

π 介子精细结构的 pQCD 研究

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2025 年 4 月 26 日

Overview

I Form Factors

II The perturbative QCD approach

Three-scale Factorization

The soft-transversal dynamics

III Pion EMFF and TFF

IV Conclusion

Form Factors

"In elementary particle physics and mathematical physics, in particular in effective field theory, a **form factor** is a function that encapsulates the properties of a certain interaction without including all of the underlying physics, but instead, providing the momentum dependence of a suitable matrix elements. Its further measured experimentally in confirmation or specification of a theory."

Momenta Redistribution



QCD is believed to exhibit confinement

hadron structures $\otimes$ hard scattering



decoupling of LD and SD interactions

factorisation theorem, EFT; CKM, $g-2$, B anomalies

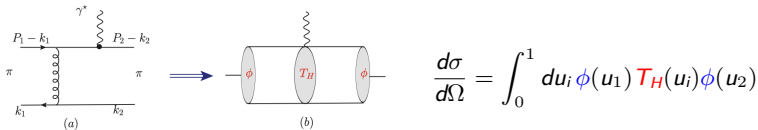
Form Factors

PION is the lightest Glodstone boson and one of the most simplest hadrons, hence the ideal probe to extremely rich QCD dynamics.

- (spacelike) electromagnetic form factor

$$\langle \pi^-(p_2) | \mathcal{J}_\mu^{\text{em}} | \pi^-(p_1) \rangle = e_q (p_1 + p_2)_\mu F_\pi(Q^2)$$

- the interaction distance of $\mathcal{J}_\mu^{\text{em}}$ is decided by the external reason Q^2
- Separate the **hard partonic physics** out of the **hadronic physics** (soft, nonperturbative objects) in exclusive processes **Factorization**



- The **universal nonperturbative objects**, studied by QCD-based analytical (QCDSRs, χ PT, DSE, instanton) and numerical approaches (LQCD)
- also by data-driven method, CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

The perturbative QCD approach

- i Three-scale factorization
- ii The soft-transversal dynamics

Three-scale Factorization

Exclusive Processes in Perturbative Quantum Chromodynamics

#1

G.Peter Lepage (Cornell U., LNS), Stanley J. Brodsky (SLAC) (Mar, 1980)

Published in: *Phys.Rev.D* 22 (1980) 2157



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reference search



4,122 citations

Factorization and Asymptotical Behavior of Pion Form-Factor in QCD

#1

A.V. Efremov (Dubna, JINR), A.V. Radyushkin (Dubna, JINR) (Nov, 1979)

Published in: *Phys.Lett.B* 94 (1980) 245-250



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reference search



1,333 citations

- the first rigorous pQCD predictions to the entire domain of **larger-momentum-transfer exclusive reactions**

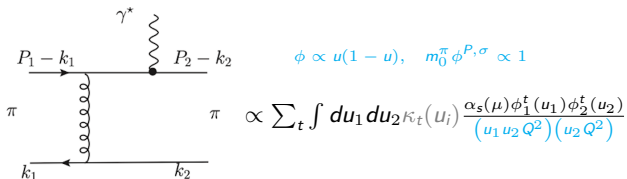
$$\mathcal{F}_\pi(Q^2) = \int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$$

- ‡ amplitudes are dominated by quark and gluon subprocesses at SDs
- ‡ evolution equations for process-independent hadron DAs $\psi(x_i, \tilde{Q})$
finding the constituents with light-cone momentum fraction x_i at transversal separations
- ‡ **leading twist DAs and α_s order calculation**
prevents anomalous contributions from the end-point $x_i \sim 1$ integration regions

Three-scale Factorization

- End-point singularities appear at high twists

$$\ddagger \quad m_{1,2}^2 \ll Q^2, \quad p_2 = (\frac{Q}{\sqrt{2}}, 0, 0_T), \quad p_3 = (0, \frac{Q}{\sqrt{2}}, 0_T), \quad k_2 = x_2 p_2, \quad \bar{k}_2 = \bar{x}_2 p_2$$



$$\phi \propto u(1-u), \quad m_0^\pi \phi^{P,\sigma} \propto 1$$

$$\pi \propto \sum_t \int du_1 du_2 \kappa_t(u_i) \frac{\alpha_s(\mu) \phi_1^t(u_1) \phi_2^t(u_2)}{(u_1 u_2 Q^2)(u_2 Q^2)}$$

- $\ddagger$ pick up k_T in the internal propagators

$$\mathcal{M} \propto \sum_{t=2,3,4} \int du_1 du_2 dk_{1T} dk_{2T} \kappa_t(u_i) \frac{\alpha_s(\mu) \phi_1^t(u_1) \phi_2^t(u_2)}{[u_1 u_2 Q^2 - (\Delta k_T)^2] (u_2 Q^2 - k_{2T}^2)}$$

- $\ddagger$ end-point singularity at leading and subleading powers

$$\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \alpha_s(\mu) \frac{k_T^2}{(u_1 u_2 Q^2)^2} + \dots$$

- $\ddagger$ the power suppressed TMD terms becomes important at the end-points

Three-scale Factorization

A Study of the Applicability of Perturbative {QCD} to the Pion Form-factor #1

Tao Huang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Qi-xing Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Peter Kroll (Wuppertal U.) (Aug, 1989)

Published in: *Z.Phys.C* 50 (1991) 139-144 • Contribution to: [Quarks 90](#)

 DOI  cite  claim

 reference search  75 citations

Analysis of the pion wave function in light cone formalism #1

Tao Huang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Bo-Qiang Ma (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Qi-Xing Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.) (Mar 8, 1994)

Published in: *Phys.Rev.D* 49 (1994) 1490-1499 • e-Print: [hep-ph/9402285](#) [hep-ph]

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 reference search  132 citations

微扰量子色动力学应用到遍举过程中的几个问题	曹俊	1998	黄涛	博士
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- introduce k_T to regularize the end-point singularity

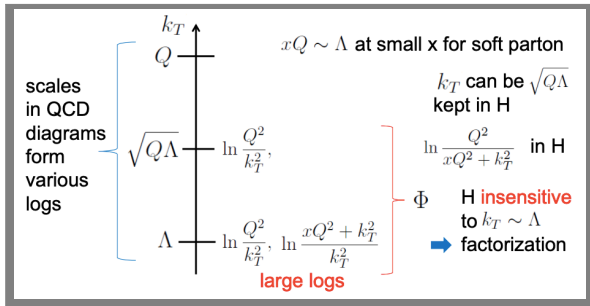
$$\mathcal{F}_\pi(Q^2) = \int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} du_2 \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T})$$

‡ constraints on the integration region $b = (1 - \sqrt{1 - 4a})/2$, $a = \langle k_T^2 \rangle / Q^2$, $\langle k_T \rangle \sim 300$ MeV

‡ leading twist DAs within different b -dependent models, also at α_s order

Three-scale Factorization

- k_T varies within three scales [stolen from H.N Li]



- ‡ large single and double logarithms from QCD corrections, ie., $\alpha_s(\mu) \ln^2 \frac{k_T^2}{m_B^2}$
- ‡ k_T resummation for T to obtain $S(u_i, b_i, Q)$ suppresses the large transversal distances (small k_T) interactions by decreasing q^2 power in denominator
- integrating over k_T , $\ln^2(x_i)$ resides when the internal parton is on shell
- threshold resummation for ψ to obtain $S_t(x_i, Q)$ suppresses the small x_i regions, repairs the self-consistency between $\alpha_s(t)$ and hard log $\ln(u_1 2_3 Q^2/t^2)$

Three-scale Factorization

Hard Elastic Scattering in QCD: Leading Behavior

#1

James Botts (SUNY, Stony Brook), George F. Sterman (SUNY, Stony Brook) (Mar 20, 1989)

Published in: *Nucl.Phys.B* 325 (1989) 62-100

 DOI  cite  claim

 reference search  598 citations

The Perturbative pion form-factor with Sudakov suppression

#1

Hsiang-nan Li (SUNY, Stony Brook), George F. Sterman (SUNY, Stony Brook) (Mar, 1992)

Published in: *Nucl.Phys.B* 381 (1992) 129-140

 DOI  cite  claim

 reference search  527 citations

Unification of the $k(T)$ and threshold resummations

#1

Hsiang-nan Li (Taiwan, Natl. Cheng Kung U.) (Dec, 1998)

Published in: *Phys.Lett.B* 454 (1999) 328-334 · e-Print: [hep-ph/9812363](https://arxiv.org/abs/hep-ph/9812363) [hep-ph]

 pdf  DOI  cite  claim

 reference search  89 citations

$$\mathcal{F}_\pi(Q^2) = \psi(u_1, \mu_{r_1}) T_H(u_i, b_i, Q) S_t(u_i) e^{-S(u, b, Q)} \psi(u_2, \mu_{r_2})$$

- threshold-suppressed hard amplitude $T_H S_t$
- sudakov-multiplied light-cone distribution amplitudes ψe^{-S}
- leading twist & QCD leading order & resolution of endpoint singularities

Three-scale Factorization

- ‡ H.N. Li, Y.L. Shen, Y.M. Wang and H. Zou, PRD 83.054029 (2011)
twist 2@NLO+twist 3@LO
- ‡ SC, Y.Y. Fan and Z.J. Xiao, PRD 89.054015 (2014)
twist 2@NLO+twist 3@NLO
- ‡ SC, PRD 100.013007 (2019)
twist 2@NLO+twist 3@NLO+twist 4@LO, scale revolutions
- ‡ H.N. Li, Y.L. Shen and Y.M. Wang, JHEP 01(2014)004
joint resummation for $\ln(x_1 x_2 Q^2 b^2)$ for large b & small $x_1 x_2 Q^2$
- ‡ H.N. Li and Y.M. Wang, JHEP 06(2015)013
rapidity singularity and pinch singularity, non-dipolar Wilson links for TMDWFs

$$\mathcal{F}_\pi(Q^2) = \sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1}) T_H^{t_i, \text{LO+NLO}}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mu_{r_2})$$

- high twist contributions and more fruitful hadron structures
- NLO QCD corrections in hard kernel and TMDWFs
- hard scale choice [PMC, Majaza, Brodsky and Wu, PRL 109.042002(2012), 110.192001(2013)]
- N²LO from QCD collinear factorization leading twist
[Chen², Feng and Jia, PRL 132. 201901(2024)], [Ji, Shi, Wang³ and Yu, arXiv:2411.03658[hep-ph]]

Three-scale Factorization $\mathcal{F}_\pi(Q^2)$

$$\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2) \quad \boxed{1980s}$$

$\Downarrow$

$$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T}) \quad \boxed{1990s}$$

$\Downarrow$

$$\psi(u_1, \mu_{r_1}) T_H(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b_i, Q)} \psi(u_2, \mu_{r_2}) \quad \boxed{2000s}$$

$\Downarrow$

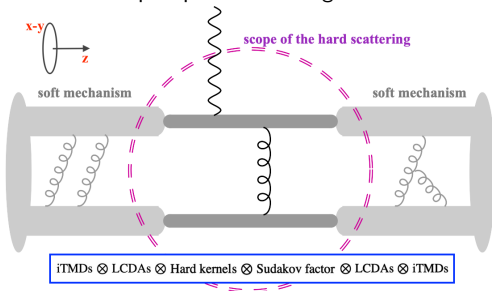
$$\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mu_{r_2}) \quad \boxed{2010s}$$

- $T_H(u_i, b_i, Q) S_t(x_i, Q)$ **threshold-suppressed hard scattering amplitude**
including both the longitudinal and transversal dynamics
- $e^{-S(u_i, b_i, Q)} \psi(u_i, \mu_r)$ **sudakov-multiplied LCDAs** wave functions at zero transversal separations $b_i \sim 0$, only the soft longitudinal dynamics, **oversight of the soft transversal dynamics**
(intrinsic transverse momentum distributions) $\Downarrow$

$$\sum_{t_i} \psi^{t_1}(u_1, \mathbf{b}_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mathbf{b}_2, \mu_{r_2}) \quad \boxed{2020s}$$

The soft-transversal dynamics

the sketch map of pion electromagnetic form factor



‡ **central region of the e.m potential field**

picks up the hard radiations of partons on the transversal plane

‡ **outside the scope of hard scattering**

energetic pions move fast along the z direction accompanied by soft bremsstrahlung radiations absorbed into the effects of high twist LCDAs

‡ **in the exterior region**, the soft radiations

in the transversal plane are notably absent from the definition of LCDAs

[J. Chai and SC, PRD 111. L071902]

- the soft pion wave function is generally to a product of **LCDA** and **iTMDs**

$$\langle 0 | \bar{u}(x) \Gamma [x^-, x_\perp; 0, 0_\perp] d(0) | \pi^-(p) \rangle \propto \int du dk_\perp^2 e^{i u p^+ x^- - i k_\perp \cdot x_\perp} \psi(u, k_T),$$

$$\psi(u, k_T) = \frac{f_P}{2\sqrt{6}} \varphi(u, \mu_r) \Sigma(u, k_T), \quad \int_0^1 du \varphi(u, \mu_r) = 1, \quad \int \frac{d^2 k_\perp}{16\pi^3} \Sigma(u, k_T) = 1.$$

$$\mathcal{F}_\pi(Q^2) = \sum_{t_i} \psi^{t_1}(u_1, \mathbf{b}_1, \mu_{r_1}) T_H^{t_i, \text{LO+NLO}}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mathbf{b}_2, \mu_{r_2})$$

The soft-transversal dynamics

- a simple gaussian function with preserving rotational invariance

$$\Sigma(u, k_T) = 16\pi^2 \frac{\beta^2}{u(1-u)} e^{-\frac{\beta^2 k_T^2}{u(1-u)}} \Rightarrow \hat{\Sigma}(u, b_T) = 4\pi e^{-\frac{b_T^2 u(1-u)}{4\beta^2}}. \quad [\text{Jakob, Kroll, PLB 315(1993)463}]$$

- iTMDs associated to two-particle twist three LCDAs

$$\begin{aligned} \psi^{p,\sigma}(u, \mu) &= \int \frac{d^2 k_T}{16\pi^3} \varphi_{2p}^{p,\sigma}(u, \mu) \Sigma(u, k_T) + \int \frac{d^2 k_{1T} d^2 k_{2T}}{64\pi^5} \rho_+ \varphi_{3p}^{p,\sigma}(u, \mu) \int \mathcal{D}\alpha_i \Sigma'(\alpha_i, k_{iT}), \\ \int \frac{d^2 k_{1T} d^2 k_{2T}}{64\pi^5} \int \mathcal{D}\alpha_i \Sigma'(\alpha_i, k_{iT}) &= 1, \quad \int_0^1 du \varphi_{2p}^{p,\sigma}(u, \mu) = 1, \quad \int_0^1 du \varphi_{3p}^{p,\sigma}(u, \mu) = 0, \\ \hat{\Sigma}'(\alpha_i, b_1, b_2) &= 4\pi e^{-\frac{2\alpha_3(b_1^2 + b_2^2) + (\alpha_1 + \alpha_2)(b_1 - b_2)^2}{16\beta'^2}}. \end{aligned}$$

- two transversal-size parameters β^2 and β'^2

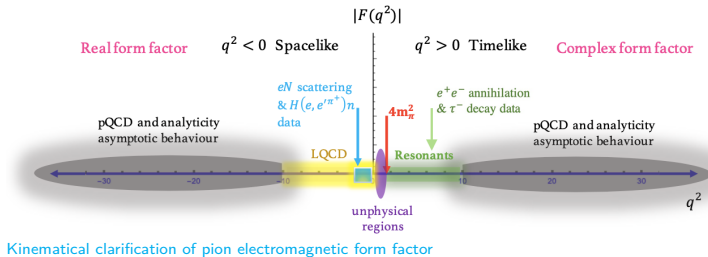
$$\ddagger \text{ asymptotic behavior of } F_{\pi\gamma\gamma^*}: \beta_\pi^2 = \frac{1}{8\pi^2 f_\pi^2 (1 + a_2^\pi + a_4^\pi + \dots)} = 0.51 \pm 0.04 \text{ GeV}^{-2}$$

$$\ddagger \text{ corresponds to the mean transversal momentum } [\langle k_T^2 \rangle]^{\frac{1}{2}} \equiv \left[\frac{\int du d^2 k_T k_T^2 |\psi(u, k_T)|^2}{\int du d^2 k_T |\psi(u, k_T)|^2} \right]^{\frac{1}{2}} = 358 \pm 15 \text{ MeV, revealing the soft transversal dynamics in the soft wave function.}$$

$$\ddagger \beta_K^2 = 0.30 \pm 0.05 \text{ GeV}^{-2} \text{ is obtained by fitting to the data of FFs, } [\langle k_T^2 \rangle]_K^{\frac{1}{2}} = 0.55 \pm 0.07 \text{ MeV}$$

Pion EMFF and TFF

Pion electromagnetic form factor

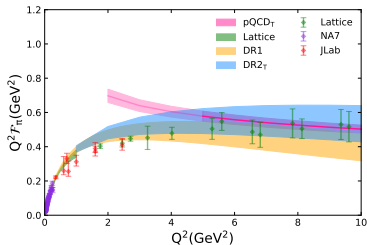


- mismatch between the QCD based calculation and the available data
- could be restored by employing the dispersion relation
- The standard dispersion relation and The modulus representation

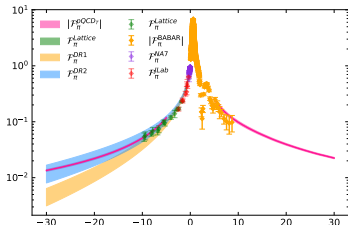
$$F_\pi(q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} F_\pi(s)}{s - q^2 - i\epsilon}, \quad q^2 < s_0 \quad \Downarrow$$

$$F_\pi(q^2) = \exp \left[\frac{q^2 \sqrt{s_0 - q^2}}{2\pi} \int_{s_0}^{\infty} \frac{ds \ln |F_\pi(s)|^2}{s \sqrt{s - s_0} (s - q^2)} \right] \quad [\text{SC, Khodjamirian, Rosov, PRD 102.074022 (2020)}]$$

Pion electromagnetic form factor



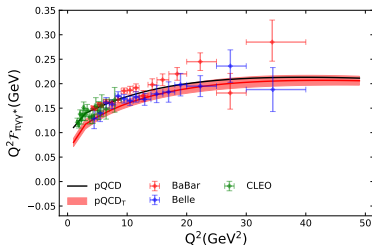
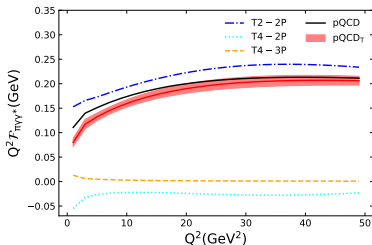
[J. Chai, **SC**, PRD 111. L071902]



- take the modular DR to fit chiral mass, obtain $m_0^\pi(1 \text{ GeV}) = 1.84 \pm 0.07 \text{ GeV}$
larger than the previous pQCD result $\sim 1.37 \text{ GeV}$ [J. Chai, **SC** and J. Hua, EPJC 83. 556(2023)]
consists with the ChPT $\sim 1.79 \text{ GeV}$ [H. Leutwyler PLB 378(1996)313-318]
a significant decrease of the FF due to the soft transversal dynamics in the small and intermediate q^2 .
- the power of pQCD prediction is impressively improved down to a few GeV^2 after considering the iTMDs effect

Pion transition form factor

- $F_{\pi\gamma\gamma^*}$ is the theoretically most clean observable $\propto a_n^\pi$
- **Hadronic light-by-light scattering (HLbL) contribution to $a_\mu^{HLbL;\pi^0}$**
- In 2009, BaBar collaboration reported the measurement exceeding the asymptotic QCD prediction $Q^2 \mathcal{F}_{\pi\gamma\gamma^*}(Q^2) = \sqrt{2}f_\pi$ in $q^2 \leq 10 \text{ GeV}^2$
- flat DAs ? fruitful structures (polynomials) in leading twist LCDAs ?
- The attractive pion TFF is heat off with the measurement from Belle collaboration in 2012, which shows a consistent with the asymptotic QCD limit
- settle down the "fat pion" issue at Belle II, BESIII, JLab and future colliders ?



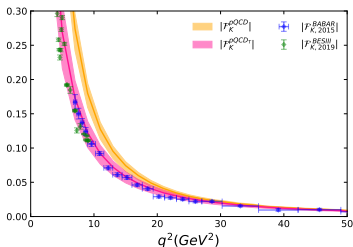
[J. Chai, SC, 2501.08783]

Conclusion

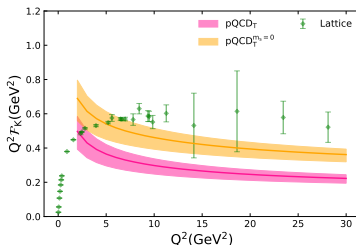
- pQCD is a powerful approach to study an exclusive QCD process
- the LCDAs description of hadron overlooks the soft transversal dynamics
- the universal soft function is actually a product of LCDAs and iTMDs
- we study the electromagnetic and transition form factors of light pseudoscalar mesons in the iTMDs-improved pQCD approach
- find the better agreements with the data and improve the prediction power down to a few GeV^2
- highly precise measurements are highly anticipated

Thank you for your patience.

Backup slides Kaon EMFF



[J. Chai, SC, PRD 111. L071902]



- $m_0^K(1 \text{ GeV}) = 1.90 \pm 0.09 \text{ GeV}$ is well-known from the CHPT relation without involving light quark masses
- fit the transversal-size parameter $\beta_K^2 = 0.30 \pm 0.05 \text{ GeV}^{-2}$ from timelike data settle for the second best and take $\beta_K^2 = \beta_K'^2$
- the iTMDs is indispensable to explain the data in the intermediate q^2
- iTMDs-improved pQCD result of spacelike FF is small than the lattice data
agrees with results obtained from the DSE approach and the collinear QCD factorization
 large $SU(3)$ flavor breaking emerges an additional term proportional to m_s in the twist-three LCDAs

Backup slides Dispersion relations

- Introducing an auxiliary function $g_\pi(q^2) \equiv \frac{\ln F_\pi(q^2)}{q^2 \sqrt{s_0 - q^2}}$ [Geshkenbein 1998]

- Cauchy theorem and Schwartz reflection principle

$$g_\pi(q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im } g_\pi(s)}{s - q^2 - i\epsilon}$$

- At $s > s_0$ on the real axis, the imaginary part of g_π reads as

$$\text{Im } g_\pi(s) = \text{Im} \left[\frac{\ln(|F_\pi(s)| e^{i\delta_\pi(s)})}{-is\sqrt{s - s_0}} \right] = \frac{\ln |F_\pi(s)|}{s\sqrt{s - s_0}},$$

- Substituting $g_\pi(q^2)$ and $\text{Im } g_\pi(s)$ into the dispersion relation, for $q^2 < s_0$

$$\frac{\ln F_\pi(q^2)}{q^2 \sqrt{s_0 - q^2}} = \frac{1}{2\pi} \int_{s_0}^{\infty} \frac{ds \ln |F_\pi(s)|^2}{s \sqrt{s - s_0} (s - q^2)}$$

- **The modulus representation** [SC, Khodjamirian, Rosov PRD 102.074022 (2020)]

$$F_\pi(q^2 < s_0) = \exp \left[\frac{q^2 \sqrt{s_0 - q^2}}{2\pi} \int_{s_0}^{\infty} \frac{ds \ln |F_\pi(s)|^2}{s \sqrt{s - s_0} (s - q^2)} \right]$$