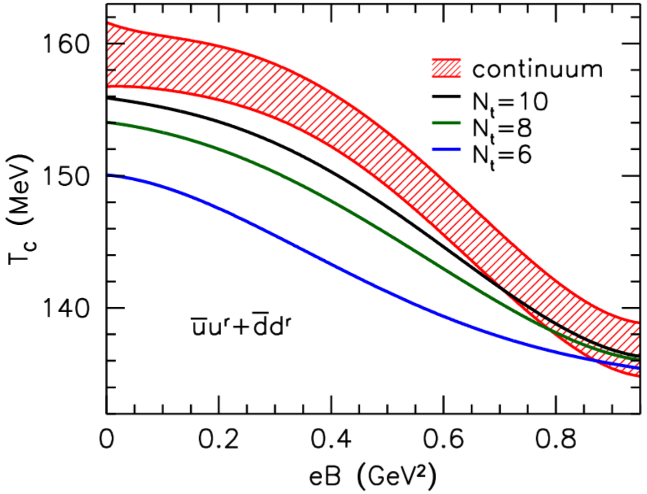
Toia A 2013 CERN Courier April 31

early universe

magnetars

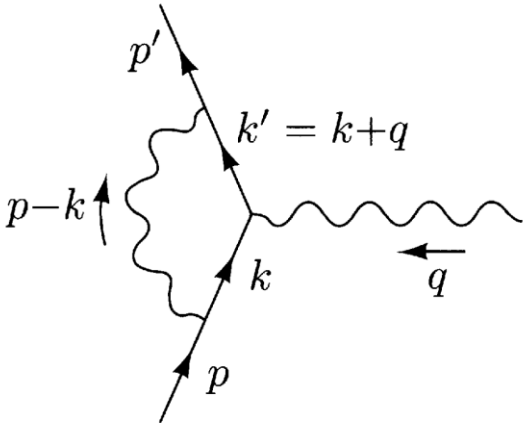
G. S. Bali, F. Bruckmann et. al. , (2012) Phys. Rev. D 86, 071502 arXiv:1206.4205





G. S. Bali, F. Bruckmann et. al. (2011) JHEP 02, 044, arXiv:1111.4956

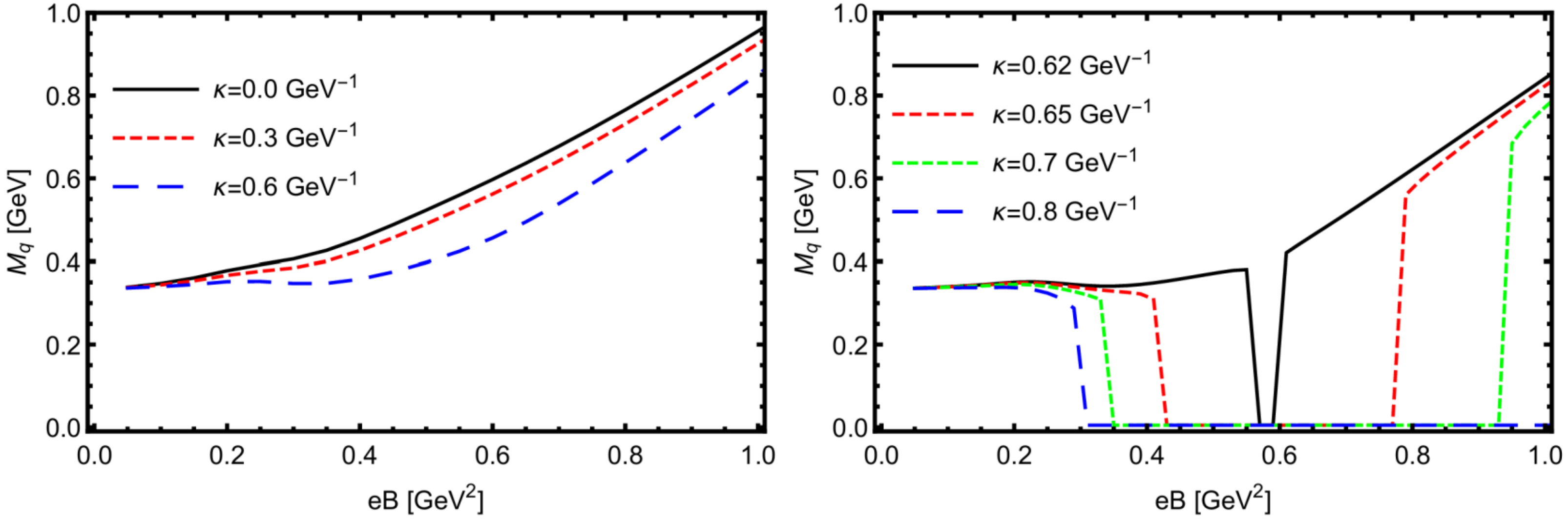
Introduction



$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2),$$

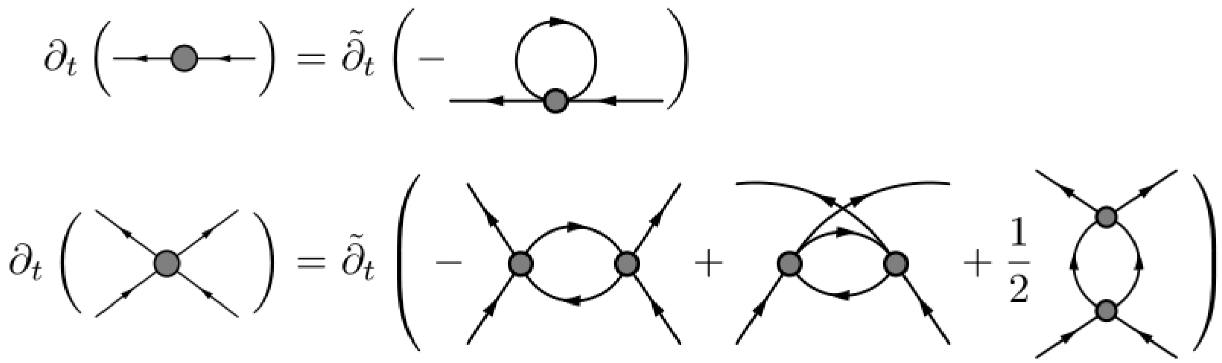
An Introduction To Quantum Field Theory Michael E. Peskin Daniel V. Schroeder

$$\mathcal{L} = \bar\psi(i\gamma^\mu D_\mu - m_0 + \kappa_f q_f F_{\mu\nu}\sigma^{\mu\nu})\psi + G_S\left\{(\bar\psi\psi)^2 + (\bar\psi i\gamma^5\vec\tau\psi)^2\right\}$$



Phys. Rev. D 103.076015 K. Xu, J. Chao, and M. Huang

The effective action



The effective action of 2-flavor Nambu-Jona-Lasinio type low energy effective theory in Euclidean space

$$\Gamma_k[\bar{q},q] = \int_x \bar{q}(\gamma_\mu D_\mu + m)q + \sum_\alpha \lambda^{(\alpha)} \mathcal{T}^{(\alpha)}_{4q,ijlm} \bar{q}_i q_j \bar{q}_l q_m$$

The quark fields $q = (u,d)^T$ $\quad Q = diag(2/3, -1/3)e$

The covariant derivative of quark fields $D_\mu \equiv \partial_\mu - iQA_{\mu,0}$

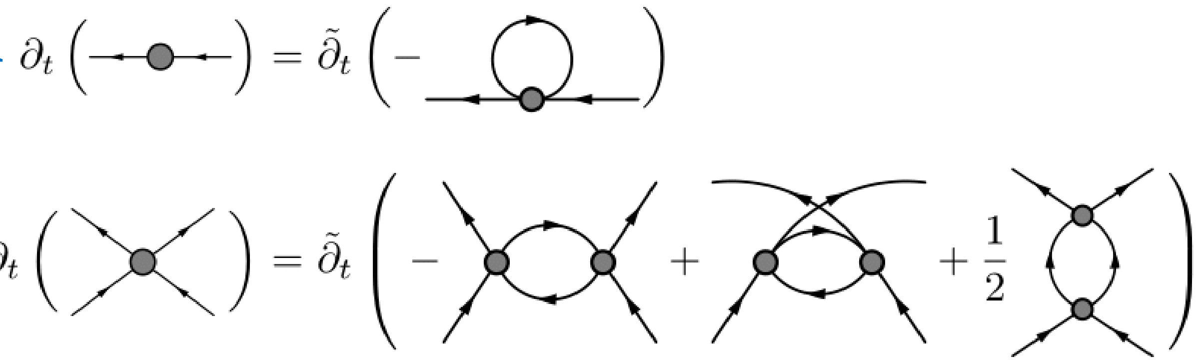
A background homogeneous magnetic field along z-direction

$$A_{\mu,0} = (0,0,xB,0)$$

Four-quark scatterings.

$$\alpha \in \{\pi, \sigma, \eta, a, (V \pm A), (V-A)^{adj}, (S \pm P)^{adj}_{-}, (S+P)^{adj}_{+}\}.$$

The Schwinger formalism of quark propagator



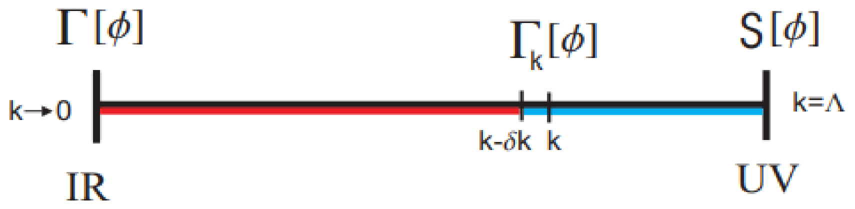
$$G(x,y;q_f) = \Phi(x,y)\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}\tilde{G}(p;q_f).$$

Translation invariance part

$$\tilde{G}(p;q_f) = \int_0^\infty ds \exp\left[-s\left((1+r_{f,s})^2\left(p_\parallel^2 + p_\perp^2 \frac{\tanh(q_f Bs)}{q_f Bs}\right) + m^2\right)\right]$$
$$\cdot\left[(-ip_\parallel\gamma_\parallel(1+r_{f,s}) + m)(1 + i\gamma_1\gamma_2\tanh(q_f Bs))\right.$$
$$\left.- ip_\perp\gamma_\perp(1+r_{f,s})(1-\tanh^2(q_f Bs))\right].$$

4d regulator

$$r_{f,s} = \frac{e^{-x}}{x}, \qquad x = \frac{p_\parallel^2 + p_\perp^2 \tanh(q_f Bs)/(q_f Bs)}{k^2}$$



t=log(k/Λ)

W.-j. Fu, C. Huang, J. M. Pawlowski, and Y.-y. Tan, SciPost Phys.14.4.069(2022)

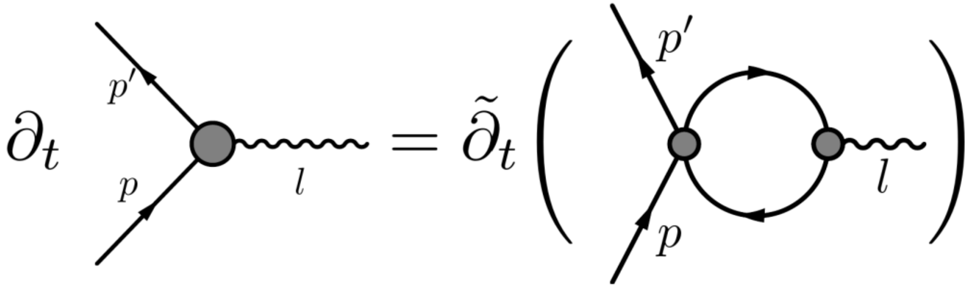
Anomalous magnetic moment

Full quark-photon vertex

$$\Gamma_{A\bar{q}q,\mu} = \delta(p+p'+l)Q \otimes \sum_{i=1}^{8} \Gamma^{(i)}_{A\bar{q}q,\mu}$$

$$\Gamma^{(i)}_{A\bar{q}q,\mu} = \lambda^{(i)}_{A\bar{q}q}[\mathcal{T}^{(i)}]_\mu(p,p',l)$$

Friederike Ihssen, Jan M. Pawlowski, Franz R. Sattler, Nicolas Wink arXiv:2408.08413



The classical tensor structure

$$[\mathcal{T}^{(1)}]_\mu(p,p',l) = \Pi^\perp_{\mu\nu}(l) i\gamma_\nu ,$$

tensor structure related to the anomalous magnetic moment

$$[\mathcal{T}^{(4)}]_\mu(p,p',l) = i\sigma_{\mu\nu}(l)_\nu .$$

We define the form factor

$$F_{2,f}(p,p',l) \equiv 2m\lambda^{(4)}_{A\bar{q}_f q_f}(p,p',l) , \quad f \in \{u,d\},$$

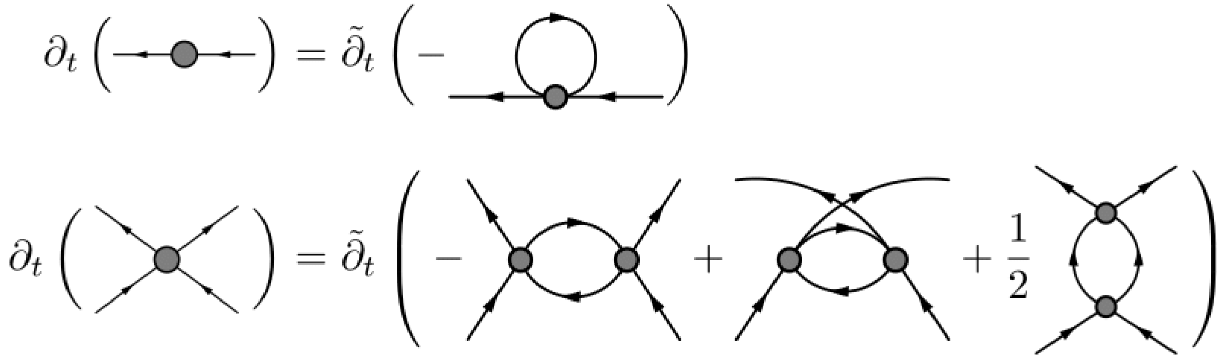
And quark AMM

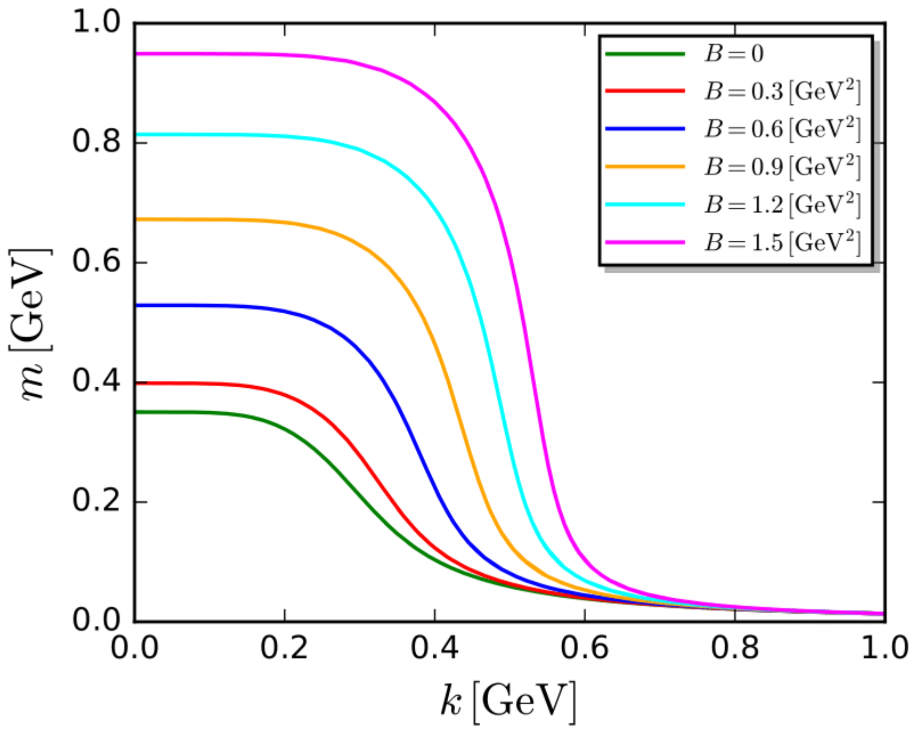
$$\kappa_f \equiv \lim_{l\to 0} \lambda^{(4)}_{A\bar{q}_f q_f}(p,p',l) , \quad f \in \{u,d\},$$

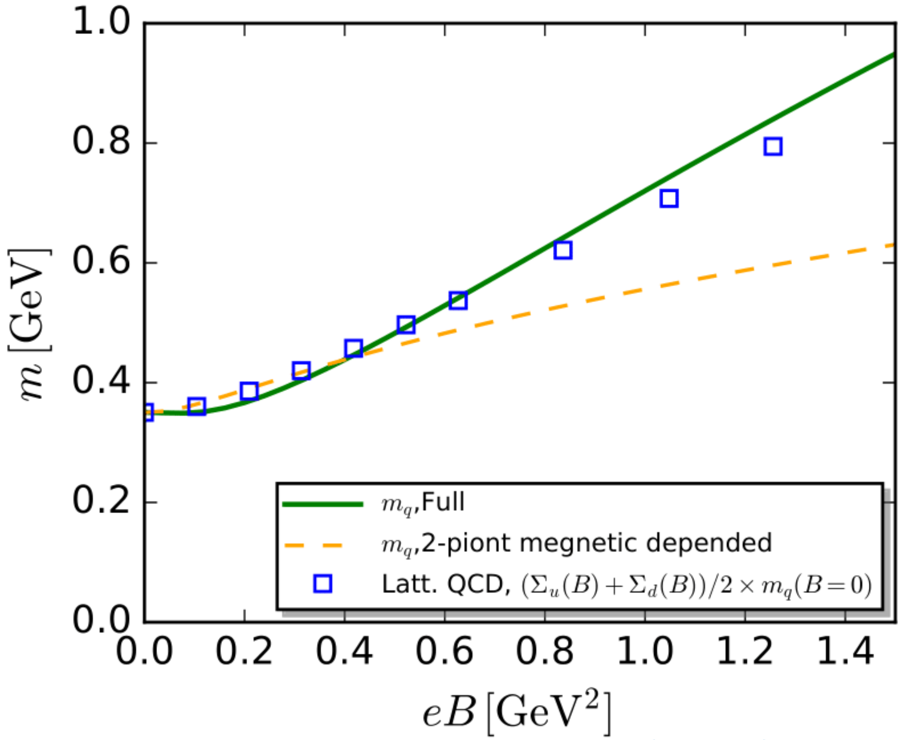
The quark masses as functions of magnetic fields

$\Lambda = 1\text{GeV},\ m_\Lambda = 13.5\text{MeV}$
$\lambda_{\pi,\Lambda} = \lambda_{\sigma,\Lambda} = 10.6\Lambda^{-2}$

$m(T=0, B=0) = 350\ \text{MeV}$
$m_\pi = 141\text{MeV}$

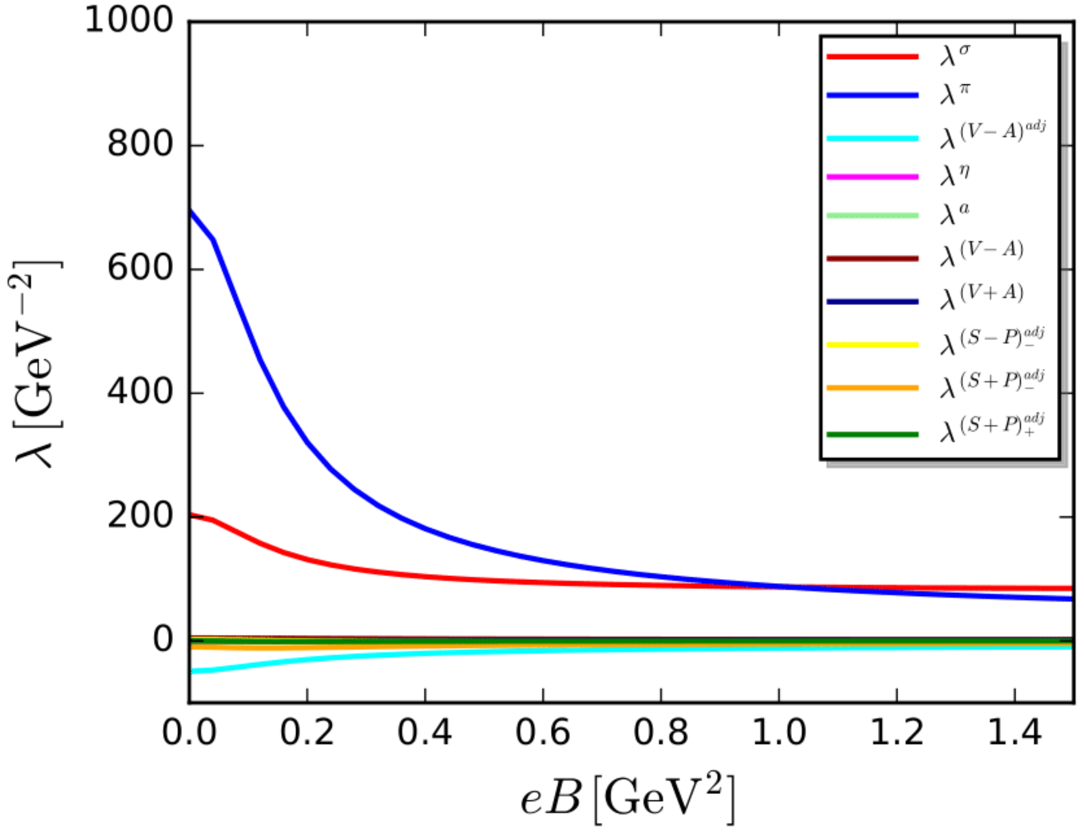


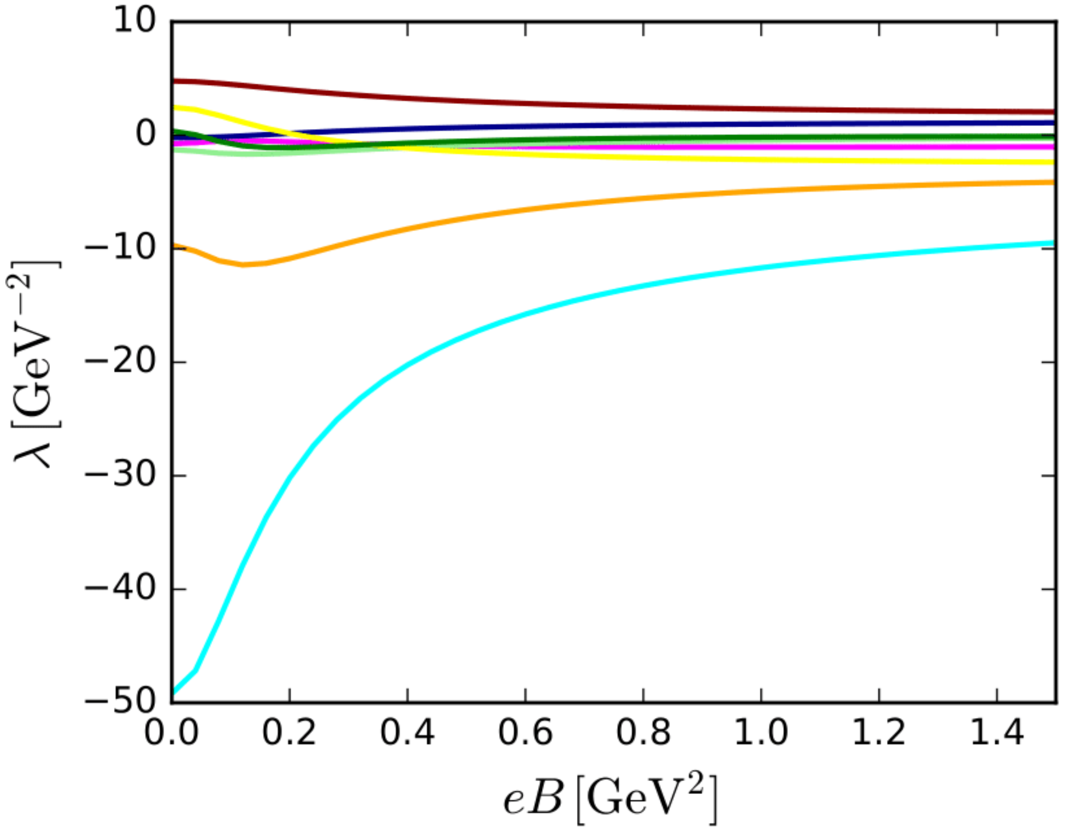




H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang, and Y. Zhang Phys. Rev. D 105, 034514 (2022)

Four-point quark correlation functions





$$\lambda_{\pi,\Lambda} = \lambda_{\sigma,\Lambda} = 10.6\Lambda^{-2}$$

Form factors

The magnetic moments of quarks

$$\mu_f = q_f(1+F_2(0))\frac{m_N}{m}\mu_N\,, \quad f\in\{u,d\}$$

The constituet quark model

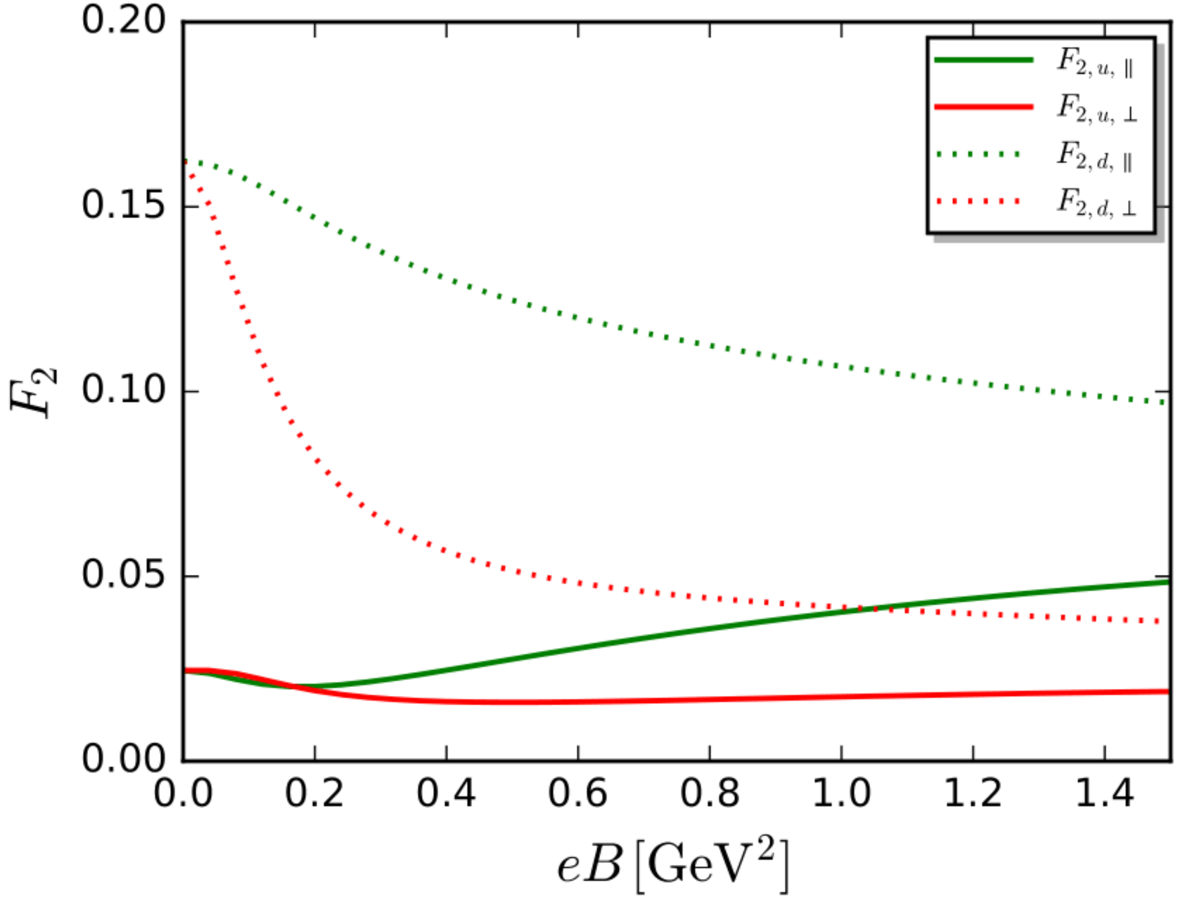
$$\mu_{\text{proton}} = \frac{1}{3}(4\mu_u - \mu_d),$$
$$\mu_{\text{neutron}} = \frac{1}{3}(4\mu_d - \mu_u).$$

Our results with the constituet quark model

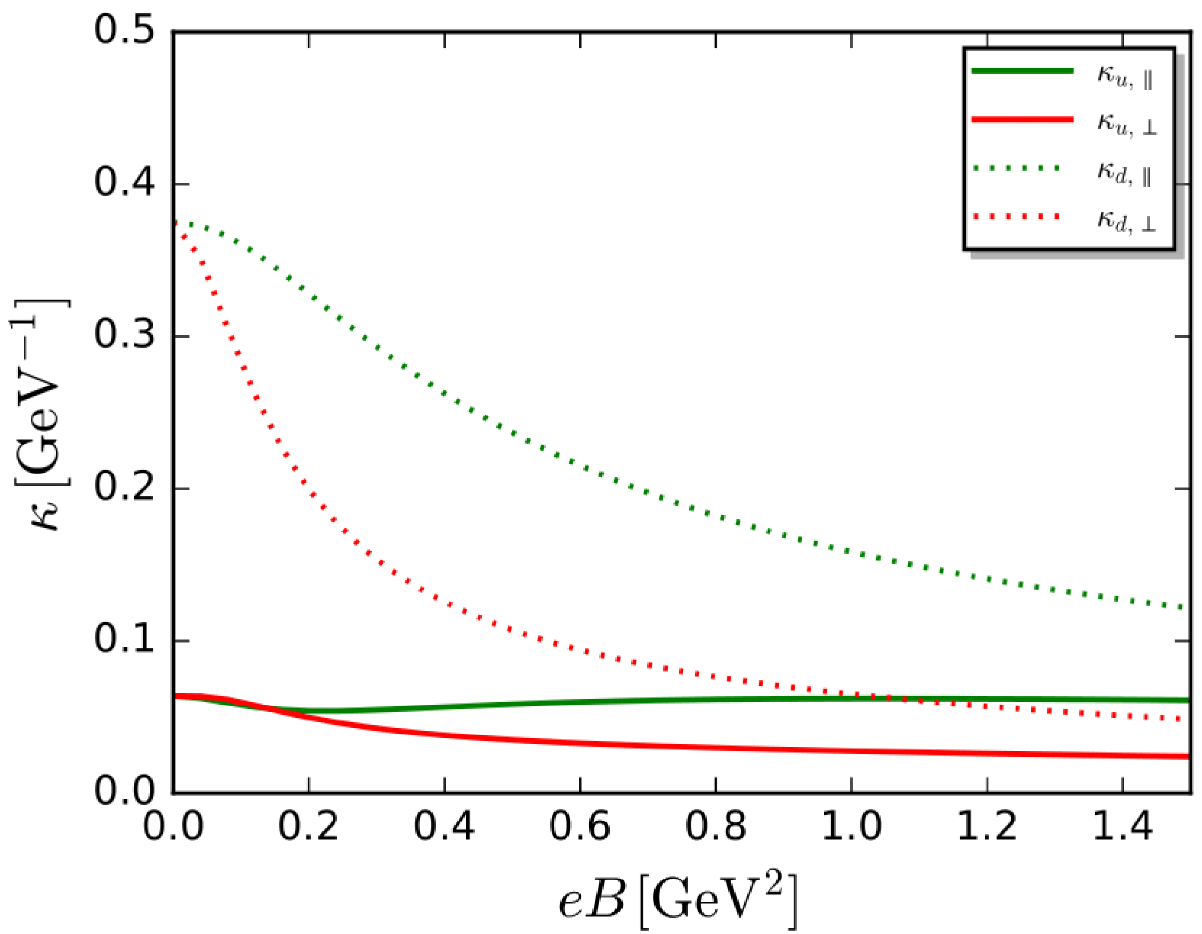
$$\mu_{\text{proton}}/\mu_N = 2.7877$$
$$\mu_{\text{neutron}}/\mu_N = -1.9967$$

Committee on Data of the International Science Council (CODATA) 2022 arXiv:2409.03787

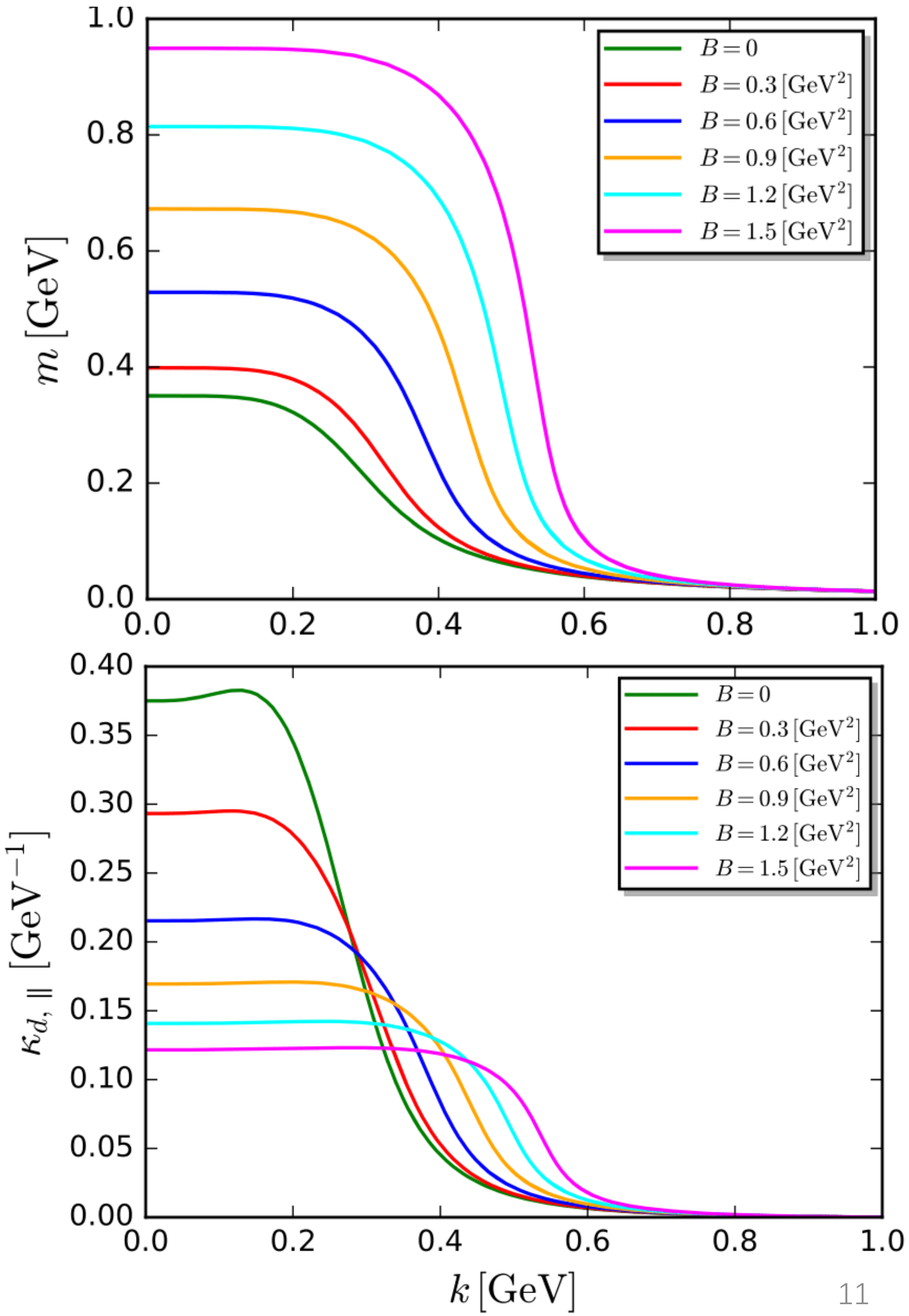
$$\mu_{\text{proton}}/\mu_N = 2.7928$$
$$\mu_{\text{neutron}}/\mu_N = -1.9130$$



Quarks AMM



$\kappa_{u,\parallel}$
$\kappa_{u,\perp}$
$\kappa_{d,\parallel}$
$\kappa_{d,\perp}$
$\kappa\,[\mathrm{GeV}^{-1}]$
$eB\,[\mathrm{GeV}^2]$
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



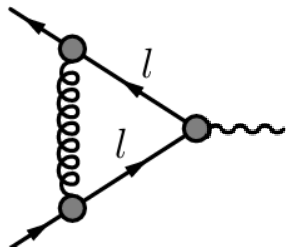
$B=0$
$B=0.3\,[\mathrm{GeV}^2]$
$B=0.6\,[\mathrm{GeV}^2]$
$B=0.9\,[\mathrm{GeV}^2]$
$B=1.2\,[\mathrm{GeV}^2]$
$B=1.5\,[\mathrm{GeV}^2]$
$m\,[\mathrm{GeV}]$
$\kappa_{d,\parallel}\,[\mathrm{GeV}^{-1}]$
$k\,[\mathrm{GeV}]$

Summary

1. We build a 2-flavor Nambu-Jona-Lasinio type low energy effective theory under magnetic fields, which self-consistently include the Fierz-complete four-quark scatterings through RG flows. The quark mass as a function of the strength of magnetic field are in agreement with the lattice QCD results.

2. We calculate quark AMM in this LEFT. We find the quark AMM is dynamically generated with chiral symmetry breaking. The quark AMM in the direction perpendicular to the magnetic field monotonically decreases with increasing magnetic field, while u-quark AMM in the parallel direction slightly increases with the magnetic field.

We will calculate quark AMM in QCD within in future work.



We hope lattice QCD groups could provide a calculation of the quark AMM.

Thanks for your attention