SIDIS in the Target Fragmentation Region

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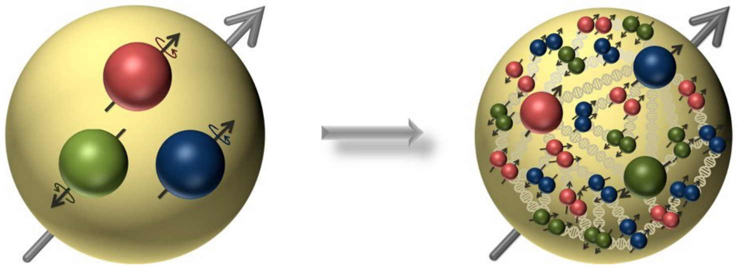
Based on: KBC, J.P. Ma and X.B. Tong, JHEP 05 (2024) 298,
JHEP 08 (2024) 227,
PRD 108 (2023) 094015.

Contents

Contents

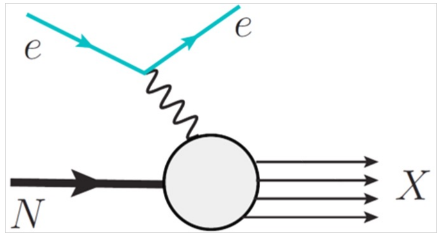
Introduction

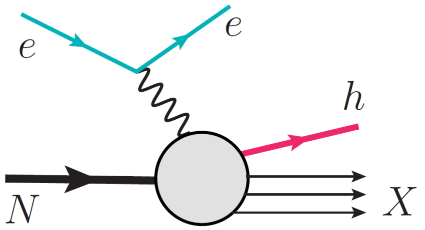
■ Probing nucleon structure



Images taken from Front. Phys. 16(6), 64701

Lepton-nucleon deep inelastic scattering (DIS)





Inclusive DIS → Semi-inclusive DIS

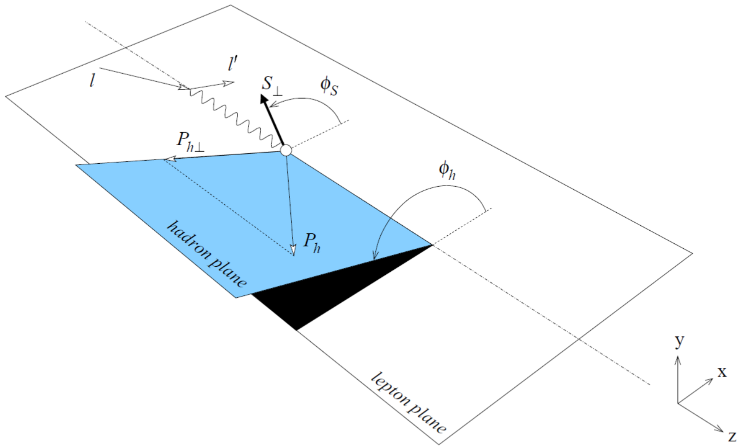
Why nucleon structure important?

- Fundamental components of matter
- Properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for new physics researches
- … …

- Three-dimensional imaging of the nucleon
- Accessing fragmentation functions
- Azimuthal and/or spin asymmetries
- Flavor separation
- … …

Introduction

- **Semi-inclusive DIS (SIDIS)**



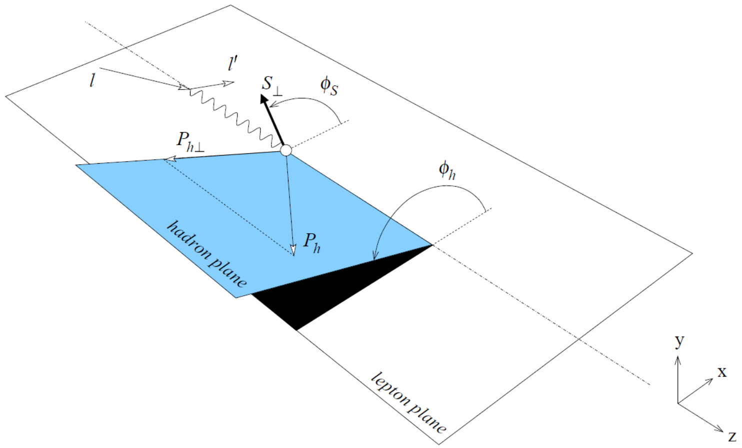
A. Bacchetta et al., JHEP 02, 093 (2007)

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$
$$\Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+\,|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$
$$+\,|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

18 structure functions for SIDIS with polarized lepton and nucleon

Introduction

■ Semi-inclusive DIS (SIDIS)



A. Bacchetta et al., JHEP 02, 093 (2007)

$$P_{h\perp} \sim M \ll Q$$

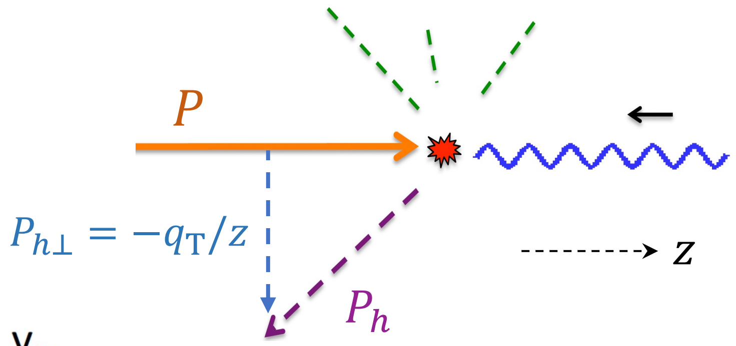
TMD factorization applies.

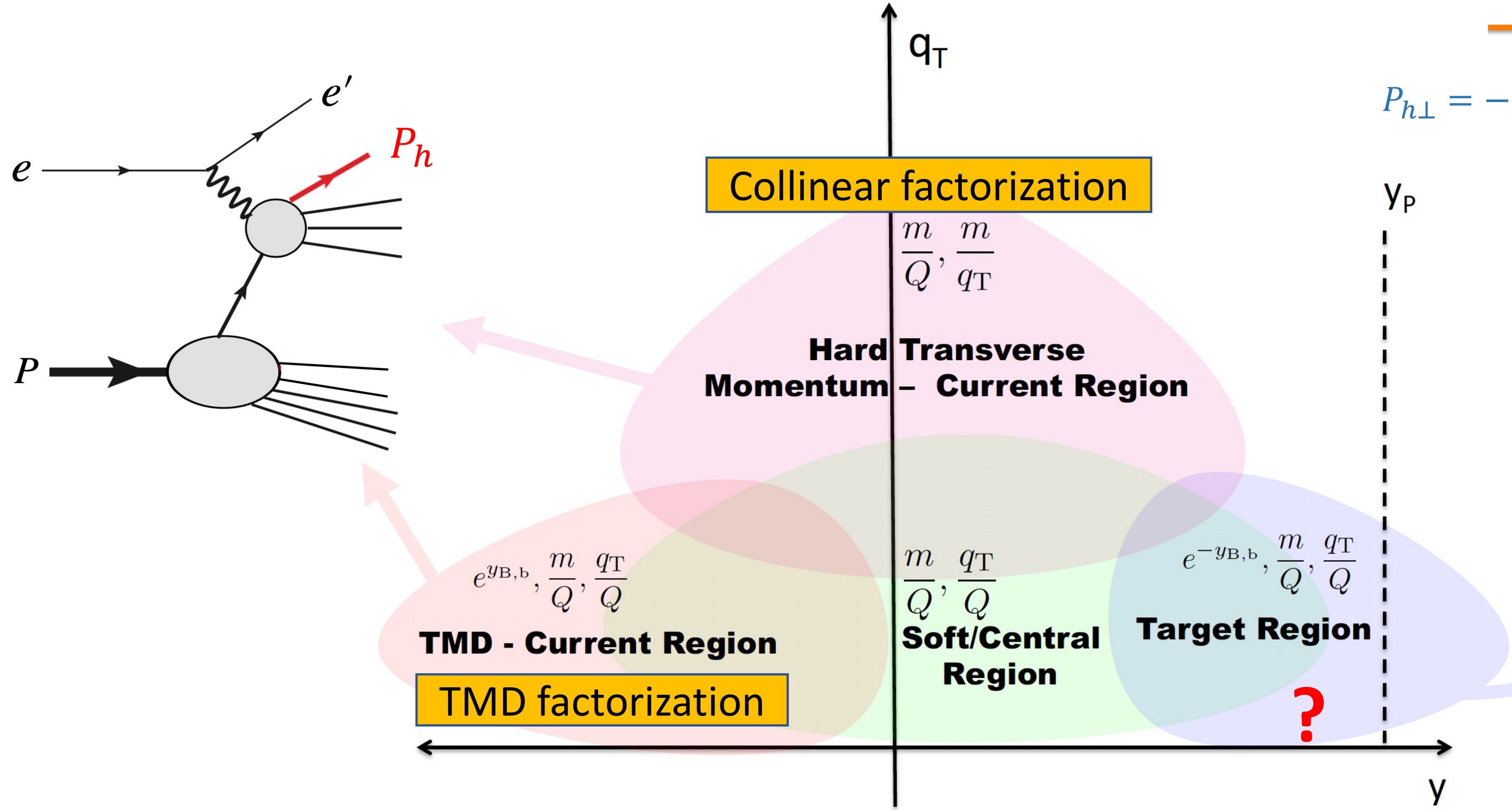
Structure functions ~TMD PDFs ⊗ TMD FFs

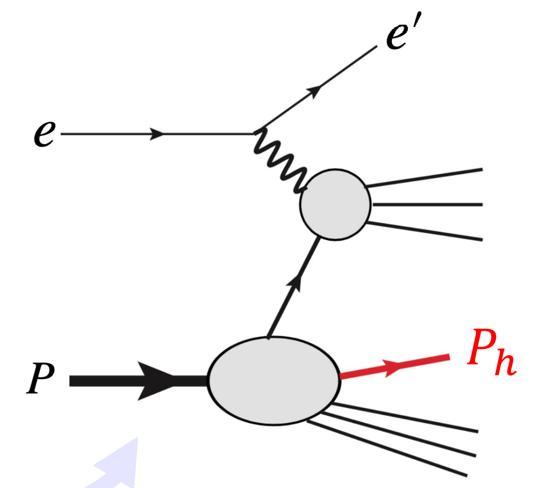
$F_{UU,T} \sim f_1 \otimes D_1$	Number density
$F_{LL} \sim g_{1L} \otimes D_1$	Helicity
$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$	Sivers
$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$	Worm-gear L-T
$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$	Transversity
$F_{UU}^{\cos 2\phi_h} \sim h_1^{\perp} \otimes H_1^{\perp}$	Boer-Mulders
$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$	Worm-gear T-L
$F_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$	Pretzelosity

8 leading structure functions

■ Kinematic regions of SIDIS







Kinematical regions of SIDIS in terms of the produced hadron's **rapidity** and **transverse momentum** in the Breit frame

M. Boglione et al. JHEP 10 (2019) 122

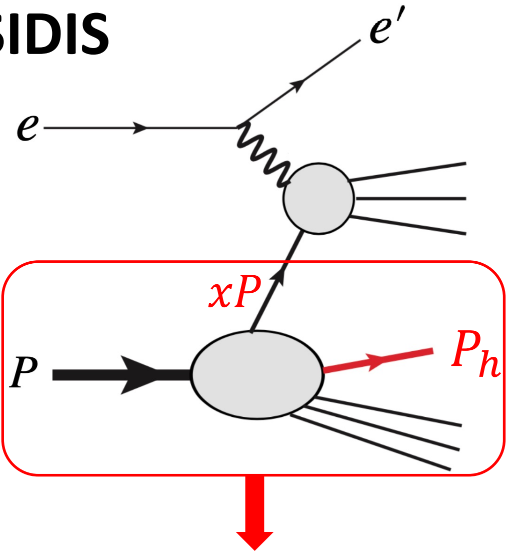
Introduction

■ Fracture functions and target fragmentation region (TFR) SIDIS

- Fracture function: $F(x, \xi_h)$

Parton distributions in the presence of an almost collinear particle observed in the final state.
(conditional probability, hybrid of PDFs and FFs)

L. Trentadue and G. Veneziano, Phys. Lett. B323, 201 (1994)



$P_h \approx \xi_h P$

$d\sigma \sim F(x, \xi_h) \otimes H$

- TFR SIDIS with hadron transverse momentum

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\, dy\, d\zeta\, d^2 \boldsymbol{P}_{h\perp}\, d\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\Bigg\{\left(1 - y + \frac{y^2}{2}\right)\sum_a e_a^2 \left[M(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp| \frac{|\boldsymbol{P}_{h\perp}|}{m_h} M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2)\sin(\phi_h - \phi_S)\right]$$

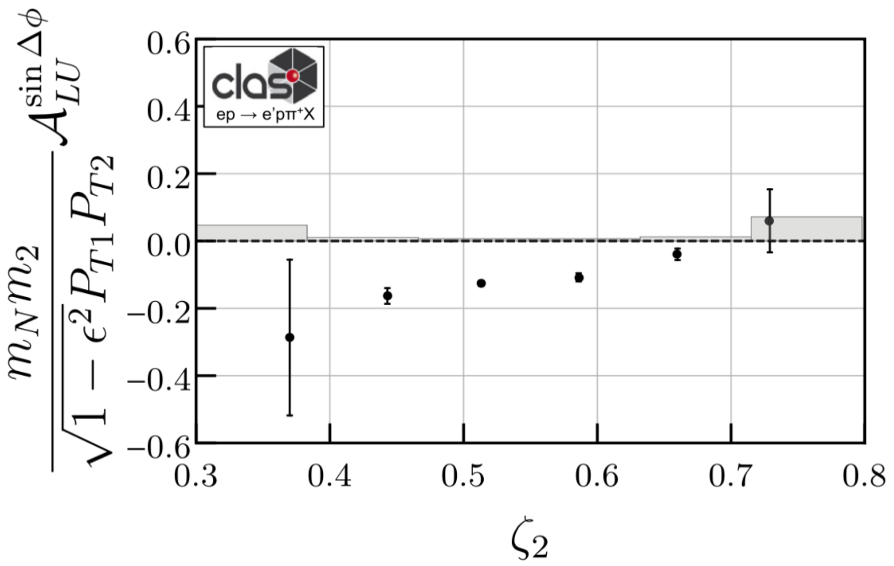
$$+ \lambda_l y\left(1 - \frac{y}{2}\right)\sum_a e_a^2 \left[S_\parallel \Delta M_L(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) + |\boldsymbol{S}_\perp| \frac{|\boldsymbol{P}_{h\perp}|}{m_h} \Delta M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2)\cos(\phi_h - \phi_S)\right]\Bigg\}.$$

4 structure functions at twist-2

Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).

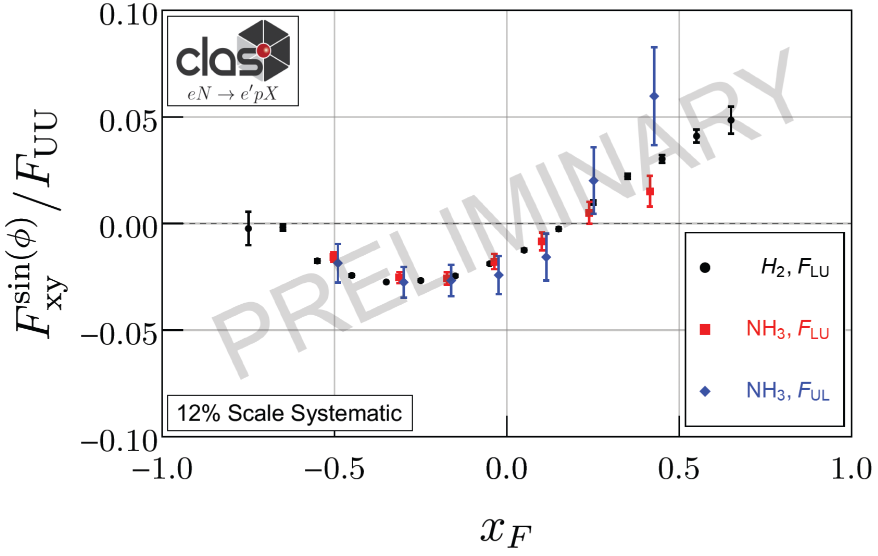
SIDIS in the target fragmentation region (TFR)

- Recent experimental studies on TFR hadron production



Beam-spin asymmetry for back-to-back dihadron production

H. Avakian et al., PRL 130, 022501 (2023)



Structure functions from CFR to TFR

A. Accardi et al., EPJA (2024) 60:173

Contents

Twist-3 contributions for TFR SIDIS

Kinematics

$$e(l,\lambda_e) + h_A(P,S) \to e(l') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2 y}{4\xi_h Q^4} L_{\mu\nu}(l,\lambda_e,l') W^{\mu\nu}(q,P,S,P_h)$$

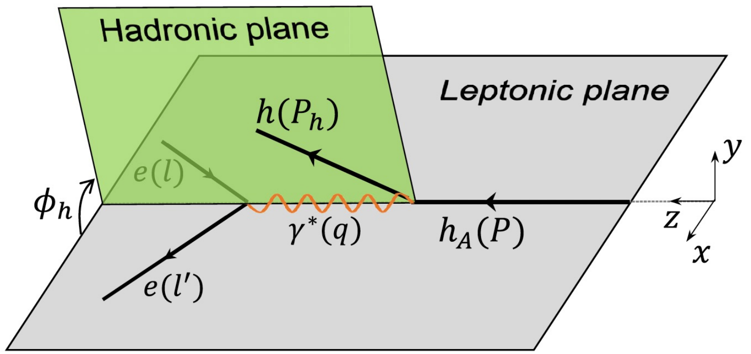
$$L^{\mu\nu}(l,\lambda_e,l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - l\cdot l' g^{\mu\nu}) + 2i\lambda_e \epsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma$$

$$W^{\mu\nu}(q,P,S,P_h) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x} \langle S; h_A | J^\mu(x) | hX \rangle \langle Xh | J^\nu(0) | h_A; S \rangle$$

$$P^\mu \approx (P^+, 0,0,0) \qquad l^\mu = (\frac{1-y}{y} x_B P^+, \frac{Q^2}{2x_B y P^+}, \frac{Q\sqrt{1-y}}{y}, 0)$$

$$P_h^\mu = (P_h^+, P_h^-, \vec{P}_{h\perp}) \qquad q^\mu = (-x_B P^+, \frac{Q^2}{2x_B P^+}, 0,0)$$

$$S^\mu = (\frac{S_L P^+}{M}, -\frac{S_L M}{2P^+}, \vec{S}_\perp)$$



$$Q^2 = -q^2,$$

$$x_B = \frac{Q^2}{2P\cdot q},$$

$$y = \frac{P\cdot q}{P\cdot l},$$

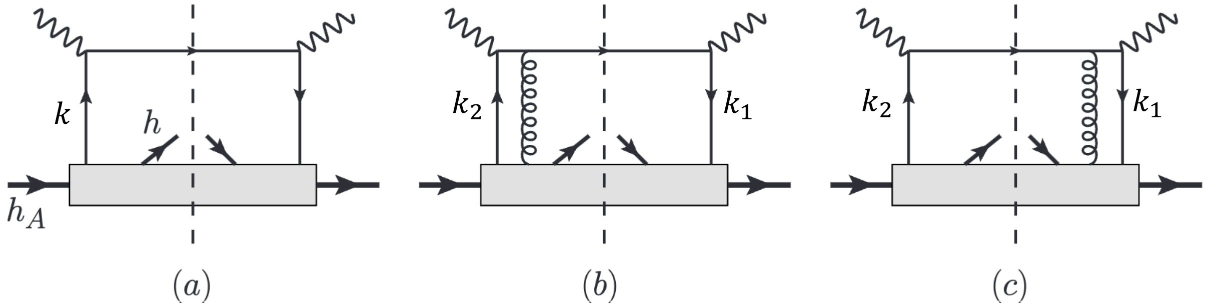
$$z_h = \frac{P\cdot P_h}{P\cdot q} \ll 1.$$

$$\xi_h = \frac{P_h\cdot q}{P\cdot q} \approx \frac{P_h^+}{P^+},$$

Twist-3 contributions for TFR SIDIS

■ Hadronic tensor

Hadronic tensor at the tree level



……

Multiple gluon scattering in the final state

$$W^{\mu\nu}\Big|_a = \int \frac{d^3k}{(2\pi)^3}\left[\left(\gamma^\mu(\not k+\not q)\gamma^\nu\right)_{ij} 2\pi\delta\big((k+q)^2\big)\right]\sum_X\int\frac{d^3\eta}{(2\pi)^4}e^{-ik\cdot\eta}\langle h_A|\bar\psi_i(\eta)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

$$W^{\mu\nu}\Big|_b = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu(\not k_1+\not q)\gamma_\alpha\frac{i(\not k_2+\not q)}{(k_2+q)^2+i\epsilon}\gamma^\nu\right)_{ij} 2\pi\delta\big((k_1+q)^2\big)\right]$$

$$\times(-ig_s)\sum_X\int\frac{d^3\eta d^3\eta_1}{(2\pi)^4}e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar\psi_i(\eta)|hX\rangle\langle Xh|G^\alpha(\eta_1)\psi_j(0)|h_A\rangle,$$

$k_i \sim Q(1,\lambda^2,\lambda)$, with $\lambda \sim \frac{\Lambda_{QCD}}{Q}$

$$W^{\mu\nu}\Big|_c = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu\frac{i(\not k_1+\not q)}{(k_1+q)^2-i\epsilon}\gamma_\alpha(\not k_2+\not q)\gamma^\nu\right)_{ij} 2\pi\delta\big((k_2+q)^2\big)\right]$$

$$\times(-ig_s)\sum_X\int\frac{d^3\eta d^3\eta_1}{(2\pi)^4}e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar\psi_i(\eta)G^\alpha(\eta_1)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

Collinear expansion for the hard parts … …

Twist-3 contributions for TFR SIDIS

■ **Hadronic tensor**

twist-2 *twist-3*

$$W^{\mu\nu} = \xi_h(\gamma^\mu\gamma^+\gamma^\nu)_{ij}\mathcal{M}_{ji}(x_B) + \left[\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\mathcal{M}^\alpha_{\partial,ji}(x_B) + (\mu\leftrightarrow\nu)^*\right]$$
$$+\left\{\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\int dx_2\left[\mathrm{P}\frac{1}{x_2-x_B} - i\pi\delta(x_2-x_B)\right]\mathcal{M}^\alpha_{F,ji}(x_B,x_2) + (\mu\leftrightarrow\nu)^*\right\}$$

$$\mathcal{M}_{ij}(x) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\mathcal{L}_n(0)\psi_i(0)|h_A\rangle,$$

$$\mathcal{M}^\alpha_{\partial,ij}(x) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\partial_\perp^\alpha(\mathcal{L}_n\psi_i)(0)]|h_A\rangle,$$

$$\mathcal{M}^\alpha_{F,ij}(x_1,x_2) = \int \frac{d\eta^-d\eta_1^-}{4\pi\xi_h(2\pi)^4}e^{-ix_1P^+\eta^- - i(x_2-x_1)P^+\eta_1^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)|hX\rangle\langle Xh|g_sF^{+\alpha}(\eta_1^-)\psi_i(0)|h_A\rangle.$$

$$\mathcal{L}_n(x) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, G^+(\lambda n + x)\right\}$$

Gauge invariant matrix elements!

$$g_sF^{+\alpha} = g_s[\partial^+G^\alpha_\perp - \partial^\alpha_\perp G^+] + \mathcal{O}(g_s^2)$$

Twist-3 contributions for TFR SIDIS

■ **Fracture functions defined via the correlation matrices**

$$\mathcal{M}_{ij}(x) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1 - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h\right) + \frac{1}{P^+}\left(P_{h\perp}^\rho u^h - M\tilde{S}_\perp^\rho u_T - S_L\tilde{P}_{h\perp}^\rho u_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_T^h\right)\right]$$

$$- \frac{(\gamma_\rho\gamma_5)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h\right) + \frac{1}{P^+}\left(\tilde{P}_{h\perp}^\rho l^h + MS_\perp^\rho l_T + S_L P_{h\perp}^\rho l_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_T^h\right)\right] + \cdots,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \frac{(\gamma^-)_{ij}}{2N_c}i\left(-P_{h\perp}^\alpha u_\partial^h + M\tilde{S}_\perp^\alpha u_{\partial T} + S_L\tilde{P}_{h\perp}^\alpha u_{\partial L}^h + \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_{\partial T}^h\right)$$

$$+ \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha l_\partial^h + MS_\perp^\alpha l_{\partial T} + S_L P_{h\perp}^\alpha l_{\partial L}^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_{\partial T}^h\right) + \cdots,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1,x_2) = \frac{(\gamma^-)_{ij}}{2N_c}\left(P_{h\perp}^\alpha w^h - M\tilde{S}_\perp^\alpha w_T - S_L\tilde{P}_{h\perp}^\alpha w_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}w_T^h\right)$$

$$- \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha v^h + MS_\perp^\alpha v_T + S_L P_{h\perp}^\alpha v_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}v_T^h\right) + \cdots,$$

Fracture functions up to twist-3

Red: twist-2.

Green: twist-3

$$\tilde{a}_\perp^\mu \equiv \varepsilon_\perp^{\mu\nu}a_{\perp\nu}$$

$$P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle} \equiv P_{h\perp}^\alpha P_{h\perp}^\beta + g_\perp^{\alpha\beta}\vec{P}_{h\perp}^2/2$$

- Only chiral-even terms contribute
- Similar form and naming rules as TMDs, $(k_\perp \to P_{h\perp})$

Twist-3 contributions for TFR SIDIS

■ Fracture functions defined via the correlation matrices

These twist-3 fracture functions are not independent from each other!

QCD equation of motion $i\gamma \cdot D\psi = 0$, gives:

$$x[u_S^K(x) + il_S^K(x)]$$
$$= u_{\partial S}^K(x) + il_{\partial S}^K(x) + i\int dy \left[\mathrm{P}\frac{1}{y-x} - i\pi\delta(y-x)\right][w_S^K(x,y) - v_S^K(x,y)]$$

The relations take a unified form!

Four sets of equations

S	K
null	h
L	h
T	null
T	h

The hadronic tensor will be expressed by those fracture functions defined only from the quark-quark correlator $\mathcal{M}_{ij}$.

Twist-3 contributions for TFR SIDIS

Results for structure functions and azimuthal asymmetries

Structure functions

Azimuthal/spin asymmetries

Twist-2

$$F_{UU,T} = x_B u_1, \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

$$\langle \sin(\phi_h-\phi_S)\rangle_{UT} = \frac{|\vec{P}_{h\perp}|}{2M}\frac{u_{1T}^h}{u_1},$$

$$\langle \cos(\phi_h-\phi_S)\rangle_{LT} = \frac{|\vec{P}_{h\perp}| C(y)}{2M A(y)}\frac{l_{1T}^h}{u_1}.$$

Twist-3

$$F_{UU}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u^h, \qquad F_{LU}^{\sin\phi_h} = \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l^h,$$

$$F_{UL}^{\sin\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u_L^h, \qquad F_{LL}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l_L^h,$$

$$F_{UT}^{\sin\phi_S} = -\frac{2M}{Q} x_B^2 u_T, \qquad F_{LT}^{\cos\phi_S} = -\frac{2M}{Q} x_B^2 l_T,$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 u_T^h, \qquad F_{LT}^{\cos(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 l_T^h.$$

$$\langle \cos\phi_h\rangle_{UU} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u^h}{u_1}, \qquad \langle \sin\phi_h\rangle_{LU} = \frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l^h}{u_1},$$

$$\langle \sin\phi_h\rangle_{UL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u_L^h}{u_1}, \qquad \langle \cos\phi_h\rangle_{LL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l_L^h}{u_1},$$

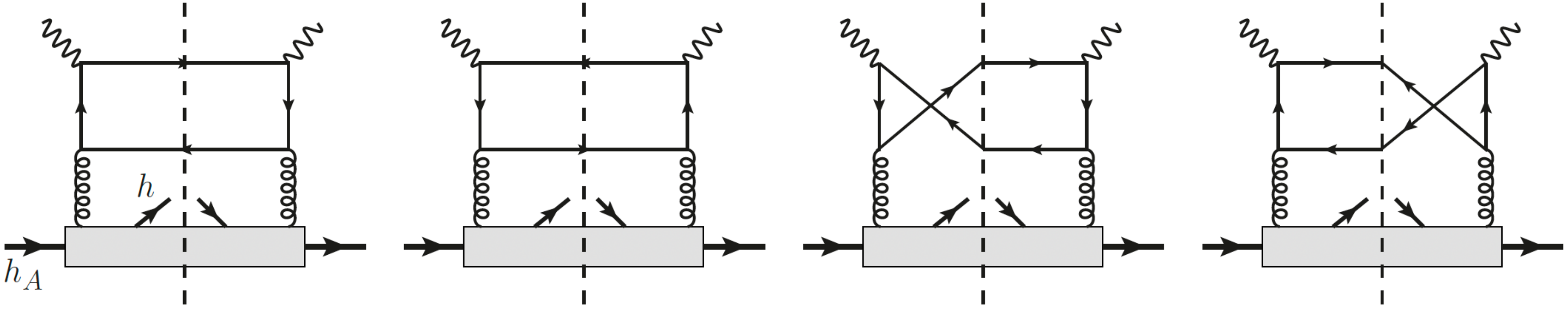
$$\langle \sin\phi_S\rangle_{UT} = -\frac{M}{Q}\frac{B(y)}{A(y)}\frac{x_B u_T}{u_1}, \qquad \langle \cos\phi_S\rangle_{LT} = -\frac{M}{Q}\frac{D(y)}{A(y)}\frac{x_B l_T}{u_1},$$

$$\langle \sin(2\phi_h-\phi_S)\rangle_{UT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{B(y)}{A(y)}\frac{x_B u_T^h}{u_1}, \qquad \langle \cos(2\phi_h-\phi_S)\rangle_{LT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{D(y)}{A(y)}\frac{x_B l_T^h}{u_1}.$$

Contents

One-loop contributions for TFR SIDIS

■ Gluonic contribution



$$W^{\mu\nu}(q,P,S,P_h) = \alpha_s T_F \sum_f e_f^2 \int \frac{dx}{x} \int d\Phi_{k_1 k_2} H^{\mu\nu\alpha\beta}(k_g,k_1,k_2) \mathcal{M}_{G,\alpha\beta}(x,\xi_h,P_{h\perp}),$$

$$\mathcal{M}_G^{\alpha\beta}(x,\xi_h,P_{h\perp}) = \frac{1}{2\xi_h(2\pi)^3} \frac{1}{xP^+} \int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \sum_X \langle h_A(P)|(G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n))^a|X h(P_h)\rangle$$
$$\times \langle h(P_h)X|(\mathcal{L}_n(0)G^{+\beta}(0))^a|h_A(P)\rangle,$$

$$\int d\Phi_{k_1 k_2} \equiv \int \frac{d^D k_1}{(2\pi)^D} \int \frac{d^D k_2}{(2\pi)^D} (2\pi)\delta(k_1^2)(2\pi)\delta(k_2^2)(2\pi)^4 \delta^{(4)}(q+k_g-k_1-k_2)\theta(q^0+k_g^0)$$

Gluonic contribution

Gluon fracture functions

$$\mathcal{M}_G^{\alpha\beta} = -\frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} u_{1g} + \frac{1}{2M^2}\left(P_{h\perp}^{\alpha} P_{h\perp}^{\beta} + \frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} P_{h\perp}^2\right) t_{1g}^h + S_L\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gL} + \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gL}^h\right]$$
$$+ \frac{g_\perp^{\alpha\beta}}{2-2\epsilon}\frac{P_{h\perp}\cdot\tilde{S}_\perp}{M} u_{1gT}^h + \frac{P_{h\perp}\cdot S_\perp}{M}\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gT}^h - \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gT}^{hh}\right] + \frac{\tilde{P}_{h\perp}^{\{\alpha} S_\perp^{\beta\}} + \tilde{S}_\perp^{\{\alpha} P_{h\perp}^{\beta\}}}{8M} t_{1gT}^h + \cdots .$$

The hard parts:

$$H_{(a)+(b)}^{\mu\nu\alpha\beta}(k_g, k_1, k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g - \not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\alpha \frac{(\not{k}_g - \not{k}_2)}{(k_g-k_2)^2}\gamma^\mu\right] + (k_1 \leftrightarrow k_2),$$

$$H_{(c)+(d)}^{\mu\nu\alpha\beta}(k_g, k_1, k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g - \not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\mu \frac{(\not{k}_1 - \not{k}_g)}{(k_1-k_g)^2}\gamma^\alpha\right] + (k_1 \leftrightarrow k_2).$$

One-loop contributions for TFR SIDIS

■ Gluonic contribution

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1g}^h(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1gL}^h(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UT}^{\sin(3\phi_h-\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^3}{4M^3}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1gT}^{hh}(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}}{2M}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2\left[t_{1gT}^h(x_B/z,\xi_h,P_{h\perp})+\frac{P_{h\perp}^2}{2M^2}t_{1gT}^{hh}(x_B/z,\xi_h,P_{h\perp})\right].$$

Generated uniquely by gluon fracture functions!

One-loop contributions for TFR SIDIS

Quark and gluon contribution

$$F_{UU,L} = \frac{\alpha_s}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[4T_F z\bar{z} u_{1g}(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \frac{\alpha_s}{2\pi} \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[4T_F z\bar{z} u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right].$$

Twist-4 if at the tree level

$$F_{UU,T} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1g}(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LL} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\Delta\mathcal{H}_g(z) l_{1gL}(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1L}(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\Delta\mathcal{H}_g(z) l_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

LO + NLO corrections

$$\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[2\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - (1+z)\ln\bar{z} - \frac{1+z^2}{\bar{z}}\ln z + 3 - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \ln\frac{Q^2\bar{z}}{\mu^2 z} - T_F(1-2z)^2 \right],$$

$$\Delta\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ \Delta P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[(1+z^2)\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - \frac{1+z^2}{\bar{z}}\ln z + 2 + z - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\Delta\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[\Delta P_{qg}(z) \left(\ln\frac{Q^2\bar{z}}{\mu^2 z} - 1 \right) + 2T_F\bar{z} \right].$$

One-loop contributions for TFR SIDIS

Structure functions	Twist	$\mathcal{O}(\alpha_S)$
$F_{UU,T}$	✓ (2)	✓ (α_S^0)
$F_{UU,L}$		✓ (α_S^1)
$F_{UU}^{\cos\phi_h}$	✓ (3)	
$F_{UU}^{\cos 2\phi_h}$		✓ (α_S^1)
$F_{LU}^{\sin\phi_h}$	✓ (3)	
$F_{UL}^{\sin\phi_h}$	✓ (3)	
$F_{UL}^{\sin 2\phi_h}$		✓ (α_S^1)
F_{LL}	✓ (2)	✓ (α_S^0)
$F_{LL}^{\cos\phi_h}$	✓ (3)	

Structure functions	Twist	$\mathcal{O}(\alpha_S)$
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	✓ (2)	✓ (α_S^0)
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$		✓ (α_S^1)
$F_{UT}^{\sin(\phi_h+\phi_S)}$		✓ (α_S^1)
$F_{UT}^{\sin\phi_S}$	✓ (3)	
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	✓ (3)	
$F_{UT}^{\sin(3\phi_h-\phi_S)}$		✓ (α_S^1)
$F_{LT}^{\cos\phi_S}$	✓ (3)	
$F_{LT}^{\cos(\phi_h-\phi_S)}$	✓ (2)	✓ (α_S^0)
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	✓ (3)	

All 18 structure functions are non-zero up to twist-3 or $\mathcal{O}(\alpha_S^1)$ level.

Contents

Connecting NEEC with fracture function

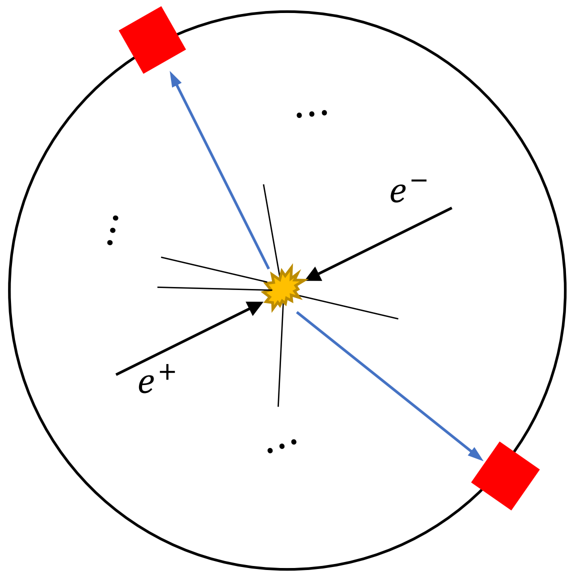
■ Energy-energy correlator (EEC)

EEC was proposed as an event-shape observable in e^+e^- annihilation in 1970's.

$$\mathrm{EEC} \sim \sum_{a,b} d\sigma(e^+e^- \to h_a h_b X)\frac{E_a E_b}{s}\delta(\Omega_{ab} - \Omega)$$

Basham, Brown, Ellis and Love, PRL 41 (1978) 1585

Energy weighted cross section, infrared safe, precise test of pQCD, easy to implement in experiments ……

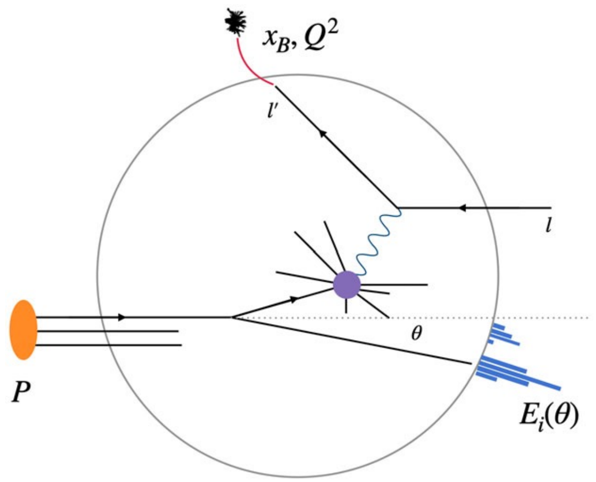


Applications to DIS process and others,

developing fast recently……

Connecting NEEC with fracture function

■ **Nucleon energy-energy correlator (NEEC)**



X.H. Liu and H.X. Zhu, PRL 130 (2023) 091901
H.T. Cao, X.H. Liu and H.X. Zhu, PRD 107 (2023) 114008

NEEC: $f_{EEC}(z,\theta) = \int \frac{\mathrm{d}y^-}{4\pi} e^{-izP^+y^-} \left\langle P \middle| \bar{\psi}(y^-) \frac{\gamma^+}{2} \hat{\mathcal{E}}(\theta) \psi(0) \middle| P \right\rangle$

The energy flow operator: $\hat{\mathcal{E}}(\theta)|X\rangle \equiv \sum_{i\in X} \delta(\theta^2 - \theta_i^2) \frac{E_i}{E_P} |X\rangle$

$$\Sigma(x_B, Q^2, \theta) \equiv \sum_i \int \frac{E_i}{E_P} \mathrm{d}\sigma(x_B, Q^2, p_i) \delta(\theta - \theta_i)$$

$\theta \ll 1$ ⟹

$$= \int \frac{\mathrm{d}z}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{EEC}(z, \theta, \mu)$$

Notice: Hadrons with $\theta \ll 1$ are almost collinear with the target nucleon.
They are produced in the **Target Fragmentation Region**!

Connecting NEEC with fracture function

■ Relation between NEEC and fracture functions

Matrix elements for fracture functions and NEECs:

$$\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp}) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4} e^{-ixP^+\eta^-} \sum_X \int \frac{d^3\boldsymbol{P}_X}{2E_X(2\pi)^3} \times \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^\dagger_n(\eta^-)|P_h X\rangle\langle X P_h|\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\sum_X \int \frac{d^3\boldsymbol{P}_X}{2E_X(2\pi)^3}|P_h X\rangle\langle X P_h| = a^\dagger_h a_h$$

$$\mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^\dagger_n(\eta^-)\mathcal{E}(\theta,\phi)\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\mathcal{E}(\theta,\phi) = \sum_h \int \frac{d^3\boldsymbol{P}_h}{2E_h(2\pi)^3}\frac{E_h}{E_N}\delta(\theta^2-\theta_h^2)\delta(\phi-\phi_h)a^\dagger_h a_h$$

$$\Rightarrow \quad \mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \sum_h \int_0^{1-x} d\xi_h\, \xi_h \frac{\boldsymbol{P}^2_{h\perp}}{2\theta^2}\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp})\Big|_{\boldsymbol{P}_{h\perp}=\frac{\xi_h\theta P^+}{\sqrt{2}}\boldsymbol{n}_t}$$

$$\boldsymbol{n}_t \equiv (\cos\phi, \sin\phi)$$

Fracture functions act as generating functions of NEECs.

Connecting NEEC with fracture function

Relation between NEEC and fracture functions

$$\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp}) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1^q - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^{h,q}\right)\right] + \frac{(\gamma_5\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L}^q - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^{h,q}\right)\right] + \cdots$$

$$\mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(\frac{1}{2\pi}f_1^q - n_t\cdot\tilde{S}_\perp\ f_{1T}^{t,q}\right)\right] + \frac{(\gamma_5\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(\frac{1}{2\pi}S_L\ g_{1L}^q - n_t\cdot S_\perp\ g_{1T}^{t,q}\right)\right] + \cdots$$

$$f_1^q(x,\theta) = 2\pi \sum\!\!\!\!\!\!\!\int u_1^q(x,\xi_h,\boldsymbol{P}_{h\perp}^2), \quad f_{1T}^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}u_{1T}^{h,q}(x,\xi_h,\boldsymbol{P}_{h\perp}^2),$$

$$g_{1L}^q(x,\theta) = 2\pi \sum\!\!\!\!\!\!\!\int l_{1L}^q(x,\xi_h,\boldsymbol{P}_{h\perp}^2), \quad g_{1T}^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}l_{1T}^{h,q}(x,\xi_h,\boldsymbol{P}_{h\perp}^2), \quad \cdots$$

One to one correspondence.

$$\sum\!\!\!\!\!\!\!\int \mathcal{A} \equiv \sum_h \int_0^{1-x} d\xi_h\ \xi_h \frac{\boldsymbol{P}_{h\perp}^2}{2\theta^2}\ \mathcal{A}\Big|_{|\boldsymbol{P}_{h\perp}|=\frac{\theta\xi_h P^+}{\sqrt{2}}}$$

Connecting NEEC with fracture function

■ Accessing NEEC properties through fracture functions

Example:

Matching of Sivers-type fracture function $u_{1T}^{h,q}(x,\xi_h,P_{h\perp})$ at $\Lambda_{QCD} \ll P_{h\perp} \ll Q$

➡

Matching of Sivers-type NEEC $f_{1T}^{t,q}(x,\theta)$ at $\Lambda_{QCD} \ll \theta Q \ll Q$

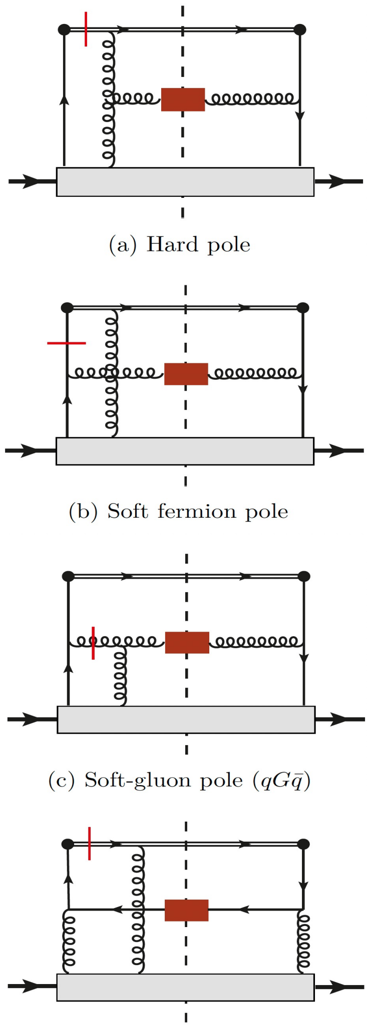
$$f_{1T}^{t,q}(x,\theta)\Big|_{\rm HP} = \frac{\alpha_s N_c}{2(2\pi)^2\theta^3 E_N}\int_x^1 \frac{dy}{y}\Big[T_\Delta(y,x)-(1+2x/(y-x))T_F(y,x)\Big],$$

$$f_{1T}^{t,q}(x,\theta)\Big|_{\rm SFP} = \frac{\alpha_s}{2(2\pi)^2 N_c\theta^3 E_N}\int_x^1 dy\frac{x}{y^3}\left[(2x-y)T_F(y,0)-yT_\Delta(y,0)\right],$$

$$f_{1T}^{t,q}(x,\theta)\Big|_{{\rm SGP},qG\bar{q}} = \frac{\alpha_s N_c}{2(2\pi)^2\theta^3 E_N}\int_x^1 \frac{dy}{y^3}\left[\frac{1}{y-x}(y^3+3x^2y-2x^3)T_F(y,y)-y(y^2+x^2)\frac{dT_F(y,y)}{dy}\right],$$

$$f_{1T}^{t,q}(x,\theta)\Big|_{\rm SGP,3G} = \frac{\alpha_s}{2\pi\theta^3 E_N}\int_x^1 dy\frac{y-x}{y^5}\Big\{2(4x^2-3xy+y^2)\left[N(y,y)-O(y,y)\right]$$
$$-2(8x^2-5xy+y^2)\left[N(y,0)+O(y,0)\right]+y(2x-y)^2\frac{d}{dy}\left[N(y,0)+O(y,0)\right]$$
$$-y(2x^2+y^2-2xy)\frac{d}{dy}\left[N(y,y)-O(y,y)\right]\Big\}.$$

Diagram examples



(a) Hard pole

(b) Soft fermion pole

(c) Soft-gluon pole ($qG\bar{q}$)

(d) Soft-gluon pole ($3G$)

Contents

Summary

- SIDIS in the TFR is factorized with fracture functions.
- TFR SIDIS is calculated up to twist-3 at the tree level. Structure functions and azimuthal asymmetries are obtained using the gauge-invariant fracture functions.
- By adding one-loop contributions at twist-2, all 18 structure functions for TFR SIDIS are non-zero.
- Connection between NEEC and fracture functions is established, which will facilitate the study of NEEC through fracture functions.

谢谢！